starting points in mathematics 6

teacher's edition

Teacher's Edition for

starting points in mathematics

Mathematics Team

Level 6

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Contents

In each Teacher's Edition of *Starting Points in Mathematics*, the pages for the student's book are referred to by numeral only, while pages in the teacher's edition are designated by the letter T and a numeral.

Authors and Consultants

Trudy Baker

Trudy is currently employed at the Joy in Learning Curriculum Development and Teacher Training Centre in Toronto. She studied at Calvin College in Michigan and at the University of Alberta in Calgary. Her background in mathematics includes teaching in elementary schools in Alberta, British Columbia, and Ontario. She is active in curriculum development for the primary and junior grades and conducts mathematics workshops for teachers. She has worked on the program as an author of material for Grades 3 to 6.

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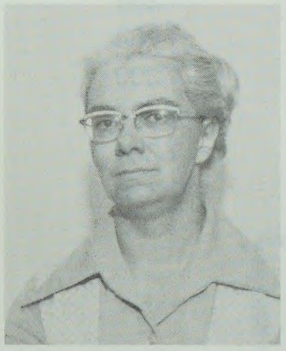
Currently Associate Professor of Mathematics Education at the University of Saskatchewan, Jim has worked closely with the Saskatoon Separate and Public School Boards in developing the mathematics curriculum. He has a Bachelor of Science degree from Parsons College, a Master of Science degree from the University of Notre Dame, and a Doctor of Education degree from the University of Nebraska. He is the author of a number of research papers and publications in North America and has made a major contribution to teacher-in-service work in his home province. He has worked on the program from its beginning, planning the development and evaluating manuscript for the primary phrase.

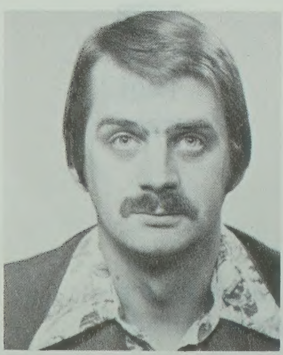
Donald L. Bornhold

Don has a Bachelor of Arts degree from the University of Western Ontario, a Bachelor of Pedagogy degree from the University of Toronto, and courses toward a Ph.D. at Columbia University. His background in mathematics has included teaching elementary and junior-high school in Kitchener and Simcoe; principal in Sherbrooke, Quebec; superintendent in Kirkland Lake, Ontario; inspector and assistant area superintendent of schools for the North York Board of Education; lecturer for the Ontario Department of Education at State College of Victoria, Melbourne, Australia; senior author of another mathematics series. He is the senior author of the program.



Grace Dilley

Grace is currently a Helping Teacher of Mathematics in School District 36, Surrey, British Columbia. She has Bachelor of Education and Master of Arts in Education degrees from the University of British Columbia. Prior to her present position, she was a classroom teacher, and a lecturer at the University of British Columbia. She is active in the British Columbia Association of Mathematics Teachers and is an author of another mathematics series. She has evaluated manuscript for the primary phase of the program.



Emery Dosdall

Emery is currently the Director of Program Supervision (K-12) for the Edmonton Public Schools. He has a Bachelor of Education degree and a diploma in Educational Administration from the University of Alberta, and a Master of Education degree from the University of Oregon. Prior to his present appointment, he was Supervisor of Mathematics (K-12), lecturer at the University of Alberta, assistant principal, and consultant. He has worked extensively in elementary mathematics curriculum development for the Edmonton Public Schools and has participated throughout his home province and nationally in the development of mathematics. He has worked on the program from its beginning, evaluating manuscript, consulting at all grade levels, and coordinating and evaluating field-test material prior to publication.

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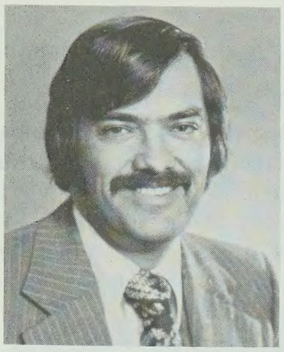
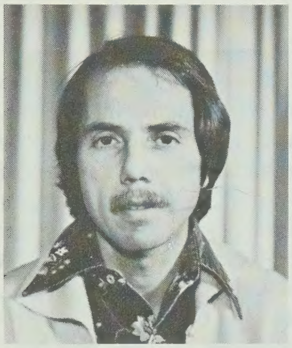
Ralph Gardner

Ralph is currently Principal of Seaview Elementary School in Port Coquitlam, British Columbia, where he is actively teaching mathematics. He has returned to the classroom following a position as Supervisor of Instruction and a position as Mathematics Consultant with the School District of Coquitlam. He has been active in mathematics in a number of capacities — serving on the Provincial Mathematics Committee, developing curriculum in the province, as an author of another mathematics program. He has worked on the program in planning the development, evaluating manuscript, and in field-testing the new approaches.



Robert Gutcher

Bob is currently an Assistant Superintendent with the Metropolitan Toronto Separate School Board, having previously been its Mathematics Coordinator. He has a Bachelor of Arts degree from the University of Western Ontario, a Master of Mathematics degree from the University of Waterloo, and a Master of Education degree from the Ontario Institute for Studies in Education. Prior to his most recent positions, he was head of a mathematics department with the Wellington County R.C.S.S. Board and earlier worked with the Etobicoke and Waterloo County Boards of Education. He has worked on the program in planning the development, evaluating manuscript, consulting at all grade levels, and as the senior author of the Grade 3 material.



Jack A. Hope

Jack is currently an Assistant Professor with the Department of Curriculum Studies at the University of Saskatchewan in Saskatoon. He is also a part-time consultant with the Saskatoon Separate and Public School Boards. He has Bachelor of Science and Master of Arts in Education degrees from the University of British Columbia and courses toward a Doctor of Education degree. His background in mathematics has included teaching elementary school in Port Alberni, British Columbia, teaching methods courses at the University of British Columbia, and conducting summer school courses at the University of Regina and the University of Victoria. He has worked on the Grades 4 to 6 phase of the program in planning the development and evaluating manuscript.

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Jean is currently School Supervisor of School District 15, Moncton, New Brunswick. She has Bachelor degrees in Education and Elementary Education from the University of Moncton. She has taught extensively in elementary and junior-high schools and was an elementary school principal prior to her present appointment. She is a Past President of the New Brunswick School Supervisors Organization and represented New Brunswick at N.C.T.M. meetings for a number of years. She continues to be active in curriculum development and is an author of other mathematics publications. She has worked on the program in planning the development and evaluating manuscript for the primary phase.

Authors and Consultants

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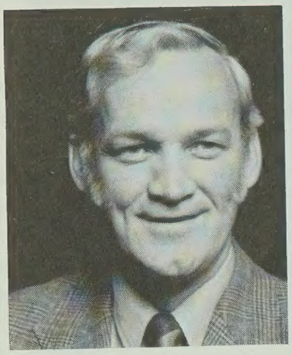
Murray is currently Professor and Head, Department of Curriculum: Mathematics and Natural Sciences, Faculty of Education at the University of Manitoba. He has Bachelor of Science and Master of Education degrees from the University of Manitoba, and a Ph.D. from Michigan State University. Prior to his present appointment, he was a teacher and head of the department of mathematics at St. John's High School, Winnipeg, and a mathematics teacher at Dauphin Collegiate. He is Past President of the Manitoba Association of Mathematics Teachers and has spoken at a number of N.C.T.M. meetings. He has worked on the program in planning its development and evaluating manuscript.



Edward B. Murrin

Ed is currently teaching mathematics at Antigonish Regional High School. Prior to his present teaching responsibilities, he has been an elementary school principal for sixteen years, a Past President of the Nova Scotia Mathematics Teachers Association, and a lecturer in elementary mathematics at St. Francis Xavier University. He was the Canadian representative for the National Council of Teachers of Mathematics from 1972 to 1975 and has been active in mathematics throughout his home province of Nova Scotia for many years. He has worked on the program in planning its development and evaluating manuscript.

Paul Pogue

Paul is currently teaching mathematics at Barrie North Collegiate Institute in Barrie, Ontario. A graduate of Lakeshore Teachers College, Toronto, he has many years of experience in elementary and high school mathematics classrooms. He has been active in curriculum planning as project director of "Recommendations for Intermediate Mathematics for the Province of Ontario", a Ministry of Education publication, and as a member of the writing team for the Intermediate guidelines established by the Ministry of Education. At the present time, he is a director of the Ontario Association of Mathematics Education. He has worked on the program in planning the development, evaluating manuscript, and as an author of material for Grades 4 to 6.

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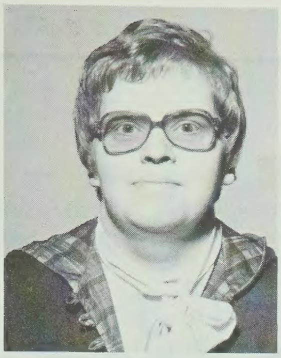
Trudy has a Bachelor of Arts degree from York University. She has been an elementary teacher and is currently a General Consultant for the North York Board of Education and Program Leader in Language Arts and Mathematics. She developed the themes for the primary phase of the program.

Stella Tossell

Stella has a Bachelor of Arts degree from the University of Toronto. Her background in mathematics has included teaching at the secondary level with the Lincoln County and York County Boards of Education. After teaching at the American School in Athens, Greece, she resumed her duties with the York County Board of Education and then joined the North York Board of Education in an advisory position at the junior-high level. More recently she has been a Mathematics Consultant mainly at the elementary level for the North York Board of Education. She has worked on the program in evaluating and writing manuscript, and writing teaching suggestions and activities for the teachers.

Authors and Consultants

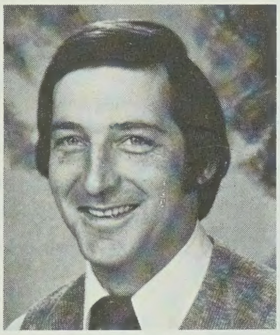


Cathie Traynor

Cathie is a Consultant with the Metropolitan Toronto Separate School Board. A graduate of Lakeshore Teachers College, Toronto, she has continued her education at York University and, through the Ontario and British Ministries of Education, in a number of specialized categories, namely, early childhood education, language development, speech therapy, and special education. She taught in the elementary grades for the Metropolitan Toronto Separate School Board and also the Ontario Ministry curriculum and mathematics courses. She has lectured at York University. She is an author on the program and has coordinated and evaluated the field-test material prior to publication.

Stewart West

Stewart has Bachelor degrees in Arts and Education from the University of New Brunswick and is currently teaching at the Magnetic Hill School in Moncton. He has served in many capacities in the province: as a member of the Provincial Committees in Mathematics and Science; as Conference Chairman for the Association Subject Council Workshop; as President and Workshop Coordinator for Elementary Teachers' Council. He continues to remain active in numerous presentations in his home province, in other centers in Canada, and in the United States. He has worked on the Grade 3 to 6 phase of the program in planning the development, evaluating manuscript, and coordinating and evaluating field-test material prior to publication.



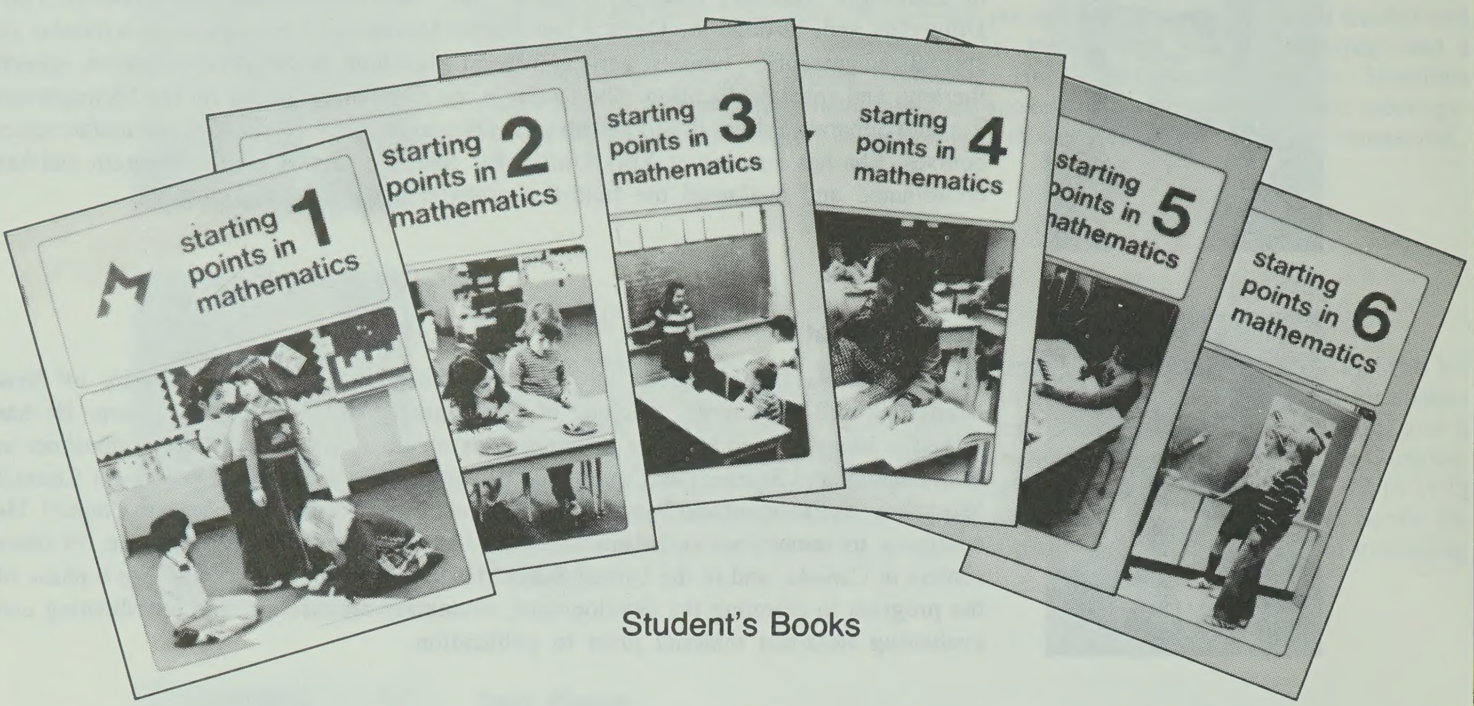
Gary White

Gary is currently Principal of Robert Meek Public School in Kingston, Ontario. He has a Bachelor of Arts degree from Queen's University and is a graduate of Peterborough Teacher's College. His background in mathematics includes teaching at the primary, junior, and intermediate levels. He is active in curriculum development and writing and presenting suggestions and activities for classroom teachers. He has worked on the program as an author of material for Grade 4.

Mary Wright

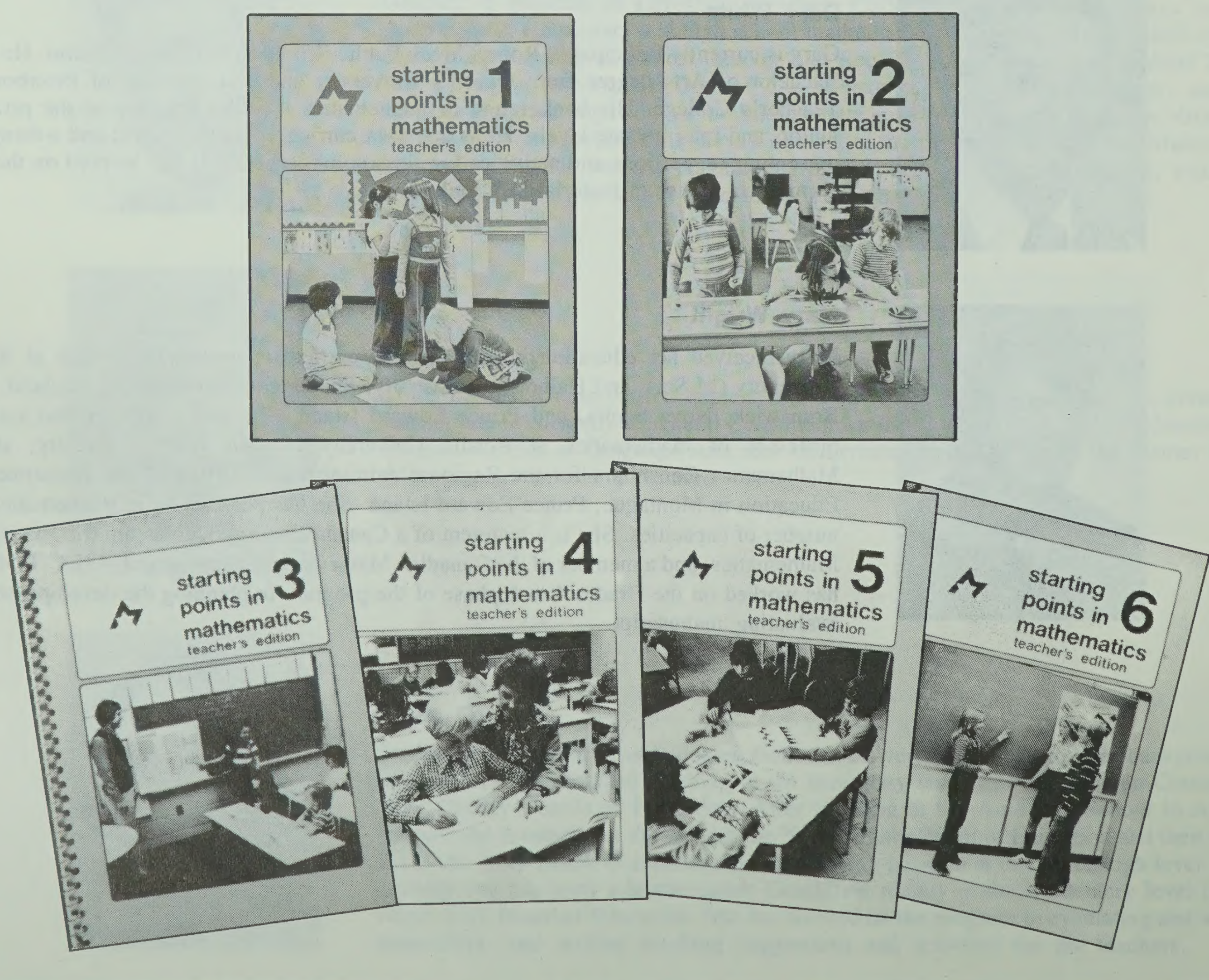
Mary received her education in England and carried on postgraduate work at Acadia University (M.Sc.), and Dalhousie University. She taught high school in England, New Brunswick, Nova Scotia, and Prince Edward Island. She was a lecturer and assistant professor of mathematics at Acadia University for ten years. Currently, she is Mathematics Consultant for the Regional Administrative Office of the Department of Education in Montague, Prince Edward Island. She has been active in mathematics in a number of capacities. She is a recipient of a Canada Council Scholarship for Teachers of Mathematics, and a member of the Canadian Mathematics Congress and of N.C.T.M. She has worked on the Grades 4 to 6 phase of the program in planning the development and evaluating manuscript.

starting points in mathematics



Student's Books

Teacher's Editions



Program Highlights

Content

- Computation strands that maintain a balance between concepts and skills
- A metric Measurement strand using units and symbols in accordance with the National Standards of Canada
- A Decimals and Fractions strand that reflects the more significant role of decimals in a metric world
- A Geometry strand that introduces transformation geometry topics in addition to the more traditional topics
- A Problem Solving strand that identifies specific problem solving skills and strategies
- Lessons on using a calculator to reinforce the understanding of number operations and as an aid for checking results

Development

- Computational concepts and skills built upon the basic facts, the continued manipulation of concrete materials, place value, systematic development of the algorithms, and practice
- Measurement concepts and skills introduced using non-standard units; refined and developed using only approved metric units
- Decimals introduced with the parts-of-a-whole concept and developed by extending the place-value concepts of whole numbers
- Corresponding ideas among the Numeration, Computation, Measurement, and Decimals strands treated as mutually supportive concepts for both development and reinforcement
- Problem Solving strand integrated with the other strands
- Material provided for maintenance of computational skills

For the Student

- A highly visual program placing mathematics ideas and experiences within meaningful settings of real-life objects and situations
- Uniform lesson structure with completed examples to illustrate each objective
- A variety of types of exercises
- Problems that provide reasons for learning mathematics
- *Special Features* showing mathematics in use in real-life situations and providing opportunities to be individually creative with mathematical skills in problem solving and enrichment activities

For the Teacher

- Manageable units for the development of concepts and skills
- Overviews that provide mathematics background and summarize the content of each unit
- Concise statements of lesson objectives
- Suggestions for activities to precede and follow each lesson in the book; suggestions for teaching each lesson in the book
- Uniform lesson structure that is adaptable to a variety of classroom strategies
- Unit themes that support the integration of mathematics with other areas of study, and suggestions on how this integration can be achieved
- Component skills necessary for achieving lesson objectives identified
- Assessment materials included in the book and the teacher's edition

Features of the Student's Book

Lesson outcome is stated.

Color and design are used to assist understanding.

A lesson begins with a worked example of what is to be learned.

7 MULTIPLYING AND DIVIDING DECIMALS

Multiplying Decimals and Whole Numbers

The camera store has 27 rolls of one type of film. The price of each roll is $3.15. What is the value of the 27 rolls?

Multiply 27 and 3.15.

$3.15 has the same value as 315 cents. Think of 3.15 as 315 hundredths.

For the product,

3 . 1 5	3 1 5 hundredths
2 7	2 7

you need to know how to multiply 7 and 315,

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5

and how to multiply 2 and 315.

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5
6 3 0 0	6 3 0 0

Then add and place the decimal point.

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5
6 3 0 0	6 3 0 0
8 5 . 0 5	8 5 0 5 hundredths

The value of the 27 rolls of film is $85.05.

134

The development of a concept enables the student to move from the use of concrete materials to the use of abstract number sentences and algorithms.

Special Features

Practice with addition, subtraction, multiplication, and division

KEEPING SHARP

Some interesting ideas for fun and enrichment

try this

Lessons and activities to help in learning the skills needed for solving problems

PROBLEM SOLVING

Working Together

Complete each multiplication.

1.

481	481 tenths	48.1
7	7	7
	tenths	

2.

145	145 hundredths	1.45
22	22	22
	hundredths	

3.

2875	2875 thousandths	2.875
35	35	35

4.

13	13 thousandths	0.013
6	6	6

Multiply.

5. 3.15 × 8
6. 14.3 × 79
7. 4.756 × 12
8. 0.67 × 15
9. 0.004 × 23
10. 20.3 × 375

Exercises

Multiply.

1. 57.7 × 28
2. 5.77 × 28
3. 2.695 × 9
4. 0.48 × 75
5. 375.6 × 237
6. $123.45 × 67

7. 13 × 1.07
8. 6 × 58.6
9. 39 × 0.003
10. 10 × 20.72
11. 40 × 3.7
12. 76 × 0.085
13. 41 × 74.8
14. 5 × $97.34

Solve.

15. The sports store has 7 tennis rackets in stock at $43.75 each. What is the value of the tennis rackets?

16. The supermarket had a delivery of 144 packages of butter which were to be priced at $1.39 each. What was the value of the butter?

When a number is multiplied by 10, each digit moves one place to the left.

Example:

	tens	ones	tenths	hundredths		tens	ones	tenths	hundredths
10 ×		3	6	7	=	3	6	7	

Multiply each number by 10. Write only the products. Do these in your head.

17. 1.14
18. 28.62
19. 3.588
20. 670
21. 0.016
22. 7.5

135

Working Together shows the steps of what is to be learned.

Exercises give practice and provide applications of what has been learned.

Word problems whose solutions incorporate the skills taught are included.

Special * exercises provide more practice with problem solving.

Checking Up

End-of-unit lessons provide a check of the understanding of the work of the unit.

Checking Skills

Special reviews provide a check of skills with addition, subtraction, multiplication, and division.

Features of the Teacher's Edition

A lesson outline may include some or all of the following:

1 The page references to the student's book

2 The outcome(s) for the lesson

3 Some of the materials that would be desirable for introducing and developing the lesson

4 A reference to a page that may be copied to provide cutouts for the students

5 Mathematical terms used for the first time and other words useful for discussing the development of the topic on the page

6 Identification of concepts and skills that students should be able to perform prior to the lesson

7 Suggested tasks for assessing prerequisite skills

8 Comments about the content of the lesson

9 Activities for developing the lesson concepts, and suggestions for introducing new words and symbols

10 Suggestions for using the pages

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1 **Pages 262-263**

2 **LESSON OUTCOME**

Write percents; convert among ratios out of 100, fractions with denominators of 100, decimal hundredths, and percents

3 **Materials**

models for 0.72 and 0.08 prepared from a copy of page T 394 or from two

4 copies of page T 382 as described on page T 89

5 **Vocabulary**

percent (%)

6 **Prerequisite Skills**

Write decimal hundredths; write a ratio in three ways

7 **Checking Prerequisite Skills**

Write the decimal.

1. three-hundredths 0.03
2. twenty-hundredths 0.20
3. ninety-two hundredths 0.92

Complete the chart to show each ratio in three ways.

4.	1 to 100	1:100	$\frac{1}{100}$
5.	24 to 100	24:100	$\frac{24}{100}$
6.	37 to 100	37:100	$\frac{37}{100}$

8 **Background**

Reference to *percent* meaning "out of a hundred" will help to emphasize that expressions such as 42%, 0.42, $\frac{42}{100}$, and 42:100 are equivalent.

13 PERCENT

Writing Percents

In a poll, 48 out of 100 students knew all the words for "O Canada"

This result can be shown as a ratio

48 : 100 $\quad \frac{48}{100}$

This result can be shown as a decimal

0.48

Of the students asked, 0.48 knew all the words for "O Canada"

This result can be shown as a **percent**

48%

Of the students asked, 48% knew all the words for "O Canada"

Question

Can you recite the words of "O Canada"?

O Canada! Our home and native land!
True patriot love in all thy sons command
With glowing hearts we see thee rise
The True North strong and free;
And stand on guard, O Canada,
We stand on guard for thee

O Canada! Glorious and free!
We stand on guard, We stand on guard for thee
O Canada! We stand on guard for thee

Knows all words	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 III
Knows most of words	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 𝍸
Knows about half of words	𝍸 𝍸 IIII
Does not know the words	III

Percent shows how many out of 100

For the symbol %, you would say "percent"

Working Together

How many students knew most of the words for "O Canada"? Show the result in five ways

1. ■ out of 100 (35) 2. ■ : 100 (35) 3. $\frac{■}{100}$ (35) 4. 0.■ (35) 5. ■ % (35)

Show the other results of the poll

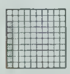
6. ■ % of the students knew about half of the words (14)
7. ■ % of the students did not know the words (3)

262

LESSON ACTIVITY

9 **Before Using the Pages**

- Display a model for 0.72 and ask what number is represented. Ask students to write the fraction and the decimal for "seventy-two hundredths" on the board. Point out that 72 of 100 parts are blue and ask what ratio can be written to express this. Have a student write the ratio using colon notation. Emphasize that 72:100, $\frac{72}{100}$, and 0.72 are different ways to represent the part of the model that is blue. Use a similar procedure for a model for 0.08, paying particular attention to the zero in the tenths' place of the decimal.



eight-hundredths

8:100 $\quad \frac{8}{100} \quad$ 0.08

Tell the students that there is another way to express "72 out of 100" and "8 out of 100". If no student suggests a percent, have the students turn to page 262.

Using the Pages

- The photograph on page 263 can motivate a discussion about making a survey and using a tally chart to record the information. Direct the students' attention to the tally chart on page 262. Ask students to count the tallies and to explain the results of the survey. Ask how many students in all were questioned for the survey.

  Ask a student to read the introductory statement at the top of page 262. Note that "48 out of 100" is shown as a ratio using the symbol :, as a fraction, and as a decimal. Introduce the term *percent* and the symbol %. If you wish, tell the students that the word comes originally from the Latin *per centum* meaning "out of a hundred". Thus, 48% means 48 out of 100. Ask in what way the symbol % suggests 100. The students will likely say that the oblique line and the two small 0's suggest the digits 1, 0, and 0 for 100. Return to the examples presented in *Before Using the Pages* and ask students to write each number as a percent.

T 286

10

Overviews

The overview at the beginning of each unit includes a list of the prerequisite skills that are required for successful completion of the unit, the outcomes for the developmental lessons in the unit, mathematical background, comments about the content and how the unit fits with the other units in the program, teaching strategies, materials, and vocabulary.

Exercises

Complete.

1.	75 out of 100	75 ■ 100	$\frac{■}{100}$ 75	75 0.■	75 ■%
2.	9 out of ? 100	9:100	$\frac{9}{100}$?	0.09 ?	9%?
3	50 out of ? 100	? 50:100	$\frac{50}{100}$	0.50 ?	50%?
4.	1 out of ? 100	? 1:100	$\frac{1}{100}$?	0.01	1% ?
5.	95 out of ? 100	? 95:100	$\frac{95}{100}$?	0.95 ?	95%

For each of these, write a sentence using %. Answers for Ex. 6–9 are given below

6. In the poll, 83 out of 100 students knew most or all of the words for "O Canada".
7. 70 out of the 100 students in the poll had to use the tune to help them remember the words.

For each of these, write a sentence using a ratio.

8. 7% of the students said that they had the song on a record at home.
9. 90% of the students knew the tune for "O Canada".

263

Working Together: The students are required to refer to the tally chart to complete Ex. 1-7. Point out that Ex. 1-5 show different ways of representing the same result.

Exercises: For Ex. 1, the students are guided as they show "75 out of 100" in different ways. Pay particular attention to the use of zeros in Ex. 2-4. For example, 9:100 for Ex. 2 is changed to 0.09 and to 9%, whereas $\frac{50}{100}$ for Ex. 3 is changed to 0.50 and to 50%.

6. In the poll, 83% of the students knew most or all of the words for "O Canada".
7. 70% of the students in the poll had to use the tune to help them remember the words.
8. 7 out of 100 students said that they had the song on a record at home.
9. 90 out of 100 students knew the tune for "O Canada".

Assessment

Complete.

1.	22 out of 100	22:100	$\frac{22}{100}$	0.22	22%
2.	31 out of 100	31:100	$\frac{31}{100}$	0.31	31%
3.	78 out of 100	78:100	$\frac{78}{100}$	0.78	78%
4.	5 out of 100	5:100	$\frac{5}{100}$	0.05	5%
5.	8 out of 100	8:100	$\frac{8}{100}$	0.08	8%

RELATED ACTIVITIES (13)

- Have the students prepare a display for the topic of percent. Draw attention to the photograph on the cover of *Starting Points in Mathematics 6*. Students can find examples of percents on coupons, on food containers, and so on. Some banks or stores may provide signs no longer needed, showing examples of percents. The display can provide a reference, and it can be used to generate word problems during this unit.
- For reinforcement, have students color copies of page T 394 to represent some of the percents shown on pages 262 and 263.
- For enrichment, provide students with copies of page T 394. Have them color part of a model blue, write the percent to show what part is blue, write the percent to show what part is not blue, and note that the sum of the two percents is 100%.
- Have students prepare pairs of cards such as the following for the game "Concentration" described on page T 379.

48%	0.48	$\frac{30}{100}$	30%

(14)

- To review graphing and to relate percents and graphs, have students draw a pictograph or a bar graph to show the results of the survey on page 262.

T 287

(11) (12)

13 Suggested activities and games which may be used to reinforce, extend, or enrich a particular topic

14 Illustrations of suggested materials

11 Reduced pages from the student's book with answers indicated

12 Exercises to evaluate the learning outcome

Answers

Answers for the exercises are given on each reduced page from the student's book. Answers that did not fit on the reduced pages or beside them appear on pages T 368 - T 377.

Other Materials

- Games and Teaching Aids
- Pages for Reproduction
- Year-End Evaluation Chart
- Index for Student's Book

Teaching Strategies

Using the Introductory Material

Knowing what the *Starting Points in Mathematics* program can do and what it cannot do is an important place to begin. There are two features to help you. First, the Scope and Sequence chart shows the content by grade level and allows you to locate particular topics in the overall development. Secondly, each unit begins with an overview which summarizes the content of the unit, includes mathematics background for the teacher, and suggests strategies for class organization and teaching.

Presenting the Lessons

The organization of the teaching suggestions for each lesson has built-in strategies to motivate and teach, and for practice and application. The identification of *Prerequisite Skills* and concepts are stated in terms of tasks students should be able to perform prior to the lesson. Teachers will want to assess prerequisite skills in certain instances; for example,

- for topics that have proven traditionally difficult,
- for new mathematics topics in the curriculum,
- with students who have missed school or been transferred,
- at the beginning of the year when the mathematical levels of the students may be uncertain,
- as a starting point to work with students who have not been successful with the previous lesson.

In the *Before Using the Page(s)* section, most lessons are initiated with concrete material to provide a review, an appropriate warm-up, or activities designed to lead students to discover a concept or a skill.

The teaching suggestions in the *Using the Page(s)* section emphasize the key aspects of the teaching example and, at times, suggest a possible sequence. When students are ready to work the exercises independently, they will benefit from a full explanation of the teaching example as a guide for their work. The teaching suggestions follow naturally from the preliminary activities and allow you to develop the lesson outcomes from the students' page(s).

In all teaching lessons, the *Working Together* section provides an examination of the sub-skills and/or the sequence of steps leading to the outcome. This section allows for immediate feedback of the students' understanding of the lesson. In this regard, this section may be completed orally or on the board. Students experiencing difficulty with the exercises can be referred to the appropriate example. *Working Together* provides another opportunity to see how all the steps of the lesson combine and, consequently, provides a means for diagnosing the students' understanding.

Each teaching lesson provides material in the *Assessment* section to evaluate the learning outcome. If the material is not used following the lesson, teachers may consider using it for pretesting or reviewing.

Most of the *Related Activities* can be used for all the students. The suggestions in this section try to provide a balance between reinforcement, enrichment, and review.

Grouping for Instruction

Knowing what to teach is one thing, knowing how to adapt a program for individual differences in ability and capacity for achievement is the ongoing role of all teachers.

It is possible to work with lower achievers and higher achievers by using the same material but by altering the teaching procedure. Lower achievers, as in other subjects, need a slower pace to provide for maximum use of concrete materials and pictorial representations as well as varied activities to ensure understanding.

With higher achievers you will often wish to move at a faster pace. This does not mean a more rapid movement through the lessons, but rather a change in approach. All students need the benefits derived from the use of concrete materials for both present and future understanding, but higher achievers tend to move more readily from the concrete to the abstract levels of mathematical thinking. They grasp concepts and skills quickly and will benefit from exploration and challenges that will allow them to use and broaden their newly acquired abilities in different settings.

Grouping for instruction is dependent on a number of factors, including teacher preference, teaching strategy, social and academic needs of students, abilities and skills of the students, the need to vary instruction, the organization of the classroom. Some possible ways for grouping are given below.

The Whole Class

Instruction of the whole class is appropriate for the introduction of new topics or class projects.

Skill Groups

For this grouping the teacher selects students having similar needs for the teaching of a specific skill. When the skill is mastered, the group is dissolved.

Interest Groups

For this grouping a student chooses to be a member of the group based on interest in the activity being offered. For example, while one concept is being explored by the whole class, a student may have the choice of working at the board or with activity cards. Interest groups may be formed for the study of a theme or for a group project. This kind of grouping promotes sharing among students and offers opportunities for students to display leadership.

Random Groups

This type of grouping may be as arbitrary as the grouping of all students wearing something red or as open as to include groups of friends. It is especially suited to situations involving games, experiments, and making models.

Often students may be part of a group, but they may work independently within the group. It is here that the teacher may observe and plan for individual needs.

Providing for Individual Differences

Use the daily performance and test results to assess the needs and abilities of the students.

For lower achievers, plan to make more extensive use of the *Prerequisite Skills* assessment before starting a lesson, the *Working Together* section, and the appropriate selection of activities from the *Related Activities*. The *Keeping Sharp* features, *Checking Skills* pages, and *Skill Practice* exercises provide continued practice for the basic operations.

Higher achievers will benefit from the flexibility and variation of the teaching model and a wider exposure to the special features *Try This* and *Problem Solving*. The *Related Activities* provide suggestions for enrichment topics.

All students will benefit from the formal lesson on problem solving found at the end of each of the sixteen units.

Problem Solving

The problem-solving strand is integrated and interspersed throughout *Starting Points in Mathematics 6*. There are sixteen teaching lessons on specific skills, eighteen special feature sections presenting problems, often as extensions of material under study, and starred problems that require special attention or integrate what has been learned in the problem-solving lessons into the exercises.

Students will benefit from specific instruction on problem solving, the concrete presentation of concepts and skills in all the lessons where this is possible, and meeting problem-solving requirements in a meaningful context. The material in this strand, however, is by no means exhaustive. Teachers will capture the right moments in their daily contact with the students to provide the insights and skills to develop better problem-solving techniques.

Testing and Evaluation

The *Checking Up* feature at the end of each unit reviews the skills taught in the unit and helps to evaluate the students' progress. It could also be used as a pretest for the unit.

The chart in the teacher's edition for each *Checking Up* feature is designed to help locate strengths and weaknesses. The *Skills* section of the chart lists the skills taught in the unit. The *Exercises* section lists the exercises in *Checking Up* corresponding to each skill identified. The *Related Pages* section lists the pages in the teacher's edition where the skill is taught. You may wish to refer to these pages for reviewing or reteaching.

The comments below the chart discuss special aspects of the exercises. They point out possible difficulties and give suggestions for remedying the difficulties.

The *Checking Skills* pages at the end of Units 6, 9, 12, and 14 evaluate students' ongoing performance in the four basic operations for whole numbers and decimals.

As well as the *Keeping Sharp* features and the *Checking Skills* pages provided throughout the text, the *Skill Practice* exercises on pages 328 to 337 provide another resource for maintaining and evaluating computational skills during the year. Suggestions are given in the *Related Activities* of appropriate lessons for assigning exercises on these pages, but they may be assigned at any time when skill maintenance is required. The *Skill Practice* exercises are also useful for diagnosing difficulties in a skill taught earlier in the year and for providing practice after a skill has been retaught.

If evaluation is to be an ongoing process, it is important to keep complete and accurate records of the achievement of each student. A file containing remarks on progress based on the observation of the teacher and samples of the student's work is recommended. The remarks can be dated and are an excellent reference when reporting to parents. The samples of work can be selected by both the teacher and the student. If the student plays an active part in contributing work which indicates her/his mastery of a concept, she/he also recognizes that learning is important. This is an essential factor in assuring future success.

The comprehensive evaluation chart on pages T 401 and T 402 is intended for use at the end of the school year, but it may be adapted for other uses. For example, if the indicated program is too ambitious for all the students in a class, the chart may be used as a guide for obtaining a minimum program or an average program for the students. The format of the chart may also be adapted as a report to show parents their children's progress.

You may wish to adopt a code for recording the stages of development in each student's mastery of concepts and skills. Slashes and dots may be used in such a way that a box marked ⧄ means that the student has been introduced to the concept or skill; a box marked ⊠ means that the student understands the concept or skill; a box marked ⊠ means that the student has mastered the concept or skill. Examples are shown below.

Finds a percent of a number	⧄ (Exposure)
Relates millilitres and litres	⊠ (Understanding)
Expresses fractions as decimals	⊠ (Mastery)

Using Calculators

Because calculators are so popular today, special lessons on the use of the calculator as an instructional aid have been included in *Starting Points in Mathematics 6*.

Some educators and parents oppose the use of calculators in the classroom. They fear a decline in computational skills, a dependence on calculators for even the simplest computations, and a premature introduction of numbers beyond the cognitive levels of the students.

Proper use of calculators does not allow these concerns to materialize. Basic skills and facts are developed first, estimation and mental calculations are encouraged, and the more frequent contacts with decimals on digital displays strengthen students' understanding of place value and their skills in rounding numbers. It has been found that students who use calculators to supplement their own computations generally develop better skills than those who do not.

The fundamental objective of all mathematics programs is the development of skills and the ability to solve problems. Problem-solving programs are strengthened when calculators are used. They save time in performing the necessary computations and, hence, more problems can be solved. By removing concerns about computation, more attention can be directed to the selection of pertinent data and to the implied relationships between numbers.

In summary, calculators can strengthen a mathematics program, but they should be used only after basic skills have been developed. They may be used

- to check basic understandings and skills;
- to support basic skills;
- to develop further insight into basic operations;
- to develop estimating skills;
- to solve real-life problems;
- to save computation time in solving problems;
- to verify mental calculations and procedures;
- to provide insights into higher mathematical concepts;
- to search for interesting number patterns.

A Thematic Approach to Mathematics

Teachers who attempt to provide an integrated curriculum frequently encounter difficulty interweaving the mathematics curriculum with other subjects and in creating sufficient real-life situations to make the association a meaningful one. Several units in *Starting Points in Mathematics 6* are theme oriented so that teachers who have never tried integrating mathematics may wish to experiment with one or more themes, for example, the situations and photographs in Unit 2 suggest "Occupations" as a theme. Similarly, the themes "Pastimes" and "Hobbies" are predominant in Units 4 and 10.

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Scope and Sequence

	Grade 4	Grade 5	Grade 6
NUMBER AND NUMERATION	Read, write numerals to 999 999 Place value for six-digit numbers Compare, order numbers to 999 999 Ordinal number concepts to 999 Round in tens' place, in hundreds' place, in thousands' place Roman numerals for 1 to 100	Read, write numerals to 999 999 999 Place value for nine-digit numbers Compare, order numbers to 999 999 Round in ones' place, in tens' place, or in hundreds' place of either of the first two periods Roman numerals for 1 to 2000	Read, write numerals to 999 999 999 999 Place value for twelve-digit numbers Compare, order numbers to 999 999 999 Round in ones', tens', or hundreds' place of any period to millions Roman numerals for 1 to 3999 Read, write, compare, order integers
ADDITION	Algorithm, four-digit addends Algorithm, three or more addends Add decimals, to hundredths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with like denominators, regrouping for whole-number sums	Algorithm, five-digit and six-digit addends Algorithm, three or more addends Add decimals, to thousandths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with like denominators	Algorithm, six-digit addends Algorithm, three or more addends Add decimals, to ten-thousandths Round addends, estimate sum; compute result and compare sums Add amounts of money Add proper and mixed-form fractions with unlike denominators Add integers
SUBTRACTION	Algorithm, four-digit minuends, four-digit subtrahends Check by addition Subtract decimals, to hundredths Subtract amounts of money Subtract proper and mixed-form fractions with like denominators, regrouping of whole-number minuends	Algorithm, five-digit and six-digit minuends Subtract decimals, to thousandths Round and subtract to estimate the difference; compute difference and compare with estimate Subtract amounts of money Subtract proper and mixed-form fractions with like denominators	Algorithm, six-digit minuends Subtract decimals, to ten-thousandths Round and subtract to estimate the difference; compute difference and compare with estimate Subtract amounts of money Subtract proper and mixed-form fractions with unlike denominators Subtract integers, using temperatures as a model
MULTIPLICATION	Algorithm, multiplicand to three digits, multiplier to two digits Multiply up to four one-digit numbers Determine missing factor in multiplication fact Multiply one-place decimal by one-digit whole number Round three-digit multiplicand in its greatest place and multiply by one-digit multiplier to estimate product Round decimal multiplicand to nearest whole number and multiply to estimate product Multiply amounts of money Identify multiples of numbers	Algorithm, multiplicand to five digits, multiplier to three digits Identify prime numbers, prime factors, common multiples of a pair of whole numbers Multiply decimal tenths, hundredths, thousandths by whole numbers Multiply decimal tenths by decimal tenths Round factors, estimate product; compute result and compare products Multiply amounts of money Multiply a fraction and a whole number which is a multiple of the denominator Find equivalent fractions, ratios, rates	Algorithm, multiplicand to seven digits, multiplier to three digits Identify prime numbers, composite numbers, prime factors, common multiples of a pair of whole numbers Multiply decimal and whole number, products to ten-thousandths Multiply decimal by decimal, products to ten-thousandths Round factors, estimate product; compute result and compare products Multiply amounts of money Multiply fraction by fraction Multiply fraction and whole number Find equivalent fractions, ratios, rates
DIVISION	Algorithm, dividend to three digits, one-digit divisor Check by multiplying Divide whole-dollar amounts Find average of set of numbers	Algorithm, dividend to six digits, divisor to two digits Check by using multiplication Divide to thousandths by one-digit or two-digit whole numbers Round dividend, divisor to estimate quotient; compute quotient and compare with estimate Divide amounts of money Find equivalent fractions, ratios, rates Find decimal equivalent to a fraction Find average of a set of numbers and compare it with the given numbers	Algorithm, dividend to six digits, divisor to three digits Check by using multiplication Divide decimal by whole number and by decimal, extra zeros in dividend, divisor to three digits Round dividend, divisor to estimate quotient; compute quotient and compare with estimate Round decimal quotients Divide amounts of money Divide fraction by fraction Divide fraction by whole number Divide whole number by fraction Find equivalent fractions, ratios, rates Find decimal equivalent to a fraction Find average of a set of numbers and compare it with the given numbers

Scope and Sequence

	Grade 4	Grade 5	Grade 6
DECIMALS AND FRACTIONS	Proper and mixed-form halves, thirds, fourths, fifths, tenths, hundredths for part-of-whole and part-of-set models Equivalent decimal tenths, hundredths with part-of-whole model Place value for decimals to hundredths; regrouping Equivalent decimals and proper and mixed-form fractions (100, 10, 4, 2 as denominators) Compare, order decimals to hundredths Add, subtract decimals, to hundredths Multiply one-place decimal by one-digit whole number Round decimal to nearest whole number, estimate sum or product Compare and order fractions using decimal equivalents and models Add, subtract proper and mixed-form fractions with like denominators, regrouping to/from whole numbers	Proper and mixed-form fractions for part-of-whole, part-of-set models Decimal thousandths Equivalent decimal tenths, hundredths, and thousandths Place value for decimals to thousandths; regrouping Compare, order decimals to thousandths Round decimal to nearest whole number, to nearest tenth Add, subtract decimals, to thousandths Multiply decimal tenths, hundredths, thousandths by whole number Multiply decimal tenths by decimal tenths Round decimal factor(s) to whole number(s) and multiply to estimate product Divide decimal by whole number, "terminating" quotients Find equivalent fractions Find the missing term in a pair of equivalent fractions Compare fractions (one denominator not necessarily a multiple of the other) Convert between decimals and fractions (halves, fourths, fifths, eighths, tenths) Convert between improper fractions and mixed-form fractions Add, subtract proper and mixed-form fractions with like denominators Multiply a fraction and a whole number which is a multiple of the denominator	Proper and mixed-form fractions for part-of-whole, part-of-set models Decimal ten-thousandths Find equivalent decimals Place value for decimals to ten-thousandths; regrouping Compare, order decimals Round decimals Add, subtract decimals, to ten-thousandths Estimate sums and differences Multiply decimal and whole number, products to ten-thousandths Multiply decimal by decimal, products to ten-thousandths Divide decimal by whole number and by decimal, extra zeros in dividend, divisor to three digits Round decimal quotients Estimate products and quotients Find equivalent fractions Find the missing term in a pair of equivalent fractions Find like denominators Compare, order fractions Convert between decimals and fractions Convert between improper fractions and mixed-form fractions Add, subtract proper and mixed-form fractions with unlike denominators Multiply fractions by fractions Multiply fractions and whole numbers Identify, find reciprocals Divide fractions by fractions, fractions and whole numbers
PROBLEM SOLVING	Use models to obtain solutions Choose the operation needed Identify relevant, irrelevant, missing information Write an equation for a word problem Multiple-step solutions; addition, subtraction, multiplication, comparison Estimate answers Recognize answers as reasonable Guess and test Multiple solutions Read scales Organize data Logical thinking	Identify relevant, irrelevant, missing information Recognize that different situations affect answers Find the number of possibilities Collect, organize, and display data Give the most reasonable answer Use models to obtain solutions Write and solve an equation for a word problem Multiple-step solutions Find and continue patterns Estimate answers Recognize incorrect results Logical thinking Guess and test Multiple solutions Use a calculator	Consider ways of estimating Identify relevant, irrelevant, missing information Find needed information Recognize that different situations affect answers Give reasonable measurements Find the number of possibilities Multiple-step solutions Write and solve an equation for a word problem Logical thinking Restate word problems Consider the chances Use models to obtain solutions Solve problems without pencil and paper Consider possible solutions Use a calculator
MEASUREMENT	Estimate and measure; choose the preferred linear unit Convert between kilometres and metres; metres and centimetres Measure and add to find perimeter Count, estimate, and calculate area in square centimetres Calculate area in square centimetres, square decimetres, or square metres Count, estimate the number of centimetre cubes; compare objects to centimetre cube, decimetre cube, metre cube	Estimate, measure and record lengths in appropriate units Convert among units of length Find the perimeter of a polygon, of a square, of a rectangle Measure circumference of circular objects Multiply to find area of a rectangle Create rectangular shapes having given perimeter or area Count, calculate to find volume in cubic centimetres Investigate metric prefixes Estimate capacity in millilitres, litres	Estimate, measure and record lengths in appropriate units Convert among units of length Find the perimeter of a polygon, of a square, of a rectangle Measure circumference, radius, diameter of circular objects Multiply to find area of a rectangle, a parallelogram, and a triangle Multiply to find the volume of a rectangular prism in cubic units Understand metric prefixes Estimate capacity in millilitres, litres

	Grade 4	Grade 5	Grade 6
MEASUREMENT (continued)	Compare small amounts of time, of length, of mass, of capacity Classify capacities in comparison with 1 mL, 500 mL, 1000 mL Classify masses in comparison with 1 g, 500 g, 1000 g Convert among units of time Tell and record time to the nearest minute using 12-hour and 24-hour clock; add, subtract time using 24-hour clock	Estimate mass in grams, kilograms Convert between millilitres and litres, grams and kilograms Find capacity in litres, millilitres, by finding volume in cubic centimetres Relate capacity, mass, and volume units for water Numeric dating; time to the second; add, subtract time, no regrouping	Estimate mass in grams, kilograms Convert between millilitres and litres, grams and kilograms Relate capacity, mass, and volume units for water Numeric dating; time to the second; convert among time zones in Canada; standard time, daylight saving time Read and record temperatures, including negative-number readings for temperatures below 0°C Find differences between temperatures
GEOMETRY	Identify, name, and draw lines, line segments and their end points Identify, name, and draw angles; identify and draw right angles Compare angles to right angles Identify, name, and draw triangles; identify and name sides and angles of triangles Identify and draw polygons; identify and name sides and angles of polygons Identify and draw circles; identify and name parts of circle Recognize patterns for, and properties of, solids Use tracing paper to test for congruency in general and under a slide, flip, or turn on a grid Identify and check line of symmetry as a flip line; identify multiple lines of symmetry (on a grid)	Identify, name, and draw lines, rays, and line segments Identify and draw parallel, intersecting, or perpendicular lines, rays, and line segments Identify, name, measure, and draw angles; identify congruent angles by tracing Classify angles as acute, right, or obtuse Classify plane shapes according to number of sides, relationships between sides and lines of symmetry; equilateral, isosceles, and scalene triangles; square, rhombus, rectangle, parallelogram, trapezoid Classify pyramids and prisms according to their faces Recognize cylinders, spheres, cones Identify and sketch lines of symmetry (not on grid) Use tracing paper to test for or help to draw slide, flip, or turn image (on or not on grid) Turn triangles to form regular polygons Fit polygons together to make a pattern without spaces Copy picture from one grid onto another grid (including distortions)	Identify, name, and draw lines, rays, and line segments Identify and draw parallel, intersecting, or perpendicular lines, rays, and line segments Identify, name, measure, and draw angles Classify angles as acute, right, obtuse, or straight Classify plane shapes according to number of sides; equilateral, isosceles, and scalene triangles; square, rhombus, rectangle, parallelogram, trapezoid, regular polygons Identify parts of a circle; relate radius, diameter, and circumference Classify pyramids and prisms Recognize cylinders, spheres, cones Identify and sketch lines of symmetry Identify congruent shapes Identify and draw similar shapes Interpret and make scale drawings Use tracing paper to test for or help to draw slide, flip, or turn image (on or not on grid) Draw slide image using a rule Draw flip image by counting Test for rotational symmetry Make tiling patterns Identify slide, flip, and turn images in tiling patterns Copy picture from one grid onto another grid (including distortions)
GRAPHING	Interpret pictographs and bar graphs Gather and organize information in pictographs and bar graphs Associate ordered pairs of numbers (including 0 as a coordinate) and points on a grid	Interpret pictographs and bar graphs Gather and organize information in pictographs and bar graphs Relate graphing of ordered pairs of numbers to graphing of ordered pairs of data Interpret and draw line graphs	Interpret, draw pictographs, bar graphs, line graphs, and broken-line graphs Gather and organize information for graphs Relate graphing of ordered pairs of numbers to graphing of ordered pairs of data Interpret circle graphs
RATIO AND PERCENT		Write ratios using colon and fraction notation Multiply, divide to find equivalent ratios, rates Find the missing term in a pair of equivalent ratios, rates Use the symbol % for percent Convert among decimals, fractions, percents, and ratios	Write ratios using colon and fraction notation Find equivalent ratios, rates Find the missing term in a pair of equivalent ratios, rates Find unit rates, unit prices Use the symbol % for percent Convert among decimals, fractions, percents, and ratios Find a percent of a number Calculate interest, discount

Contents of Student's Book

The Mathematics Centre

A mathematics centre, like centres for other subject areas, is a place for the storage of certain specific materials and an area for the students to become involved in activities. With careful planning and involvement of the students, the mathematics centre can become a stimulating environment. The students will enjoy bringing materials from home to supplement and add variety to those in the centre. The mathematics centre is an ideal place to display the students' work. If a thematic approach is used for teaching mathematics, the centre can be adapted as a setting for each new theme.

When the students have finished their regular assignments, they may engage in extra activities and projects in the mathematics centre. Activity cards, puzzles, games, and homemade as well as commercial materials will lead the students to broader understanding as well as provide opportunities for the teacher to observe and question the students, and to evaluate their progress. Consideration of the students' interaction in the mathematics centre will suggest adaptations to make this strategy an important part of the learning experience.

STORAGE OF MATERIALS

Materials should be stored where students can have easy access to them. Open shelves and small tables can be used in a pleasing and practical arrangement for holding containers. Containers for materials should be both sturdy and colorful. Vinyl coverings, spray enamel, wallpaper, and fabric will increase the durability of the containers as well as increase the appeal of the mathematics centre. Some ideas for suitable containers are pails (formed by cutting the top off a large plastic bottle), trays (from corrugated boxes in which canned goods are sold), baskets (in which fruit and vegetables are sold), boxes (sturdy ones such as those from small appliances), and other containers (ice cream containers, large cans, plastic tubs).

MATERIALS

The materials used for teaching mathematics need not be expensive commercial materials; simple everyday objects can be used effectively as learning and teaching aids.

A list of the materials suggested for each unit is given in the unit overview. When a unit is almost completed, look ahead to the next unit and begin to collect and have the students help to collect the materials.

The following materials will be helpful for developing the various concepts and skills.

Number

- counters such as buttons, beans, pebbles, bingo chips
- objects for grouping, such as pipe cleaners, drinking straws, stirrers, beans in plastic bags, Unifix cubes
- models for ones, tens, hundreds, and thousands
- flash cards for basic addition, subtraction, multiplication, and division facts
- dominoes and domino cards, playing cards, dice
- shapes marked to show halves, thirds, fourths, fifths, eighths, and tenths of a whole
- set holders such as plastic tubs, egg cartons, plastic hoops, Styrofoam trays, paper or foil plates
- models for tenths and hundredths

Geometry

- a collection of three-dimensional shapes, such as balls, boxes, cores from rolls of paper, funnels
- wooden, plastic, or cardboard plane shapes (triangles, rectangles, squares, pentagons, hexagons, octagons, circles)
- commercial wooden or plastic plane shapes (triangles, rectangles, squares, pentagons, hexagons, octagons, circles)
- geoboards, rubber bands, geopaper from page T 395
- parquetry blocks, gummed shapes for forming patterns
- pictures of symmetrical objects and shapes
- cutouts of plane shapes for showing slides, flips, and turns
- felt, plastic, or cardboard tangram pieces
- materials for constructing models of three-dimensional shapes (straws, pipe cleaners, toothpicks, plasticine, clay)
- square tiles (ceramic tiles, floor tiles)
- samples of fabric, wallpaper, and gift wrapping paper

Measurement

- real money, play money
- non-standard units for measuring length (straws, paper clips, ribbons), capacity (jars, paper cups, milk cartons), mass (washers, plasticine balls)
- unmarked metre sticks, metre sticks and tapes marked in millimetres, centimetres, and decimetres
- rulers or straight edges
- centimetre cubes, decimetre cubes
- containers for comparing capacity (jars, bottles, cans, boxes)
- materials for filling containers (sand, water, rice)
- one-litre containers, other containers marked in litres (juice cans, pails) and in millilitres (soft-drink cans, canned goods)
- objects for comparing masses (pebbles, stones, books)
- one-kilogram masses, objects with masses marked in kilograms (boxes of detergent, bags of sugar) and in grams (boxes of cereal, pasta, or crackers)
- balance scales, kitchen scales, step-on scales
- thermometers

There are many other teaching aids that will be useful in a mathematics program, for example, D-Stix, Multibase Arithmetic Blocks. They may be acquired over a number of years, but the following aids are basic for the program.

Display Board: This may be a flannel board, a magnetic board, or a bulletin board for use with cutouts of objects, numerals, and symbols in demonstrating new concepts.

Attribute Blocks: These are sets of wooden or plastic blocks that show likenesses and differences in color, shape, size, and thickness. One set usually includes 48 pieces made up of four shapes (circle, rectangle, square, triangle), three colors (red, blue, yellow), two sizes, and two thicknesses. (Some sets also include the hexagon.)

If commercial attribute blocks are not available, you may wish to make your own blocks by using the patterns on pages T 383 to T 385. The blocks may be made from plywood of two thicknesses, one of which is about three times as thick as the other.

Number Line: A number line on the chalkboard or display board may be permanently displayed at a level where students can see it and also reach it, if necessary; for example, with number strips in showing extensions of basic addition facts. Each student should have an individual number line.

Other teaching aids are described on page T 378.

Timing Schedule

The following information will guide you in planning your schedule for working through *Starting Points in Mathematics 6* in one year. Depending on the abilities of the students in your class, you will find sufficient material in Book 6 for a minimum program, an average program, and an enriched program.

For this book, most lessons are developed over two pages and these are referred to as "double page" lessons in the table below. There are also "single page" lessons, for example, the *Problem Solving* lessons and most *Checking Up* lessons that appear at the end of each unit.

There are approximately 175 days in the school year. The number of lessons in Book 6 is 185. This number excludes the *Checking Skills* exercises provided after the *Checking Up* pages for Units 6, 9, 12, and 14 and the *Skill Practice* exercises on pages 328 to 337. No suggestions have been made for these and other maintenance lessons because the need for these can be determined only by you for the students in your particular class.

The number of days required to complete a unit will depend on the level of the students in your class. For example, students who have completed the first five books of *Starting Points in Mathematics* may be able to complete Unit 1 in 8 days or fewer than 8 days. If much of the work is new for students, you may plan to spend as many as 10 days to consolidate most of the concepts of Unit 1 at this time. Other alternatives would be to omit certain lessons and teach them at a later time. For example, the work of comparing and ordering numbers can be taught during Unit 2 (addition and subtraction), Unit 6 (measurement), or Unit 9 (geometry).

Since the work of Unit 2 (addition and subtraction) will likely be review for most students, you may plan to begin the program with this unit. At the same time the work of Unit 1 may be introduced gradually. In this way students will have ample opportunity to practice skills with which they are already familiar and still be provided with the challenge of learning new concepts.

Teachers wishing to provide a minimum program may plan to omit part or parts of certain units and, depending on the students' progress, return to some of these later in the year.

A unit or part of a unit that is not required by the curriculum guidelines may be omitted to allow extra time for topics with which students require more help, such as multiplication and division with decimals. For skill-developmental lessons, it may not be necessary to assign all the exercises, but only sufficient exercises to ensure that the skills have been mastered. For example, you might assign the odd-numbered exercises and if these are reasonably well completed, there is probably no need for further practice. If, on the other hand, there seem to be difficulties, reteaching and review should be provided before assigning the even-numbered exercises. In planning any schedule, it should also be kept in mind that certain topics such as measurement (Units 6 and 8) and geometry (Units 9 and 14) require more time than others as they involve more activity with concrete objects and manipulative materials.

Unit	Number of Lessons		Lessons in Unit	Number of Days	My Schedule
	Double Page	Single Page			
1	6	2	8	7-10	
2	9	2	11	8-11	
3	6	2	8	7- 9	
4	14	4	18	15-18	
5	10	4	14	11-14	
6	13	1	14	12-14	
7	9	4	13	11-14	
8	8	2	10	9-12	
9	11	1	12	12-14	
10	6	6	12	10-13	
11	13	4	17	16-18	
12	6	3	9	7-10	
13	6	2	8	7- 9	
14	10	1	11	11-13	
15	5	8	13	11-14	
16	5	2	7	6- 7	
Total	137	48	185	160-200	

Unit 1 Overview

Numeration

In this unit, the importance and the use of both large and small numbers in everyday life are considered. In connection with large numbers, place values are studied in numerals with up to twelve digits and the names *billions*, *millions*, and *thousands* are used in reading the periods of numerals in standard notation. Expanded form is included to provide another look at place value. Comparing and ordering numbers emphasizes the need to examine digits in numerals from left to right. Skills in rounding numbers are reviewed and extended to rounding to the nearest hundred million. Notation for decimals to three decimal places (thousandths) is reviewed and the need for this is related to very small measurements. The problem-solving skill in this unit deals with the use of estimates in real life and various methods of making estimates are explored. One *Try This* feature challenges students to write numerals given certain conditions and another shows how Roman numerals can be converted to Hindu-Arabic numerals through expanded notation. The unit concludes with exercises to assess the concepts and skills involved in numeration.

Prerequisite Skills

- read and write standard numerals for numbers to 999
- interpret place value in numerals to 999
- write the expanded form for numbers to 999
- compare and order numbers to 999

Unit Outcomes

- read and write standard numerals for numbers to 999 999 999 999
- interpret place value of digits in numerals to 999 999 999 999
- write the expanded form for numbers to 999 999 999 999
- compare and order numbers to 999 999 999
- round to the nearest one, ten, or hundred, in any period to millions
- read and write numerals and words for one-place, two-place, and three-place decimals
- identify methods of estimating

Background

Our familiar Hindu-Arabic system of numeration is a place-value system. Ancient civilizations, such as the Egyptian, Babylonian, Greek, and Roman, had numeration systems but they were not place-value systems. A place-value system is based on a set of digits that can be used and repeated to represent both very large numbers and very small numbers. Our numeration system is a decimal system because it is based on the number ten. The word *decimal* comes from the Latin word *decem* meaning ten. In our decimal system there are ten digits, 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. Of these, 0 is special. Although it is not a digit for a counting number as the others are, it is essential as a place holder in our place-value numeration system. The earlier civilizations had no such symbol and accurate interpretation of numerals was either difficult or impossible.

A place-value numeration system is one in which the value of any digit in a numeral is determined in part by its position in the numeral. Since ours is a base-ten system, the positions have values represented by powers of ten which increase from right to left. A digit, therefore, represents a number which is the product of its own value and the value of the place in which it appears. In the numeral 125 745 200, the digits 2, 5, and 0 are repeated. The first 2 from the right represents 2 hundreds (2 × 1 hundred), whereas the other 2 represents 2 ten millions (2 × 1 ten million). Similarly, the first 5 from the right has a value of 5 thousands (5 × 1 thousand), and the other 5 has a value of 5 millions (5 × 1 million). The two zeros are place holders in the ones' and the tens' positions to ensure that the 2 to their left is interpreted as hundreds.

h	t	o	h	t	o	h	t	o
millions			thousands					
10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
1	2	5	7	4	5	2	0	0

125 745 200
100 000 000
20 000 000
5 000 000
700 000
40 000
5 000
200

A thorough understanding of place values is needed to interpret numerals in standard form. The expanded form of a numeral, such as 600 000 + 30 000 + 4 000 + 200 + 5 for the standard numeral 634 205, shows the values of the digits and their places clearly. Expanded form also shows the addition of values which is implied, but not shown, between the digits in the standard form. In summary, the value of any digit in a numeral is dependent upon three things: the *base* of the system, the *face* value of the digit, and the *place* value of the digit.

In the base-ten numeration system, digits are considered in groups of three starting from the right. Each group is called a *period* with hundreds, tens, and ones recurring in each period. In the past, commas have been written between the periods, but now it is customary to leave a space between two consecutive periods, except in cases where not more than four digits are involved.

The reading of large numbers is easy if each period is read and named in turn, from left to right. The numeral 365 is read "three hundred sixty-five" wherever it occurs. Thus, in 365 784 210 the 365 is read "three hundred sixty-five *million*". Similarly, 784 is read "seven hundred eighty-four *thousand*". It should be emphasized that the word "and" is not inserted between hundreds and tens in reading a three-digit numeral. For instance, for 365 it is incorrect to say "three hundred *and* sixty-five". The word "and" is reserved as a separator between the whole-number and fractional parts in reading a decimal.

The basic structure of our numeration system can be extended to the right to show powers of ten less than one. From the name of our system these numbers are called *decimals*. Proceeding to the right, the value of each place is one-tenth of the value on the left.

hundreds	tens	ones		tenths	hundredths	thousandths
10^2	10^1	10^0		10^{-1}	10^{-2}	10^{-3}
2	0	4	.	3	7	5

The numeral shown in the chart is read "two hundred four and three hundred seventy-five thousandths".

As stated earlier, the Roman system of numeration is not a place-value system. It has no symbol for zero and it cannot use the same digit to represent different powers of ten, although it is primarily a base-ten system. Both addition and subtraction of the values of digits are employed in using the seven symbols I, V, X, L, C, D, and M. If a symbol of equal or lesser value is on the right of another symbol, their values are added; but if a symbol of lesser value is on the left of another symbol, the value of the expression is their difference. For instance, in MC the value of M (1000) and the value of C (100) are added (1100), but in CM the number represented is the difference of the two values (900). Symbols may be repeated up to three times, but may be used on the left of a greater value only once. To interpret a Roman numeral it is necessary to separate it into groups of symbols which represent specific values, starting at the left. The Roman numeral MCMLXXXIV may be considered as shown.

M CM LXXX IV
1000 + 900 + 80 + 4

Teaching Strategies

Numeration involving numerals with up to twelve digits to show billions may be too advanced for some students at the beginning of the school year or may even be beyond the scope of the program in some schools. In either of these situations, it is suggested that exercise assignments for each lesson (beginning with pages 6 and 7) omit the exercises that involve billions at the end of the exercise set. An alternative that would allow for even more gradual introduction of large numbers would be to begin the year's program with Unit 2, in which the familiar operations of addition and subtraction are reviewed and extended. In Units 2 and 4, the work in addition, subtraction, multiplication, and division is limited to numbers not greater than hundred thousands, with a few exceptions involving multiplications having large products. Thus, Unit 1 could be introduced gradually as a change of pace from work in other units.

The wide use of numbers in everyday life is generally overlooked. People in the business world, of course, use numbers constantly in their transactions, but even the everyday experiences of students involve numbers. A list might be made of numbers they encounter to impress upon them the importance of comparatively small numbers. At the same time, reading the financial section of a major newspaper will reveal the frequent occurrence of large numbers. For instance, the following large numbers appeared on one page of a large daily newspaper: 100 000, 5 500 000, 47 500 000, 49 600 000, 300 000 000, 2 500 000 000, 3 500 000 000, and 6 500 000 000. Rounded numbers are often used in reporting financial matters and the following appeared on the same page: 560 200 and 689 800, 4 027 600 and 4 082 700, 4 304 000 and 4 351 000, 30 630 000 and 30 680 000. In some of these pairs of numbers, rounding was to the nearest hundred, and in others to the nearest thousand and to the nearest ten thousand. Such numbers are ideal for comparing and ordering numbers.

Newspaper articles may also be used for creating exercises in reading and writing numerals in standard form. Students may be given selected clippings and instructed to write in words all the numerals that appear in standard form, and to write in standard form all the numbers indicated in other ways.

For the study of numeration, a place-value pocket chart with the names *thousands* and *millions* (and *billions* if warranted) at the top is useful. Instructions for preparing a chart similar to the one illustrated below are given on page T378. If the pocket chart has more than one level of pockets, two numbers may be compared. Each period in the numerals is examined in turn from left to right.

h	t	o	h	t	o	h	t	o	h	t	o
billions			millions			thousands					
				1	6	4	5	6	0	0	0
				1	6	4	5	3	0	0	0

As shown here, in the millions' period the numbers are the same, but in the thousands' period 456 is greater than 453, so 16 456 000 is greater than 16 453 000. If the pocket chart and sets of numeral cards are available on a table or in a mathematics centre, students may use their spare time in displaying and interpreting numerals.

Rounding numbers is a comparatively easy skill, but for some people it is difficult because they attempt to consider all the digits at once and they become confused. To round a number to any value it is necessary to focus attention on only two places, on the place to which the number is to be rounded and then on the next place to the right. If the digit in this next place is 0, 1, 2, 3, or 4, the number is rounded down; that is, all the digits to the right of the place being rounded are changed to 0. On the other hand, if the digit in that place is 5, 6, 7, 8, or 9, the number is rounded up; that is, the digit in the desired place of rounding is increased by one and all the digits to its right are changed to 0.

Round to the nearest ten thousand.

$\boxed{3\ 67}$3 498 ⟶ 3 670 000 (rounded down)

$\boxed{3\ 67}$5 106 ⟶ 3 680 000 (rounded up)

Increase by 1.

Materials

- a card showing the word "thousand" and a card showing the word "million" (and a card showing the word "billion," if warranted).
- numeral cards showing three-digit numerals, including some with one or two zeros
- real coins (optional)
- models for ones, tenths, and hundredths made from copies of pages T 392 - T 394 as described on page T 89
- a ruler marked in millimetres

Vocabulary

millions	kilometre, km
digit	millimetre, mm
expanded form	decimal
standard form	decimal point
place-value chart	diameter
billions (optional)	Roman numerals
is greater than (>)	kilograms, kg
is less than (<)	names for numbers to 999 999 999 999
round to the nearest	names of the planets
round down	
round up	

OBJECTIVE

Identify the use of numbers in situations outside the classroom

1 NUMERATION

Thinking About Numbers

How many meals have you eaten?

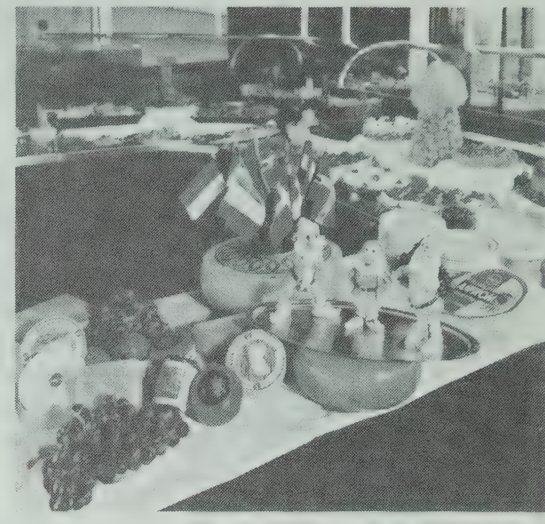
How long have you slept?

How many words do you know?

How long have you spent reading?

How much money have you spent?

Answers will vary



How many trees are in Canada?

How many kinds of trees are in Canada?

How many needles are on a spruce tree?

How thick is a maple leaf?

How tall is the tallest tree?

2

LESSON ACTIVITY

Before Using the Pages

- Pages 2 and 3 deal with the use of numbers in situations outside the classroom. As a contrast, begin with a discussion of the use of numbers within the school environment. Suggest that it would be difficult to live without numbers and present examples to illustrate this. For instance, ask the students what answers they would give for these questions.

  "How old are you?"
  "How many students are in the class?"
  "How many days of this month are not school days?"
  "How much time have you spent at school today?"

  After a few examples, ask students to suggest similar questions. Then have them answer a few of the questions and write the numerals on the board. Discuss the methods used to find the answers. For example, for the second question, they may have counted by ones, twos, or fours, depending on the arrangement of the desks in the classroom. The third question may require the use of a calendar and addition or subtraction. The fourth question can involve more than one unit of time, for example, hours and minutes. Summarize that the answers to these questions depend on numbers and that the numbers can be determined with relative ease. Then tell the students that there are questions whose answers are very large numbers or very small numbers, which are not so easily determined as in the examples just considered. You may wish to have one or two students suggest a question of this type.

Using the Pages

- The photographs on these pages were selected to motivate questions related to numbers. Five such questions accompany each photograph. The intent is not that the students provide specific numbers as answers, but rather that they develop an awareness of the use of numbers.

  Ask what is shown in the first photograph and have students read the accompanying questions aloud. Ask them to suggest answers to the questions. For example, they may

How many times have you looked at the stars?

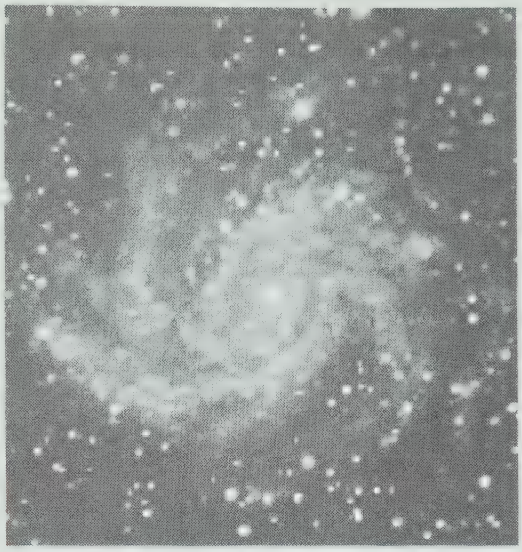
How many stars are in the sky?

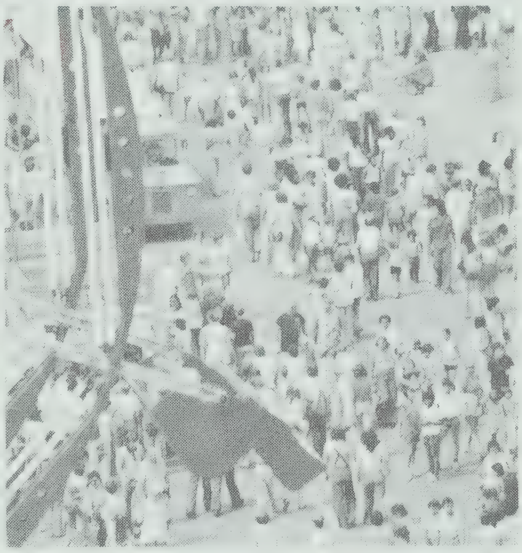
How far can you see on a clear night?

How far away is the sun?

How heavy is the earth?

Answers will vary





How many people have you seen at one time?

How many people have you met?

How many people do you know?

How many places have you visited?

How many cities would you like to visit?

3

RELATED ACTIVITIES

- Obtain large pictures similar to the photographs shown on pages 2 and 3 and display them for several days. They can motivate questions involving numbers, similar to the questions on pages 2 and 3. Have students write questions to accompany each picture.
- Have students list ways they make use of numbers in their activities during one day. Display the lists and provide opportunities for comparing and discussing the information.
- Adapt the previous activity for one or more of the following suggestions: the use of numbers by adults in the home (in following a recipe, planning a budget, and so on); the use of numbers by people at work (in a store, in a factory, in a hospital, in a bank, on a farm, and so on).
- Have students use a procedure similar to the one described in *Before Using the Pages* to determine an approximate answer to the question "How many meals have you eaten?" or "How long have you slept?" Some students may be interested in researching answers to the questions "How far away is the sun?" and "How heavy is the earth?"

suggest that they have eaten hundreds of meals, have slept thousands of hours, know thousands of words, and so on. For such answers, have students write corresponding numerals on the board. Some students may suggest a method for finding an approximate answer to the first question; for example, express one's age as a number of days, assume that one has eaten three meals each day of one's life, and use multiplication. After discussing the five questions provided, you may wish to have students ask other questions suggested by the photograph. Some examples are How many grapes are on a grapevine? How many meals have you eaten at a restaurant? and How many people have eaten at the restaurant since it opened?

Each of the remaining photographs and questions may be discussed in a similar manner. Note that the question "How thick is a maple leaf?" suggests a very small number. Some students may suggest the use of a decimal to represent a number less than one. Others may respond to this question by saying that a maple leaf is about as thick as a piece of paper or that it is thinner than a piece of cardboard.

The answers suggested by the students can reflect an understanding of various concepts in mathematics, and the extent of their knowledge can be helpful in preparing lessons that follow. Some of the questions can encourage recall of units for the measurement of time, money, length, and mass. For such questions, have students write on the board the names and symbols for units they recall. Then ask them to suggest which unit is most appropriate for the situation in the question. For example, they may recall that millimetres (mm), centimetres (cm), metres (m), and kilometres (km) are units of length. The most appropriate unit for "How far away is the sun?" is the kilometre.

LESSON OUTCOME

Read and write standard numerals for numbers to 999 999; interpret place value of digits in numerals to 999 999; write the expanded form for numbers to 999 999

Materials

a card showing the word "thousand"; numeral cards for the numbers 251, 512, and several other numbers to 999, including such examples as 042, 007, and 000

Vocabulary

names for numbers to 999 999, digit, expanded form, standard form, place-value chart

Prerequisite Skills

Read and write standard numerals for numbers to 999; interpret place value in numerals to 999; write the expanded form for numbers to 999

Checking Prerequisite Skills

Write the standard numeral.

1. five hundred thirty-two 532

2. 800 + 90 + 6 896

What does each 7 mean?

3. 371 7 tens **4.** 617 7 ones

Write the expanded form.

5. 425 400 + 20 + 5 **6.** 903 900 + 3

Numbers to 999 999

In one year recently, 140 394 passenger cars were registered in Newfoundland.

hundreds	tens	ones	hundreds	tens	ones
thousands					
1	4	0	3	9	4

In a numeral with four, five, or six digits, the digits in these three places show **thousands**.

140 thousand 394 passenger cars were registered in Newfoundland.

If you know three-digit numerals and the word "thousands", you can read any numeral with up to six digits.

The **standard form** for 140 thousand 394 is 140 394 .

In **expanded form**, 140 394 = 100 000 + 40 000 + 300 + 90 + 4.

Working Together

Use the place-value chart above to help you answer these questions.

Example: The 1 in 316 257 means 1 ten thousand.

1. What does the 7 mean in 47 632? 7 thousands
2. What does the 2 mean in 250 388? 2 hundred thousands
3. What does the 5 mean in 452 984? 5 ten thousands

Complete. Leave a space after the thousands.

4.	366 thousand 164	366 164
5.	48 thousand	48 000 ?
6.	?19 thousand 8 ?	19 008
7.	?502 thousand ?73	502 073

Write each in expanded form.

8. 12 345
9. 306 000
10. 70 302

Write each in standard form.

11. eight hundred two thousand 802 000
12. 700 000 + 50 000 750 000
13. 7 ten thousands 5 thousands 75 000

8. 10 000 + 2 000 + 300 + 40 + 5
9. 300 000 + 6 000
10. 70 000 + 300 + 2

4

LESSON ACTIVITY

Before Using the Pages

- Display the numeral card for 251 and then place the card for 512 below it. Have students read the numerals, pointing out that the word "and" is not used. For instance, 512 is read "five hundred twelve". Ask how the numerals are alike and how they are different, leading the students to recall and use the terms *digit*, *ones' place*, *tens' place*, and *hundreds' place*. Ask what is the greatest number that can be represented by a three-digit numeral (999). Then ask what is needed to show a number greater than 999. Establish that the numeral requires more than three digits.
- Have a student select two cards, for example, one for 368 and one for 251, read the numerals, and then place the cards side by side to form a six-digit numeral.

368	thousand	251

Ask how the numeral is read. When a student says the word "thousand", place the corresponding word card between the two numeral cards. This helps to emphasize that reading a six-digit numeral is similar to reading two three-digit numerals. Repeat the procedure with other numeral cards. Ask what is the greatest number that can be represented by a six-digit numeral.

Using the Pages

- Draw attention to the picture on page 5. Ask the students to identify the license plate for their province, for neighbouring provinces, and the remaining license plates. Have a student read the statement at the top of page 4. Ask how many digits there are in 140 394. Note the space that separates groups of three digits. Associate each group with the corresponding digits in the place-value chart. Point out that the words "ones", "tens", and "hundreds" are repeated across the top of the place-value chart, but that the meaning is different in each grouping. For example, the word "tens" refers to ten ones or to ten thousands

Exercises

Complete.

1.	736 thousand 418	736 418
2.	492 thousand 255	429 255
3.	92 thousand 29	92 029
4.	102 thousand 700	102 700
5.	75 thousand 8	75 008

What does each 4 mean?

6. 47 632 4 thousands
7. 366 164 4 ones
8. 452 984 4 hundred thousands, 4 ones
9. 140 394 4 ten thousands, 4 ones

Write each in expanded form.

10. 6067 6000 + 60 + 7
11. 250 388 200 000 + 50 000 + 300 + 80 + 8
12. 308 020 300 000 + 8 000 + 20
13. 50 403 50 000 + 400 + 3

Write each in a standard form.

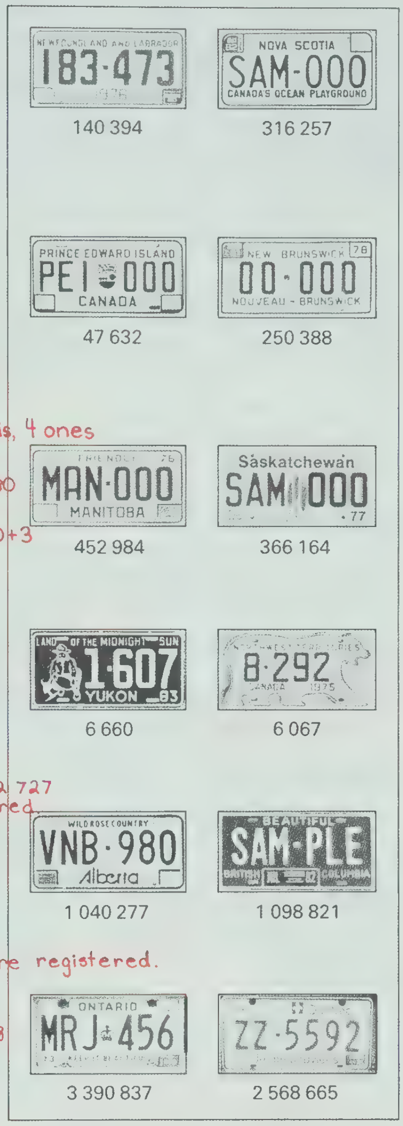
14. 80 thousand 80 000
15. forty thousand forty 40 040
16. 20 000 + 3 000 + 60 23 060
17. 5 hundred thousands 6 ten thousands 560 000

Write each sentence using a numeral in standard form.

18. In the territories, twelve thousand seven hundred twenty-seven cars were registered.
In the territories, 12 727 cars were registered.
19. In Atlantic Canada, seven hundred fifty-four thousand six hundred seventy-one cars were registered.
In Atlantic Canada 754 671 cars were registered.

How many thousands of passenger cars were registered in

*20. Alberta? 1040
*21. British Columbia? 1098
*22. Ontario? 3390
*23. Quebec? 2568



5

RELATED ACTIVITIES

- Students who require more experience with concrete materials may work in small groups. Provide them with a list of word names for numbers, beginning with small numbers and gradually introducing larger numbers. Have them use an abacus and a place-value pocket chart prepared as described on page T 378 to represent the numbers. Also, have students represent numbers for other students to identify.
- Prepare cards similar to the following for multiples of 1, 10, 100, and so on, to multiples of 100 000.

60 000	300	5

Have students overlap the appropriate cards to show the standard form for a number and then separate the cards to show the expanded form.

- Provide the students with newspapers and magazines and ask them to search for examples of large numbers and small numbers. Have them copy the numerals and, if applicable, the sentences involving them to indicate their use. The examples may be used for practice in reading numerals and interpreting place value.

according to its position in the chart. Have students name, for example, the digit in the hundred thousands' place and the digit in the hundreds' place. Review the name of each place value from right to left: ones, tens, hundreds; (*one*) thousands, *ten* thousands, *hundred* thousands. (The word "one" shown in parentheses is usually not spoken.) Ask a student to explain the concept indicated by the statement in the "thought cloud". Have the students note the *standard form* and the *expanded form* for 140 394. Emphasize that they are different forms for the same number.

Working Together: Ex. 1-3 deal with interpreting place value. Have the students read the example silently. Then point out that the 1 is in the ten thousands' place, and therefore, it represents 1 ten thousand. Ask what the 3 and the 6 mean in the same numeral before assigning Ex. 1-3. Ex. 4-7 deal with reading numerals and writing the standard form. Thus, in Ex. 6, for example, 19 008 is read "19 thousand 8". Draw attention to the instruction in the "thought cloud" and point out how the space is helpful in reading the numeral. Complete one of Ex. 8-10 on the board to help the students understand the procedure.

Exercises: Allow students to discuss their responses to Ex. 20-23.

Assessment

What does each 2 mean?

1. 721 000 2 ten thousands
2. 62 430 2 thousands

Write each in expanded form.

3. 43 270
4. 105 980

Write each in standard form.

5. twenty-five thousand six hundred 25 600
6. 500 000 + 6 000 + 20 506 020
7. 6 hundred thousands 3 ten thousands 630 000

3. 40 000 + 3000 + 200 + 70
4. 100 000 + 5000 + 900 + 80

Pages 6-7

LESSON OUTCOME

Read and write standard numerals for numbers to 999 999 999; interpret place value of digits in numerals to 999 999 999; write the expanded form for numbers to 999 999 999 (to 999 999 999 999 for all of the above is optional)

Materials

numeral cards and word cards prepared for pages 4 and 5, a card showing the word "million" (one showing "billion" is optional)

Vocabulary

millions, (billion, optional), names for numbers to 999 999 999 (to 999 999 999 999)

Prerequisite Skills

Read and write standard numerals for numbers to 999 999; interpret place value of digits in numerals to 999 999; write the expanded form for numbers to 999 999

Checking Prerequisite Skills

What does each 8 mean?

1. 768 400 8 thousands **2.** 800 000 8 hundred thousands

Write each in expanded form.

3. 612 000)0 000 + 10 000 + 2000 **4.** 30 400 30000 + 400

Write each in standard form.

5. seven thousand ten 7010

6. 90 000 + 8 000 98 000

7. 3 hundred thousands 300 000

Numbers to 999 999 999 (and Beyond)

In August, there are about 6 200 000 wild geese in Canada before they fly south.

hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
millions			thousands					
		6	2	0	0	0	0	0

In a numeral with seven, eight, or nine digits, the digits in these three places show **millions**.

In August, there are about 6 million 200 thousand wild geese in Canada.

If you know three-digit numerals and the words "thousands" and "millions", you can read any numeral with up to nine digits.

The **standard form** for 6 million 200 thousand is 6 200 000.

In **expanded form**, 6 200 000 = 6 000 000 + 200 000

Working Together

Use the place-value chart shown above to help you answer these questions.

Example: The 1 in 4 150 000 means 1 hundred thousand.

1. What does the 4 mean in 2 140 000? 4 ten thousands

2. What does the 8 mean in 870 300 000? 8 hundred millions

3. What does the 365 mean in 365 480 000? 365 millions

Leave a space after the millions and after the thousands.

Complete.

4.	758 million 300 thousand	758 300 000
5.	97 million	97 000 000
6.	?9 million ?2 thousand	9 002 000
7.	646 million ?30 thousand	646 030 000

Write each in expanded form.

8. 831 530 000

9. 7 050 000

10. 54 706 000

Write each in standard form.

11. four hundred three million 403 000 000

12. 2 000 000 + 900 000 2 900 000

13. 5 hundred millions 7 ten millions 570 000 000

8. 800 000 000 + 30 000 000 + 1 000 000 + 500 00 + 30 000

9. 7 000 000 + 50 000

10. 50 000 000 + 4 000 000 + 700 000 + 6 000

6

LESSON ACTIVITY

Before Using the Pages

- Display the numeral cards and word cards prepared for the lesson described on pages T4 and T5. Have each of three students select a numeral card and read the numeral. Have two of them arrange their cards side by side to form a six-digit numeral. Ask what number is shown. Have the third student place her/his card to the left of the other two cards to form a nine-digit numeral. Ask what number is shown. Ask a student to place the word cards for *thousand* and *million* between the appropriate cards to emphasize that reading a nine-digit numeral is similar to reading three three-digit numerals.

470	million	368	thousand	251

Ask what is the greatest number that can be represented by a nine-digit numeral. If you wish to introduce "billions", ask what is needed to show a number greater than 999 999 999. Have a student select a numeral card and place it to the left of the cards that form the nine-digit numeral, leaving space for a word card. Ask how many digits there are in all for the new numeral. Then ask what word is read after the new group of three digits. Place the word card for *billion* in the space provided and have a student read the twelve-digit numeral.

Have students form nine-digit numerals (and twelve-digit numerals, if appropriate) using other cards and read the numerals. After several examples, the word cards can be removed and the numeral cards placed closer together, leaving spaces to correspond to those left when writing such numerals.

Using the Pages

- Draw attention to the photograph on page 7. Have a student read the statement at the top of page 6. Remind the students that the words "ones", "tens", and "hundreds" are repeated in the place-value chart. Review the relationships among the places from right to left, for example,

Exercises

Complete.

1.	402 million 760 thousand	402 760 000
2.	538 million 410 thousand	538 410 000
3.	27 million	27 000 000
4.	1 million 70 thousand	1 070 000
5.	340 million 6 thousand	340 006 000
6.	80 million	80 000 000

What does each 6 mean?

7. 165 340 000 6 ten millions
8. 20 607 000 6 hundred thousand

Write each in expanded form.

9. 619 085 000
10. 93 408 000
11. 5 102 000

9. 600 000 000 + 10 000 000 + 9 000 000 + 80 000 + 5 000
10. 90 000 000 + 3 000 000 + 400 000 + 8 000
11. 5 000 000 + 100 000 + 2 000

Write each in standard form.

12. fifty million 50 000 000
13. seven million ten thousand 7 010 000
14. 40 000 000 + 6 000 40 006 000
15. 200 000 000 + 3 000 000 203 000 000
16. 1 ten million 5 hundred thousands 10 500 000
17. 9 hundred millions 2 millions 902 000 000

In a numeral with 12 digits, the digits in the first three places show **billions**.

124 billion 480 million.

hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
billions			millions			thousands					
1	2	4	4	8	0	0	0	0	0	0	0

What does each 9 mean?

18. 49 217 000 000 9 billion
19. 96 000 000 000 9 ten billions

Write in expanded form.

20. 503 005 000 000 500 000 000 000 + 3 000 000 000 + 5 000 000

Write each in standard form.

21. 59 billion 74 million 59 074 000 000
22. 8 billion 9 million 60 thousand 8 009 060 000
23. 8 000 000 000 + 4 000 000 8 004 000 000
24. 6 ten billions 8 hundred millions 60 800 000 000

7

RELATED ACTIVITIES

- Adapt the activities described in *Related Activities* on page T5 for millions.
- Have pairs of students use the cards listed under *Materials* in the following ways.
  1. One student shows a numeral. The other names the number.
  2. One student names a number. The other shows the numeral.

Along with each of the above suggestions, the students may write the word name or show the expanded form for the number.

- Ask each student to draw a place-value chart. Have students help to generate a number by indicating the digit for each place using statements such as "In the millions' place, the digit is eight. The digit in the place to the right of the ten thousands' place is six. There are no hundreds." As each digit is described, ask the students to record it in their place-value charts. Then have them write the word name for the number. Ask students to read the number represented.
- For enrichment, discuss different ways for expanding a number. For example, three ways of expanding 43 156 are shown.

43 156 = 43 000 + 156
43 156 = 40 000 + 3 000 + 156
43 156 = 40 000 + 3 000 + 100 + 50 + 6

Have students select numbers from pages 4-7 and expand each in three different ways.

10 ones = 1 ten, 10 tens = 1 hundred,
10 hundreds = 1 thousand,
10 ten millions = 1 hundred million.

Ask "What digit is the hundreds' place? the ten thousands' place? the millions' place?"

Compare the numeral 6 200 000 with the form 6 million 200 thousand shown in the concluding statement, to emphasize where the words "million" and "thousand" are used when reading the numeral.

Working Together: Ex. 1-3 deal with interpreting place value. Have a student read the example. Explain that the 1 represents 1 hundred thousand because it is in the hundred thousands' place. Ask students to tell the place value of other digits in the example. Ex. 4-7 help students to associate the word name and the standard numeral for a number. Note, for instance, in Ex. 6, that the standard numeral shows three digits (002) in the thousands' period, but this is read "two thousand".

Exercises: Assign Ex. 18-24 only if the concept of billions has been introduced in your class.

After the students have completed the exercises, have them read some of the numerals. Summarize that a number can be represented by several different numerals. Emphasize that the numerals differ but that the number represented does not change.

Assessment

What does each 7 mean?

1. 7 600 00 7 millions
2. 532 700 000 7 hundred

Write each in expanded form.

3. 246 000 000
4. 10 405 800

3. 200 000 000 + 40 000 000 + 6 000 000
4. 10 000 000 + 400 000 + 5000 + 800

Write each in standard form.

5. six hundred seventy-three million 673 000 000
6. 500 000 000 + 6 000 000 506 000 000
7. 2 hundred millions 1 thousand 200 001 000

Pages 8-9

LESSON OUTCOME

Compare and order numbers to 999 999 999

Materials

numeral cards prepared for the lesson on pages 4 and 5

Vocabulary

is greater than (>), is less than (<), kilometre, km, names of the planets

Prerequisite Skills

Compare and order numbers to 999; read and write standard numerals for numbers to 999 999 999; interpret place value of digits in numerals to 999 999 999

Checking Prerequisite Skills

Tell which is greater,

1. 405 or 450. 450 **2.** 827 or 821. 827
3. 604 or 406. 604 **4.** 99 or 103. 103

List from least to greatest.

5. 404, 40, 444, 44, 400, 440
40, 44, 400, 404, 440, 444

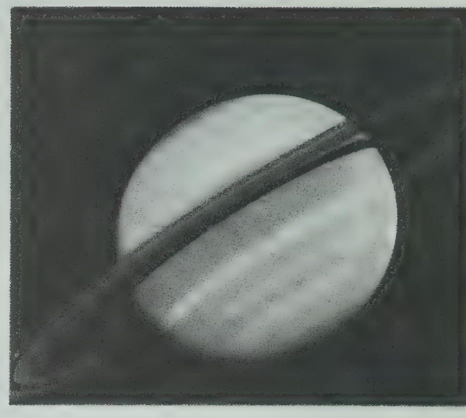
What does each 1 mean?

6. 1 203 000 1 million
7. 4 196 800 1 hundred thousand
8. 615 347 000 1 ten million

Write each in standard form.

9. three million sixty-one thousand 3 061 000
10. 5 millions 3 ten thousands 5 030 000

Comparing and Ordering Numbers



Titan, the largest moon of Saturn, is 1 161 000 km (kilometres) from the planet. Hyperion, one of the smallest moons, is 1 442 000 km from the planet. Which moon is farther from Saturn?

1 161 000
1 442 000 show 1 million.

1 161 000 shows 161 millions.
1 442 000 shows 442 millions.

442 is greater than 161, so
1 442 000 is greater than 1 161 000.

1 442 000 > 1 161 000

Hyperion is farther from Saturn than Titan.

Working Together

Both numerals show the same number of millions. Which shows the greater number of thousands?

1. 693 428 000 / 693 528 000
2. 7 964 000 / 7 946 000
3. 53 713 000 / 53 317 000

Which is greater,

4. 2 833 290 or 2 843 290?
5. 81 346 000 or 81 344 000?

List from least to greatest.

6. 801 577 000 / 811 597 000 / 801 597 000 / 801 957 000
6. 801 577 000, 801 597 000, 801 957 000, 811 597 000

List from greatest to least.

7. 382 300 000 / 38 800 000 / 328 300 000 / 382 800 000
7. 382 800 000, 382 300 000, 328 300 000, 38 800 000

Use >, <, or = to make true statements.

8. 363 920 000 < 373 920 000 9. 98 433 000 > 98 343 000

8

LESSON ACTIVITY

Before Using the Pages

- Use the numeral cards prepared for the lesson described on pages T4 and T5. Have a student form a numeral using three cards, for example, $\boxed{251}$ $\boxed{378}$ $\boxed{460}$, and read the number represented. Have another student form a numeral using two cards, for example, $\boxed{999}$ $\boxed{000}$, and read the number represented. Then ask which number is greater. Establish that it is easy to identify the greater number because the first numeral has more digits. (Note that this procedure can be used for whole numbers but not necessarily for decimals, as in 1.4 and 0.25.)

- On the board, write pairs of numerals that have the same number of digits. Some examples are given below. For each pair, ask which is the greater number and have students describe the procedures they use to determine this. Discuss whether one procedure is more efficient.

1. 412 899	2. 62 842 017	3. 1 504 263
416 898	62 824 019	1 540 263

- Ask the students to think of a situation for which it would be necessary to compare large numbers. Two such situations would be comparing the attendance at an annual exhibition for different years and comparing annual sales of a store or a company.

Using the Pages

- Explain to the students that the Voyager I spacecraft passed Saturn at a distance of 126 000 km on 1980 11 12. It took a lot of photographs and allowed scientists to make many new discoveries about Saturn, its rings, and its moons (now totalling 15). One activity related to the wealth of numerical information provided by events such as this is organizing the information. Being able to compare and order numbers is a skill that is essential for this activity.

  Have a student read the information at the top of page 8, drawing attention to the term *kilometres* and the symbol km. Point out that the symbol is written without a period and can be singular or plural.

  Point out that the two numerals to be compared have the same number of digits. Discuss that the comparison takes place from left to right. The spaces that separate groups of

Exercises

Which is greater,

1. 29 374 or 280 374?
2. 92 400 or 93 000?
3. 31 526 000 or 31 536 000?
4. 8 080 000 or 8 008 000?
5. 1 298 000 or 1 289 000?

Use >, <, or = to make true statements.

6. 6 996 > 6 969
7. 780 087 000 = 780 087 000
8. 40 004 000 < 40 040 000
9. 7 720 000 > 7 702 000
10. 90 900 000 < 99 900 000

List from least to greatest.

11.

5 055 000	50 555 000
50 550 000	505 501 000
550 051 000	5 050 000

11. 5 050 000, 5 055 000, 50 550 000, 50 555 000, 505 501 000, 550 051 000

List from greatest to least.

12.

628 368 000	6 826 000
628 382 000	626 862 000
62 886 000	628 386 000

12. 628 386 000, 628 382 000, 628 368 000, 626 862 000, 62 886 000, 6 826 000

List the planets and their distances from the Sun in order—from the one closest to the Sun to the one farthest from the Sun.

13.

Planet	Average distance from the Sun in kilometres
Earth	149 500 000
Jupiter	778 000 000
Mars	227 800 000
Mercury	57 900 000
Neptune	4 497 000 000
Pluto	5 900 000 000
Saturn	1 427 000 000
Uranus	2 869 000 000
Venus	108 100 000

*14. How far is Halley's Comet from the Sun? Research project. See "Related Activities."

13. Mercury 57 900 000, Venus 108 100 000, Earth 149 500 000, Mars 227 800 000, Jupiter 778 000 000, Saturn 1 427 000 000, Uranus 2 869 000 000, Neptune 4 497 000 000, Pluto 5 900 000 000

Four two-digit numerals can be made with the digits 5 and 9.

59, 95, 55, 99

1. List the two-digit numerals that can be made with the digits 4 and 6. 44, 46, 64, 66
2. List the two-digit numerals that can be made with the digits 4 and 6 without using a digit twice in a numeral. 46, 64
3. List the two-digit numerals that can be made with the digits 4, 6, and 7. 44, 46, 47, 64, 66, 67, 74, 76, 77
4. List the two-digit numerals that can be made with the digits 4, 6, and 7 without using a digit more than once in a numeral. 46, 47, 64, 67, 74, 77
5. List the three-digit numerals that can be made with the digits 4, 6, and 7 without using a digit more than once in a numeral. 467, 476, 647, 674, 746, 764
6. List the three-digit numerals that can be made with the digits 4, 6, and 7.

try this

444 446 447 464 466 467 474 476 477 644 646 647 664 666 667 674 676 677 744 746 747 764 766 767 774 776 777

9

three digits in a numeral suggest comparing not by individual places but by groups (periods). In the worked example, the corresponding groups are shown in red to facilitate the comparison at each step. Emphasize that each step is similar to comparing two numbers having from one to three digits. For instance, the first step is similar to comparing 1 and 1; the second step of the example involves comparing 161 and 442. Ask what the symbol > represents. Ask a student to show the symbol for "is less than" on the board (<).

Working Together: For Ex. 6, some students may compare the digits from left to right to find the least number first. Others may recognize that the greatest number can be identified first quite easily.

Exercises: After the students have completed the exercises, you may wish to have them read some of the answers aloud. Note that Ex. 13 includes numbers to billions.

Try This: These exercises reinforce place-value concepts. It may be necessary to help students organize their approach to Ex. 3-6 so that numerals are neither repeated nor omitted. For example, they may start by writing all the numerals that begin with a particular digit as in 44, 46, 47 (Ex. 3). Students who are having difficulty may benefit from writing the digits on small pieces of paper and arranging them to form numerals which may then be copied.

RELATED ACTIVITIES

- For each exercise in *Try This*, have the students order the numbers from least to greatest.
- Prepare a work sheet of exercises similar to the following. If it is not possible to tell which number is greater, ask the students to cross out the two numbers. If it is possible to tell which number is greater, ask them to write > or < to make a true statement.

4□ 1 0□ ○ 4□ 9 8 4

□□□ □□□ ○ □ □□□ □□□

1 0□□ 6 3 7 ○ □□□ □□□

- Groups of two to four students may play the game "Number Sequence" described on page T 379.
- Have students list the provinces and territories in order according to the number of passenger cars registered in each. Use the information given in the picture on p. 5.
- Have students research the flight of Voyager I and Voyager II (or any other spacecraft), and present to the class large numbers that relate to the flight.
- Have students research Halley's Comet. It returns to the vicinity of Earth in late 1985, passing closest to this planet on 1986 04 11.

Assessment

Which is greater,

1. 42 870 or 442 870? 442 870
2. 399 339 000 or 393 993 000? 399 339 000

Use >, <, or = to make true statements.

3. 283 454 ○ 283 445 >
4. 18 081 290 ○ 18 081 290 =
5. 667 677 000 ○ 767 677 000 <

List from least to greatest.

6. 355 335 000 353 335 000 353 353 000 355 535 000

353 335 000
353 353 000
355 335 000
355 535 000

LESSON OUTCOME

Round to the nearest one, ten, or hundred, in any period to millions

Materials
real coins (optional)

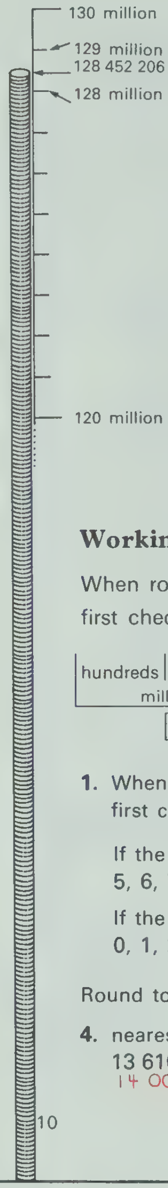
Vocabulary
round to the nearest, round down, round up

Prerequisite Skills
Interpret place value in numerals with up to nine digits

Checking Prerequisite Skills
What does each 3 mean?

1. 634 200 — 3 ten thousands
2. 983 412 000 — 3 millions
3. 952 360 000 — 3 hundred thousands

Rounding



In 1977, Canada minted 128 452 206 dimes.

Round 128 452 206 to the nearest million.

128 452 206 is closer to 128 million than to 129 million.
128 452 206 rounded to the nearest million is 128 000 000.

In 1977, Canada minted about 128 000 000 dimes.

Working Together

When rounding to $\boxed{\text{this place}}$,
first check the digit in (this place).

hundreds	tens	ones	hundreds	tens	ones	hundreds	tens	ones
millions			thousands					

1. When rounding to the nearest hundred million, first check the digit in the ten millions place.

If the digit you check is 5, 6, 7, 8, or 9, round up.

If the digit you check is 0, 1, 2, 3, or 4, round down.

Would you round down or up to the nearest ten million?

2. 453 621 000 down
3. 415 903 000 up

Round to the

4. nearest million.
13 610 000
14 000 000

5. nearest ten million.
46 000 000
50 000 000

6. nearest hundred million.
654 530 000
700 000 000

10

LESSON ACTIVITY

Before Using the Pages

- Draw a number line marked into ten equal parts on the board. Ask questions and direct the students as suggested below, labeling the number line according to their answers. Sample answers are given in parentheses.

"Name a multiple of one hundred." (400)
"Name the next multiple of one hundred." (500)
"Name a number between 400 and 500." (448)
"Is the number closer to 400 or to 500?" (400)
"We say that 448 rounded to the nearest hundred is 400."
"Round 458 to the nearest hundred." (500)
"Is 450 closer to 400 or to 500?"
"By agreement, we round 450 up to 500, rather than down to 400."

Follow a similar sequence to review rounding numbers to other given places. Use exercises similar to the following to lead students toward rounding larger numbers.

Round 428 to the nearest ten.
Round 428 163 to the nearest ten thousand.
Round 428 163 000 to the nearest ten million.

- The following activity can prepare the students for focusing on the place values necessary for rounding to a given place. Say, "I am thinking of a number, but I am not going to show you all the digits." Write 13 7 _ _ on the board and ask, "Can you round my number to the nearest thousand?" Replace the 7 in the hundreds' place with a different digit, such as 2, and repeat the question. Discuss why they do not need to know the digits that appear to the right of the hundreds' place. Use other similar examples.

Using the Pages

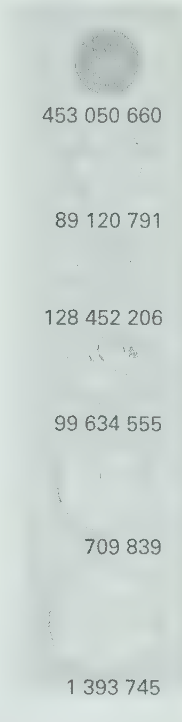
- Have a student read the two statements at the top of page 10. Tell the students that rounding the number to the nearest million would give an easier number to remember and still

Exercises

Complete.

	Round to the nearest	million	ten million
1.	63 106 000	63 000 000	60 000 000
2.	756 480 000	756 000 000	760 000 000
3.	99 977 000	100 000 000	100 000 000
4.	204 852 000	205 000 000	200 000 000

This shows how many coins were minted in 1977.



For the coins minted in 1977, round

to the nearest million, the number of

5. nickels. 89 000 000
6. one-dollar coins. 1 000 000

to the nearest hundred thousand, the number of

7. dimes. 128 500 000
8. fifty-cent coins. 700 000

to the nearest hundred million, the number of

9. pennies. 500 000 000
10. dimes. 100 000 000

to the nearest ten million, the number of

11. nickels. 90 000 000
12. quarters. 100 000 000
13. pennies. 450 000 000
14. dimes. 130 000 000

In 1977, 1 066 735 698 coins were being used in Canada. Round this number to the nearest

15. million. 1 067 000 000
16. ten million. 1 070 000 000
17. hundred million. 1 100 000 000
18. billion. 1 000 000 000

Canadian coins are minted in six different values.

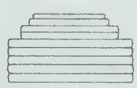
1. Show the value of each coin in three ways.

1. 1¢, $0.01, one cent
5¢, $0.05, five cents
10¢, $0.10, ten cents
25¢, $0.25, twenty-five cents
50¢, $0.50, fifty cents
100¢, $1.00, one dollar

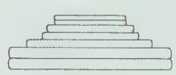
Give the value of each stack of coins.

2. $2.78

3. $3.11



4. $3.00



5. How tall would the stack on p. 10 be?

PROBLEM SOLVING

5. A dime is about 1 mm thick. A stack of 128 452 206 dimes would be about 128 000 000 mm, or 128 km, tall.

11

RELATED ACTIVITIES

- Ask students to find examples in newspapers and magazines of numbers that appear to have been rounded to a particular place and others that are probably exact. Have them cut out the examples and organize them into two groups according to whether or not they have been rounded. Have them suggest why some numbers have been rounded and others not.
- Have the students prepare a list of examples that describe occasions when they have used rounded numbers.
- Students having difficulty rounding large numbers should practice rounding numbers less than 1000 to the nearest ten and to the nearest hundred. Have them use copies of page T 390 to mark some number lines in multiples of 10 and others in multiples of 100. A number to be rounded may be located between two multiples on the appropriate number line and then rounded. The activity may be adapted for rounding larger numbers to places in the thousands' period.
- The following activity can challenge some students. Say, for example, "A number rounded to the nearest hundred thousand is 7 400 000. What may the number be?" Have each student write five possible numbers. Discuss the numerous possibilities and the limits on the possibilities. For example, the least possible number in this instance is 7 350 000 and the greatest possible number is 7 449 999.

indicate closely the number of dimes that were minted. Point out how a number line shows that the number of dimes is closer to 128 million than to 129 million.

Working Together: Have a few students explain the statement above the place-value chart. Emphasize that the task of rounding a number to a given place is simplified by concentrating on the digit in the place to the immediate right of the given place.

After discussing Ex. 1, have the students read the information in the box. Ask what is meant by "round up" and "round down". Discuss Ex. 2 and 3 and then assign Ex. 4-6.

Exercises: Have students name the coins shown in the photograph and tell the value of each coin.

For Ex. 3, pay particular attention when rounding to the nearest million: the 9 in the millions' place (99 977 000) is increased by one so that the number is rounded to 100 millions, or 100 000 000. When rounding the same number to the nearest ten million, the 9 in the ten millions' place is increased by one so that the number is rounded to 10 ten millions, or 100 000 000. A similar situation occurs in Ex. 12.

Problem Solving: Ex. 1 may indicate whether students need to review writing numerals for amounts of money. For Ex. 2, the students are required to recognize, for example, that the top coin in the pile must be a dime because dimes are smaller than pennies. In the diagram for Ex. 2, the six kinds of coins are represented, but this is not the case for Ex. 3 and 4. Students will have to refer to Ex. 2 to help them decide which coins are represented in Ex. 3 and 4. They may also refer to the photograph or work with real coins.

Assessment

Round to the

1. nearest million.
4 213 000 4 000 000
2. nearest hundred thousand.
12 683 000 12 700 000
3. nearest ten million.
895 701 000 000
895 700 000 000
4. nearest hundred million.
607 438 000
600 000 000

Pages 12-13

LESSON OUTCOME

Read and write numerals and words for one-place, two-place, and three-place decimals

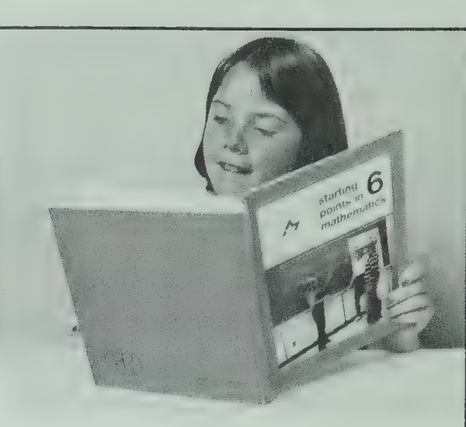
Materials

models for ones, tenths, and hundredths made with copies of pages T392-T394 as described on page T89, a ruler marked in millimetres

Vocabulary

millimetre, mm, decimal, decimal point, diameter, Roman numerals

Decimals



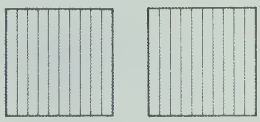
One leaf of this book is 0.075 mm (millimetre) thick.

0.075 is a **decimal**.
The . is a **decimal point**.

0.075 is read seventy-five thousandths.
0.075 means 75 of 1000 equal parts.

Working Together

How many wholes are there?
How many tenths are there?
Write the decimals.

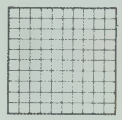
1.  1 whole, 1 tenth, 1.1

2.

ones	tenths
3	6

3 wholes, 6 tenths, 3.6

How many wholes are there?
How many hundredths are there?
Write the decimals.

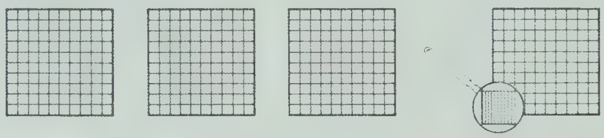
3.  0 wholes, 43 hundredths, 0.43

4.

tens	ones	tenths	hundredths
9	0	0	7

90 wholes, 7 hundredths, 90.07

How many wholes and thousandths are there?
Write the decimals.

5.  3 wholes, 4 thousandths, 3.004

6.

ones	tenths	hundredths	thousandths
0	1	7	9

0 wholes, 179 thousandths, 0.179

Write the decimals.

7. forty and five-hundredths 40.05
8. three hundred six-thousandths 0.306

When you read or write a decimal in words, use "and" for the decimal point.

Write the words.

9. 0.8 eight-tenths
10. 2.308 two and three hundred eight-thousandths
11. 1.46 one and forty-six hundredths
12. 0.005 five-thousandths

12

LESSON ACTIVITY

Before Using the Pages

- Display a model for 7 tenths, establishing that the amount shown in blue is less than one whole. Develop that there are 10 equal parts, 7 of which are blue, and that the number represented is read "seven-tenths". Write the numeral 0.7 on the board and elicit the words *decimal* and *decimal point* from the students. Display models for other examples of tenths, such as 0.1 and 3.9. Have students name the numbers and write the numerals on the board. Emphasize the need for 0 ones in a numeral such as 0.7. Note that the word *and* is used in naming decimals greater than one, for example, "three and nine-tenths" for 3.9. Summarize that a decimal tenth shows one digit to the right of the decimal point. In a similar manner, use models of hundredths to review that a decimal hundredth shows two digits to the right of the decimal point. Ask what is shown by a decimal with three digits to the right of the decimal point.

Using the Pages

- The example at the top of page 12 introduces the concept of thousandths. Direct the students' attention to the photograph and then to the statements to the left of the photograph. Briefly discuss the term *millimetre* and the symbol mm. Have the students observe a length of 1 mm on a ruler. Establish that 0.075 represents a number less than one whole, reviewing the need for a zero to the left of the decimal point. Point out that the decimal point separates wholes from parts of a whole. Have the students observe the thickness of page 12 of their books to emphasize a thickness that is less than 1 mm. Summarize that 0.075 represents a number of wholes and the number of thousandths of another whole.

Working Together: In preparation for Ex. 2, 4, and 6, draw on the board a place-value chart showing hundreds, tens, and ones. Discuss the place values from left to right, emphasizing the relationships 1 hundred = 10 tens, and 1 ten = 10 ones. Then extend the chart to thousandths,

Exercises

Write the decimals.

1. hundreds | tens | ones ▌ tenths
   2 3 0 3 — 230.3
2. tens | ones ▌ tenths | hundredths
   1 8 9 4 — 18.94
3. ones ▌ tenths | hundredths | thousandths
   0 2 0 7 — 0.207
4. seven-tenths 0.7
5. eleven and three-thousandths 11.003
6. nineteen-hundredths 0.19
7. eight hundred one-thousandths 0.801
8. sixty and two-hundredths 60.02
9. five hundred sixty-six thousandths 0.566
10. five hundred and sixty-six thousandths 500.066

What does each 5 mean?

11. 1.58 5 tenths
12. 6.35 5 hundredths
13. 1.005 5 thousandths
14. 25.2 5 ones

Copy and complete.

15. 10.16, 10.17, 10.18, , , 10.19, 10.20, 10.21
16. 0.302, 0.301, 0.300, , , 0.299, 0.298, 0.297
17. 0.96, 0.97, 0.98, , , 0.99, 0.100, 0.101

Write each numeral in words.

	Coin	Diameter of coin in millimetres
18.	Penny	19.05
19.	Nickel	21.21
20.	Dime	18.034
21.	Quarter	23.88

18 nineteen and five-hundredths
19 twenty-one and twenty-one hundredths
20 eighteen and thirty-four thousandths
21 twenty-three and eighty-eight hundredths

On many buildings, the date is written with Roman numerals.

1	I	50	L
4	IV	90	XC
5	V	100	C
6	VI	400	CD
9	IX	500	D
10	X	900	CM
40	XL	1000	M

Examples:

M CM LXX IX

1000, 900, 70, 9

1000 + 900 + 70 + 9 = 1979

DCC VIII

700, 8

700 + 8 = 708

Write the Roman numerals.

1. 1832 2. 3009
3. 78 4. 666
5. 2404 6. 1930

1. MDCCCXXXII 2. MMMIX 3. LXXVIII 4. DCLXVI 5. MMCDIV 6. MCMXXX

Write the standard form.

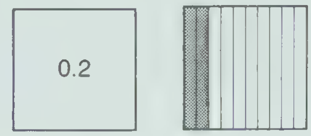
7. MDLXXXIII 1583
8. CCXLIX 249
9. XCIII 93
10. LXX 70

try this

13

RELATED ACTIVITIES

- Encourage the students to search newspapers for examples of decimals and bring the examples to school for display and discussion.
- To reinforce the concept of decimals, have students color models made from copies of pages T 392 - T 394.
- Students can use copies of pages T 392 and T 393 to prepare cards similar to the following for the game "Concentration" described on page T 379.



- Have students locate points on a number line for decimals. Use copies of page T 390.
- Ask students to write a decimal with a value between two other decimals such as 3.2 and 3.5. Doing this for decimals such as 5.1 and 5.2 presents a challenge because the students are required to write a decimal showing hundredths.
- Prepare work sheets similar to the following to relate decimals and money.

Amount	Dollars		
	ones	tenths	hundredths
$7.43			
	9	0	6

- Ask students to write the year of their birth or other memorable dates in Roman numerals.

having students provide the names to complete the following statements.

1 one = 10 _____
1 tenth = 10 _____
1 hundredth = 10 _____

hundreds | tens | ones ▌ tenths | hundredths | thousandths

Compare the names of place values to the left and to the right of the ones' place, for instance, tens and tenths. In Ex. 5, point out that each hundredth can be divided into ten equal parts, as shown with the magnifying glass. Each tenth of a hundredth is one-thousandth.

In Ex. 7, emphasize that the word "and" separates the whole number from the decimal part. Because there is no "and" in Ex. 8, the decimal is written as 0.306. Point out that three hundred *and* six-thousandths (300.006) is not the same as three hundred six-thousandths (0.306).

Exercises: Ex. 11-14 emphasize place value. For Ex. 18-21, review the meaning of the term *diameter*.

Try This: Ask the students where they have seen Roman numerals. Point out the Roman numerals I, V, X, L, C, D, M, and the number each represents. Discuss the significance of placing the letter X to the right of C (CX) as opposed to the left of C (XC), for example. Discuss the examples 1979 and 708.

Assessment

Write the decimals.

1. ones ▌ tenths | hundredths | thousandths
   4 1 8 3 — 4.183
2. five hundred seven-thousandths 0.507
3. four and nine-tenths 4.9

What does each 3 mean?

4. 0.34 3 tenths
5. 1.203 3 thousandths

Write the words.

6. 2.036 two and thirty-six thousandths
7. 0.721 seven hundred twenty-one thousandths

OBJECTIVE

Identify methods of estimating

Background

Page 14 is the first of the special problem-solving pages in this book. See page xv for comments on the approach to problem solving in *Starting Points in Mathematics*.

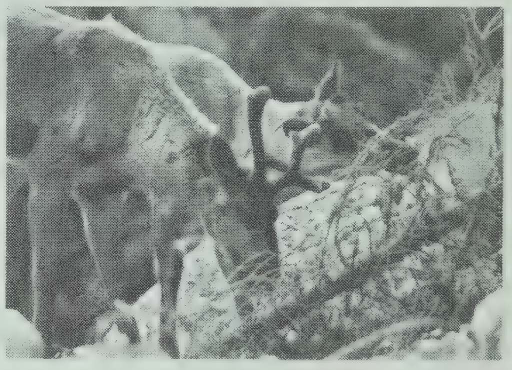
RELATED ACTIVITIES

- Some students may be able to write problems similar to those on the page. Have small groups of students discuss the problems.
- Ask students to describe situations in which it was necessary for them to estimate, for example, a length or a number.
- Encourage the students to listen to news reports because they frequently involve estimates of numbers. For example, they may hear a statement such as "Damage, as a result of the fire, was estimated at $175 000."
- Some students may be interested in carrying out procedures suggested for estimating answers to problems on page 14. Provide opportunities for them to do so and to report their results to the other students.

Thinking About Estimating

How would you estimate Answers will vary

1. the number of hairs on your head?
2. your height a year from now?
3. the number of words in a book?
4. the number of letters on a page?
5. the number of minutes it would take you to read a book?
6. the number of seconds it would take you to copy a page?
7. the number of people in a crowd?
8. the number of birds in a flock?
9. the speed of a flying bird?
10. the length of a beetle?
11. the height of a mountain?
12. the depth of a river?
13. the number of drops of water in a pail?
14. the number of grains of sand in a pail?
15. the number of blades of grass in a football field?
16. the number of ants in an anthill?
17. the number of steps you walk in a day?
18. the length of time it would take you to climb a hill?
19. the number of streets in a city?
20. the number of sheep on a farm?
21. the distance around a large lake?
22. the number of snowflakes in a snowball?
23. the number of trees in a forest?
24. the number of animals in a forest?
25. the number of insects in a forest?



PROBLEM SOLVING

14

LESSON ACTIVITY

Before Using the Page

- An estimate is not a haphazard guess but a carefully considered opinion. Tell the students that often it is helpful to be able to judge a length without using a ruler or to judge the number of objects in a group without counting all the objects. Explain that this is called "making an estimate" and that an estimate of an answer is an approximate value of the exact answer. As an example, ask the students what they think the height of the classroom is. Have several students suggest an answer. Ask them to explain how they arrived at their answers. For example, one student may suggest that the height is a little more than twice her/his own height, which is known. Another may know that the distance from the floor to the doorknob is about one metre and use that to estimate the height of the room. Then, if your school has more than one floor, have students use the first estimate to estimate the height of the school.

Using the Page

- Allow the students a few moments to read some of the problems silently and to consider approaches that might be used. Then have a student choose a problem of particular interest to her/him and describe a method for making an estimate. Repeat this procedure for two or three of the problems. For example, the number of letters on a page (Ex. 4) can be estimated by counting the letters in one line, counting the lines, and then multiplying the two numbers. Emphasize that the answers obtained are estimates. For the remaining problems, divide the class into groups of about four students. Have each student, in turn, read a problem and lead the discussion for that problem. Encourage the students to explore more than one way of making an estimate to answer each problem.

You may wish to have the students read the problems and begin the discussion on one day and then continue the discussion on another day. This may give them an opportunity to think about some of the problems.

Checking Up

23. 6 850 230
6 850 320
63 804 300
63 840 200

24. 35 287 600
35 287 060
35 270 600
35 208 600

What does each 2 mean?

1. 372 468 500 — 2 billions
2. 47 320 000 — 2 ten thousands
3. 48.02 — 2 hundredths
4. 0.312 — 2 thousandths

Write each in expanded form.

5. 672 003 040
6. 1 700 000
7. 73 055 420

5. 600 000 000 + 70 000 000 + 2 000 000 + 3 000 + 40
6. 1 000 000 + 700 000
7. 70 000 000 + 3 000 000 + 50 000 + 5 000 + 400 + 20

Write each in standard form.

8. two hundred twenty million eighty-nine thousand 220 089 000
9. forty-one million one hundred seventeen thousand 41 117 000
10. fifteen and four-hundredths 15.04
11. seven hundred and two-thousandths 700.002
12. 700 000 000 + 90 000 000 + 4 000 000 794 000 000
13. 30 000 000 + 100 000 + 200 30 100 200
14. 4 ten millions 3 hundred thousands 6 ten thousands 40 360 000
15. 8 billions 5 millions 6 thousands 8 005 006 000

Write the words.

16. 0.6 six tenths
17. 40.07 forty and seven-hundredths
18. 1.203 one and two hundred three-thousandths

Use >, <, or = to make true statements.

19. 7 630 000 < 44 630 000
20. 142 398 700 = 142 398 700
21. 20 404 300 < 20 440 300
22. 9 500 500 000 > 9 500 050 000

List from least to greatest.

23. 4 63 840 200 1 6 850 230
2 6 850 320 3 63 804 300

List from greatest to least.

24. 4 35 208 600 3 35 270 600
1 35 287 600 2 35 287 060

Round to the

25. nearest hundred thousand.
49 980 000
50 000 000

26. nearest ten million.
630 400 000
630 000 000

27. nearest billion.
340 572 800 000
341 000 000 000

Write each using a numeral in standard form.

28. From 1968 to 1975, two million forty thousand kilograms (kg) of maple sugar were produced in Canada.
From 1968 to 1975, 2 040 000 kg of maple sugar were produced in Canada.

29. The value of maple syrup produced in Canada in 1975 was eleven million one hundred fifty-five thousand dollars.
The value of maple syrup produced in Canada in 1975 was $11 155 000.

15

Page 15

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Vocabulary
kilograms, kg

RELATED ACTIVITIES

- Students may enjoy being tested on how quickly they can write the standard numerals for the following.

1. 2 000 000 + 400 2 000 400
2. CCCIX 309
3. CMIV 904
4. six hundred seven million 607 000 000
5. 6 542 000 rounded to the nearest million 7 000 000
6. 2 ten millions 6 hundred thousands 20 600 000
7. forty million two thousand
8. sixty million fifty thousand
9. XVI 16 — 7. 40 002 000
10. XCIII 93 — 8. 60 050 000
11. 400 000 + 700 + 20 400 720
12. XL 40
13. 79 rounded to the nearest ten 80
14. 2748 rounded to the nearest hundred 2700
15. 14 536 rounded to the nearest thousand 15 000

Skills	Exercises	Related Pages
Interpret place value in numerals	1	T6-T7
	2	T4-T5
	3, 4	T12-T13
Write numerals in expanded form	5-7	T6-T7
Write numerals in standard form	8	T4-T5
	9, 12-14, 28, 29 (15)	T6-T7
	10, 11	T12-T13
Write decimals in words	16-18	T12-T13
Compare numbers	19-21 (22)	T8-T9
Order numbers	23, 24	T8-T9
Round numbers	25-26 (27)	T10-T11

Comments

The chart shows the skill required for each exercise on page 15 and indicates the pages from T4 to T13 where the corresponding lessons are presented. Note that Ex. 15, 22, 27 involve the concept of billions which may be optional for your class. Refer to the chart to help determine areas of difficulty and skills that may require reteaching or review. Students who demonstrate an understanding of the concepts and skills may enjoy a related activity selected from lessons in this unit.

If students have difficulty with Ex. 1-14, ask them to draw a place-value chart on a piece of lined paper turned sideways. The digits may be written in the appropriate columns as each number is interpreted. For example, for Ex. 14, a student might show 4 in the ten millions' place, 3 in the hundred thousands' place, and 6 in the ten thousands' place. Then the necessary 0's would be written to complete the numeral.

h	t	o	h	t	o	h	t	o	
millions			thousands						
	4	0	3	6	0	0	0	0	40 360 000

You may wish to have the students use an abacus for more practice in interpreting place value in numerals.

For Ex. 28, point out the symbol kg for *kilograms*, a unit of mass.

Unit 2 Overview

Addition and Subtraction

This unit begins with a review of basic addition facts which encourages students to group addends having sums of 10 wherever possible. The process of regrouping sums is reviewed with attention to the place values involved in the changes. Addition is performed using numbers with up to six digits and related word problems are included. Subtraction is reviewed first with no regrouping. Regrouping minuends is reviewed with emphasis on place values. Regrouping minuends with zeros is given special attention with an emphasis on interpreting minuends and parts of them in terms of more than one place value. Students are shown how to check subtraction by adding the difference and the subtrahend. Problems are included as a part of each lesson. This unit includes the first lesson on the use of the calculator and students learn to identify the keys that perform the operations of addition and subtraction.

Prerequisite Skills

- complete the basic addition and subtraction facts
- interpret place value in numerals with up to six digits

Unit Outcomes

- use sums of 10 to add one-digit numbers
- add two numbers with regrouping, addends with up to six digits
- round addends and add to estimate the sum, then compare the estimate of the sum with the exact sum
- add more than two numbers, sums with up to six digits
- subtract numbers with no regrouping, minuends with up to six digits; use addition to check subtraction
- subtract with regrouping, minuends with up to six digits
- round the minuend and the subtrahend and subtract to estimate the difference, then compare the estimate of the difference with the exact difference
- subtract with regrouping, zero in one or more places in the minuend, minuends with up to six digits
- solve word problems using addition or subtraction
- relate additive and subtractive situations to the use of the $\boxed{+}$ and $\boxed{-}$ keys on a calculator
- identify unnecessary information in a word problem

Background

Addition is the most basic operation with numbers, and the other three, subtraction, multiplication, and division, can all be derived from it directly or inversely.

The basic addition facts are those which have two one-digit addends, such as in the facts $6 + 3 = 9$, $0 + 4 = 4$, and $9 + 8 = 17$. Using the ten digits 0 to 9 in both positions for addends produces a set of 100 addition facts. Of these, 10 are "doubles", such as $3 + 3 = 6$, and the other 90 facts may be reduced to 45 basic relationships through the *commutative property of addition*. Because of this property, the order of two addends may be changed without affecting their sum. Thus, for every two facts in which the addends are different, only one basic relationship needs to be known, as in $7 + 6 = 13$ and $6 + 7 = 13$. This set of 55 basic facts may be reduced to a set of 45 if one rule for zero as an addend is applied; namely, the sum of any number and zero is that number. Mastery of the 45 basic addition facts and the rule for zero is essential for success in adding numbers.

The inverse relationship between addition and subtraction, by which one operation "undoes" the other, permits the same set of basic facts to be used in both operations. For example, the basic subtraction facts $11 - 8 = 3$ and $11 - 3 = 8$ can be derived from the basic addition fact $8 + 3 = 11$ and its commuted fact $3 + 8 = 11$. Sets of four facts for unequal addends and sets of two facts for equal addends are called "families" of facts. Efficiency and accuracy in both operations can be improved if families of facts are learned and mastered.

The four basic operations with numbers are *binary* in nature; that is, only two numbers can be combined at any one time and with one unique result. In addition the result is called the *sum*, and in subtraction it is called the *difference*. If three or more numbers are involved in one or more operations, two combine to form a new number which is then combined with another number, and so on, until all the numbers have been used in the operation(s).

In addition, three or more addends may be considered in any order with no effect on their sum. The first lesson in this unit provides an application of this *associative property of addition* and the formation of tens. The students are challenged to first combine addends which have sums of 10 and then add the others. Regardless of how the addends are grouped, the sums are unaffected in each case because of the associative property of addition; but grouping for sums of 10 is an efficient approach to the addition of several one-digit numbers. The associative property of addition is also used in checking the accuracy of column addition by adding in opposite directions.

The commutative and associative properties do not apply in subtraction. However, because of the inverse relationship between subtraction and addition, addition may be used to check accuracy in subtraction, as shown.

$$\begin{array}{r} 6804 \\ -\ 1975 \\ \hline 4829 \end{array} \qquad \begin{array}{r} 4829 \\ +\ 1975 \\ \hline 6804 \end{array}$$

The regrouping that occurs in both addition and subtraction is basically the same skill. The inverse relationship is again apparent, for in addition the regrouping proceeds from right to left, from a lesser to a greater place value, and in subtraction it occurs from left to right, from a greater to a lesser place value. While regrouping in the addition of three or more numbers may involve two, three, or more greater place values, regrouping in subtraction involves a change from one unit of a value to ten of a lesser value. In the example shown, the sum of the ones is 31 (regrouped as 3 tens 1 one), the sum of the tens is 26 (regrouped as 2 hundreds 6 tens), and the sum of the hundreds is 12 (regrouped as 1 thousand 2 hundreds). Note that partial sums are encountered and these become unseen addends. Extensions of basic addition facts are often required. Both of these features contribute to difficulties in addition and deserve special attention.

$$\begin{array}{r} {\scriptstyle 2\,3\,\,} \\ 257 \\ 369 \\ 456 \\ +\ 179 \\ \hline 1261 \end{array}$$

Students may be helped with unseen addends by responding to the oral presentation of examples, such as "four plus nine" and "sixteen plus three". The latter illustrates a simple extension of the basic fact $6 + 3 = 9$ which may also be applied to such additions as $16 + 3$, $26 + 3$, $56 + 3$. Another type of extension involves basic facts having sums from 10 to 18, for exam-

ple, 4 + 9 = 13. This fact may be applied to such additions as 14 + 9, 24 + 9, 64 + 9. These are examples of extensions with bridging, in which the sums are in the next decade above that of the first addend. In the column addition shown above, when adding downward the unseen addends in the ones' place are 16 and 22; 16 + 6 = 22 and 22 + 9 = 31 are extensions of the basic facts 6 + 6 = 12 and 2 + 9 = 11, respectively. It is important that students have facility in thinking of unseen addends and in using extensions in column addition.

Students often experience considerable difficulty in subtracting from minuends with zeros. Difficulties can be minimized by reducing the number of steps in the regrouping process. Specific techniques are offered in the suggestions for teaching pages 26 to 29. It should be pointed out that a numeral can be interpreted in a variety of ways. For example, a three-place numeral such as 375 shows hundreds, tens, and ones, and may be interpreted as 3 hundreds 7 tens 5 ones, as 37 tens 5 ones, or as 375 ones. Similarly, a number such as 40 000 may be interpreted in the following ways.

$\boxed{4}0\,000$	4 ten thousands
$\boxed{40}\,000$	40 thousands
$\boxed{40\,0}00$	400 hundreds
$\boxed{40\,00}0$	4 000 tens
$\boxed{40\,000}$	40 000 ones

In the subtraction of 23 495 from 40 000 it is immediately apparent that regrouping must take place. To subtract ones, a ten must be regrouped as 10 ones. If the minuend 40 000 is interpreted as 4000 tens, the regrouping can occur in one step, changing it to 3999 tens 10 ones (A). With older teaching methods regrouping usually proceeded one step at a time, from right to left, and with two changes in each place (B). The methods outlined in the lessons in this unit can lead to improved efficiency, especially in subtraction with zeros in the minuend.

$$\text{A}\quad \begin{array}{rrrrr} 3 & 9 & 9 & 9 & 10 \\ \cancel{4} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ -\,2 & 3 & 4 & 9 & 5 \\ \hline 1 & 6 & 5 & 0 & 5 \end{array} \qquad\qquad \text{B}\quad \begin{array}{rrrrr} & 9 & 9 & 9 & \\ 3 & \cancel{10} & \cancel{10} & \cancel{10} & 10 \\ \cancel{4} & \cancel{0} & \cancel{0} & \cancel{0} & \cancel{0} \\ -\,2 & 3 & 4 & 9 & 5 \\ \hline 1 & 6 & 5 & 0 & 5 \end{array}$$

The exercises which appear in the lesson on the use of the calculator may be completed without a calculator, but the benefits of using a calculator should not be overlooked. (See the comments about the use of calculators on page xv.)

Solving problems requires, besides computational skills, an ability to identify whether the relationships are additive, subtractive, multiplicative, or divisive. Although a calculator can compute rapidly, it cannot choose the operations. Additive situations are relatively easy to spot because the operation is performed only for the purpose of finding the total number when two or more groups are combined. On the other hand, subtraction may be used for several purposes: to find how many are left, to find how many are gone, to find which group is larger (smaller), and to find how many more are needed. Of the first two, which may be referred to as "separations", the "how many left" is the more common. The third type of subtractive situation compares two groups or two numbers. In this type there is no separation; two sets are matched to see which has more (fewer) members and two numbers are dealt with by subtraction to find their difference. The type which asks, "How many more are needed?" is actually an incomplete additive situation for which the inverse operation of subtraction is used.

Teaching Strategies

Because the students have performed addition and subtraction in previous years, this unit is primarily a review and an extension of the operations. It is suggested, therefore, that a survey be taken prior to beginning the unit to determine aspects which might require extra attention. Examples in the survey should include addends and minuends with up to five digits, some of which are zeros. From these results, plans may then be made to group students with similar needs for instruction.

Students who already have a good mastery of the basic facts, who have a good understanding of the regrouping processes, and who are reasonably accurate in their work may proceed on their own through most of the lessons in this unit. These students may mark their own work from prepared sets of answers and thereby assume an increased responsibility for their own progress and improvement. The teacher may wish to assign and check the assessment exercises for each lesson to ensure that each student has reached the desired level of competence. Since these more capable students will probably proceed more quickly than the other students, they will have time to engage in some of the *Related Activities* suggested for the lessons, especially those which provide some form of enrichment and enjoyment.

Slower students usually require more exercises and more time to achieve satisfactory levels of competence. However, they can enjoy and get satisfaction from mathematics and should have opportunities to explore some of the *Related Activities* for the lessons. The activities provide valuable practice in the form of games and other devices.

If there is a limited number of calculators, present the lesson on pages 32 and 33 relatively early in the unit so that all the students may have opportunities to use a calculator. In their free time some students may use calculators to check their answers for some of the exercises.

Results of the *Checking Up* at the end of the unit should be analyzed using the chart of skills listed on page T 37. Appropriate follow-up work should be planned to help those students whose work indicates the need for reteaching and more practice.

Materials

an abacus
models for thousands, hundreds, tens, and ones
a red and a blue pencil for each student
a map of the Atlantic provinces (optional)
atlases or road maps for British Columbia
a six-by-ten section of page T 396 for each student
a copy of page T 382, tracing paper, and colored pencils for each student
calculators (optional)

Vocabulary

addition	difference
add	decrease
sum	operations
basic addition fact	parentheses
addends	calculator
vertical form	digit display
regroup	tonne, t
estimate	names of the ten provinces and their abbreviations
subtraction	names of farm machines in the table on page 33
subtract	
basic subtraction fact	

Pages 16-17

LESSON OUTCOME

Use sums of 10 to add one-digit numbers

Vocabulary

addition, add, sum, basic addition fact

2 ADDITION AND SUBTRACTION

Using Sums of 10

Anna and her friends listed the kinds of work their mothers do. How many mothers are on the list?

Education	3
Business	8
Sales	2
Health services	7
Community work	1

Add 3, 8, 2, 7, and 1.

Anna used **sums** of 10 to help her add.

3
8
2
7
1

$10 + 10 + 1 = 21$

Eric added this way.

3
8
2
7
1

$10 + 11 = 21$

21 mothers are on Anna's list.

Working Together

Find sums of 10.

1. 4, 0, 7, 3 — 10
2. 6, 5, 5, 4 — 10, 10, 10
3. 2, 1, 8, 7 — 10, 10

Add. Use sums of 10.

4. 5, 9, 0, 1 — 15
5. 6, 3, 1, 2 — 12
6. 4 + 4 + 1 + 0 + 2 11
7. 5 + 8 + 2 + 3 + 7 25
8. 8 + 6 + 1 + 2 + 3 20

16

LESSON ACTIVITY

Before Using the Pages

- Begin a review of basic addition facts by asking what the sum of 7 and 9 is. Have the student who answers name two other addends from 0 to 9. Ask the student who states their sum to name another pair of addends. Continue this procedure for several examples. Encourage the students to vary the addends from 0 to 9.
- Ask students to list all the pairs of numbers having a sum of 10, for example, 9 + 1, 8 + 2, 7 + 3, and so on. Then ask them to list numbers in threes for a sum of 10, for example, 7 + 2 + 1 and 5 + 3 + 2.

Using the Pages

- Ask students to identify the kinds of jobs suggested in the illustrations on pages 16 and 17. They may know persons who have similar jobs, or they themselves may be interested in similar jobs. Read the information at the top of page 16. Note that there are five categories on Anna's list. Ask students to name an example for each category. The illustration may suggest some examples. Point out that to answer the question, the sum of five numbers must be found.

  Have the students compare Anna's method with Eric's method. Then ask one student to add the numbers in an upward direction aloud and another to add in a downward direction. Establish that finding sums of 10 makes it easier to add mentally. The two examples demonstrate that there may be more than one way to find sums of 10 in an exercise. Read the concluding statement. Remind the students that a word problem such as the one at the top of page 16 should always be answered with a concluding statement.

Working Together: For Ex. 1, ask which addend does not affect the sum. Discuss different ways of finding sums of 10 for Ex. 2 and 3. For Ex. 4-8, have students give the sums orally to indicate the combinations they used for sums of 10.

Exercises

Add. Write only the sums. Use sums of 10 to help you add.

1.	2.	3.	4.
3	8	7	6
5	0	3	1
5	2	4	4
13	10	14	11

5.	6.	7.	8.
5	2	5	7
1	6	7	6
9	0	5	3
4	2	2	16
19	10	19	

9.	10.	11.	12.
8	9	8	5
4	3	7	3
2	5	6	9
4	0	4	2
2	1	3	2
20	18	28	21

13. 6 + 4 + 7 + 1 18
14. 3 + 0 + 7 + 2 12
15. 1 + 1 + 8 + 1 11
16. 7 + 5 + 8 + 2 + 3 25
17. 6 + 9 + 1 + 4 + 7 27
18. 5 + 2 + 1 + 3 + 9 20
19. 5 + 4 + 9 + 6 + 0 24
20. 1 + 6 + 2 + 4 + 7 20
21. 8 + 5 + 1 + 1 + 7 22
22. 3 + 2 + 1 + 8 + 7 21
23. 8 + 5 + 4 + 5 + 2 24
24. 1 + 8 + 0 + 9 + 3 21
25. 7 + 0 + 8 + 1 + 5 + 9 + 2 + 6 + 3 + 1 + 5 + 2 + 0 + 1 + 4 + 2 + 4 + 1 + 5 66

Solve.

26. The flight crew on an airplane has 3 pilots, 1 supervisor, 9 attendants, and 1 purser. How many are in the crew? 14

27. 4 assistant accountants, 7 tellers, 5 accounting clerks, 3 typists, 1 assistant manager, and 1 manager work at a bank. How many work at the bank? 21

28. In this card game the players tried to find groups of cards having a sum of 10. What groups can you find in each hand?

Derek: 7+3, 1+9, 5+1+4

Rosa: 2+6+2, 3+6+1, 3+7, 2+7+1

Brian: 1+6+2+1, 5+5

17

Exercises: Remind the students to write a concluding statement for each of Ex. 26 and 27. For each player in Ex. 28, have the students list as many different combinations for sums of 10 as possible.

Assessment

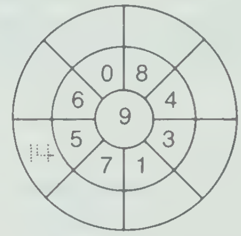
Add. Write only the sums.

1.	2.
1	4
8	5
2	2
3	5
7	6
21	22

3. 1 + 9 + 0 + 2 12
4. 2 + 6 + 1 + 3 + 5 17
5. 8 + 9 + 1 + 0 + 1 19

RELATED ACTIVITIES

- To practice basic addition facts, provide students with number wheels and tables prepared from copies of page T 391.



+	8
5	13
7	
4	

- On one or two copies of page T 382, have the students write a one-digit number in each of the squares. Cut the copies into strips of 10 squares, for example. Give each student a number strip to find the sum of the numbers, using sums of 10. Have the students exchange number strips and repeat the procedure.
- Have each student complete an addition table on a copy of page T 382 and search for patterns in the table.
- Have the students play the game "Total Action" described on page T 379. The numeral cards for the game consist of five cards for each digit from 0 to 9. The required sum is 10.
- Prepare several numeral cards for each digit from 0 to 9 or use the cards prepared for the game in the preceding activity. Have a student draw eight cards and then arrange them, as many as possible, in groups having sums of 10. Then have the student record the sum.

Pages 18-19

LESSON OUTCOME

Add two numbers with regrouping, addends with up to six digits; solve related word problems; round addends and add to estimate the sum, then compare the estimate of the sum with the exact sum

Materials

an abacus or models for thousands, hundreds, tens, and ones; a red pencil and a blue pencil for each student

Vocabulary

addends, vertical form, regroup, estimate

Prerequisite Skills

Complete the basic addition facts; interpret place value in numerals with up to six digits

Checking Prerequisite Skills

Add.

1.	2.	3.	4.
5	9	2	9
8	3	6	7
13	12	8	16

What does each 6 mean?

5. 68 934 — 6 ten thousands

6. 210 769 — 6 tens

Adding Two Numbers

In one week, 21 768 bicycles were made in a factory. The next week, 23 361 bicycles were made. How many bicycles were made in the two weeks?

Add 21 768 and 23 361.

```
 2 1 7 6 8    Show the addends
 2 3 3 6 1    in vertical form.
         9    Add ones.

       1
 2 1 7 6 8    Add tens.
 2 3 3 6 1    Regroup
       2 9    12 tens as
              1 hundred 2 tens.

   1 1
 2 1 7 6 8    Add hundreds.
 2 3 3 6 1    Regroup
     1 2 9    11 hundreds as
              1 thousand
                1 hundred.

   1
 2 1 7 6 8    Add thousands.
 2 3 3 6 1
   5 1 2 9

 2 1 7 6 8    Add ten thousands
 2 3 3 6 1
 4 5 1 2 9
```

45 129 bicycles were made in the two weeks.

To estimate the number of bicycles made, round each addend to the nearest thousand and add.

21 768 → 22 000
23 361 → 23 000
45 000

The sum is about 45 000.

About 45 000 bicycles were made in two weeks.

Working Together

Add by following the steps.

1.
```
  11 11
  41 872
  38 659
  80 531
```

Add ones and regroup.
Add tens and regroup.
Add hundreds and regroup.
Add thousands and regroup.
Add ten thousands.

Round to the nearest thousand.

2. 6532 7000
3. 2342 2000

Estimate each sum. Estimates may vary for Ex. 4-9.

4. 726 + 573 (1300)
5. 68 928 + 1 630 (71 000)

Round and add to estimate each sum. Then find the exact sum.

6. 63 215 + 24 103 87 318 (87 000)
7. $436.58 + 54.90 $491.48 ($490)
8. 7 850 + 84 265 92 115 (92 000)
9. $39 876 + $72 364 $112 240 ($112 000)

18

LESSON ACTIVITY

Before Using the Pages

- Write the addition 2105 + 6384 on the board. Have students show the numbers in a place-value chart on the board.

thousands	hundreds	tens	ones
2	1	0	5
+ 6	3	8	4

Have students explain the steps of the addition as one student shows the steps using an abacus or models and another records the steps in the place-value chart. Emphasize place value in each step; for example, the basic fact 1 + 3 = 4 is applied for adding hundreds in the third step (1 hundred plus 3 hundreds equal 4 hundreds).

Repeat this procedure for 1387 + 3296, paying particular attention to the necessary regrouping. For example, 7 ones plus 6 ones equal 13 ones, and since 10 ones equal 1 ten, 13 ones can be regrouped as 1 ten 3 ones. Similarly, 18 tens can be regrouped as 1 hundred 8 tens since 10 tens equal 1 hundred. Have students explain the steps as in the preceding example. Establish that regrouping occurs when the sum for a column is greater than nine.

thousands	hundreds	tens	ones
	1	1	
1	3	8	7
+ 3	2	9	6
4	6	8	3

Using the Pages

- Draw the students' attention to the photograph at the top of page 19 and ask what kind of factory is shown. Then have a student read the word problem at the top of page 18. Discuss the use of addition to find the answer.

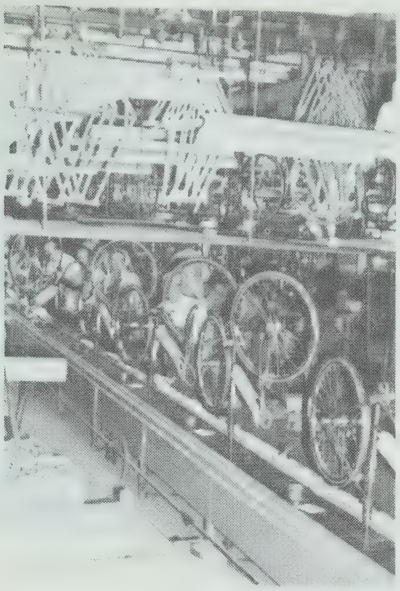
  Ask what is meant by the terms *addend* and *vertical form*. Ask why the addends are written in vertical form. For the worked example, each step of the addition is highlighted in red and then described. Discuss each step and pay particular attention to the regrouping. Have a student read the concluding statement.

Solve.

13. Ingrid's bicycle cost $139.95. Her sister's bicycle cost $119.94. How much did the two bicycles cost together? $259.89

14. A store sold 3267 ten-speed bicycles and 1398 five-speed bicycles one year. How many of these kinds of bicycles did it sell in that year? 4665

15. Noel has a ten-speed bicycle that cost $149.95. His brother has a coaster bicycle that cost $79.95. How much did the two bicycles cost together? $229.90



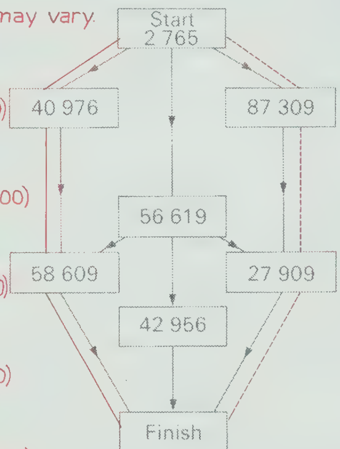
Copy the diagram and color a path

16. red for the numbers that have the sum 102 350.

17. blue for the numbers that have the sum 117 983.

Exercises

Round and add to estimate each sum. Then find the exact sum. Estimates may vary.



1. 1841 + 5746 = 7587 (8000)
2. 3279 + 3805 = 7084 (7000)
3. 2657 + 4379 = 7036 (7000)
4. 6284 + 395 = 6679 (6700)
5. 4679 + 8773 = 13 452 (14 000)
6. 4536 + 9886 = 14 422 (15 000)
7. 81 214 + 7 051 = 88 265 (88 000)
8. 980 246 + 89 667 = 1 069 913 (1 070 000)
9. $859 673 + 349 528 = $1 209 201 ($1 210 000)
10. $26 539 + 28 934 = $55 473 ($60 000)
11. $873.09 + 589.37 = $1 462.46 ($1500)
12. $9568.96 + 8594.66 = $18 163.62 ($19 000)

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 1-10 on page 328.
- For practice with extensions of basic addition facts, provide the students with copies of page T 382. Have them write addends in the table as shown and then complete it.

+	14	18	16	12	11
2					
8					
5					

- Students having difficulty may benefit from showing some of the exercises on page 19 using an abacus, a place-value chart, or, where practical, models of thousands, hundreds, tens, and ones.
- Provide other exercises similar to Ex. 16 and 17. Students may help to draw the diagrams and provide the numbers. Have the students find the sum for each path or find the path for a specific sum.
- Have the students prepare a list of examples for occasions outside the classroom when they have estimated sums, for instance, the number of students in all on two school buses. Display the list for them to add new examples as they think of them.

- Discuss the procedure for estimating the sum, shown at the top of page 18. Explain that the estimate can be used to check the sum.

Working Together: Ex. 1 emphasizes the right-to-left order of adding, regrouping as needed. For Ex. 2 and 3, the students are directed to round the numbers to a particular place. However, to estimate the sums for Ex. 4-9, the students must decide which place is preferable for rounding the two addends of each exercise. It is important to discuss what influences this choice. For instance, because one of the addends in Ex. 5 has only four digits, rounding to the nearest thousand is likely preferable.

Exercises: It is desirable that eventually a quick, mental calculation be carried out to estimate a sum. At this time, however, the students are required to show two additions for each exercise, one for the estimate of the sum and one for the exact sum. This will enable you to assess their ability to round numbers, to note whether all the addends of an exercise are rounded to the same place, and to discuss the choice of place for rounding the addends.

Ensure that the students understand that they should estimate each sum first, and then add. For Ex. 1-12, have them compare the estimate and the exact sum to check the addition. Remind the students to show their work for Ex. 13-15 and to answer each with a concluding statement. Provide each student with a red pencil for Ex. 16 and a blue pencil for Ex. 17. You may wish to have the students attempt to find each path by rounding the numbers and estimating each sum.

Assessment

Estimates may vary.

Round and add to estimate each sum. Then find the exact sum.

1. 9781 + 5893 = 15 674 (16 000)
2. 26 937 + 15 706 = 42 643 (50 000)
3. 43 645 + 89 824 = 133 469 (130 000)
4. 10 284 + 38 739 = 49 023 (49 000)

Solve.

5. Martha's bicycle cost $142.95 and Matthew's bicycle cost $138.90. How much did the two bicycles cost together? $281.85

LESSON OUTCOME

Add more than two numbers, sums with up to six digits; solve related word problems

Materials

a map of the Atlantic Provinces (optional)

Vocabulary

tonne, t

Prerequisite Skills

Add more than two one-digit numbers; add two numbers with regrouping

Checking Prerequisite Skills

Add.

1.	2.	3.
7	6	8
8	1	5
3	0	1
4	2	5
22	3	9
	12	28

4.	5.
7614	97 026
8089	86 483
15 703	183 509

Adding More Than Two Numbers

The catch of eels was 91 t for Prince Edward Island, 25 t for Nova Scotia, 119 t for New Brunswick, and 12 t for Newfoundland. How many tonnes of eels were caught in all?

The symbol t stands for **tonne**.

1 t = 1000 kg

```
   1
   9 1
   2 5
 1 1 9
   1 2
 -----
     7
```

Show the addends in vertical form. Add ones. Regroup 17 ones as 1 ten 7 ones.

Remember to watch for sums of 10.

```
 1 1
   9 1
   2 5
 1 1 9
   1 2
 -----
   4 7
```

Add tens. Regroup 14 tens as 1 hundred 4 tens.

```
 1 1
   9 1
   2 5
 1 1 9
   1 2
 -----
 2 4 7
```

Add hundreds.

247 t of eels were caught in all.

Working Together

Add by following the steps.

1.
```
  12
 325
 473
 869
 417
----
2084
```

Add ones and regroup.
Add tens and regroup.
Add hundreds.

Add.

2.	3.
24 565	$32.48
61 830	0.73
50 324	81.21
67 245	0.06
31 978	3.27
235 942	$117.75

4. 2 568 + 792 + 36 520 + 15 + 4   39 899

5. 25 + 6 249 + 312 + 32 701   39 287

20

LESSON ACTIVITY

Before Using the Pages

- Write the addition 7289 + 6438 + 2095 + 3761 on the board and ask how many addends there are. Draw a place-value chart for thousands, hundreds, tens, and ones on the board and have students show the addends in the chart.

thousands	hundreds	tens	ones
7	2	8	9
6	4	3	8
2	0	9	5
+ 3	7	6	1

Have students explain each step of the addition and write the sum on the board. For example, in adding the ones, a student would explain that the 23 ones are regrouped as 2 tens 3 ones, that the 3 ones are recorded in the ones' place of the sum, and that the 2 tens are recorded in the tens' place above the addends.

Using the Pages

- A brief discussion of the photograph can introduce the word problem. Ask, for example, what industry is shown and in what parts of Canada such an industry would be found. You may wish to have students locate the four provinces named in the problem on a map of Canada. Discuss that the amount of fish in a catch is given by mass, and that the kilogram is a unit too small for this purpose. Introduce the word *tonne* and the symbol t. Ask students to name objects for which the masses would be measured in tonnes, for example, trucks, ships, and large animals such as elephants.

Review the reason for writing the addends in vertical form. Have students perform the addition aloud, reminding them to watch for sums of 10. Ask a student to read the concluding statement.

Exercises

Add.

Remember to watch for sums of 10.

1.	2.	3.	4.	5.
341	445	2069	32 421	$8403
826	69	5347	593	6569
519	251	6203	10 987	2427
		5672	4 389	5521
1686	765	19 291	48 390	$22 920

6.	7.	8.	9.	10.
29 031	24 692	30 456	63 789	$890.32
11 094	61 345	125	6 037	8.47
24 570	98	80 747	27 822	29.81
69 308	27 352	24 813	961	0.54
70 541	6 513	71 635	98 331	612.36
204 544	120 000	207 776	196 940	$1541.50

11. 4987 + 6153 + 8523 + 2641 22 304
12. 87 + 59 246 + 103 + 8 59 444
13. $372 + $49 + $3826 + $3 + $53 $4303
14. $5.63 + $0.92 + $84.71 + $0.08 $91.34

This chart shows the catch in tonnes for 1976.

	Prince Edward Island	Nova Scotia	New Brunswick	Newfoundland
Cod	1 880	50 952	3 938	119 565
Haddock	27	19 001	42	249
Mackerel	1 363	7 984	656	5 350
Herring	667	96 286	75 296	48 921
Eels	91	25	119	12
Scallops	1 480	89 572	1 250	30
Lobsters	3 858	5 904	2 755	2 255
Crabs	6	499	6 172	2 666

15. How many tonnes of scallops were caught in all? 92 332
16. What was the total catch for New Brunswick? 90 228 t
17. What was the catch of haddock and herring for Newfoundland and Nova Scotia together? 164 457 t
18. How many tonnes of lobsters and crabs were caught in all? 24 115
19. Which kind of fish gave the heaviest catch in all? herring
20. Which province had the greatest catch? Nova Scotia

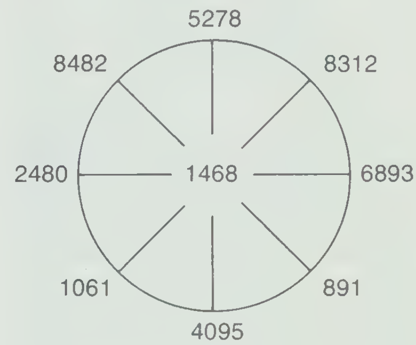
21

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 11-28 on page 328.
- For practice with regrouping, prepare a work sheet with exercises similar to the following. Have the students regroup for each exercise and then write the standard form for the number.

thousands	hundreds	tens	ones
13	10	26	17
10	15	20	11

- Challenge students to prepare "Magic Circles" similar to the following. For a Magic Circle, the sums of three numbers in line are equal.



- Have students write and solve addition problems based on the chart on page 21.
- To provide practice in estimating sums for more than two numbers, give road maps to the students. Ask them to select two cities and to estimate the distance between them by rounding intermediate distances and then adding the rounded distances.

Working Together: Have students explain each step for Ex. 1. For Ex. 3, point out the symbol $ and how the decimal points are aligned. Review that the zero to the left of the decimal point is needed to show zero dollars, as in $0.73. For Ex. 4 and 5, pay close attention to lining up the addends in vertical form when each addend shows a different number of digits.

Exercises: Before the students begin the exercises, draw attention to the chart on page 21. To help them interpret the chart, ask them to name the kinds of fish with which they are familiar. Then ask questions such as "How many tonnes of lobsters were caught in New Brunswick?" and "Which province had the greatest catch of cod?" Remind the students to write Ex. 11-14 in vertical form and to answer Ex. 15-20 in complete sentences.

Assessment

Add.

1.	2.	3.	4.
1751	2419	62 598	16 281
4289	5823	15 344	47 990
6372	1304	8 613	17 567
12 412	2581	55 012	84 123
	12 127	0 726	59 342
		142 293	225 303

Solve.

5. In Newfoundland, 119 565 t of cod, 5350 t of mackerel, 48 921 t of herring, and 249 t of haddock were caught. How many tonnes of these four kinds of fish were caught? 174 085

OBJECTIVE

Demonstrate competence in adding and in estimating sums; solve related word problems

Materials

atlases or road maps of British Columbia, a six-by-ten section of graph paper (use copies of page T396) for each student

Practice

Copy and complete this puzzle.

1.

a 1	b 3	c 2	8	■	■	d 9	e 1	f 4	3
■	2	4	■	g 2	0	■	0	0	■
h 2	5	8	2	■	■	i 3	1	7	6
j 1	3	1	3	■	■	k 1	1	6	4
■	9	7	■	l 2	2	■	6	7	■
m 2	0	1	5	■	■	n 8	2	0	7

Across

a. 336 + 487 + 362 + 143
d. 5704 + 2193 + 1246
g. 4 + 8 + 1 + 1 + 6
h. 23 + 746 + 1809 + 4
i. 483 + 7 + 2061 + 625
j. 755 + 123 + 435
k. 98 + 513 + 64 + 489
l. 2 + 5 + 7 + 3 + 5
m. 1676 + 5 + 320 + 14
n. 944 + 136 + 7 + 7120

Down

b. 67 328 + 59 240 + 70 375 + 34 732 + 93 715
c. 50 436 + 89 673 + 48 734 + 59 328
e. 47 389 + 6 422 + 37 591 + 5 267 + 4 493
f. 57 326 + 81 784 + 99 387 + 76 021 + 93 152

Remember to watch for sums of 10 to help you add.

First, estimate the sum without doing any work on paper. Then add. Compare the sum with your estimate. Estimates may vary.

2. ($100 000)
$35 805
64 942
$100 747

3. ($900 000)
$475 634
419 287
$894 921

4. ($1 000 000)
$154 462
783 668
$938 130

5. $637.59
196.74
$834.33 ($800)

6. $684.32
758.79
$1443.11 ($1500)

Use <, >, or = to make true statements.

7. 7 + 9 + 3 + 1 + 2 + 5 + 9 > 6 + 5 + 3 + 4 + 3 + 6 + 8
8. 5 + 7 + 4 + 8 + 3 + 1 + 8 = 9 + 4 + 0 + 9 + 5 + 9
9. 46 789 + 26 411 + 16 676 < 54 367 + 8 245 + 27 364
10. 98 521 + 67 584 + 52 863 > 95 770 + 73 312 + 48 886
11. $89 632 + $42 759 + $73 401 > $43 849 + $76 214 + $85 719
12. $780.97 + $463.52 + $791.28 = $779.11 + $988.40 + $268.26

22

LESSON ACTIVITY

Using the Pages

- Give each student a six-by-ten section of graph paper on which to copy and complete the cross-number puzzle. Have the students shade the unused squares and place the letters in the other squares before writing the numerals. Students who are not familiar with a cross-number puzzle will need to be shown the procedure for completing one. Point out that some numerals are written across and some are written down, so that each digit of a sum occupies one square. Review how the puzzle can reveal that an error in computation has been made. For example, the numerals for d (across) and e (down) share one digit.

  For Ex. 2-6, you may wish to have the students write their exercises using the format indicated below for Ex. 2. Emphasize that the estimate is written first and then the exact sum is found.

$$\begin{array}{lr} & 35\,805 \\ & +\ 64\,942 \\ \text{Exact sum} \longrightarrow & 100\,747 \\ \text{Estimate of the sum} \longrightarrow & \boxed{100\,000} \end{array}$$

- Ask the students in which province a scene similar to the one in the photograph would be found. Draw attention to the chart at the top of page 23. You may wish to have students locate the places named in the chart on a map of British Columbia. Then explain how the chart can be used to find the distance between two places. Have the students use their fingers to trace along the row for Kamloops and up the column for Vancouver. Note that the distance between Kamloops and Vancouver is given in the box where the row and the column meet. To check whether the students can read the chart, ask, for example, "What is the distance between Victoria and Dawson Creek?" Ex. 17 is starred because answers will vary according to the places selected and the route traveled. An atlas or a road map will be

Distances in British Columbia Measured in Kilometres

Dawson Creek	937	412	1143	1061	533	1194	1239	652
Kamloops		525	1255	183	404	425	460	285
Prince George			731	649	117	792	827	240
Prince Rupert				1379	851	1522	1558	970
Princeton					526	288	323	407
Quesnel						671	707	119
Vancouver							68	552
Victoria								587
Williams Lake								

This chart shows that the distance between Kamloops and Vancouver is 425 km.

Solve.

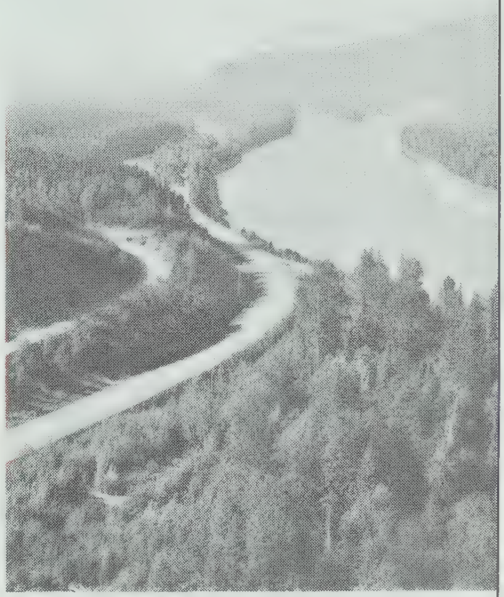
13. Jason drove from Victoria to Vancouver and then from Vancouver to Prince George. How far did he drive? 860 km

14. Adrian's father drove from Kamloops to Quesnel and from Quesnel to Williams Lake. Then he returned to Kamloops. How far did he travel? 808 km

15. Jane Berry lives in Victoria. She is planning a trip to Kamloops and then to Dawson Creek before returning home. How far will she travel on the trip? 2 636 km

16. How far is it in all from Prince Rupert to Princeton to Prince George to Quesnel to Dawson Creek to Prince Rupert? 3821 km

*17. Use an atlas or a road map. Find one of the places named in the chart above. Plan a trip to four of the other places named. Tell how far you would travel in all. Answers will vary.



23

required to help determine the route. If only a few are available, have some students use the maps while others work on the other exercises.

RELATED ACTIVITIES

• To practice addition and to prepare for subtraction, provide a work sheet with additions that have missing digits as shown.

$$\begin{array}{r} 3\,\boxed{4}\,7\,\boxed{8}\,6 \\ +\,\boxed{4}\,8\,\boxed{3}\,9\,\boxed{5} \\ \hline 8\,3\,1\,8\,1 \end{array}$$

$$\begin{array}{r} 8\,\boxed{9}\,\boxed{5}\,6\,2 \\ +\,3\,4\,7\,\boxed{8}\,\boxed{1} \\ \hline 1\,2\,4\,3\,4\,3 \end{array}$$

$$\begin{array}{r} \boxed{6}\,\boxed{4}\,\boxed{3}\,\boxed{7} \\ +\,1\,9\,5\,8 \\ \hline 8\,3\,9\,5 \end{array}$$

$$\begin{array}{r} 5\,8\,2\,8\,2 \\ +\,\boxed{1}\,\boxed{8}\,\boxed{2}\,\boxed{6}\,\boxed{1} \\ \hline 7\,6\,5\,4\,3 \end{array}$$

• Students may enjoy making cross-number puzzles on copies of page T 382 for others to complete.

• Provide students with a road map of an area near their homes or of another area that is of interest to them. Have them write addition problems similar to those on page 23 for other students to solve.

• Students may enjoy playing the game "Greatest Sum" described on page T 379. The number of digits for each addend in the charts will depend on the ability of the students. You may wish to draw charts for more than two addends.

LESSON OUTCOME

Subtract numbers with no regrouping, minuends with up to six digits; use addition to check subtraction; solve related word problems

Materials

an abacus or models for thousands, hundreds, tens, and ones; a copy of page T 382, tracing paper, and colored pencils for each student

Vocabulary

subtraction, subtract, basic subtraction fact

Prerequisite Skills

Complete basic subtraction facts, minuends to 9

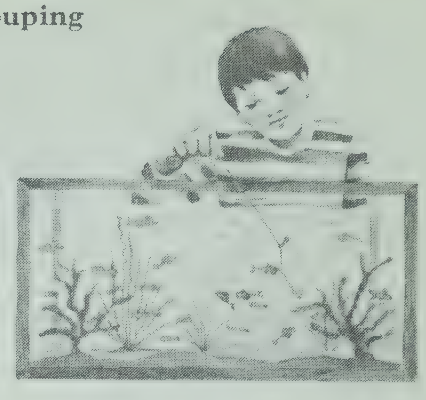
Checking Prerequisite Skills

Subtract.

1. $\begin{array}{r} 5 \\ \underline{2} \\ 3 \end{array}$ 2. $\begin{array}{r} 8 \\ \underline{6} \\ 2 \end{array}$ 3. $\begin{array}{r} 7 \\ \underline{3} \\ 4 \end{array}$ 4. $\begin{array}{r} 9 \\ \underline{4} \\ 5 \end{array}$

5. $\begin{array}{r} 6 \\ \underline{5} \\ 1 \end{array}$ 6. $\begin{array}{r} 8 \\ \underline{8} \\ 0 \end{array}$ 7. $\begin{array}{r} 4 \\ \underline{0} \\ 4 \end{array}$ 8. $\begin{array}{r} 9 \\ \underline{7} \\ 2 \end{array}$

Subtraction with No Regrouping



A pet store had 256 goldfish. 145 were sold. How many goldfish are left?

Subtract 145 from 256.

Show 256 - 145 in vertical form. Subtract ones.

$\begin{array}{r} 2\ 5\ 6 \\ \underline{1\ 4\ 5} \\ 1 \end{array}$

Subtract tens.

$\begin{array}{r} 2\ 5\ 6 \\ \underline{1\ 4\ 5} \\ 1\ 1 \end{array}$

Subtract hundreds.

$\begin{array}{r} 2\ 5\ 6 \\ \underline{1\ 4\ 5} \\ 1\ 1\ 1 \end{array}$

Add to check.

$\begin{array}{r} 111 \\ \underline{145} \\ 256 \end{array}$

111 goldfish are left.

This number should match the first number used in the subtraction. If the two numbers do not match, there is a mistake in your work.

Working Together

Subtract by following the steps.

1. $\begin{array}{r} 5984 \\ \underline{3721} \\ 2263 \end{array}$

Subtract ones.
Subtract tens.
Subtract hundreds.
Subtract thousands.

Add to check the subtraction.

2. $\begin{array}{r} 47\ 369 \\ \underline{44\ 136} \\ 3\ 233 \end{array}$ $\begin{array}{r} 3\ 233 \\ \underline{44\ 136} \\ 47\ 369 \end{array}$

Subtract. Add to check.

3. $\begin{array}{r} 974 \\ \underline{202} \\ 772 \end{array}$ 4. $\begin{array}{r} \$8397 \\ \underline{8041} \\ \$\ 356 \end{array}$ 5. 68 951 - 14 650 54 301 6. \$378.65 - \$360.54 \$18.11

24

LESSON ACTIVITY

Before Using the Pages

- Present a situation that involves the use of a basic subtraction fact. Say, for example, "There are seven tonnes of eels and three tonnes of crabs. How many more tonnes of eels than crabs are there?" Discuss why addition is not used. Lead the students to recall the term *subtraction*. Have a student write the subtraction sentence 7 − 3 = 4 on the board, noting that the symbol − is read "minus". Use other examples. Include minuends to 18, if you wish.
- Write the subtraction 7246 − 3125 on the board. Have students show the numbers in a place-value chart on the board.

thousands	hundreds	tens	ones
7	2	4	6
− 3	1	2	5

Question the students to obtain as much information as possible from them to explain each step of the subtraction. Have one student show the steps using an abacus or models while another writes the numerals in the place-value chart. Emphasize place value; for example, have a student say, "Four tens minus two tens equal two tens." Review that subtraction is completed from right to left.

Using the Pages

- Have a student read the word problem at the top of page 24. Ask why subtraction is used to find the solution. Point out that writing the numerals in vertical form facilitates subtraction, and note that the greater number is written above the lesser number. Each step of the subtraction is highlighted in red and explained in words. The explanation emphasizes the right-to-left order of subtraction and the application of a basic subtraction fact in each place. Discuss each step and have a student read the concluding statement.

Ask why addition can be used to check subtraction. Lead the students to express the inverse relationship between addition and subtraction in their own words. Point out that if the sum is different from the minuend, there is a mistake.

Exercises

Subtract. Add to check.

1. 74
   30
   44

2. 683
   652
   31

3. 942
   340
   602

4. 5168
   2143
   3025

5. 27 469
   27 308
   161

6. 97 685
   92 360
   5325

7. 59 362
   47 151
   12 211

8. 90 835
   10 231
   80 604

9. $49 876
   25 212
   $24 664

10. $678.39
    157.02
    $521.37

11. 895 − 403 492
12. 5978 − 5608 370
13. 9427 − 4123 5304
14. 88 516 − 70 214 18 302
15. 78 941 − 65 930 13 011
16. 467 981 − 234 511 233 470
17. $7278 − $7026 $252
18. $57 982 − $35 342 $22 640
19. $988.64 − $427.01 $561.63
20. $792.94 − $201.50 $591.44

Solve.

21. 55 guppies are in an aquarium. 41 guppies are in another. How many fewer guppies are in the second aquarium? 14
22. A swordtail costs $1.99. A molly costs $1.75. How much more does the swordtail cost? $0.24
23. A store sold 102 of its 213 guppies. How many guppies are left? 111
24. A guppy costs $1.15. A larger guppy costs $1.98. How much less does the smaller guppy cost? $0.83

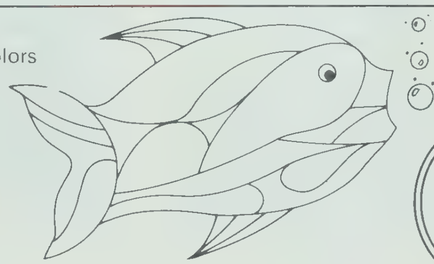
Copy and complete this chart.

25.

+	9	12	6	23	18	24	18
16	25	28	22	39	34	40	34
7	16	19	13	30	25	31	25
8	17	20	14	31	26	32	26
15	24	27	21	38	33	39	33
20	29	32	26	43	38	44	38
23	32	35	29	46	41	47	41
31	40	43	37	54	49	55	49

Trace the fish. Color it using as few colors as possible so that no touching sections are the same color.

How many colors did you need? Not more than four colors are needed



try this

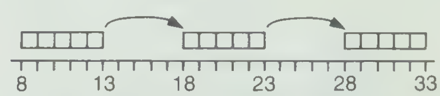
25

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 29-38 on page 328.
- For practice with basic subtraction facts, provide students with number wheels and tables similar to the following on copies of page T 391.

−	9
15	6
17	
13	
18	

- Provide drawings similar to the fish in the *Try This* feature or maps for the students to color using four or fewer colors. Some students may wish to draw outlines for themselves or for other students to color with four or fewer colors.
- Many students can benefit from practice with extensions of basic addition and subtraction facts. Provide exercises similar to the following. Demonstrate some of the patterns by placing a number strip along the number line as shown for extensions of 8 + 5 = 13.



8 + 5 = _____ 12 − 8 = _____
18 + 5 = _____ 22 − 8 = _____
28 + 5 = _____ 32 − 8 = _____

If the sum is the same as the minuend, the work is likely correct, but it is still possible for there to be a mistake. For example, the same error may be made in a basic subtraction fact and in the corresponding addition fact, such as 14 − 9 = 6 and 6 + 9 = 14.

Working Together: For Ex. 1, have the students follow the steps for the subtraction. For Ex. 2, ask what numbers should be added to check the subtraction. Then ask what the sum will be if the work is correct. Have the students add to check the subtraction. Demonstrate that an addition exercise need not be written because the addition can be performed mentally by adding upward in the completed subtraction exercise. Point out how the symbol $ and decimal points are recorded for subtraction with money.

Exercises: Since Ex. 1-24 involve subtraction without regrouping, only basic subtraction facts with minuends to 9 are used. Provide each student with a copy of page T 382 on which to copy and complete the chart for Ex. 25. Ex. 25 involves basic addition and subtraction facts and their extensions.

Try This: Provide the students with tracing paper. Emphasize that they are to use as few colors as possible and yet to have no touching sections the same color. Although the question asked is "How many colors did *you* need?" it is important to display the results and discuss the number of colors used in each diagram. If no student used as few as four colors, challenge them to try the activity again using fewer colors. For any diagram, no more than four colors are needed.

Assessment

Subtract. Add to check.

1. 726
   314
   412

2. 3598
   1084
   2514

3. 76 416
   23 201
   53 215

4. 86 491 − 74 371 12 120

5. 592 378 − 540 248 52 130

Solve.

6. One swordtail costs $2.05. A larger swordtail costs $2.39. How much more does the larger swordtail cost? $0.34

Pages 26-27

LESSON OUTCOME

Subtract with regrouping, minuends with up to six digits; solve related word problems; round the minuend and the subtrahend and subtract to estimate the difference, then compare the estimate of the difference with the exact difference

Materials

an abacus or models for thousands, hundreds, tens, and ones

Vocabulary

difference, decrease

Prerequisite Skills

Complete the basic subtraction facts; subtract with no regrouping, minuends with up to six digits

Checking Prerequisite Skills

Subtract.

1. 14 − 5 = 9   **2.** 17 − 8 = 9

3. 16 − 7 = 9   **4.** 12 − 9 = 3

5. 3581 − 1260 = 2321   **6.** 92 643 − 51 530 = 41 113

7. 81 569 − 41 433   40 136

8. 276 897 − 226 590   50 307

Subtraction with Regrouping

In a ten-year period, the number of farms in Manitoba decreased from 39 747 to 29 963. How many fewer farms were there at the end of the ten years?

Subtract 29 963 from 39 747.

```
  3 9 7 4 7
  2 9 9 6 3
          4
```

Show 39 747 − 29 963 in vertical form. Subtract ones. Then try to subtract tens.

```
      6 14
  3 9 7 4 7
  2 9 9 6 3
        8 4
```

To subtract tens, regroup 7 hundreds 4 tens as 6 hundreds 14 tens, and subtract.

```
      16
    8  6 14
  3 9 7 4 7
  2 9 9 6 3
      7 8 4
```

To subtract hundreds, regroup 9 thousands 6 hundreds as 8 thousands 16 hundreds, and subtract.

```
   18 16
  2 8  6 14
  3 9 7 4 7
  2 9 9 6 3
    9 7 8 4
```

To subtract thousands, regroup 3 ten thousands 8 thousands as 2 ten thousands 18 thousands, and complete the subtraction.

There were 9784 fewer farms.

To estimate the **difference**, round each number to the nearest thousand and subtract.

39 747 → 40 000
29 963 → 30 000
10 000

The difference is about 10 000.

There were about 10 000 fewer farms.

26

LESSON ACTIVITY

Before Using the Pages

- Use an abacus or models of thousands, hundreds, tens, and ones to represent a number in different ways as follows. Display 2 thousands 4 hundreds 3 tens 5 ones (A) and ask what number is shown (2435). Regroup 1 ten as 10 more ones and ask whether the number represented is the same as before or different (B). Regroup 1 hundred as 10 more tens and repeat the question (C). Regroup 1 thousand as 10 more hundreds and ask the question again (D). Remind the students that the value of the number does not change when the name for the number is changed. Thus, a number may have many different names.

	th	h	t	o
A	2	4	3	5
B	2	4	2	15
C	2	3	12	15
D	1	13	12	15

Write the subtraction 2435 − 798 in vertical form on the board. Review that subtraction is carried out place by place from right to left. Establish that 8 ones cannot be subtracted from 5 ones. Lead the students to suggest that 1 ten must be regrouped as 10 more ones. Review how the regrouping is shown and then subtract the ones. Use a similar procedure to complete the subtraction.

Using the Pages

- Have a student read the word problem at the top of page 26. Relate the words "how many fewer" to the use of subtraction for the solution. Read the statements that accompany the first step in the subtraction. Note that 3 ones can be subtracted from 7 ones. Ask the students what difficulty arises when they try to subtract tens and how the difficulty is overcome. Similarly, discuss the regrouping that is required in each of the remaining steps of the subtraction. Have a student read the concluding statement.
- Draw attention to the word *difference* and ask which number in a subtraction is called the difference. Have a student read

Working Together

Subtract by following the steps.

1.
$$\begin{array}{r} \overset{5\ \overset{14}{4}\ \overset{12}{2}\ \overset{11}{1}\ 17}{65\,327} \\ 29\,658 \\ \hline 35\,669 \end{array}$$

Regroup and subtract.
Regroup and subtract.
Regroup and subtract.
Regroup and subtract.
Subtract.

Estimate the difference. Estimates may vary for Ex. 2-5

2. 58 364 − 17 431 40 000

Round and subtract to estimate each difference. Then find the exact difference.

3. 93 641 − 47 237 = 46 404 (40 000)

4. $14 595 − 5 096 = $ 9 499 ($10 000)

5. $512.65 − $494.37 $18.28 ($20)

Exercises

Round and subtract to estimate each difference. Then find the exact difference. Estimates may vary

1. 8746 − 6391 = 2355 (3000)
2. 4532 − 2815 = 1717 (2000)
3. 73 851 − 50 141 = 23 710 (24 000)
4. 83 975 − 48 320 = 35 655 (30 000)
5. 964 394 − 915 676 = 48 718 (40 000)
6. $11 842 − 6 348 = $ 5494 ($6000)
7. $56 829 − 18 998 = $37 831 ($40 000)
8. $939 539 − 764 579 = $174 960 ($180 000)
9. $924.81 − 879.85 = $ 44.96 ($40)
10. $262.10 − 69.47 = $192.63 ($190)

This chart shows the change in the number of farms in the Prairie Provinces.

	Number of farms		
	1966	1971	1976
Manitoba	39 747	34 981	29 963
Saskatchewan	85 686	76 970	69 578
Alberta	69 411	62 702	57 310

11. What was the decrease in the number of farms in Alberta from 1966 to 1976? 12 101

12. In 1976, how many more farms were in Alberta than in Manitoba? 27 347

13. What was the decrease in the number of farms in Saskatchewan from 1966 to 1971? 8716

14. In which province was the decrease in the number of farms greatest from 1971 to 1976? Saskatchewan

15. Was the decrease in the number of Alberta farms greater from 1966 to 1971 or from 1971 to 1976? 1966 to 1971

*16. How many farms do you think there will be in Saskatchewan in 1981? Answers will vary.

27

RELATED ACTIVITIES

• For further practice, you may wish to have the students complete Ex. 39-47 on page 328.

• To provide practice with regrouping, prepare cards similar to the following for the game "Dominoes" described on page T 379.

	th	h	t	o
3684	6	18	5	14

	th	h	t	o
7864	5	11	13	7

• Have students estimate differences for exercises on page 25 and compare each estimate with the exact difference.

• To reinforce the relationship between addition and subtraction, have students write families of facts, using a set of numeral cards, two each for the numbers 0 to 9. The cards are placed in a container. Each student draws two cards and writes as many related addition and subtraction facts as possible for the two numbers. For example, two families may be derived from the numbers 4 and 7.

$$\begin{array}{ll} 4 + 7 = 11 & 7 - 4 = 3 \\ 7 + 4 = 11 & 7 - 3 = 4 \\ 11 - 7 = 4 & 3 + 4 = 7 \\ 11 - 4 = 7 & 4 + 3 = 7 \end{array}$$

• For practice with addition and subtraction, use copies of page T 391 to prepare diagrams showing addends in the circles and sums in the squares.

the difference in the worked example. Ask what procedure can be used to estimate a difference. For the example on page 26, it is stated that each number is rounded to the nearest thousand. Discuss that it would also be permissible to round the numbers to the nearest ten thousand.

Working Together: Ex. 1 emphasizes the right-to-left order of subtracting and indicates the places for which regrouping is required. For Ex. 2, ask to which places the numbers for estimating the difference may be rounded. Ask the students whether they can tell which places will require regrouping in Ex. 3 and 4.

Exercises: For Ex. 1-10, you may wish to have the students write the exercises using the format suggested on page T 24. Emphasize that the estimate is written before the exact difference is found.

Before the students begin Ex. 11-16, familiarize them with the chart by asking, for example, "How many farms were there in Saskatchewan in 1971?" Note that for each province there is a *decrease* in the number of farms from 1966 to 1976 and discuss possible reasons for this. Some students may answer Ex. 14 by using estimates rather than exact differences. Ex. 16 is starred because answers will vary. However, point out that the answer should not be a haphazard guess, but one based on information provided for previous years.

Assessment

Estimates may vary.

Round and subtract to estimate each difference. Then find the exact difference.

1. 9346 − 6827 = 2519 (2000)
2. $543.62 − 147.89 = $395.73 ($400)
3. 643 173 − 629 425 = 13 748 (10 000)
4. 139 412 − 69 508 69 904 (70 000)
5. 928 371 − 919 485 8886 (10 000)

Solve.

6. In a five-year period, the number of farms in Saskatchewan decreased from 76 970 to 69 578. How many fewer farms were there at the end of the five years? 7392

Pages 28-29

LESSON OUTCOME

Subtract with regrouping, zero in one or more places in the minuend, minuends with up to six digits; solve related word problems

Materials

models for thousands, hundreds, tens, and ones

Prerequisite Skills

Subtract with regrouping, minuends with up to six digits

Checking Prerequisite Skills

Subtract.

1. 62 526
   34 978
   27 548

2. 534 892
   156 738
   378 154

3. 47 567 − 28 580 18 987

4. 841 517 − 546 789 294 728

Subtraction, Regrouping with Zeros

Find the difference of 19 274 and 30 000.

30 000 Cannot subtract
19 274 4 ones from 0 ones.

Think
30 000 shows 3 ten thousands.
30 000 shows 30 thousands.
30 000 shows 300 hundreds.
30 000 shows 3000 tens.

Here are some other examples that show regrouping with zeros.

3 10 2 9 15 ~~40 305~~ 17 297 23 008	16 13 5 ~~7~~ 9 ~~3~~ 17 ~~67 047~~ 39 658 27 389
12 1 ~~2~~ 9 9 10 ~~23 000~~ 15 684 7 316	10 12 5 ~~0~~ 9 ~~2~~ 10 ~~61 030~~ 12 739 48 291

2 9 9 9 10
~~30 000~~
19 274

Regroup 3000 tens 0 ones as 2999 tens 10 ones.

2 9 9 9 10
~~30 000~~
19 274
10 726

Then, subtract ones, tens, hundreds, thousands, and ten thousands.

The difference is 10 726.

Working Together

Copy and complete.

1. 3 9 9 9 15
   ~~40 005~~
   19 326
   20 679

2. 69 9 17 10
   ~~70 080~~
   24 397
   45 683

Subtract by following the steps.

3. 59 11 9 13
   ~~60 203~~
   54 987
   5216

Regroup and subtract.
Subtract.
Regroup and subtract.
Subtract.
Subtract.

Subtract.

4. 304
   156
   148

5. 52 000
   49 617
   2 383

6. 80 014 − 7 268 72 746

7. 10 000 − 5 093 4907

28

LESSON ACTIVITY

Before Using the Pages

- Display 1 thousand and have students identify the number represented. Display 1 hundred, have students identify the number, and ask how many hundreds are the same as 1 thousand. Demonstrate this with the models. In a similar manner, show that 100 tens and 1000 ones are the same as 1 thousand, illustrating the difficulty of demonstrating the last two examples. Summarize the concept on the board.

one thousand		
	$\boxed{1}$ 0 0 0	1 thousand
	$\boxed{1\ 0}$ 0 0	10 hundreds
	$\boxed{1\ 0\ 0}$ 0	100 tens
	$\boxed{1\ 0\ 0\ 0}$	1000 ones

Develop similar examples for 4000, 4003, and 60 005, for instance, without models where their use is impractical.

- Write the following on the board, saying as you regroup in each example, "One hundred tens, zero ones is equal to ninety-nine tens, tens ones". (Note that one stroke is used to "cross out" 100.)

	tens	ones
1000	~~100~~	~~0~~
	99	10

9 9 10
~~1 0 0 0~~

Use a similar procedure to develop other examples as shown.

	hundreds	tens	ones
4003	~~40~~	~~0~~	3
	39	10	
60 005	~~600~~	~~0~~	5
	599	10	5

3 9 10
~~4 0 0~~ 3

5 9 9 10
~~6 0 0 0~~ 5

Using the Pages

- Have a student read the statement at the top of page 28. Ask how it is known in which order to show the numbers for vertical form. Since 30 000 is greater than 19 274, 19 274 is subtracted from 30 000. Point out that in each place value except the ten thousands' place, the digit in the subtrahend is greater than the digit in the minuend. Suggest that although several regroupings are required, it is possible to deal with many in one step by considering which name

Exercises

Subtract.

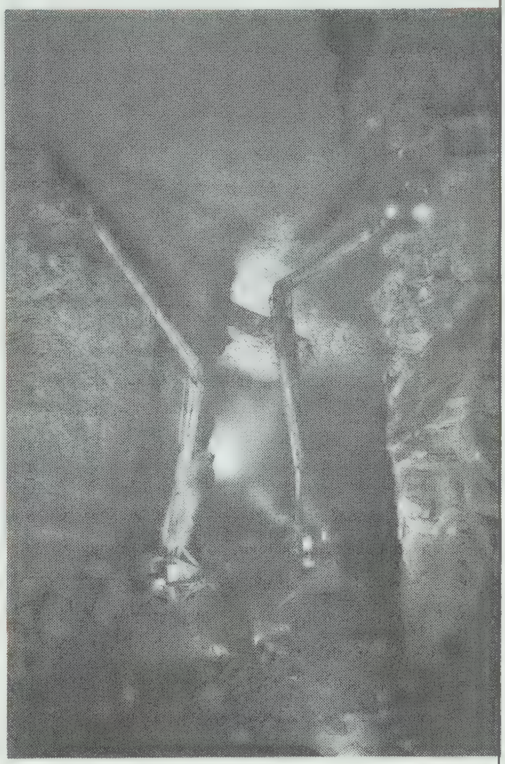
You can check the subtraction by adding or by estimating.

1. 600 327 273	2. 501 289 212	3. 5030 384 4646	4. 9005 8636 369	5. \$8300 1979 \$6321
6. 64 031 44 907 19 124	7. 80 303 45 715 34 588	8. 71 007 3 948 67 059	9. 90 000 45 107 44 893	10. \$400.30 375.68 \$ 24.62

11. 70 806 − 69 745 1061 12. 30 106 − 26 248 3858 13. \$40 906 − \$6 549 \$34 357

14. 740 000 − 311 920 428 080 15. 950 001 − 812 086 137 915 16. \$270.09 − \$78.54 \$191.55

15 002 people are employed for a mine in Quebec. 3365 of them are miners. 4103 people process the metal. 771 people work in the laboratories. There are 1603 people in the office for the mine and 5160 work on maintenance and construction.



Solve.

17. How many of the people employed for the mine are not miners? 11 637
18. How many more people work in the office than in the laboratories? 832
19. How many fewer people process the metal than work in maintenance and construction? 1057
20. What is the difference of the numbers for the largest group of employees and the smallest group of employees? 4389
21. If 8 people stopped working in the office, how many would be left in the office? 1595
How many people would still be employed by the mine? 14 994

29

RELATED ACTIVITIES

• For further practice, you may wish to have the students complete Ex. 48-57 on page 328.

• For practice with regrouping when zero appears in the minuend, present numbers such as 70 000 and 60 408 and have the students regroup in the following ways.

For 70 000, show	
10 more ones.	6 9 9 9 10 ~~7 0 0 0 0~~
10 more tens.	6 9 9 10 ~~7 0 0 0~~ 0
10 more hundreds.	6 9 10 ~~7 0 0~~ 0 0
10 more thousands.	6 10 ~~7 0~~ 0 0 0

For 60 408, show	
10 more ones.	3 9 18 6 0 ~~4 0 8~~
10 more hundreds.	5 9 14 ~~6 0 4~~ 0 8
10 more thousands	5 10 ~~6 0~~ 4 0 8

• Some students may need to practice naming the number that is one less than such numbers as 40, 700, and 5000. This skill is applied in regrouping in subtractions similar to the following.

$$\begin{array}{r} {\scriptstyle 3\ 9\ 10} \\ \cancel{4}\,\cancel{0}\,\cancel{0} \\ \underline{1\ 2\ 3} \end{array} \qquad \begin{array}{r} {\scriptstyle 6\ 9\ 9\ 10} \\ \cancel{7}\,\cancel{0}\,\cancel{0}\,\cancel{0} \\ \underline{4\ 6\ 8\ 5} \end{array} \qquad \begin{array}{r} {\scriptstyle 4\ 9\ 9\ 9\ 14} \\ \cancel{5}\,\cancel{0}\,\cancel{0}\,\cancel{0}\,\cancel{4} \\ \underline{1\ 2\ 3\ 4\ 5} \end{array}$$

for 30 000 is most appropriate for the situation. Draw the students' attention to the "thought cloud" showing four ways of interpreting 30 000. Emphasize that each represents the same number, and ask which name is the most appropriate (3000 tens). Point out that one stroke is used through the four digits of 3000 tens in the subtraction exercise.

The exercises in the box show other subtractions that require regrouping because of zeros in the minuend. You may wish to discuss each example on the board or have the students try the subtractions independently and check their solutions with those on the page. They may also use addition to check their subtractions.

Working Together: For each of Ex. 1 and 2, discuss the regrouping that is shown before having the students complete the subtraction. Ex. 3 guides the students by indicating where the regrouping is required. After they have completed Ex. 4-7, have a few students show and explain their work on the board.

Exercises: Before the students begin the exercises, draw their attention to the photograph showing the interior of a copper mine. Explain that a mine employs many people who do not work as miners. Have students read the statements above Ex. 17-21 aloud. Tell them that Ex. 17-21 refer to the information given. Ex. 17 may be solved in two ways — by adding the numbers of those who are not miners or by subtracting the number of miners from the number of people employed.

Assessment

Subtract.

1. 4000 1478 2522	2. 60 200 3 912 56 288	3. 500 020 175 846 324 174

4. 30 709 − 24 807 5902 5. 200 102 − 108 436 91 666

Solve.

6. If 16 035 people are employed in a mine and 3879 of them are miners, how many of the employees are not miners? 12 156

Pages 30 - 31

OBJECTIVE

Demonstrate competence in subtracting and in estimating differences; solve related word problems

Vocabulary
operations, parentheses

Practice

First, estimate the difference without doing any work on paper. Then subtract. Compare the difference with your estimate. Estimates may vary

1. 5837 − 1820 = 4017 (4000)
2. 2462 − 1959 = 503 (500)
3. 17 634 − 8 967 = 8 667 (9000)
4. $68 521 − 39 407 = $29 114 ($30 000)
5. $434.20 − 346.86 = $ 87.34 ($80)
6. 90 000 − 7 603 = 82 397 (82 000)
7. 206 034 − 156 849 = 49 185 (50 000)
8. 500 818 − 424 967 = 75 851 (80 000)
9. $80 007 − 49 129 = $30 878 ($30 000)
10. $509.00 − 217.50 = $ 291.50 ($300)

Copy and complete.
Add across. Subtract down.

11.

48 736	40 497	89 233
27 940	38 763	66 703
20 796	1 734	22 530

This block is a check.

12.

50 000	30 700	80 700
12 847	28 362	41 209
37 153	2 338	39 491

13.

24 004	48 074	72 078
19 876	15 784	35 660
4 128	32 290	36 418

14.

50 738	40 086	90 824
42 474	24 296	66 770
8 264	15 790	24 054

Ida and Al played a game of X's and O's. Each exercise with a difference greater than 32 352 shows where Ida placed an X. Each exercise with a difference less than 32 352 shows where Al placed an O. Each exercise with a difference of 32 352 shows where there was neither an X or an O.

36 035 − 3 483	70 101 − 37 749	60 060 − 27 908
51 037 − 19 775	66 300 − 34 047	90 001 − 57 649
74 010 − 40 668	48 043 − 15 591	81 234 − 48 783

15. Copy the diagram and find the differences.

32 552	32 352	32 152
31 262	32 253	32 352
33 342	32 452	32 451

X		O
O	O	
X	X	X

16. Who won the game? Ida

30

LESSON ACTIVITY

Using the Pages

- For Ex. 1-10, you may wish to have the students write their exercises using the format referred to on page T 29 for similar exercises, so that the estimate is seen below the exact difference. However, ensure that the students write their estimates before they find the exact differences. You may wish to work a simpler version of Ex. 11 on the board with the students to ensure that they understand the procedure required for Ex. 11-14. The following example may be used. Have students explain what the arrows with the operation symbols indicate.

+ →
− ↓

12	17	29
10	9	
2		

Ask how the number in the bottom right block can indicate whether or not their work is correct. Ex. 15 and 16 review subtraction and comparison of numbers. Most students are likely familiar with the game known as X's and O's. To clarify the instructions, ask a student to read them aloud and ask others to explain the procedure in their own words. Remind the students to answer Ex. 17-21 with concluding statements.

Try This: Ask what is meant by the terms *operations* and *parentheses*. Ask what two operations are shown in the exercises and what two other operations there are besides addition and subtraction. Have a student write the symbols for the four basic operations and parentheses on the board (+, −, ×, ÷). Some students may be able to describe the use of parentheses in non-mathematical situations. Explain that in mathematics, parentheses are used to indicate that an operation is to be performed first. Point out that each sentence in the example shows 13 − 7 + 5, but the parentheses enclose a different pair of numbers. To

Solve.

17. There were 87 603 logs at a pulp mill until 27 795 logs were made into pulp. How many logs were left at the mill? 59 808

18. The number of logs at a pulp mill was 29 765. Then 83 719 more logs came and 49 695 logs were cut into boards. How many logs remained? 63 789

	Number of employees	
	1970	1975
Saw mills	42 540	40 788
Logging	44 814	45 533

19. How many more people were employed in logging in 1975 than in 1970? 719

20. How many fewer people were employed in saw mills in 1975 than in 1970? 1752

21. In 1975, how many more people were employed in logging than in saw mills? 4745

When there are two **operations**, **parentheses** tell which operation comes first.

first

(13 − 7) + 5 = 11

13 − (7 + 5) = 1

first

Work inside the parentheses first. Then work from left to right.

1. (472 + 4836) + 2764 8072
2. 472 + (4836 + 2764) 8072
3. (2000 − 728) − 649 623
4. 2000 − (728 − 649) 1921
5. (6598 + 9321) − 1728 14 191
6. 6598 + (9321 − 1728) 14 191
7. (13 095 − 5 231) + 6 893 14 757
8. 13 095 − (5 231 + 6 893) 971
9. 18 002 − (7 904 − 5 962) + 961 − (6 005 − 437) 11 453

try this

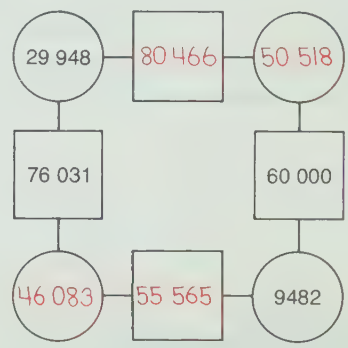
31

demonstrate how this influences the result, have students explain the sentences.

After the students have completed the exercises, have them compare the results for pairs of exercises that show the same numbers, for example, Ex. 5 and 6. Ex. 1 and 2 present an opportunity to review the associative (grouping) property of addition, which states that the way in which the addends are grouped does not affect the sum.

RELATED ACTIVITIES

- Divide the students into groups of two. Have each pair play a game of X's and O's. Then have each student create subtraction exercises to show the result in each place in a way similar to Ex. 15 and 16 on page 30. These could be displayed for others to determine where X's and O's were written during the game.
- Adapt the rules of the game "Greatest Sum" on page T 379 for a game called "Least Difference" for which the players subtract to find the difference. The winner is the player with the least difference. Remind the students that the digit in the column for the greatest place value in the minuend must be greater than the digit having the same place value in the subtrahend, to enable them to subtract.
- Have each student start with \$90 000 and subtract \$4286 repeatedly until the difference obtained is less than \$4286. Repeat this with other amounts of money.
- Use copies of page T 391 to prepare diagrams similar to the following. Addends are shown in the circles and sums are shown in the squares.



OBJECTIVE

Relate additive and subtractive situations to the use of the $\boxed{+}$ and $\boxed{-}$ keys on a calculator

Materials

calculators (optional)

Vocabulary

calculator, digit display, names of the ten provinces and their abbreviations, names of farm machines in the table on page 33

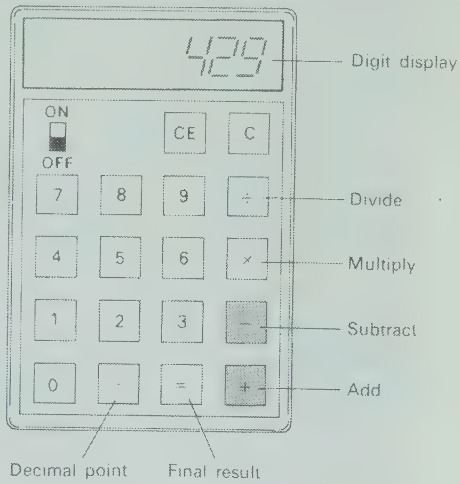
Background

This is the first of four lessons in *Starting Points in Mathematics 6* on the use of the calculator. Comments about using calculators in the classroom are given on page xv. Note that although it is desirable that students have access to calculators for these lessons, the lessons are designed so that they can be used to teach calculator skills with or without calculators. These lessons also reinforce and extend concepts and skills presented in the unit in which they appear. Because of the differences among calculators, it may be necessary to adapt some of the lesson suggestions for the calculators available.

Using a Calculator to Add or Subtract

A calculator can be used to find answers quickly.

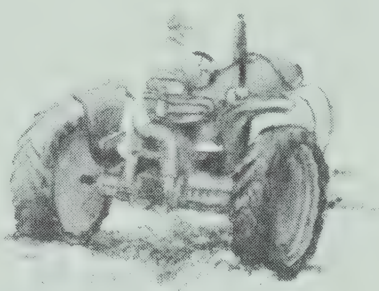
To use a calculator, you need to know which keys to press.



Calculator keys are not always in the same place on different calculators.

How would you use a calculator to find the number of tractors used on farms in Canada in 1971?

I would add the numbers in the column for tractors using the $\boxed{+}$ key and the $\boxed{=}$ key.



32

LESSON ACTIVITY

Before Using the Pages

- Question the students to determine their knowledge of and experience with calculators. Ask, for example, whether there is a calculator in the home and if so, who uses it. Ask for what purposes a calculator can be used. Emphasize the need to understand numbers and operations in order to use a calculator efficiently.

Using the Pages

- Have a student read the statements at the top of page 32. Ask how many keys there are for the calculator illustrated on the page. Have students identify the keys for the digits 0 to 9 and relate them to the ''window'' labeled *digit display*. Discuss the use of the remaining keys that are labeled. Note that the $\boxed{CE}$ (Clear Entry) and $\boxed{C}$ (Clear) keys are not referred to at this time. Draw attention to the statement in the ''thought cloud''. If students have calculators, have them compare theirs with the calculator illustrated.
- Direct the students' attention to the table on page 33. Have students identify the names of the provinces from the abbreviations shown. Ask questions such as ''How many combines were there in Manitoba in 1976?'' and ''Which province had the most tractors?'' If possible, have students describe the various kinds of farm machines named. A hay baler (illustrated on page 33) compresses and ties hay into bales. A swather is a mowing machine. A combine is a machine for harvesting and threshing grain, and a forage crop harvester cuts green crops like clover and alfalfa.

  Read the word problem at the bottom of page 32. Ask students whether they agree or disagree with the answer provided and have them explain why. In this example, it is not necessary to name the specific numbers. To provide an example for which specific numbers must be named, ask, ''How many more tractors were there in Ontario than in Saskatchewan?'' Write the answer on the board to illustrate the written format for such an example: Subtract 139 487 from 165 623 using the $\boxed{-}$ key and the $\boxed{=}$ key. Note that the exercises on page 33 require the students to tell *how*

This table shows the farm machines used in Canada in 1976.

	Trucks	Tractors	Combines	Swathers	Hay balers	Forage crop harvesters
Nfld.	411	429	1	6	108	13
P.E.I.	3 369	5 366	1 248	121	1 834	353
N.S.	3 916	5 631	310	170	2 005	368
N.B.	3 991	5 862	759	263	1 953	324
Que.	21 713	80 017	5 925	6 719	30 012	5 979
Ont.	73 374	165 623	24 914	11 598	37 481	15 674
Man.	48 323	65 176	23 174	23 526	13 950	1 569
Sask.	140 684	139 487	61 126	62 544	30 860	3 275
Alta.	109 694	116 316	42 689	43 944	31 093	5 099
B.C.	17 106	21 352	1 704	3 119	4 927	2 062

How would you use a calculator to find the answers to each of these questions for farms in Canada in 1976? Answers are given below.

1. the number of farm machines in Quebec
2. how many fewer combines there were in Manitoba than in Alberta
3. the number of hay balers in Ontario, Saskatchewan, and British Columbia
4. the difference of the number of combines and the number of forage crop harvesters
5. the number of hay balers and swathers in Newfoundland and Prince Edward Island together
6. the difference of the numbers of farm machines in Nova Scotia and in New Brunswick
7. the province with the most farm machines
8. the number of trucks
9. the number of farm machines

Calculator

33

1. Add the six numbers in the row for Quebec using the $\boxed{+}$ key and the $\boxed{=}$ key.
2. Subtract 23 174 from 42 689 using the $\boxed{-}$ key and the $\boxed{=}$ key.
3. Add 37 481, 30 860, and 4 927 using the $\boxed{+}$ key and the $\boxed{=}$ key.
4. Add the ten numbers in the column for combines using the $\boxed{+}$ key and the $\boxed{=}$ key. Write the sum. Add the ten numbers in the column for forage crop harvesters using the $\boxed{+}$ key and the $\boxed{=}$ key. Write the sum. Find the difference of the two sums using the $\boxed{-}$ key and the $\boxed{=}$ key.
5. Add 6, 108, 121, and 1834 using the $\boxed{+}$ key and the $\boxed{=}$ key.
6. Add the six numbers in the row for Nova Scotia using the $\boxed{+}$ key and the $\boxed{=}$ key. Write the sum. Add the six numbers in the row for New Brunswick using the $\boxed{+}$ key and the $\boxed{=}$ key. Write the sum. Find the difference of the two sums using the $\boxed{-}$ key and the $\boxed{=}$ key.
7. Add the six numbers in the row for each province using the $\boxed{+}$ key and the $\boxed{=}$ key. Write each sum. Compare the ten sums to find the greatest number.
8. Add the ten numbers in the column for trucks using the $\boxed{+}$ key and the $\boxed{=}$ key.
9. Add all the numbers in the table using the $\boxed{+}$ key and the $\boxed{=}$ key.

they would use a calculator; they are not required to solve the problems. This is suggested as a follow-up activity in *Related Activities*.

- For some exercises such as Ex. 4, it is necessary to find two sums and then find the difference of the two sums because both addition and subtraction are required. Thus, it would be necessary to record the first sum before continuing to use the calculator. Students can indicate this by writing a statement such as "Write the sum" after the procedure for the first sum is described, and then proceed to describe the procedure for the second sum. After the students have completed the exercises, have them form small groups to discuss their answers. With small groups each student has more opportunity to participate in the discussion. Results of their discussions may be reported later to the whole class.

RELATED ACTIVITIES

- If calculators are available, have the students use them to complete the exercises on page 33. Ex. 9 would present an opportunity to discuss what happens when a numeral has more digits than the digit display allows.
- Have students use calculators to complete exercises encountered earlier in this unit. Have them compare the results obtained using a calculator with the results obtained without using a calculator.
- Have students explain how they would use a calculator to solve some of the word problems in this unit.
- Students may be able to find pictures of calculators in catalogs and consider the similarities and differences.

OBJECTIVE

Identify unnecessary information in a word problem

RELATED ACTIVITIES

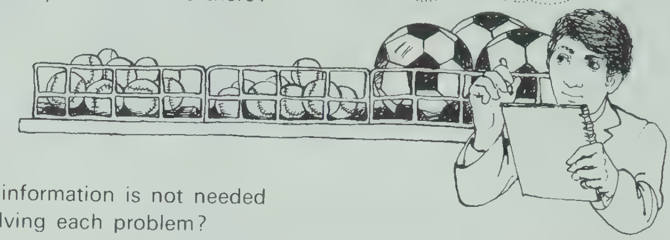
- Ask the students to rewrite some of the word problems in this unit to show too much information. Display together different versions of a problem for the students to compare.
- You may wish to have the students solve the problems on page 34.
- Encourage the students to be more aware of problem-solving situations that present more information than is needed to solve the problem. These problems need not involve mathematics. Provide opportunities for them to describe the situations to the rest of the class.
- Some students may enjoy writing problems similar to those on the page for other students to identify the unnecessary information.

Too Much Information

Sometimes there is more information than is needed for solving a problem.

Mr. Da Costa is counting the stock in his sports store. He counted 87 baseballs of one type, 12 of another, 38 of another, and 33 soccer balls. How many baseballs are there?

This is not needed for solving the problem.



What information is not needed for solving each problem?

1. Kevin saved $53.50. He bought a volleyball for $28.99 and a volleyball net for $15.98. He plans to buy a soccer ball for $8.98. How much did he spend?

2. Mr. Da Costa had 75 tennis rackets. He sold 27 for $19.95 each and 32 for $25.95 each. Then he bought 17 more. How many did he have then?

3. Carla went to five stores looking for a basketball. She bought one that regularly cost $12.95, but was on sale for $9.99. She also bought a basketball hoop for $5.98. How much less did she spend because of the sale?

4. Mr. Da Costa has 213 pairs of one type of hockey skates and 394 of another. He also has 412 pairs of one type of figure skates and 362 of another. How many pairs of hockey skates are there?

5. Mr. Da Costa had 76 badminton rackets. He sold 35 of them in April and 21 in May. The others are on sale for $14.99. How many are left?

6. The store opens at 9:00 a.m. every day except Sunday. It closes at 6:00 p.m. on Monday, Tuesday, Wednesday, and Saturday, at 8:00 p.m. on Thursday, and at 9:00 p.m. on Friday. How many more hours is it open on Friday than on Tuesday?

PROBLEM SOLVING

34

LESSON ACTIVITY

Using the Page

- Begin by asking a student to read the statement at the top of the page. Have the students read the word problem silently. Ask why the information about the soccer balls is not needed to solve the problem. Explain that it is important to read the problem carefully, more than once, to be certain about the question and the unnecessary facts.
- For the exercises, point out to the students that they are to find the information that is not needed; they are not to solve the problems. After they have completed the problems, ask them to explain how they decided which information is not needed for solving each problem. They will likely relate the information given in the problem to the question that is asked.

Checking Up

Add.

1. 5 + 2 + 6 + 5 = 18
2. 8 + 7 + 3 + 4 = 22
3. 5 + 9 + 4 + 1 + 6 = 25
4. 3 + 6 + 4 + 3 + 7 = 23
5. 8 + 5 + 4 + 7 + 6 30
6. 3 + 4 + 9 + 3 + 5 24
7. 2 + 0 + 9 + 8 + 1 20
8. 435 + 162 = 597
9. 1043 + 3982 = 5025
10. 52 478 + 39 618 = 92 096
11. \$79 365 + 28 796 = \$108 161
12. \$648.37 + 892.97 = \$1541.34
13. 722 + 834 + 586 + 535 = 2677
14. 139 + 72 + 238 + 4709 = 5158
15. 23 540 + 19 207 + 8 496 + 32 + 72 412 = 123 687
16. 12 004 + 18 937 + 5 343 + 894 + 5 712 = 42 890
17. \$473.68 + 315.20 + 306.42 + 425.19 + 752.24 = \$2272.73

Subtract.

18. 837 − 425 = 412
19. 7542 − 6930 = 612
20. 56 473 − 8 192 = 48 281
21. \$24 364 − 17 509 = \$ 6 855
22. \$634.20 − 487.63 = \$146.57
23. 60 000 − 2 749 = 57 251
24. 50 604 − 45 786 = 4818
25. 74 003 − 72 107 = 1 896
26. \$20 004 − 9 795 = \$10 209
27. \$800.50 − 432.66 = \$367.84

Find the result.

28. \$68 735 + \$37 396 \$106 131
29. \$542.83 − \$7.86 \$534.97
30. 43 021 − 2 986 40 035
31. 45 + 338 + 4 692 + 7 + 68 115 73 197

Solve.

32. There are 15 007 tickets for a circus. How many tickets are left after 8529 are sold? 6478
33. An artist sold one painting for \$769.50 and another for \$176.75. How much did the artist receive for the two paintings? \$946.25
34. A library signed out 2538 items on Tuesday, 997 on Wednesday, 1642 on Thursday, 3578 on Friday, and 6013 on Saturday. How many items did the library sign out that week? 14 768
35. 1674 of the 5106 employees at a hospital are nurses. How many of the employees are not nurses? 3432

Page 35

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Have students demonstrate on an abacus a few of the exercises that present difficulty.
- Provide newspapers or catalogs for students to cut out pictures of items and their prices. Display the pictures on a bulletin board or paste them on construction paper for later use. For practice in adding amounts of money, have students "purchase" two or more items and find the total cost. For practice in subtracting amounts of money, name a specific amount such as \$700.00 and have students find the amount they would have left after "purchasing" one or more items. Students may also write word problems that relate to pictures, for other students to solve.
- Have students make estimates to check their answers for some of Ex. 8-31.
- Have students write and solve subtraction problems related to the charts on pages 21 and 23.

Skills	Exercises	Related Pages
Add one-digit numbers	1-7	T 18-T 19
Add two numbers	8-12, 28	T 20-T 21
Add more than two numbers	13-17, 31	T 22-T 23
Subtract, no regrouping	18	T 26-T 27
Subtract, regrouping	19-22, 29	T 28-T 29
Subtract with zeros in the minuend	23-27, 30	T 30-T 31
Solve addition problems	33, 34	
Solve subtraction problems	32, 35	

Comments

To help locate areas of difficulty in the students' work, refer to the above chart showing the skill required for each exercise. Then determine whether errors are a result of copying numerals incorrectly, poor recall of basic facts, or regrouping incorrectly. In addition exercises, students may become careless and regroup when it is not required, or they may forget to regroup when it is required, as shown in the following examples.

$$\begin{array}{r} {\scriptstyle 1\,1} \\ 625 \\ +\ 149 \\ \hline 874 \end{array} \qquad \begin{array}{r} {\scriptstyle 1} \\ 489 \\ +\ 763 \\ \hline 1242 \end{array}$$

Generally, students have more difficulty regrouping in subtraction, particularly when the minuend has one or more zeros. The second activity described in *Related Activities* on page T 31 is particularly helpful because it enables students to concentrate on the concept of regrouping before the skill is applied in subtraction. It is helpful to have them write subtraction exercises in place-value format on graph paper. (Use copies of page T 382 or T 396.) The digits of each numeral can be spaced to allow room for showing the regrouping. Some exercises may be shown on an abacus at the same time.

1	13	7	14
~~2~~	~~3~~	~~8~~	~~4~~
1	7	4	6
	6	3	8

	1	9		9	9	14
\$	~~2~~	~~0~~		~~0~~	~~0~~	~~4~~
		9		7	9	5
\$	1	0		2	0	9

Unit 3 Overview

Graphing

This unit is relatively short and appears at the beginning of the book so that the concepts and skills associated with graphs may be used for a variety of purposes throughout the rest of the school year, not only in mathematics, but also in other subject areas. The unit begins with the use of tally charts for collecting and organizing information. This technique is used in the subsequent lessons on pictographs and bar graphs. A lesson on identifying points on a grid by means of ordered pairs of numbers provides a foundation for locating points when making line graphs and broken-line graphs. Since graphs deal with information, the *Problem Solving* lesson is concerned with developing skills in finding information on maps.

Prerequisite Skills

- match whole numbers with points on a number line

Unit Outcomes

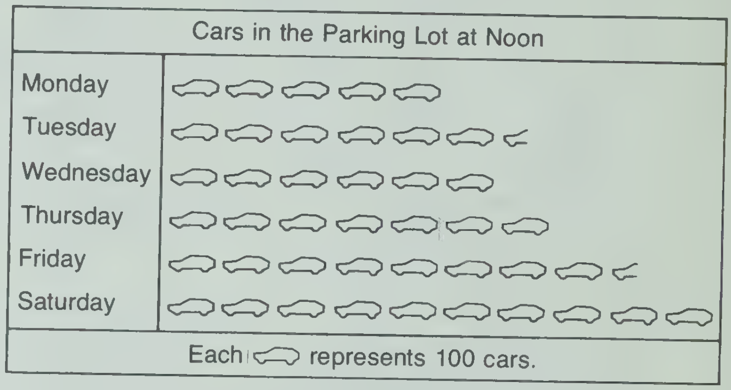
- interpret a tally chart
- use a tally chart to collect and organize information
- interpret and draw a pictograph
- interpret and draw a bar graph
- match points on a grid with ordered pairs of numbers
- write an ordered pair of numbers to show information
- interpret and draw a line graph
- interpret and draw a broken-line graph
- find information to solve problems

Background

Numbers and operations with numbers are valuable tools by which people can convey quantitative information to one another, and by which they can make important decisions concerning their activities. With numbers they can record simple information like scores in games, or they can report complex information such as the production of natural resources and manufactured goods, the purchase and sale of commodities, and other financial transactions. Numbers themselves are abstract concepts and sometimes are not as meaningful to some people as they might be. The most meaningful representation of numerical data is to be found in the objects themselves. For instance, if the number of students in a school is 983, the magnitude of that number is most obvious when all the students are assembled in one place. Similarly, an airplane may be reported to be worth \$3.5 million, but the brief way of showing that amount provides little impression of the actual size of the number. Graphs provide a visual means of showing numerical data in such a way that more meaningful concepts are given to the abstract numbers represented. Graphs also make interesting comparisons of related data possible.

Graphs offer systematic ways of organizing, classifying, and recording information. One of the first steps in constructing any graph involves collecting and organizing the data. Frequently a simple tally system is used to record the items in one or more categories. Tallies are usually grouped in fives and are counted by fives and ones to determine the number of items. A tally showing 卌 ||| would represent eight items, for which the numeral 8 would be written. For the abstract number to be shown on a graph, several things must be considered, the first of which concerns the type of graph best suited for the nature of the data.

The pictograph is probably the most realistic because pictures, or representative symbols, show not only the quantities but also the nature of the data. Pictures or symbols are used either on a one-to-one basis to real objects or on a one-to-many basis. A pictograph, therefore, requires an explanatory statement called a *key* to establish and to reveal this relationship. The key for the following pictograph indicates that each symbol represents 100 cars. The unit value of (car symbol) is 100. Multiples of the unit value of a symbol are easily shown by a number of whole symbols, and other numbers are either rounded to the nearest multiple or are represented by parts of symbols. In the pictograph, the number of cars in the parking lot on Wednesday is represented by six symbols which may indicate exactly 600 cars or about 600 cars, for example, 590. Similarly, half a symbol may represent from 38 to 62 cars. Pictographs are less precise than other types of graphs, but they are visually attractive and are usually easy to interpret.



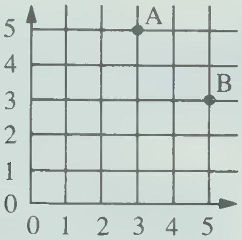
Bar graphs may be used to show various kinds of information, but they are probably best suited for showing static, or unchanging, data. They are also easily interpreted for making comparisons between two or more of the categories represented. There are both vertical and horizontal bar graphs. For a vertical bar graph, the numerical scale is on the vertical axis, increasing from the base line upward. For a horizontal bar graph, the numerical scale is on the horizontal axis at the bottom, increasing from left to right along the base line. Names of items, dates, or categories are associated with the bars on the other axis in each of the two types of bar graphs. As stated, it is relatively easy to make visual comparisons on bar graphs, because the proportionate lengths of the bars represent the numbers graphically.

A line graph is sometimes used to show a constant relationship between two items, such as distance and time, or the total cost for several items at a specific price. The line joining points on the grid in this type of graph begins at the lower left and extends to the upper right, since the two scales both begin at 0 in the lower left corner.

Broken-line graphs are used to show progressively changing relationships between two items, such as the temperature at regular intervals of a day, or the weekly sales at a store for part of a year. Usually in such graphs there is a numerical scale on the vertical axis, and dates or intervals of time on the horizontal axis. Since the relationships show variations, the progression of line segments from one point to the next point makes a broken line, rather than a straight line.

Both the line graph and the broken-line graph require skill in plotting points on a grid. Points where lines intersect on a grid can be accurately named by using pairs of numbers, one from

each of two adjacent edges. To avoid any uncertainty as to which point is intended, it is universally accepted that the first number of an *ordered pair* of numbers refers to the horizontal distance from the starting point, and the second number refers to the vertical distance from the same starting point. In an ordered pair of numbers, order is important — count *over* first and then *up*. Thus, (3,5) names point A and (5,3) names point B.



The reading of graphs requires examination of three features: the title of the graph; the names, categories, dates on one of the axes or the meaning of the numerical scale shown; and the meaning of the scale on the other axis. Sometimes it is necessary to interpret positions which lie between numbered lines on the grid, in which case the size of each interval on the scale also needs to be taken into consideration.

Drawing graphs is much more difficult than reading them, because a number of decisions have to be made after the information has been collected: What kind of graph — pictograph, bar graph, line graph, or broken-line graph — will best represent the data? What title describes the content briefly and accurately? What headings or categories should be shown and how should they be named? What scales and intervals, or what symbols are to be used? Of these, the last is probably the most difficult. On a pictograph, poorly chosen symbols and unit values for them can result in either too many (few) symbols or an inability to adapt the symbols to represent numbers between multiples of the unit values. For example, the pictograph shown would involve too many symbols if each symbol represented 50 cars. On the other hand, it would be awkward to represent 50 cars if the unit value for were 150 cars. Similarly, poorly chosen intervals on scales can result either in bar graphs and line graphs that are too large to be shown completely, or in graphs that are too small to show differences in the data. To develop skills for making graphs, students require much experience in examining and interpreting good graphs of all types and in actually making them.

Most students at this level have probably had experiences with maps, particularly road maps, but they may not have used all the features of a map. This unit includes a lesson on locating places on a map. As with graphs, maps generally have two scales along the edges which identify rows and columns of regions on them. Letters are usually associated with one scale and numbers with the other, so that by using one symbol from each scale a particular region can be located. Whereas, with a grid there is an ordered pair of numbers to identify a point, on a map the order of the two symbols is not significant and it is a region, rather than a point, that is identified.

Teaching Strategies

Although this unit on graphing is relatively short, the varied activities of collecting and organizing information and of preparing graphs to display it are time-consuming. Besides, the work with graphs requires a considerable amount of discussion to help the students to understand the basic elements of graphs and to develop the necessary skills of interpreting and making graphs.

Since any discussion is more beneficial if the number of participants is relatively small, it is recommended that the class be divided into small groups. Basic computational skills are not needed in much of the work with graphs, so students with different abilities may be grouped together. It is probably more important that members of groups be co-operative with one another, so that joint efforts and discussions may be beneficial. If there is a balance of both leaders and followers in each group, students can learn good social work habits from one another.

The guidance of the teacher in the activities and discussions is important in the development of meaningful concepts and skills in graphing. While the teacher is working with one or more of these small groups, others may be assigned exercises to maintain basic skills in addition, subtraction, and numeration, which were developed in the first two units. The *Keeping Sharp* feature on page 45 may be supplemented by other exercises, such as those on page 328. Some of the *Related Activities* from this unit and from the preceding units may also be used to organize profitable activities for the groups which are not engaged in making graphs.

Before the students reach the lesson on pages 40 and 41, it may be advisable to review briefly the two ways of recording time. On page 41 the notation shown is on a 12-hour basis, whereas on page 46 it is on a 24-hour basis. A few exercises relating the two notations should be sufficient to review the students' previous understanding of them.

Although the formal work with graphs is confined to Unit 3 in *Starting Points in Mathematics 6*, the teacher should use occasions during the rest of the school year for the students to study graphs which appear in newspapers and magazines, and also to apply their skills of graphing to portray information from other subject areas, particularly from science and social studies.

Materials

a list of the members of the class for each student (optional)
large sheets of paper and colored construction paper (optional)
a straight edge and copies of pages T 396 and T 397 for each student
an unmarked 10-by-10 display grid
chart paper (optional)
a thermometer
a red pencil, a blue pencil, and a green pencil for each student
a map of Canada, maps of your province or territory

Vocabulary

tally chart	broken-line graph
tally	line segments
survey	index
pictograph	legend
bar graph	kilometres per hour, km/h
horizontal line	hour, h
vertical line	minute, min
grid	second, s
ordered pair	metres, m
plot a point	degrees Celsius, °C
line graph	lire, drachmas, rands

Pages 36-37

LESSON OUTCOME

Interpret a tally chart; use a tally chart to collect and organize information

Materials

a list of the members of the class for each student (optional)

Vocabulary

tally chart, tally, survey

Working Together

6.

Vowel	Tally	Number of vowels
a	卌 卌 卌 II	17
e	卌 IIII	9
i	卌 卌 III	13
o	III	3
u	II	2

Exercises

2.

Mark	Tally	Number of students
0	I	1
1	I	1
2	I	1
3		0
4	III	3
5	II	2
6	IIII	4
7	卌 I	6
8	III	3
9	IIII	4
10	II	2

3 GRAPHING

Organizing Information

Reginald spent a day at the airport counting the airplanes from each airline.

He made a **tally chart** to show the information.

Tally Chart

Airline	Tally	Number of airplanes
Air Canada	卌 卌 I	11
Air France	卌	5
Quebecair	III	3
CP Air	IIII	4
Lufthansa	I	1
Royal Air Maroc	I	1
Alitalia	II	2
Eastern Provincial	II	2
Finnair	II	2
KLM	III	3
Olympic Airways	I	1

In another chart, he used "Other" to group the airlines that had just one airplane landing.

Table of Information

Airline	Number of airplanes
Air Canada	11
Air France	5
Quebecair	3
CP Air	4
Alitalia	2
Eastern Provincial	2
Finnair	2
KLM	3
Other	3

36

LESSON ACTIVITY

Before Using the Pages

- Ask students to name four methods of transportation. Write their suggestions on the board. Ask each student to name her/his favorite method of transportation. Have ten students in turn show a tally on the board for her/his preference. As they do, watch to see whether a student marks the fifth stroke across the previous four strokes. If not, introduce the symbol 卌 to show that it facilitates counting the tallies by fives. Ask students to use this method to represent the numbers eight, ten, and twenty-two on the board. Explain the meaning of *tally* and *tally chart*.

Using the Pages

- The photograph can motivate a discussion of air travel. For example, ask students to tell whether they have travelled on an airplane and if so, what airlines they have used. Ask them to name different airline companies and to indicate their country of origin, for instance, CP Air originated in Canada.

Read the statement at the top of page 36. Point out that using a tally chart to record the number of airplanes from each airline allowed Reginald to record each airplane as he saw it. Have the students read the tally chart. Then compare the tally chart and the table of information. Ask the students why they think the airlines with just one airplane landing were grouped under "Other".

Working Together: Ex. 1-4 provide practice in interpreting a tally chart and in using the information for calculations. Have several students explain their answers for Ex. 5. For Ex. 6, guide the students as they list the vowels in a tally chart and record each tally. Emphasize that making the tally chart involves beginning at the headings for the columns of the table of information and recording a tally for each vowel as it occurs, rather than counting all the a's, then all the e's, and so on.

Working Together

From Reginald's tally chart, tell

1. how many airplanes there were from Air Canada and CP Air together. 15
2. how many airplanes there were from the three airlines that used the airport most that day. 20
3. how many more airplanes there were from Quebecair than from Olympic Airways. 2
4. how many fewer airplanes there were from Finnair than from Air France. 3

From Reginald's table of information,

5. tell the meaning of "Other" in the "Airline" column. the airlines that had just one airplane landing
6. find how many there are of each vowel. Use a tally chart. A tally chart is shown on page T40.

Exercises

Freda made a survey of the favorite sports of 29 students.

1. Count the tallies and complete her tally chart.

Sport	Tally	Number of students
Soccer	𝍸 //	7
Baseball	////	? 4
Hockey	𝍸 /	? 6
Basketball	//	? 2
Track	//	? 2
Volleyball	𝍸	? 5
Swimming	//	? 2
Other	/	? 1

This chart shows the marks out of 10 that the students in Nick's class received on a test.

7	7	8	4	6	9	10	5	5
8	10	2	9	0	1	7	6	7
9	7	6	8	4	4	7	6	9

2. Make a tally chart to show this information. A tally chart is shown on page T40
3. How many students had a mark of 6 or greater than 6? 19
4. Which mark was received most often? 7
5. How many more students received a mark of 9 than a mark of 8? 1

Jay made a survey of 31 students to find out which of the following kinds of movies they like best: science fiction, western, horror, detective, spy, or comedy.

6. On a tally chart, show what you think the result would be. Answers will vary
7. Use a tally chart to make a survey of the favorite kinds of movies of the students in your class. Answers will vary

37

RELATED ACTIVITIES

- Have each student use a tally chart to make a survey of her/his choice, for example, recording the months of their birthdays or the favorite sports of their classmates. Also, have them tally the number of words with one letter, two letters, three letters, and so on, for one page in a book. Begin a display for Unit 3 with these tally charts. The charts may be used for drawing pictographs after completing pages 38 and 39 and for drawing bar graphs after completing pages 40 and 41.
- Tally charts can be used to record the results of probability experiments. Some examples are as follows: flip a coin and use a tally to record each time it lands "heads" and each time it lands "tails"; toss a die and record the result of each toss; rotate a number spinner and record the results.

Have several pairs of students try the same experiment and ask them to compare their tally charts. The information in the tally charts may be displayed in graphs after the appropriate lessons on graphing have been completed.

1.

Score	Tally	Number of scores
9	|	1
8	𝍸	5
7	||||	4
6	𝍸 |||	8
5	||	2
4	||	2
3	𝍸 |	6
2	||	2

Exercises: Ex. 1 deals with the skill of interpreting a tally chart. Ex. 2-5 involve making a tally chart from given information and interpreting the chart. You may wish to have the students complete Ex. 6 on a separate piece of paper so that the results can be displayed and compared. Because the exercise states "show what you think", differences in these tally charts are to be expected.

For Ex. 7, each student may prepare a tally chart on a separate sheet of paper showing her/his name at the top of the page. The sheets may be circulated to enable the students to mark tallies on one another's sheets. The students may enjoy preparing tally charts and then conducting surveys in which they ask each student in the class which kind of movie he/she likes best. A list of the members of the class may help students to record who they have interviewed. You may prefer to write the headings on the board and have each student, in turn, mark a tally. Students can copy and then complete the tally sheet.

Encourage the students to discuss the results of their surveys and to explain how making a tally chart facilitates conducting a survey.

Assessment

This chart shows the baseball scores for Alice's team during the last three years.

9	7	6	8	6	3	8	7	6	5
8	2	6	7	3	8	6	5	4	3
3	8	2	6	6	4	3	7	3	6

1. Make a tally chart to show this information. A tally chart is shown above.
2. Which score was achieved most often? 6
3. Which score was achieved least often? 9

Pages 38-39

LESSON OUTCOME

Interpret and draw a pictograph

Materials

large sheets of paper and colored construction paper (optional)

Vocabulary

pictograph

Prerequisite Skills

Use a tally chart to organize information; interpret a tally chart

Checking Prerequisite Skills

This chart shows the grades received by students on a test.

A	C	B	A	A	B
D	A	B	C	C	A
B	B	B	C	D	A
B	D	A	B	B	C

1. Make a tally chart to show this information.

2. Which grade was received most often? B

1.

Grade	Tally	Number of grades
A	卌 ||	7
B	卌 ||||	9
C	卌	5
D	|||	3

Pictographs

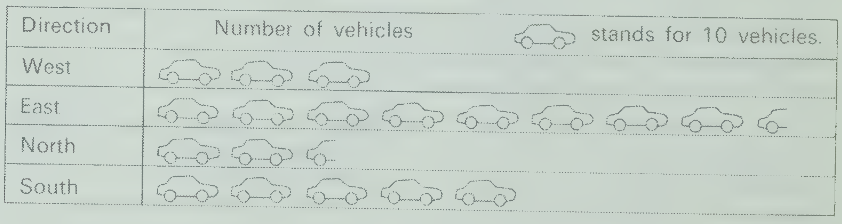
One day after school, Yolanda made a survey of the traffic at the corner near her school. She made a tally chart to show the number of vehicles coming from each direction.

Direction	Tally	Number of vehicles
West	卌 卌 卌 卌 卌 卌 ||	32
East	卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 卌 ||	87
North	卌 卌 卌 卌 |||	23
South	卌 卌 卌 卌 卌 卌 卌 卌 卌 ||||	49

Then she drew a **pictograph** to show the information in a different way.

Yolanda used to represent 3 to 7 vehicles, and to represent 8 to 12 vehicles.

Traffic on October 1



LESSON ACTIVITY

Using the Pages

- Many students have likely encountered the concept of a graph in previous work. Usually, it is easier to interpret information from a graph than from a tally chart. Have a student read the statements at the top of page 38. Then discuss why a tally chart is useful for recording the information. Ask the students to count the tallies to check the number of vehicles recorded for each direction. Introduce the word *pictograph* and point out that pictographs involve a symbol to represent a specific number of items. Discuss the use of half a symbol to represent approximately half the number represented by a whole symbol. Have the students note that the symbols in the pictograph are aligned vertically. Draw attention to the fact that a pictograph should always have a title. Have the students compare the number represented in the pictograph for vehicles from each direction with the number shown in the tally chart. Emphasize that a tally chart shows the exact number of items, but a pictograph shows an approximate number of items. Point out that one advantage of a pictograph is that numbers of items can be compared quickly.

Working Together: Remind the students to use the pictograph, not the tally chart, to complete Ex. 1-4. Discuss the reason for the use of the word "about" in Ex. 3 and 4. Emphasize that an exact number cannot necessarily be obtained from a pictograph.

You may wish to use a large sheet of paper to develop the graph for Ex. 6 and 7 with the students at the same time as they prepare their own graphs. For Ex. 6, remind the students that half a symbol can be used and discuss the advantages and disadvantages of having one symbol stand for each of the numbers given. Lead the students to realize that using one symbol to stand for 1 stamp or for 10 stamps would require too many symbols for the pictograph. Establish that representing 100 stamps with one symbol would result in difficulties for showing 225 stamps, 376 stamps, and 175 stamps. Point out that if each symbol

Working Together

From Yolanda's pictograph, tell

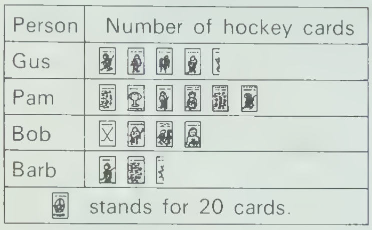
1. the direction from which the most vehicles were coming. east
2. the direction from which the fewest vehicles were coming. north
3. about how many vehicles were coming from the north and from the south in all. 72
4. about how many more vehicles were coming from the east than from the west. 55

Five students collect stamps. Ian has 251, Tim has 305, Hilda has 225, David has 376, and Betty has 175.

5. What symbol would you use to show this information in a pictograph? □ Other answers are possible
6. Would your symbol stand for 1, 10, 50, or 100 stamps? 50
7. Draw the pictograph using whole symbols and half symbols. A pictograph is shown on page T368
8. Write a title for your pictograph. Stamp Collections Other answers are possible

Exercises

Barb drew this pictograph to show how many hockey cards each student has.



From Barb's pictograph, find

1. who has the most hockey cards. Pam
2. about how many hockey cards Gus has. 90
3. about how many hockey cards Bob and Pam have together. 200
4. about how many more hockey cards Bob has than Barb. 30

This chart shows 6 countries from which immigrants came to Canada in 1973 and 1974. Each number has been rounded to the nearest 5000.

Britain	65 000	Hong Kong	25 000
U.S.A.	50 000	India	20 000
Portugal	30 000	Jamaica	20 000

5. Draw a pictograph to show this information. A pictograph is shown on page T368

Use whole symbols and half symbols.

Make a survey to find the favorite hobbies of the students in your class. Answers will vary

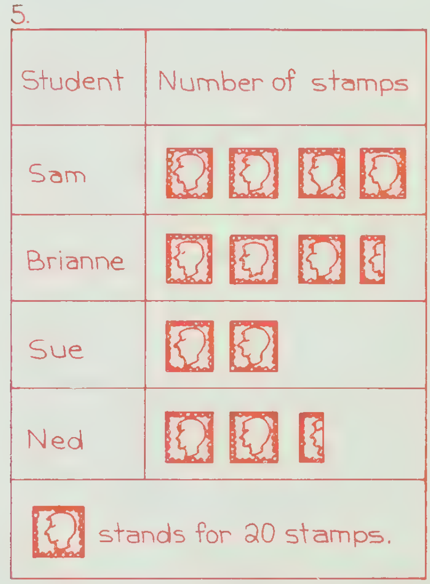
6. Use a tally chart to collect the information.
7. Draw a pictograph to show the information.

39

RELATED ACTIVITIES

- Have the students conduct a survey of their choice, or use tally charts prepared earlier for the activities on page T41, and prepare pictographs. Display the pictographs.
- Ask students to cut examples of pictographs from magazines and newspapers and to include them in the display. Help them to interpret the information shown.
- For practice in interpreting pictographs, display the pictographs made for the first activity suggested above or for Ex. 5 or 7 on page 39. Ask each student to write on a card several questions about one of the pictographs and to pin the card beside the pictograph. Other students may take cards to their desks, answer the questions, and return the cards.

5.



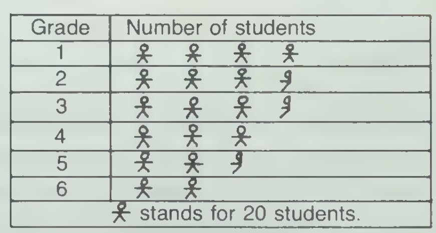
stands for 50 stamps, the number of stamps collected by each student can be represented satisfactorily with symbols and halves of symbols. Guide the students as they prepare the headings and the title of the pictograph for Ex. 7.

Exercises: For Ex. 5, discuss why using a symbol to stand for 10 000 immigrants is a better choice than 1000, 5000, or 20 000. The students may enjoy discussing the results of their survey for Ex. 6 and 7. Have several students explain the symbols they chose for Ex. 7 and their reasons for selecting the number their symbols would represent. You may wish to have the students work in small groups for Ex. 6 and 7. The graphs may be prepared on large sheets of paper, using symbols cut from colored construction paper. The pictographs can be displayed for several days.

Assessment

Anita drew this pictograph to show how many students are in each grade at her school.

Students at Briarwood School



1. Which grade has the most students? 1
2. About how many students are in Grade 5? 50
3. About how many students are in Grades 1 and 2 together? 150
4. About how many more students are there in Grade 3 than in Grade 4? 10

Four students collect coins. Sam has 80, Brianne has 71, Sue has 35, and Ned has 49 coins. A pictograph is shown above.

5. Use whole symbols and halves of symbols to draw a pictograph to show this information.

Pages 40-41

LESSON OUTCOME

Interpret and draw a bar graph

Materials

a straight edge and graph paper (use copies of page T 396 or T 397) for each student

Vocabulary

bar graph, horizontal line, vertical line

Working Together

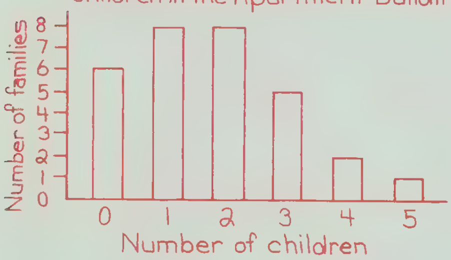
3.

Children in the Apartment Building

Number of children	Number of families
0	卌 |
1	卌 |||
2	卌 |||
3	卌
4	||
5	|

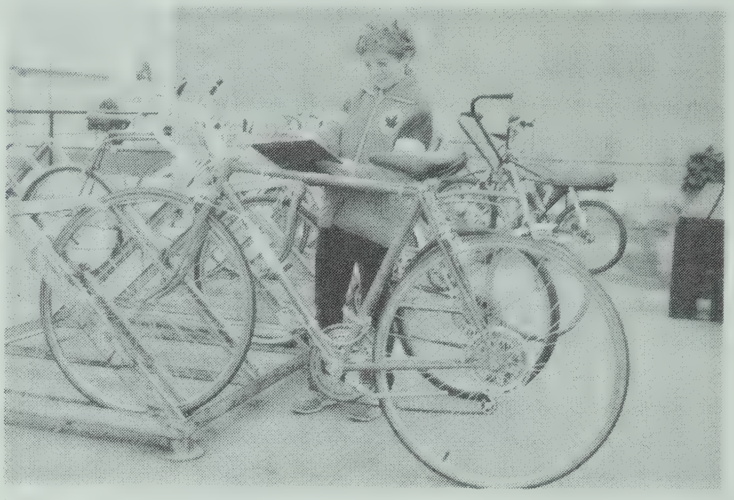
4-10.

Children in the Apartment Building



Bar Graphs

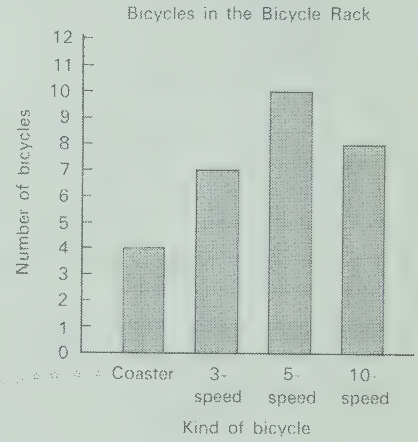
Ken counted the different kinds of bicycles in the bicycle rack.



He made a tally chart to show the number of each kind of bicycle.

Kind of bicycle	Tally	Number of bicycles
coaster	////	4
3-speed	卌 //	7
5-speed	卌 卌	10
10-speed	卌 ///	8

Then he drew a **bar graph** to show the information in a different way.



Each vertical bar stands for a different kind of bicycle.

40

LESSON ACTIVITY

Using the Pages

- Use the photograph to motivate a discussion about bicycles. Ask students whether they have bicycles and what kind of bicycles they have. Ask how many other bicycles each family owns and how the bicycles are alike or different. Introduce the statement at the top of the page. Have the students suggest why Ken might wish to count the different kinds of bicycles. Ask how the making of a tally chart would help him count the bicycles, and then discuss the tally chart. Introduce the term *bar graph* and ask the students why they think the graph on page 40 is called a bar graph. Discuss the title, the headings, the scale, and the fact that the bars are equally spaced and have the same width. Ask for the number of each kind of bicycle shown in the bar graph. Have the students note whether the information shown in the tally chart is identical to the information shown in the bar graph. Then ask them to compare bar graphs and pictographs. Lead them to realize that bar graphs can often show the number of items more accurately than pictographs.

Working Together: Ex. 1 and 2 require the students to interpret a bar graph. For Ex. 3, review that the tally chart on page 40 is organized to show the kinds of bicycles and the number of each kind. Have the students show the information given in the chart in a similar way: families with 0 children, 1 child, 2 children, and so on. Then remind them that making the tally chart involves marking a tally for each item in the chart given for Ex. 3, rather than counting the 0's, 1's, and so on.

Ex. 4-10 outline the steps for drawing a bar graph. These steps will be helpful when the students are completing other bar graphs. For Ex. 5 and 6, have the students refer to the bar graph on page 40 as a guide. Ask students to explain *horizontal line* and *vertical line* in their own words. Point out that the groups are shown along the horizontal line and the scale for the number in each group is shown along the vertical line. Ex. 10 reminds the students that, like pictographs, bar graphs also require a title.

Working Together

From Ken's bar graph, tell

1. which kind of bicycle was the most common. 5-speed
   the least common. coaster
2. how many more 10-speed bicycles there were than 3-speed bicycles. 1

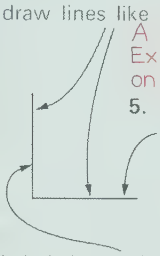
This chart shows the number of children in each of 30 units of an apartment building.

0	2	3	1	1	2	0	4	2	3
0	0	3	3	1	1	2	2	2	2
1	2	1	0	0	4	5	3	1	1

3. Make a tally chart to show this information.
   A tally chart is shown on page T44.

To draw a bar graph using the information from your tally chart,

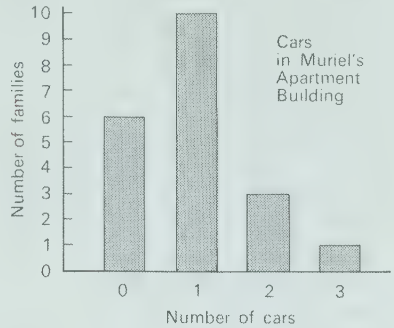
4. draw lines like this.
   A bar graph for Ex 4-10 is shown on page T44



5. Label the horizontal line to show what it represents.
6. Label the vertical line to show what it represents.
7. On the horizontal line, place the names where you will draw the bars in your graph.
8. Mark the vertical line with numbers to let you show the information in your tally chart.
9. Complete the bar graph.
10. Write a title for your graph.

Exercises

This bar graph shows the number of cars owned by each of 20 families



From this bar graph, find

1. how many families have 1 car. 10
2. how many families have no cars. 6
3. how many families have at least 1 car. 14
4. how many more families have 2 cars than have 3 cars. 2

This tally chart shows the times at which 29 students went to bed one night.

8:30	9:00	9:30	10:00
卌 ||||	卌 卌 ||	卌	|||

5. Draw a bar graph to show this information.
   A bar graph is shown on page T368

Make a survey of your classmates to find out their favorite books, TV shows, animals, or foods.

6. Use a tally chart to collect the information. Answers will vary.
7. Draw a bar graph to show the information.

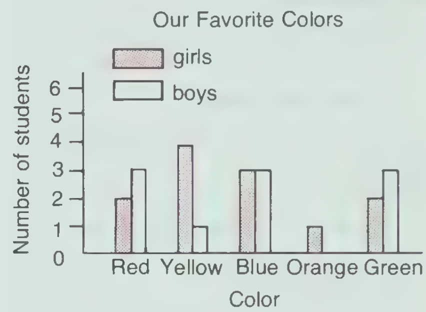
41

RELATED ACTIVITIES

• Have students draw a bar graph to display the information shown in a pictograph on pages 38 and 39 or for the first of the *Related Activities* described on page T43. Then have them compare the two kinds of graphs.

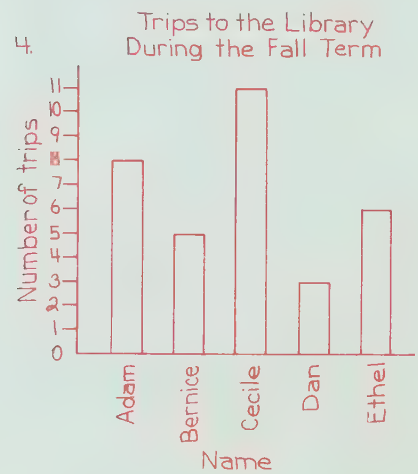
• The *Related Activities* on page T43 can be adapted for bar graphs.

• Challenge students to draw a double bar graph such as the following.



Another possible topic for a double bar graph is favorite sports.

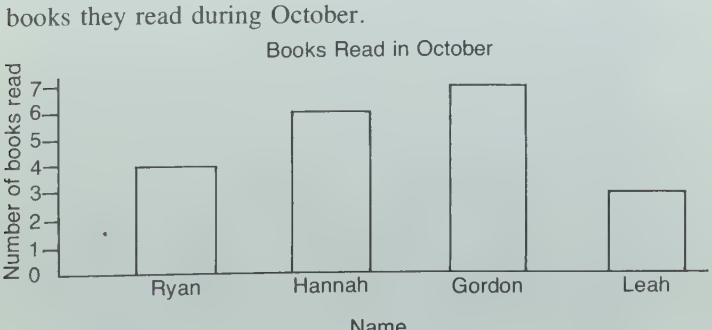
4.



Exercises: For Ex. 6, the students may be interested in predicting their classmates' favorite items before beginning the survey. During the survey, it will probably be necessary to include other choices in the lists. After making the tally chart, but before drawing the bar graph, choices that are listed only once can be grouped under "Other" as in the example on page 36.

Assessment

Ryan and his friends made a bar graph to show the number of books they read during October.



1. How many books did Hannah read? 6
2. How many books did Leah and Gordon read in all? 10
3. How many more books did Gordon read than Ryan? 3

This tally chart shows the number of times each student went to the library during the fall term.

Name	Tally
Adam	卌 |||
Bernice	卌
Cecile	卌 卌 |
Dan	|||
Ethel	卌 |

4. Draw a bar graph to show this information.
   A bar graph is shown above.

LESSON OUTCOME

Match points on a grid with ordered pairs of numbers

Materials

an unmarked 10-by-10 display grid; copies of pages T 396 and T 397 and a straight edge for each student

Vocabulary

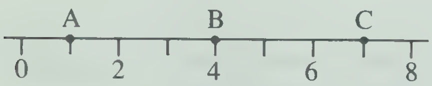
grid, ordered pair, plot a point

Prerequisite Skills

Match whole numbers with points on a number line

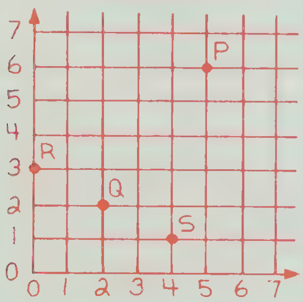
Checking Prerequisite Skills

For this number line,



1. what point matches 4? B
2. what number names point C? 7

5.– 8. (Assessment)



Ordered Pairs of Numbers and Points on a Grid

Each point on a grid can be matched with a pair of numbers.

To match point A with a pair of numbers, count over 3 and up 4.

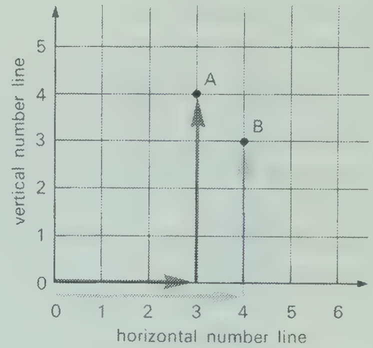
Point A matches (3, 4).

(3, 4) is an **ordered pair** of numbers.

Each ordered pair of numbers can be matched with a point on a grid.

To match the ordered pair (4, 3) with a point on the grid, count over 4 and up 3.

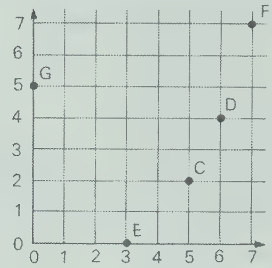
(4, 3) matches point B.



(3, 4) is the same pair of numbers as (4, 3), but it is a different ordered pair of numbers.

Working Together

For the points on this grid, complete the chart.



		Count over	Count up	Ordered pair
1.	point C	? 5	? 2	?(5,2)
2.	point D	? 6	? 4	?(6,4)
3.	point E	? 3	? 0	?(3,0)

Give the ordered pair of numbers that matches

4. point F. (7,7)
5. point G. (0,5)

Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these.

You **plot** a point when you draw a dot on a grid for an ordered pair of numbers.

6. P(5, 1) 7. Q(1, 5) 8. R(3, 3) 9. S(0, 4) 10. T(4, 0)

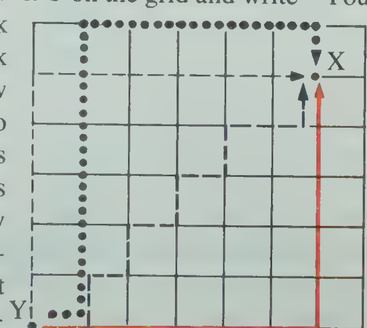
Plotted points are shown on page T368.

42

LESSON ACTIVITY

Before Using the Pages

- Display an unmarked grid and tell the students that it represents a map. Mark point Y on the grid and write "You are here". Also, mark point X on the grid. Ask students to describe how to go from point Y to point X by following lines on the grid. The students will conclude that many different paths are possible. Lead them to suggest that if there is an agreement on how to reach the point, there will be only one possible path. Introduce the procedure of always counting over and then counting up to reach point X. Label the horizontal line and the vertical line for the grid. Ask students to count "over" and "up" to describe the location for each of several points on the grid.



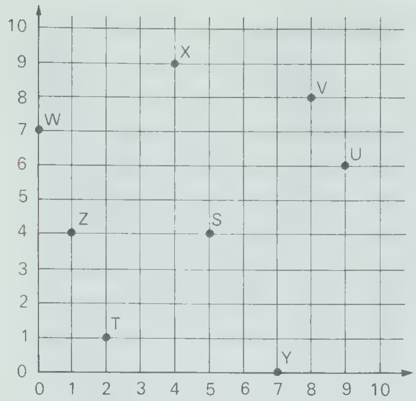
You are here.

Using the Pages

- Direct the students' attention to the grid. Point out the horizontal number line and the vertical number line. The first part of the worked example shows how to find on a grid the point corresponding to a given ordered pair of numbers. Lead the students through this portion of the example, and have them trace the path to Point A along the red arrows. Emphasize that it is important to count over first for the first number and then up for the second number. Explain that the term *ordered pair* of numbers means a pair of numbers for which the order is significant. Point out how an ordered pair of numbers is written.

  The second part of the worked example shows how to find the ordered pair of numbers corresponding to a point that is marked on a grid. Have the students begin in the lower left corner of the grid, count over, and then count up along the blue arrows to locate Point B on the grid.

Exercises



Give the ordered pair of numbers that matches

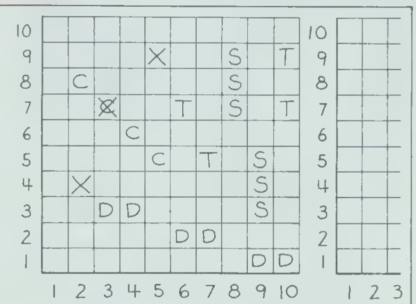
1. point S. (5,4) 2. point T. (2,1)
3. point U. (9,6) 4. point V. (8,8)
5. point W. (0,7) 6. point X. (4,9)
7. point Y. (7,0) 8. point Z. (1,4)

Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these.

9. A(4, 5)	10. B(7, 2)	11. C(8, 1)	12. D(9, 3)	13. E(2, 2)
14. F(3, 10)	15. G(0, 2)	16. H(2, 0)	17. J(8, 5)	18. K(5, 8)
19. L(8, 0)	20. M(6, 6)	21. N(0, 0)	22. P(0, 1)	23. Q(10, 9)

Plotted points are shown on page T368.

Gina and Luke have two of these charts each to play the game.

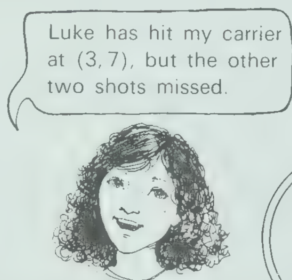


Each marks a fleet on one grid:
1 carrier (4 squares);
2 submarines (3 squares each);
3 destroyers (2 squares each); and
4 torpedo boats (1 square each).

They take turns. Each fires three shots by naming three ordered pairs. After these shots, the other player names any ships that were hit. Each uses the second grid to record the shots against the other player.

The player who names all the squares used by the other player's fleet is the winner.

Play this game with a classmate.



try this

RELATED ACTIVITIES

• Ask the students to draw a simple picture or a geometric shape on a grid so that each vertex of the picture is at an intersection of two grid lines. Then have them list the ordered pairs of numbers for the vertices in sequence. Have the students exchange the lists of ordered pairs of numbers (but not the pictures), plot the points on another grid, and join the vertices in sequence. Compare the two pictures — the original from which the ordered pairs of numbers were listed and the other drawn from the ordered pairs of numbers.

• Give the students a list of ordered pairs of numbers that form a pattern, such as (1,3), (2,1), (3,4), (4,2), (5,5), and (6,3). (Note that the pattern of the first numbers is 1, 2, 3, 4, 5, 6, and the pattern of the second numbers is 3, 1, 4, 2, 5, 3, 6, 4. Have them plot the given points, extend the pattern of points on the grid, and then record the ordered pairs for the extended pattern. Ask the students to create and then extend other patterns on a grid.

• Geoboards with nails in a square array can be labeled to represent a grid. Students may work in pairs with each student placing a rubber band around several nails. Then they may exchange geoboards and list the ordered pairs of numbers for the nails, in an order that shows the shape made with the rubber band.

Working Together: Ex. 1-5 involve finding the ordered pair of numbers that matches a point on the grid. Ex. 3 and 5 have zero as one number in the ordered pair. Explain that Ex. 3 involves counting over 3 and then counting up 0. Ask a student to explain Ex. 5 in a similar way.

For Ex. 6-10, the students are to prepare a grid and plot the point for each ordered pair of numbers. Provide each student with a copy of page T 396 or T 397 and a straight edge for Ex. 6-10 and help them prepare the grid. Ask students to explain what is meant by *plotting a point*.

Exercises: Give each student a copy of page T 396 or T 397 and ask them to use their straight edges to prepare the grid for Ex. 9-23.

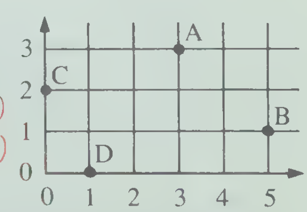
Try This: This game involves locating a square region on a grid by naming an ordered pair of numbers. It is important to note that each ordered pair of numbers refers to a square region, not to a point; for this reason, the numbers along the number lines begin at 1, not 0. Locating a square involves counting over for the first number and up for the second number.

Ask the students to point to the square for the ordered pair of numbers (8,4). Then discuss the instructions for the game. Ensure that the students understand that the squares for a ship must be in line horizontally, vertically, or diagonally.

For one game, each player needs two copies of page T 396.

Assessment

Give the ordered pair of numbers that matches



1. point A. (3,3) 2. point B. (5,1)
3. point C. (0,2) 4. point D. (1,0)

Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these. The plotted points are shown on page T46.

5. P (5,6) **6.** Q (2,2) **7.** R (0,3) **8.** S (4,1)

LESSON OUTCOME

Write an ordered pair of numbers to show information; interpret and draw a line graph

Materials

an unmarked 10-by-10 display grid; copies of page T396 or T397 and a straight edge for each student

Vocabulary

line graph, kilometres per hour, km/h, hour, h, minute, min, metres, m, second, s, lire, drachmas, rands

Prerequisite Skills

Match points on a grid with ordered pairs of numbers

Checking Prerequisite Skills

Give the ordered pair of numbers that matches

1. point A. (5,3) **2.** point B. (4,0)

Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these.

3. X (6,2) **4.** Y (0,5)

Plotted points are shown on page T368.

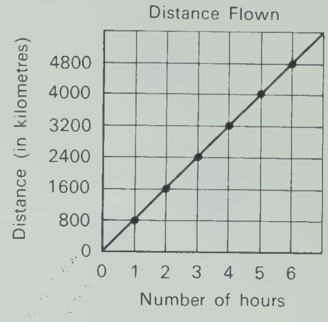
Line Graphs

Kelly and her parents went to Rome by airplane at 800 km/h (kilometres per hour).

She made a table of ordered pairs to show this information.

Number of hours	Distance in kilometres	Ordered pair
1	800	(1, 800)
2	1600	(2, 1600)
3	2400	(3, 2400)
4	3200	(4, 3200)
5	4000	(5, 4000)
6	4800	(6, 4800)

Then she drew a **line graph** to show the information.



A table of information helps to show how to mark the number lines for a line graph.

Working Together

From Kelly's line graph, tell

1. how far the plane flew in 5 h (hours). 4000 km

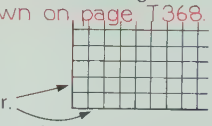
2. how long it took the plane to fly 1600 km. 2h

Kelly saved $0.50 each week for her trip.

3. Make a table of 6 ordered pairs of numbers using this information.
A table of ordered pairs is shown on page T368.

To draw a graph using the information from your table,

4. draw lines like these on graph paper.



5. Label the horizontal line to show what it represents.

6. Label the vertical line to show what it represents.

7. Mark the horizontal line with numbers that will allow you to show the information.

8. Mark the vertical line with numbers that will allow you to show the information.

9. Mark a dot to show each ordered pair. Join the dots.

10. Write a title for your line graph.

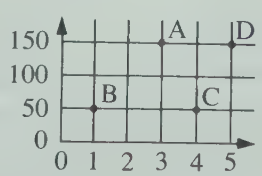
A line graph for Ex. 4-10 is shown on page T368.

44

LESSON ACTIVITY

Before Using the Pages

- Display an unmarked grid. Review that it can be used to plot ordered pairs of numbers, but it needs to be labeled. Then label it as shown and ask how the grid differs from those on pages 42 and 43.



Mark a point, such as A, and have a student name the corresponding ordered pair of numbers. Then name an ordered pair, such as (4,50), and have a student give the name of the corresponding point on the grid. Explain that grids having two different scales are useful in showing ordered pairs for certain situations.

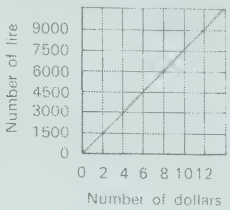
Using the Pages

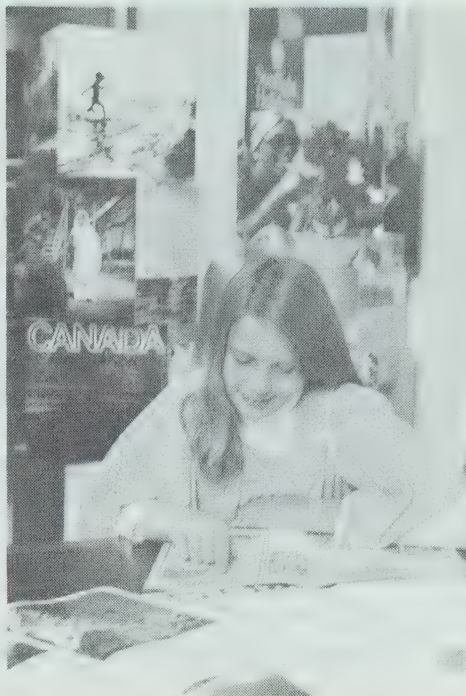
- Introduce the term *kilometres per hour* and the symbol km/h. Have students suggest where they have seen the symbol km/h. Read the first statement and explain that 800 km/h means 800 km are travelled in each hour. Ask for the number of kilometres that would be travelled in two hours and in three hours at the same rate. Point out similar information in the table and discuss how the table helps to derive the ordered pairs. Have the students read the statement above the graph. Emphasize the term *line graph* and explain that it is used because the plotted points are in a straight line. Discuss the title of the graph, the headings, the different scales, and how consecutive points are connected by line segments. Have the students compare the line graph with the table of ordered pairs. Emphasize that for the ordered pairs in the table, the order of ''over first and then up'' is followed.

Working Together: Introduce the symbol h for hours and have the students complete Ex. 1 and 2. You may wish to ask other similar questions about Kelly's line graph.

Exercises

This graph shows the number of lire Kelly received for every $2.





From this line graph, find

1. the number of lire Kelly received for $8. 6000
2. the number of dollars that would be paid for 7500 lire. 10
3. how much more 4500 lire cost than 3000 lire. $2

For each of these, make a table of 6 ordered pairs. Then draw a line graph.
Tables of ordered pairs and line graphs are shown on pages T368 and T369

4. Christine was going to visit relatives in Greece. For every dollar, she received 30 drachmas.
5. Celia went to South Africa. For every $5, she received 4 rands.
6. Paul cycled 5 km in 25 min (minutes).
7. Jane swam 15 m (metres) in 30 s (seconds).

KEEPING SHARP

Add or subtract.

1.	2.	3.	4.	5.	6.
7	17	432	5089	2451	5621
3	21	234	911	2398	4130
9	59	909	555	1126	5448
3	75	682	42	3509	3956
22	172	2257	6597	9484	19155

7. 111 − 89 22
8. 1011 − 809 202
9. 11 011 − 8 909 2102
10. 110 011 − 89 009 21002

45

RELATED ACTIVITIES

- Ask the students to write questions about the graphs they drew for Ex. 4-7 on page 45 for other students to answer.
- For enrichment, have students complete the following charts for six ordered pairs of numbers and then record the results for the charts on one graph, using a different color for each chart.

Number	Add 3	Ordered pair
1	4	(1,4)
2		
3		

Number	Subtract 1	Ordered pair
1	0	(1,0)
2		
3		

Number	Multiply by 2	Ordered pair
1	2	(1,2)
2		
3		

- Have students draw line graphs for ordered pairs having the same first number (Ex. 1), the same second number (Ex. 2), and both numbers the same in each ordered pair (Ex. 3), and note the results.

1. (0,3), (0,4), (0,5), (0,6), (0,7)
2. (3,4), (4,4), (5,4), (6,4), (7,4)
3. (0,0), (4,4), (8,8), (12,12), (16,16)

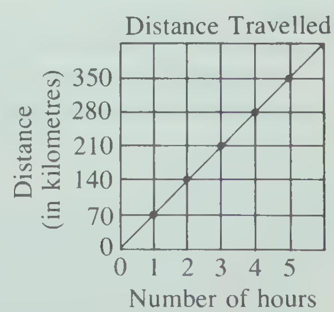
Help the students prepare the table of ordered pairs for Ex. 3. For example, help them adapt the headings of Kelly's table of ordered pairs by changing "Number of hours" to "Number of weeks" and "Distance (in kilometres)" to "Amount saved (in cents)". This will assist them in completing Ex. 5-8. Provide a copy of page T 396 or T 397 and a straight edge for each student for Ex. 4-10. Guide the students as they follow the steps outlined in Ex. 4-10 for drawing a line graph and have them refer to Kelly's graph if necessary. You may wish to develop Ex. 3-10 on the board at the same time as the students complete their own graphs.

Exercises: Discuss what is shown in the photograph to motivate students to tell what countries they have visited, what name describes the currency of each country, and the fact that one lira, for example, does not have the same value as one dollar. Discuss the various currencies used in Ex. 1-5. Note that the symbols min, m, and s are introduced in Ex. 6 and 7. Give each student a straight edge and copies of page T 396 or T 397 for Ex. 4-7.

Keeping Sharp: Remind the students to use sums of 10 whenever possible to help with the addition.

Assessment

This line graph shows the number of kilometres a train travelled during every hour.



1. How far did the train travel in 3 h? 210 km
2. How long did it take the train to travel 280 km? 4 h

Each set of books has a mass of 6 kg.

3. Make a table of 5 ordered pairs to show this information.
4. Draw a line graph to show the information.
A table and line graph are shown on page T369.

LESSON OUTCOME

Interpret and draw a broken-line graph

Materials

chart paper (optional); a thermometer; copies of page T396 or T397, a straight edge, a red pencil, a blue pencil, and a green pencil for each student

Vocabulary

broken-line graph, line segments, degrees Celsius, °C

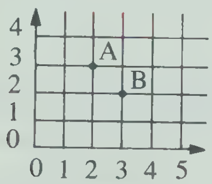
Prerequisite Skills

Match points on a grid with ordered pairs of numbers

Checking Prerequisite Skills

Give the ordered pair of numbers that matches

1. point A. (2,3) **2.** point B. (3,2)



Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these.

3. P (2,30) **4.** Q (5,50)

Plotted points are shown on page T51.

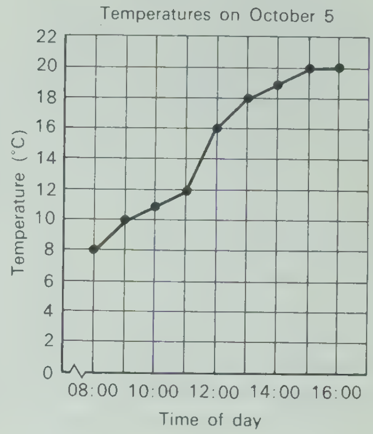
Broken-Line Graphs

On October 5, Nota's class recorded the temperature in degrees Celsius every hour from 08:00 until 16:00.

Temperatures on October 5

Time	Temperature (°C)
08:00	8
09:00	10
10:00	11
11:00	12
12:00	16
13:00	18
14:00	19
15:00	20
16:00	20

Then they drew a **broken-line graph** by plotting the points and connecting the points with line segments.



Working Together

From the broken-line graph, tell

1. the temperature at 14:00. 19°C
2. when the temperature was 12°C. 11:00

Nota recorded the temperatures from 16:00 to 22:00.

Temperatures on October 5

Time	Temperature (°C)
16:00	20
17:00	18
18:00	16
19:00	15
20:00	13
21:00	11
22:00	9

To draw a broken-line graph using Nota's information,

A graph is shown on page T369.

3. draw a horizontal line and a vertical line on graph paper starting from the same point.
4. Label the horizontal line and the vertical line.
5. Mark the lines with numbers for showing the information.
6. Mark a dot to show the temperature each hour.
7. Join the dots.
8. Write a title for the graph.

46

LESSON ACTIVITY

Before Using the Pages

- A day or two before this lesson, encourage the students to comment on the temperature and the changes in temperature that they notice during the day. Some students may be able to recall having heard or seen the "high" and "low" temperature predictions for the day.

Using the Pages

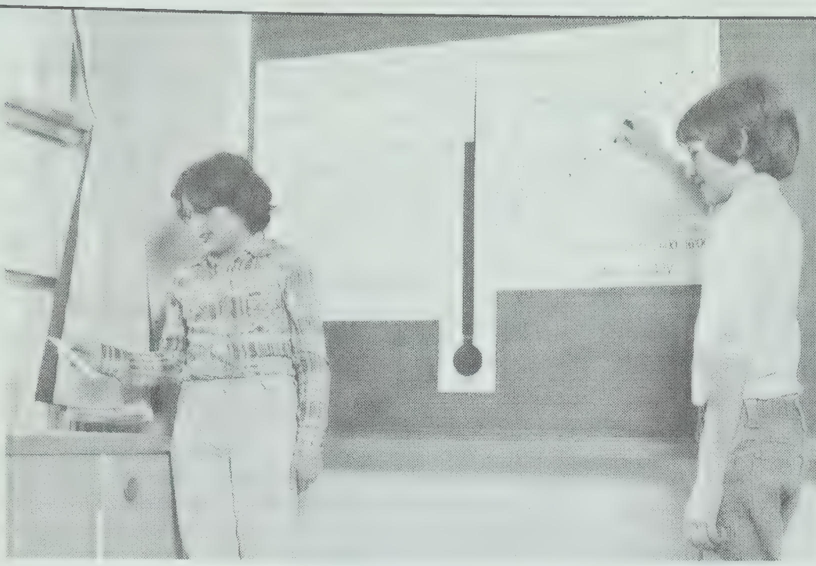
- Have the students refer to the illustration of the thermometer in the photograph on page 47. Ask what temperature is shown on the thermometer. Discuss the title and headings.

  Discuss the statement at the top of page 46 and the chart showing the same information as the chart on page 47. Point out that times are shown according to the 24-hour clock. These times are read as "eight hours" and "fourteen hours", for example. Then have the students observe that the temperature continues to rise throughout the day from 08:00 to 16:00.

  Direct the students to study the graph of the information given in the chart. Note the title, the headings, the scales, and discuss why there is a break in the horizontal number line for the graph. Discuss how the points are obtained, for example, "over" to 12:00 and "up" to 16. Note that the points do not form a straight line, but they are connected by line segments. The series of line segments is called a *broken line*, hence the name *broken-line graph*. Summarize by noting that a graph is a more effective method of displaying the temperature changes than a chart because it shows the trend more clearly.

Working Together: Ex. 1 and 2 deal with interpreting the graph. Note that Ex. 1 involves a point that is not located at the intersection of two lines on the grid. Ask questions about the graph; for example, "During which hour did the temperature change the most?"

Ex. 3-8 list the steps for drawing a broken-line graph. Provide each student with a copy of page T397 and a straight edge for drawing the graph.

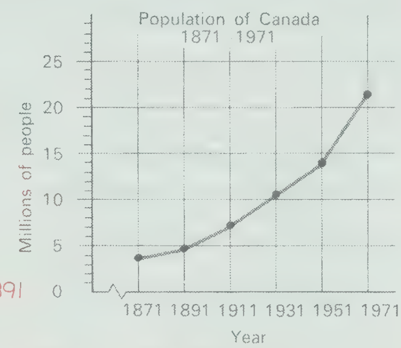


Exercises

The graph shows the population of Canada from the year 1871

Answers may vary for Ex. 1-3

1. About how many people were in Canada in 1871? 4 000 000
2. About how much did the population increase from 1911 to 1931? 3 500 000
3. In which period did the population increase the most? the least? 1951-1971 1871-1891



*4. About how many people were in Canada when you were born?

Answers will vary for Ex 4-6

*5. What is the population of Canada now?

*6. What do you think Canada's population will be in 1991?

Begin a broken-line graph to show the daily outdoor temperature.

Answers will vary for Ex 7-9

7. Show the high temperature for each day in red.
8. Show the low temperature for each day in blue.
9. Show the noon temperature for each day in green.

47

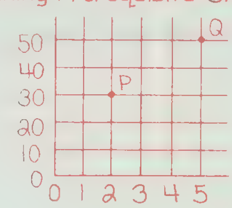
RELATED ACTIVITIES

• For each month remaining in the school year, have the students complete a graph showing the daily temperature in a way similar to that suggested for Ex. 7-9 on page 47. These graphs will help students to relate a temperature in degrees Celsius to the weather at various times of the year. They can also be used for finding averages when studying page 67 in Unit 4, for introducing positive and negative integers, and for finding differences in temperatures in Unit 16.

• Have students research the population of their province, territory, city, town, or a nearby city. Then have them use the information to draw a broken-line graph similar to the one on page 47. Encourage the students to select different places for these graphs in order to provide a variety for the display. Ask students to write and answer questions about the graphs.

• Have the students search magazines and newspapers for examples of broken-line graphs and paste the graphs on cards. Ask each student to select a card and write questions about the graph. Then have them exchange cards and answer the questions.

3. and 4.
(Checking Prerequisite Skills)



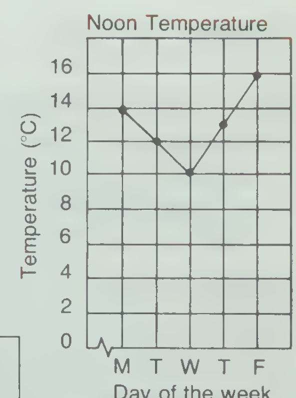
Exercises: Discuss the changes in the population of Canada during the 100 years shown in the graph. Ask why the word "about" is used in Ex. 1 and 2. Ex. 4 and 5 are starred because they may require research about the population of Canada. Encourage the students to explain the reasons for their answers for Ex. 6.

Ex. 7-9 can be completed on copies of page T 396 or T 397, on a piece of chart paper similar to that used for the graph in the photograph on page 47, or on both of these. Ask the students to find the temperatures for Ex. 7 and 8 in the newspaper or from weather reports on radio or television. For Ex. 9, have them use a thermometer to find the temperature each day at noon.

Assessment

Lorna's class drew a broken-line graph to show the noon temperature each school day last week.

1. What was the noon temperature on Tuesday? 12°C
2. On which day was the noon temperature the lowest? Wednesday
3. What was the difference of the noon temperature on Thursday and the noon temperature on Friday? 3°C



Alvin's class found the average temperature for the last five months.

Month	Average temperature (°C)
May	14
June	19
July	22
August	20
September	15

4. Draw a broken-line graph to show this information.
A graph is shown on page T 369.

Page 48

OBJECTIVE

Find information to solve problems

Materials

a map of Canada, maps of your province or territory

Vocabulary

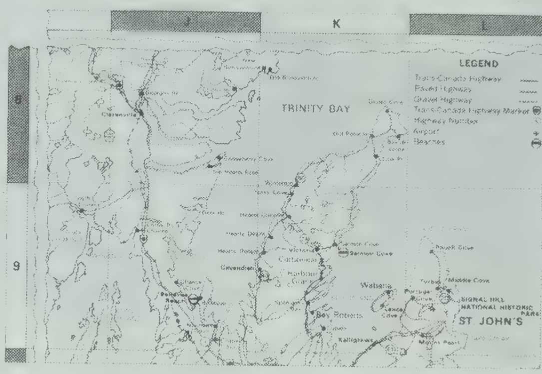
index, legend

RELATED ACTIVITIES

- For a geography lesson using ordered pairs, provide each student with an atlas and an outline of a region being studied. List several places. Have the students use the index of the atlas to find the page number and the ordered pair for locating each place. Then have them find the places in the atlas and mark them on the outline.
- Divide the class into small groups and give each student an atlas to play the following game. One player names a place that the other students may not know. The other players find the place listed in the index and then use the page number and the ordered pair to locate the place on a map. The first player to locate the place shows it to the other players, scores one point, and then names the next place to be located.

Finding Information

The *Index to Cities, Towns, and Settlements* that is printed with a road map of Newfoundland shows that Hearts Content can be located with the ordered pair (K, 9).



The map shows that there is something special at Salmon Cove.

The *Legend* that is printed with the map shows that ⊜ represents a beach.

Use a map of your province or territory.
Answers will vary

1. Does it have an Index to Cities and Towns?
2. Does it use ordered pairs to help you locate places?
3. If your map uses ordered pairs for places, give the ordered pair for your capital city.
4. Find a place on your map that is close to a border and give its ordered pair.
5. Does your map have a Legend? How many items are listed in it?
6. Find an example of each item in the Legend.

Stay close to home if possible.

7. Find a provincial or national park on your map and give the ordered pair that shows where it is located.
8. Make a list of the other information that is included on your map page along with the map itself.
9. Make up a question about your map to give to a classmate.

PROBLEM SOLVING

LESSON ACTIVITY

Before Using the Page

- Display a map of Canada and ask the students to locate the city or town in which they live, or the nearest center marked on the map. Then ask a student to locate the province of Newfoundland on the map.

Using the Page

- Have the students look at the map of Canada and find the part of the map of Newfoundland that is shown on page 48. Read the statement at the top of the page and explain that ordered pairs are used to locate places on a map. Have the students find K at the top of the map, 9 at the side of the map, and then find the section of the map indicated by (K,9). Note that the order of ''over first and then up'' is followed for locating places on a map. Lead the students to find the place called Hearts Content located in this section of the map.

  Ask for the ordered pairs for other places on this map, such as Gooseberry Cove, or name a place on the map with its corresponding ordered pair and have the students locate it.

  Read the information about the legend and direct the students' attention to the section identified by (L,8), which shows the legend. Ask them to read items listed in the legend and then to find an example of each item on the map.
- Provide the students with maps of their home province or territory for the exercises. Explain that more than one ordered pair may be given to describe the location of a place such as a provincial or a national park because it may appear in more than one section of a map.

Checking Up

This chart shows the number of students who went to each volleyball practice.

12	9	11	8	10	7	8	7
10	12	10	9	11	9	8	9
7	8	9	10	7	12	12	11

1. Make a tally chart to show how many times each number of students was at volleyball practice.
A tally chart is shown on page T369

Five students collect autographs of sports stars. Jim has 57, Lynn has 71, Raymond has 44, Olive has 65, and Jessica has 39.

2. Draw a pictograph that shows this information.
A pictograph is shown on page T369

This chart shows the average high temperature in degrees Celsius in Halifax for 6 months.

March	3	June	19
April	8	July	23
May	14	August	22

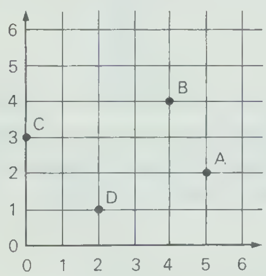
3. Draw a bar graph to show this information.
4. Draw a broken-line graph to show this information.

Graphs are shown on page T369

The price of gasoline is 30¢ for each litre.

5. Make a table of six ordered pairs of numbers using this information.
A table of ordered pairs is shown on page T369.
6. Draw a line graph to show this information.
A line graph is shown on page T370

For this grid, give the ordered pair matching



7. point A. (5,2) 8. point B. (4,4)
9. point C. (0,3) 10. point D. (2,1)

Use graph paper. Draw a horizontal number line and a vertical number line starting from the same point. Then plot each of these.
Plotted points are shown on page T370.

11. E (1, 3) 12. F (4, 0)
13. G (6, 6) 14. H (0, 0)

49

Page 49

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Materials

a copy of page T 397 and a straight edge for each student

RELATED ACTIVITIES

- Continue the work with graphs by having the students interpret and draw graphs showing information in other subjects, such as social studies, science, and language arts.
- Have each student label a horizontal line and a vertical line on a grid. They may use numbers for the lines as in the worked example on page 42, numbers for the squares as in the *Try This* feature on page 43, numbers and letters as in the example on page 48, or only letters. Have each student draw a map on the grid. The map may represent a country, a city, their neighborhood, or an imaginary area where a treasure is buried. Have each student list names of places to be marked on the map and give the ordered pair for each place. Then have the students exchange maps and use the ordered pairs to mark the places on the map.

Skills	Exercises	Related Pages
Use a tally chart to organize information	1	T 40 - T 41
Draw a pictograph	2	T 42 - T 43
Draw a bar graph	3	T 44 - T 45
Draw a broken-line graph	4	T 50 - T 51
Write an ordered pair of numbers to show information	5	T 48 - T 49
Draw a line graph	6	T 48 - T 49
Match points on a grid with ordered pairs of numbers	7 - 14	T 46 - T 47

Comments

Provide each student with a straight edge and copies of page T 397 for the graphs.

Ex. 1 - 6 check the skills required for making tally charts and drawing graphs rather than the skills for interpreting graphs. After the exercises have been completed and checked, you may wish to ask questions about the graphs that the students drew and note their abilities to interpret graphs.

Determine whether students had difficulty in deciding on the value for the symbol used in the pictograph for Ex. 2. Note also whether students had difficulty in choosing the scales for vertical or horizontal number lines of the graphs in Ex. 3 - 6. Difficulties with Ex. 7 - 14 may have been caused by confusion in following the rule ''over first and then up'' or from the tendency to start with 1 as 0, resulting in a move being one unit short.

Unit 4 Overview

Multiplication and Division

This unit reviews and extends the operations of multiplication and division from previous levels. It begins with a review of the basic multiplication facts. The steps in the standard algorithm for multiplication are reviewed with one-digit multipliers and are extended to two-digit and three-digit multipliers. Place values of digits in the multipliers are emphasized and zeros are written in the ones' and tens' places of partial products, as required. The use of multiplication in division is reviewed, as well as the vocabulary to name numbers in the algorithm. The steps in the algorithm are reviewed with one-digit divisors and are extended to division with two-digit and three-digit divisors. One lesson directs special attention to zeros in the quotient. Estimates of digits in the quotient, those which are correct and those which are too great or not great enough, are given careful treatment. Division is applied in finding an average. The *Problem Solving* lesson deals with the effect which different situations have on the answers to problems.

Prerequisite Skills

- regroup with numbers to 999 999
- express a multiple of ten (one hundred, one thousand) as a number of tens (hundreds, thousands) and vice versa
- round to the nearest ten, hundred, thousand, or ten thousand
- write the missing factor in a multiplication fact
- add more than two numbers

Unit Outcomes

- complete the basic multiplication facts
- multiply by a one-digit number, multiplicands to five digits
- multiply by a multiple of 10, 100, and 1000, multiplicands to four digits
- multiply by a two-digit number, multiplicands to six digits
- round two factors and multiply to estimate the product, then compare the estimate of the product with the exact product
- multiply by a three-digit number, multiplicands with up to four digits
- use multiplication to divide, divisors and quotients to 9, remainders
- divide by a one-digit number, dividends with up to five digits, zero as only the last digit in the quotient
- divide by a one-digit number, dividends with up to five digits, zeros in one or more places in the quotient
- find the average for a set of numbers
- divide by a multiple of ten from 10 to 90, dividends with up to five digits
- divide by a two-digit number when the trial estimates for digits in the quotient are correct, dividends to six digits
- divide by a two-digit number when trial estimates for digits in the quotient are incorrect, dividends to six digits
- round and multiply to estimate a quotient, then compare the estimate of the quotient with the quotient obtained by division
- divide by a three-digit number, dividends with up to five digits
- solve word problems involving multiplication and division
- prepare a keychart to show the order of pressing two or more of the $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$ keys on a calculator
- identify situations that affect answers

Background

Multiplication is a *binary* operation in which two numbers called *factors* are combined to produce a third number called their *product*. In division a number called a *dividend*, and another called a *divisor*, yield a *quotient*. Since the two operations are inversely related, the same basic facts are utilized as shown below.

factor		factor		product
7	×	8	=	56
8	×	7	=	56

dividend		divisor		quotient
56	÷	7	=	8
56	÷	8	=	7

Because multiplication is *commutative*, the order of two factors may be changed without affecting their product. Division, however, is not commutative.

$$7 \times 8 = 8 \times 7 \qquad 56 \div 7 \neq 7 \div 56$$

In discussing multiplication it is sometimes necessary to distinguish between the two factors, in which case one is called the *multiplier* and the other is called the *multiplicand*. Sometimes, interchanging the multiplier and the multiplicand simplifies the actual work.

48	×	9	=	432
multiplier		multiplicand		product

$$\begin{array}{r} 9 \\ \times\ 48 \\ \hline 72 \\ 360 \\ \hline 432 \end{array} \qquad \begin{array}{r} 48 \\ \times\ 9 \\ \hline 432 \end{array}$$

Multiplication has two other properties which division does not have. The *associative property of multiplication* states that the order of multiplying three or more factors does not affect the product. This property is useful in simplifying some calculations, as shown.

$$\begin{aligned} 376 \times 4 \times 25 &= 376 \times (4 \times 25) \\ &= 376 \times 100 \\ &= 37\,600 \end{aligned}$$

The *distributive property of multiplication over addition* states that if one factor is expressed as the sum of two or more numbers, each of the numbers is multiplied by the other factor and these products are then added (A). This property is applied in the multiplication algorithm. At first, separate products are recorded for each place value of the multiplicand (B). The standard algorithm reduces these steps to a single product (C). In this form, difficulties sometimes arise in the regrouping of values. For example, for 6 × 5 = 30, only the 0 is written in the ones' place, and the 3 tens of 30 must be remembered while the product 6 × 7 (tens) is obtained; then the 3 (tens) is added to the 42 (tens), making 45 (tens). Briefly, the procedure is to multiply first, then add the regrouped number. Errors in multiplication often result when students reverse these two steps.

A

$$\begin{aligned} 6 \times 375 &= 6 \times (300 + 70 + 5) \\ &= 1800 + 420 + 30 \\ &= 2250 \end{aligned}$$

B

$$\begin{array}{r} 375 \\ \times\ \ 6 \\ \hline 30 \\ 420 \\ 1800 \\ \hline 2250 \end{array}$$

C

$$\begin{array}{r} 375 \\ \times\ \ 6 \\ \hline 2250 \end{array}$$

Of all the algorithms used in the four basic operations, the one for long division usually causes the most difficulty for students, probably because it requires a meaningful combination of skills in numeration and place value, in multiplication, and in subtraction. The most commonly used algorithm involves dividing each place value of the dividend in turn from left to right. It is most important to think of place values in both the dividend and the quotient. In $8\overline{)956}$ the dividend 956 represents 9 hundreds 5 tens 6 ones, and the first digit in the quotient will be in the hundreds' place because 8 is less than 9 (hundreds). Sometimes it is necessary to reinterpret a dividend in terms of a lesser place value, as in $8\overline{)756}$. In this case, because 8 is greater than 7 (hundreds) there will be no digit in the hundreds' place of the quotient. The dividend is interpreted as 75 tens 6 ones and the first digit of the quotient will be in the tens' place.

In all their work with division, students are usually guided through different developmental stages, such as those shown in D, E, and F. The final level of the algorithm shown in G is reached in *Starting Points in Mathematics 5* and is the one used in this book.

D
$$\begin{array}{r|r} 7\overline{)1872} & \\ 1400 & 200 \\ \hline 472 & \\ 420 & 60 \\ \hline 52 & \\ 49 & 7 \\ \hline 3 & 267 \end{array}$$

E
$$\begin{array}{r} \left.\begin{array}{r} 7 \\ 60 \\ 200 \end{array}\right\} \rightarrow 267 \\ 7\overline{)1872} \\ 1400 \\ \hline 472 \\ 420 \\ \hline 52 \\ 49 \\ \hline 3 \end{array}$$

F
$$\begin{array}{r} 267 \\ 7\overline{)1872} \\ 1400 \\ \hline 472 \\ 420 \\ \hline 52 \\ 49 \\ \hline 3 \end{array}$$

G
$$\begin{array}{r} 267 \\ 7\overline{)1872} \\ 14 \\ \hline 47 \\ 42 \\ \hline 52 \\ 49 \\ \hline 3 \end{array}$$

Dividing by a two-digit or a three-digit divisor presents additional difficulties in estimating digits in the quotient. Rounding a two-digit divisor to the nearest ten and a three-digit divisor to the nearest hundred allows the basic multiplication facts (one-digit factors) to be used. Sometimes the estimated digits for the quotient are correct, but frequently they are too great and must be decreased (H), or they are too small and must be increased (I).

H
$$\begin{array}{r} \square \\ 54\overline{)4230} \end{array}$$
Round 54 to 5(0).
$5 \times 8 = 40$
$50 \times 8 = 400$
Try 54×8 (tens).

$$\begin{array}{r} 8 \\ 54\overline{)4230} \\ 432 \\ \hline \end{array}$$
Decrease 8 to 7.

$$\begin{array}{r} 7\square \\ 54\overline{)4230} \\ 378\downarrow \\ \hline 450 \end{array}$$

I
$$\begin{array}{r} \square \\ 76\overline{)4794} \end{array}$$
Round 76 to 8(0).
$8 \times 5 = 40$
$80 \times 5 = 400$
Try 76×5 (tens).

$$\begin{array}{r} 5 \\ 76\overline{)4794} \\ 380 \\ \hline 99 \end{array}$$
Increase 5 to 6.

$$\begin{array}{r} 6\square \\ 76\overline{)4794} \\ 456\downarrow \\ \hline 234 \end{array}$$

Because such steps can occur for each digit in the quotient for a division, it is obvious that long division is a complex process involving a certain amount of trial and error as well as competent use of the basic facts and operations in multiplication and subtraction.

Teaching Strategies

If any students cannot recall basic multiplication and division facts accurately and quickly, it is important that they work toward mastery, since their success with the lessons of the unit will be hampered unless they have complete mastery of the facts.

Before beginning the work in Unit 4 it may be advisable to survey the students' abilities to use their previous knowledge of and skills in multiplication and division of whole numbers. Not only should the strengths and weaknesses of the class as a whole and of individuals be noted, but the ways in which the algorithms are used should also be examined. Planning the lessons in this unit should take these observations into consideration and extra attention should be directed to overcoming any apparent deficiencies. It may be beneficial to group students with similar needs and to teach them by using appropriate materials and approaches.

It should be pointed out in the multiplication algorithm that zeros are used as place holders in the partial products when the tens' digit and the hundreds' digit of the multiplier are used. Their inclusion helps to keep the partial products in their proper places and also emphasizes the place values of the digits in the multiplier. The latter is important when products are estimated using rounded numbers. Emphasis on place values in dividends and in quotients is recommended. If the mechanical process of division is rushed without sufficient attention to place values, students frequently omit to record a zero in the quotient, when it is required, before going on to the next step. Careful attention should be given to place values in all operations, particularly in division.

The lesson on the use of the calculator in this unit involves making keycharts for solving word problems. Students are not required to solve the problems, but the more capable students should be challenged to do so. If calculators are not available, the problems on page 79 may be used for a lesson on solving one-step and two-step problems. In lieu of keycharts, students should write a number sentence for each of the steps.

Materials

copies of page T 382 or T 396 and colored pencils for each student
overhead projector (optional)
models for thousands, hundreds, tens, and ones
an abacus
16 counters, a metre stick
calculators (optional)

Vocabulary

multiply	quotient
multiplication, ×	remainder
factor, product	millilitres, mL
basic multiplication fact	day, d
divide	average
division, ÷, $\overline{)}$	square metres, m^2
divisor	hectare, ha
dividend	keychart

LESSON OUTCOME

Complete the basic multiplication facts

Materials

copies of page T382 or T396 and colored pencils for each student, overhead projector (optional)

Vocabulary

multiply, multiplication, ×, factor, product, basic multiplication fact

4 MULTIPLICATION AND DIVISION

Basic Multiplication Facts

Walter and Denise are using this chart to play the game "Lineup".

They take turns. Each names two **factors**, and then writes the **product** in the proper square. The player who first writes four products in line is the winner.

7 × 5
6 × 4
6 × 8
7 × 6
7 × 4
8 × 4
9 × 4

Second factor

First factor

×	0	1	2	3	4	5	6	7	8	9
0										
1										
2										
3										
4										
5					20					
6					24	30	36	42	48	
7				21	28		42	49		
8					32		48			
9					36					

6 × 7
6 × 6
6 × 5
8 × 6
7 × 7
7 × 3
5 × 4

Who won the game? Denise
Who do you think had the first turn? Walter

50

LESSON ACTIVITY

Before Using the Pages

- Show the following array on the board.

• • • • • • • • 8
• • • • • • • • 8
• • • • • • • • 8
• • • • • • • • 8

Ask the students to find the sum as quickly as possible. Some students may perform the addition, but there will be others who can name the sum quickly by associating the addition with multiplication. Ask a student to explain how he/she is able to name the sum so quickly. The reply might be, for example, "I remembered that four eights are thirty-two." Write the statement 4 groups of 8 are 32 on the board and have students help to "translate" it into a number sentence by replacing the underlined words with the appropriate mathematical symbols ($4 \times 8 = 32$). Have a student read the sentence to review that the symbol × is read "times". Ask what operation the symbol × indicates.

Write on the board other similar examples such as 4 sixes, 9 groups of 3, and 7 times 7, and have students express the corresponding multiplication sentences.

Using the Pages

- Have a student read the statements at the top of page 50. Ask a student to tell which numbers are the *factors* and which number is the *product* in a multiplication sentence, for example, $8 \times 9 = 72$. To ensure that the procedure for playing the game is understood, have a student explain the method in her/his own words. Draw attention to the square showing the product (35) for Denise's first turn (7×5). Ask how the correct square for a product is found, drawing attention to the headings "First factor" and "Second factor".

  Have a student read the two questions at the bottom of page 50. Ask how the answers to the questions can be determined using the information provided on the page. Students will likely suggest using the factors shown to observe the results of playing the game. Have the students

Exercises

Who won each game?
Who do you think had the first turn?

1.

	Denise	Walter	
0	0 × 8	6 × 2	12
7	1 × 7	5 × 3	15
16	4 × 4	7 × 1	7
0	8 × 0	3 × 5	15
20	5 × 4	6 × 4	24
18	6 × 3	7 × 5	35
48	8 × 6	4 × 2	8

Walter won
Denise had the first turn

2.

	Denise	Walter	
48	8 × 6	7 × 3	21
54	9 × 6	8 × 4	32
42	7 × 6	6 × 6	36
45	9 × 5	6 × 2	12
5	5 × 1	7 × 5	35
27	9 × 3	5 × 7	35

Walter won
Denise had the first turn

3.

	Denise	Walter	
4	2 × 2	1 × 3	3
9	3 × 3	2 × 3	6
1	1 × 1	4 × 4	16
6	3 × 2	0 × 0	0
12	3 × 4	4 × 2	8
3	3 × 1	3 × 5	15

Denise won
Walter had the first turn

Copy and complete.

4.

×	0	1	2	3	4	5	6	7	8	9
8	0	8	16	24	32	40	48	56	64	72

5.

×	4	9	2	0	5	7	1	6	3	8
6	24	54	12	0	30	42	6	36	18	48

6.

×	8	2	7	0	4	6	9	1	3	5
7	56	14	49	0	28	42	63	7	21	35

7.

×	2	9	5	8	0	7	4	3	1	6
9	18	81	45	72	0	63	36	27	9	54

Multiply.
Write only the products.

8. 7 × 4 28
9. 8 × 8 64
10. 9 × 3 27
11. 7 × 9 63
12. 2 × 0 0
13. 1 × 4 4
14. 8 × 3 24
15. 9 × 9 81

16. $\begin{array}{r} 8 \\ 7 \\ \hline 56 \end{array}$ 17. $\begin{array}{r} 5 \\ 2 \\ \hline 10 \end{array}$ 18. $\begin{array}{r} 9 \\ 8 \\ \hline 72 \end{array}$

19. $\begin{array}{r} 0 \\ 0 \\ \hline 0 \end{array}$ 20. $\begin{array}{r} 8 \\ 5 \\ \hline 40 \end{array}$ 21. $\begin{array}{r} 5 \\ 4 \\ \hline 20 \end{array}$

22. $\begin{array}{r} 7 \\ 7 \\ \hline 49 \end{array}$ 23. $\begin{array}{r} 4 \\ 8 \\ \hline 32 \end{array}$ 24. $\begin{array}{r} 5 \\ 5 \\ \hline 25 \end{array}$

Complete each of these in as many ways as possible.

25. 3 × 3 = 9
26. 4 × 6 = 24
27. × = 8
28. 0 × 5 = 0
29. 1 × 1 = 1
30. 9 × 9 = 81
31. × 5 = 5
32. 4 × 4 = 16
33. 8 × = 4
34. × 6 = 2
35. × 2 = 1
36. 1 × 9 = 9
37. 7 × = 4
38. × 6 = 3
39. × 3 = 1
40. 8 × = 4
41. × = 63
42. 8 × 7 = 56
43. 8 × 9 = 72
44. 6 × 9 = 54
45. 7 × 3 = 21
46. 4 × = 2

47. Play "Lineup" with a friend.

27 1 × 8, 8 × 1, 2 × 4, 4 × 2
31 3 × 5 = 15, 5 × 5 = 25, 7 × 5 = 35, 9 × 5 = 45
32 4 × 4 = 16, 4 × 9 = 36
33 8 × 5 = 40, 8 × 6 = 48
34 2 × 6 = 12, 7 × 6 = 42
35 5 × 2 = 10, 6 × 2 = 12, 7 × 2 = 14, 8 × 2 = 16, 9 × 2 = 18
37 7 × 6 = 42, 7 × 7 = 49
38 5 × 6 = 30, 6 × 6 = 36

51

Answers for Ex 39, 40, 41, and 46 are given below.

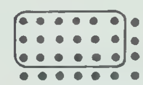
RELATED ACTIVITIES

• Students may enjoy developing and testing strategies for the game "Lineup".

• For practice with basic multiplication facts, prepare pairs of cards similar to the following for the game "Concentration" described on page T379.

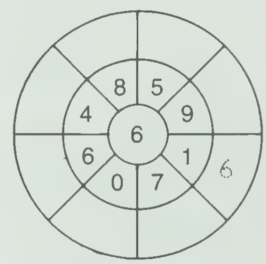
$\boxed{4 \times 8}$ $\boxed{32}$

• Some students may benefit from ringing arrays of dots to match given multiplication exercises and writing facts to match given arrays. Copies of page T395 may be helpful in presenting arrays.



3 × 6 = 18
6 × 3 = 18

• Prepare number wheels and tables on copies of page T391 to provide further practice with multiplication facts.



×	3
4	12
9	
8	

• Ask the students to create exercises similar to Ex. 25-46 on page 51 for other students to complete.

work in pairs to do this, each student taking the part of one of the players. Provide them with copies of page T382 or sections of page T396 to use as charts for the game and have each player in a game use a different color of pencil. When they have finished, determine which pairs of students played the game so that Denise had the first turn. Have one pair demonstrate the procedure on a large chart drawn on the board. (You may wish to use an overhead projector for this purpose.) Discuss the results. Then have two students demonstrate the procedure so that Walter has the first turn. Discuss the results and have students answer the questions at the bottom of page 50.

Exercises: For Ex. 1-3 (and for the example on page 50), the winner of the game has made the last entry in the chart. Because the information provided indicates that each player had the same number of turns, the player who won the game did not have the first turn. If this concept is understood by the students, it will enable them to write all the products of an exercise in a chart, ring the four products in line, and name who won the game and who had the first turn. Have them show the products, using two colors to distinguish the entries of the two players.

You may wish to have the students turn their lined paper sideways to copy and complete Ex. 4-7. Emphasize quick responses for Ex. 8-24. Note that some of the exercises for Ex. 25-46 may be completed in more than one way, and students are to show as many ways as possible. Provide the students with copies of page T382 for Ex. 1-3 and 47.

Assessment

Multiply. Write only the products.

1. $\begin{array}{r} 7 \\ 3 \\ \hline 21 \end{array}$ 2. $\begin{array}{r} 0 \\ 6 \\ \hline 0 \end{array}$ 3. $\begin{array}{r} 2 \\ 5 \\ \hline 10 \end{array}$

4. 9 × 8 72
5. 4 × 1 4

39. 4 × 3 = 12, 5 × 3 = 15, 6 × 3 = 18
40. 8 × 3 = 24, 8 × 8 = 64
41. 7 × 9 = 63, 9 × 7 = 63
46. 4 × 5 = 20, 4 × 6 = 24, 4 × 7 = 28

LESSON OUTCOME

Multiply by a one-digit number, multiplicands with up to five digits; solve related word problems

Materials

models for thousands, hundreds, tens, and ones

Vocabulary

millilitres, mL

Prerequisite Skills

Complete the basic multiplication facts; complete extensions of basic multiplication facts; regroup with numbers to 999 999

Checking Prerequisite Skills

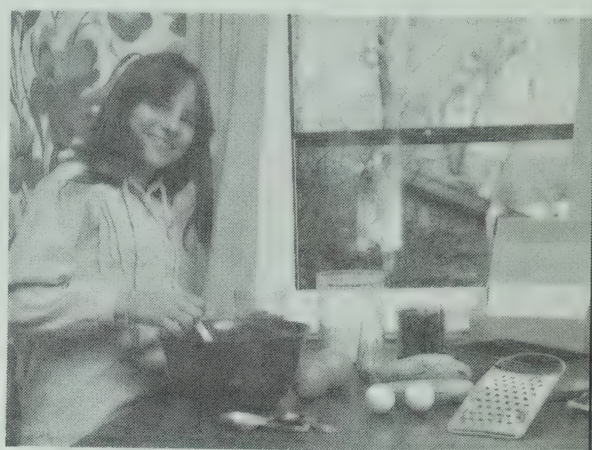
Complete.

1. 3 × 6 = 18
   3 × 6 tens = 18 tens
   3 × 60 = 180
2. 8 × 5 = 40
   8 × 5 hundreds = 40 hundreds
   8 × 500 = 4000
3. 9 × 7 = 63; 9 thousands × 7 = 63 thousands; 9000 × 7 = 63 000
4. 24 tens = 2 hundreds 4 tens
5. 63 hundreds = 6 thousands 3 hundreds
6. 48 thousands = 4 ten thousands 8 thousands

Multiplying by a One-Digit Number

375 mL (millilitres) of mashed bananas are needed for a loaf of banana bread. How many millilitres of mashed bananas are needed for 6 loaves?

Multiply 6 and 375



hundreds	tens	ones	
	3		
3	7	5	6 × 5 = 30 or
		6	3 tens 0 ones.
		0	

4	3		
3	7	5	6 · 7 tens = 42 tens
		6	3 more tens make
	5	0	45 tens or
			4 hundreds 5 tens.

	4	3		
	3	7	5	6 × 3 hundreds
			6	= 18 hundreds
2	2	5	0	4 more hundreds make
				22 hundreds or
				2 thousands 2 hundreds.

2250 mL of mashed bananas are needed for 6 loaves of banana bread

Working Together

Multiply by following the steps

1. 76 953 × 7 = 538 671 (regrouped: 46 32)

Multiply. Regroup.
Multiply. Add. Regroup.
Multiply. Add. Regroup.
Multiply. Add. Regroup.
Multiply. Add.

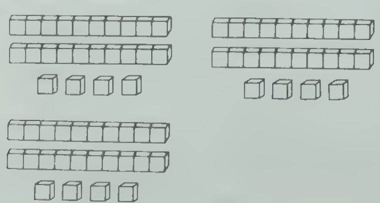
Multiply.

2. 3041 × 5 = 15 205
3. 48 007 × 8 = 384 056
4. 4 · 7490 = 29 960
5. 50 806 × 9 = 457 254

LESSON ACTIVITY

Before Using the Pages

- Review that 3 × 4 means 3 groups of 4. Write 3 × 24 on the board and ask what meaning can be given to it. Have each of 3 students represent 24 using models and display them as shown below. Emphasize that for 3 groups of 24 there are 3 groups of 4 ones and 3 groups of 2 tens. Write the exercise 3 × 24 in vertical form on the board and derive the product as described below.



tens	ones
2	4
×	3
$\boxed{1}$	2
6	0
7	2

Ask what number is represented by 3 groups of 4 ones. Have one student regroup the models of 12 ones as 1 ten 2 ones and another write the partial product on the board. Draw a square around the 1 in the tens' place to highlight the regrouping. Ask what number is represented by 3 groups of 2 tens. Have a student write the partial product 60 in the exercise and find the final product. Summarize the steps: multiply the ones first; regroup; multiply the tens; and add. Suggest that the amount of written work can be reduced from three lines to one line but that the steps of the procedure are the same. Draw an arrow to indicate that the 1 ten obtained from regrouping 12 ones can be written above the tens' place. Have students suggest how to complete the multiplication in the short form.

tens	ones
2	4
×	3
$\boxed{1}$	2
6	0
7	2

$$\begin{array}{r} {}^{1}\\ 24 \\ \times\ 3 \\ \hline 72 \end{array}$$

Write 7 × 86 on the board in vertical form and have students demonstrate the short procedure for writing the

Banana Bread

125 mL brown sugar
2 eggs
75 mL corn oil
50 mL orange juice
500 mL flour
5 mL salt
2 mL baking soda
10 mL baking powder
375 mL mashed ripe bananas
150 mL chopped walnuts
200 mL raisins
5 mL grated orange rind

Beat eggs and sugar until light and fluffy. Stir in corn oil and orange juice.

Sift together flour, salt, baking soda, and baking powder.

Add dry ingredients alternately with bananas, beating slightly after adding each. Fold in nuts, raisins, and orange rind.

Bake at 180°C about 45 min.

Exercises

Multiply.

1. 421 × 2 = 842
2. 6130 × 3 = 18 390
3. 5271 × 4 = 21 084
4. 81 526 × 3 = 244 578
5. 47 650 × 6 = 285 900
6. 45 607 × 8 = 364 856
7. 7081 × 7 = 49 567
8. 50 091 × 6 = 300 546
9. 10 509 × 5 = 52 545
10. 68 004 × 9 = 612 036
11. 4 × 9403
12. 13 657 × 9
13. 7 × 50 237
14. 8 × 80 792
15. 6002 × 6
16. 9 × 32 009
17. 5 × 20 878
18. 80 911 × 8

11 37 612 · 12 122 913
13 351 659 · 14 646 336
15 36 012 · 16 288 081
17 104 390 · 18 647 288

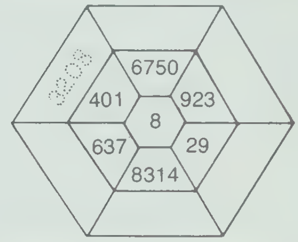
Use the recipe for banana bread to solve each of these.

19. How many millilitres of raisins are needed for 4 loaves? 800
20. How much orange juice would be used for 7 loaves? 350 mL
21. How many millilitres of flour are needed for 9 loaves? 4500
22. How much corn oil would be used for 3 loaves? 225 mL
23. How many millilitres of chopped walnuts would be in 2 loaves? 300
24. How many millilitres of brown sugar are needed for 8 loaves? 1000
25. Write the amount of each ingredient needed for 5 loaves. Amounts are given on page T370

53

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 1-10 on page 329.
- Ask students to write word problems similar to Ex. 19-24, using other recipes.
- Use copies of page T391 to prepare hexagons similar to the following for the students to complete.



- In a multiplication exercise such as 8×347, the second step involves the computation $8 \times 4 + 5$ and the third involves $8 \times 3 + 3$. To help students practice this aspect of the multiplication algorithm, provide exercises similar to the following.

$7 \times 4 + 6$ $\quad$ $5 \times 7 + 5$
$8 \times 9 + 4$ $\quad$ $8 \times 8 + 8$
$3 \times 6 + 8$ $\quad$ $2 \times 6 + 5$

- Exercises similar to the following can help students having difficulty with writing products in one step. Ensure that they work from right to left.

$$\begin{array}{r} \boxed{2}\,\boxed{3} \\ 735 \\ \times\quad 6 \\ \hline \boxed{4}\,\boxed{4}\,\boxed{1}\,\boxed{0} \end{array} \qquad \begin{array}{r} \boxed{4}\,\boxed{7}\,\boxed{2} \\ 1483 \\ \times\quad 9 \\ \hline 1\,3\,\boxed{3}\,4\,7 \end{array}$$

product. To record 60 tens, establish that the 6 is written in the hundreds' place and the 0 in the tens' place. Ask whether the short procedure can be used if the two-digit factor (86) is changed to show three, four, or five digits.

Using the Pages

- Direct the students' attention to the photograph on page 52 and the recipe on page 53. Ask what the symbol mL represents in the recipe. Ask the students to read the recipe silently. Then ask questions such as "What amount of flour is needed for the recipe?" and "What amount of orange juice is needed?" Have a student read the word problem at the top of page 52 aloud. Ask why multiplication is used for the solution.

Lead the students through the steps of the multiplication. Emphasize the place value of each digit and the regrouping. For example, in the second step, 5 tens is written in the tens' place of the product and 4 hundreds is written above the 3 in the hundreds' place. Have a student read the concluding statement.

Working Together: Ex. 1 identifies the steps required to complete the multiplication. Ex. 2-5 provide an opportunity to discuss the effect of having zero in the multiplicand.

Exercises: Remind the students to write Ex. 11-18 in vertical form so that the one-digit factor is the multiplier. For each of Ex. 19-24, remind them to show the multiplication and to write a concluding statement.

Assessment

Multiply.

1. 214 × 8 = 1712
2. 6035 × 6 = 36 210
3. 87 419 × 7 = 611 933
4. 9 × 9507 85 563
5. 5 × 40 068 200 340

Solve.

6. 500 mL of flour are needed for each cake. How many millilitres of flour are needed for 4 cakes? 2000

Pages 54-55

LESSON OUTCOME

Multiply by a multiple of 10, 100, and 1000, multiplicands with up to four digits

Materials

an abacus

Prerequisite Skills

Multiply by a one-digit number, multiplicands with up to four digits; express a multiple of ten (one hundred, one thousand) as a number of tens (hundreds, thousands) and vice versa

Checking Prerequisite Skills

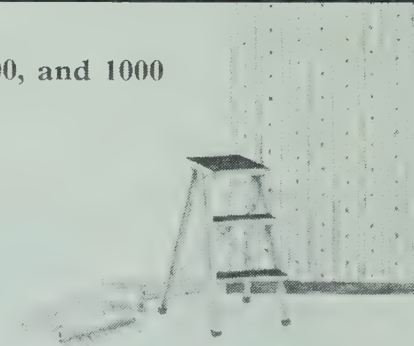
Multiply.

1. 964
 7
 6748
2. 4638
 9
 41 742
3. 8 × 7671 61 368
4. 6 × 9032 54 192

Complete.

5. 40 = 4 tens
6. 180 = 18 tens
7. 300 = 3 hundreds
8. 1200 = 12 hundreds
9. 25 000 = 25 thousands
10. 180 000 = 180 thousands

Multiplying by Multiples of 10, 100, and 1000



Alana helped her parents wallpaper their living room and dining room. They used 20 rolls of wallpaper. Each roll cost $17. How much did the wallpaper cost?

Multiply 20 and 17.

For the product 17
20 (2 tens 0 ones)

you need to know how to multiply 0 and 17,

	1	7
	2	0
		0

When 0 is a factor, the product is 0

and how to multiply 2 and 17.

	1	7
	2	0
3	4	0

2 tens × 17 = 34 tens or 340.

The wallpaper cost $340.

Working Together

Use the multiplication sentence 6 × 73 = 438 to help you complete each of these.

1. 73
 6
 438
2. 73
 6 tens
 438 tens
3. 73
 60
 438 0
4. 73
 6 hundreds
 438 hundreds
5. 73
 600
 43 8 00
6. 73
 6 thousands
 438 thousands
7. 73
 6 000
 438 000

Write 0 in the ones place. Then multiply by 9 (tens).

8. 247
 90
 22 230
9. 3406
 90
 306 540

Multiply.

10. 486
 40
 19 440
11. 329
 700
 230 300
12. 80 × 7500 600 000
13. 7060 × 3000 21 180 000

54

LESSON ACTIVITY

Before Using the Pages

- Review multiplication with zero as a factor by asking for the product of factors such as 0 and 6.
- Discuss the place values of digits for multiples of 10. For example, write 70 on the board. Ask for the number of ones and for the number of tens. Develop that all multiples of 10 show zero in the ones' place. Similarly, demonstrate that multiples of 100 show 0 tens and 0 ones, and that multiples of 1000 show 0 hundreds, 0 tens, and 0 ones.

Using the Pages

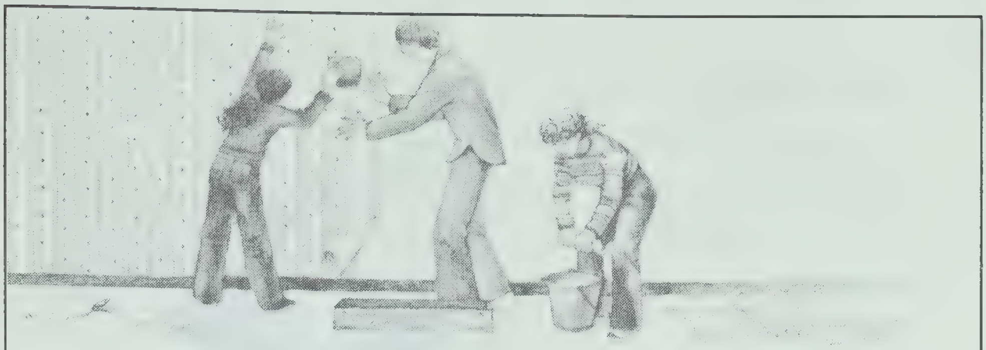
- Question the students about the illustration to introduce the word problem. Ask what is shown and explain that wallpaper is bought in rolls. Ask why multiplication is used for solving the problem.

 The worked example demonstrates that since 20 is 2 tens 0 ones, multiplication by 20 can be thought of in two steps: multiply by 0 ones; and then multiply by 2 tens. Discuss each step of the multiplication and have the students refer to the numerals highlighted in red, to the explanation written beside the numerals, and to the information in the "thought clouds". Draw attention to the fact that 20 is a multiple of ten and 340 is also a multiple of ten. Show the following exercise on the board.

$$\begin{array}{r} 17 \\ \times\ 2 \text{ tens} \\ \hline 34 \text{ tens} \end{array}$$

Emphasize that 2 tens × 17 means 34 tens, not 34 ones. Read the concluding statement and ask why the symbol $ is shown.

Working Together: Ex. 1-7 help students to relate multiplication by multiples of 10, 100, and 1000 to multiplication by a one-digit number. Have them note that multiplying by a number of tens results in a zero in the ones' place of the product. Multiplying by a number of hundreds gives a product with 0 tens and 0 ones. Finally, multiplying by a number of thousands gives a product with 0 hundreds, 0 tens, and 0 ones. Because the above results are consistent,



Exercises

Multiply.

1. 7134 × 50 = 356 700
2. 469 × 700 = 328 300
3. 9840 × 6000 = 59 040 000
4. 385 × 200 = 77 000
5. 5080 × 800 = 4 064 000

6. 10 × 916 7. 600 × 6206 8. 3000 × 368 9. 4823 · 100
10. 327 × 7000 11. 9400 × 80 12. 9000 × 409 13. 3045 · 4000
14. 200 × 5000 15. 1000 × 9476 16. 2043 · 400 17. 90 × 8203
18. 5000 × 5096 19. 6009 × 900 20. 70 × 8796 21. 486 × 8000

Study these multiplication sentences.

10 × 5 = 50	100 × 8 = 800	1 000 × 4 = 4 000
10 × 69 = 690	100 × 71 = 7 100	1 000 × 65 = 65 000
10 · 404 = 4040	100 × 395 = 39 500	1 000 × 708 = 708 000

Give a rule that helps you find the product when
For the other factor, move the digits

1. 10 is a factor. one place to the left and write one zero
2. 100 is a factor. two places to the left and write two zeros
3. 1000 is a factor. three places to the left and write three zeros

Multiply. Write only the products.

4. 10 × 607 6070 5. 1000 × 200 200 000 6. 974 × 1000 974 000 7. 1000 × 320 320 000
8. 100 × 1305 130 500 9. 460 × 10 4600 10. 1000 × 30 30 000 11. 10 × 1802 18 020
12. 5 × 1000 5000 13. 100 × 349 34 900 14. 884 × 10 8840
15. 100 × 560 56 000 16. 100 × 462 46 200
17. 7900 × 10 79 000 18. 1000 × 25 25 000

try this

6 9160 7. 3 723 600 8. 1 104 000 9 482 300
10. 2 289 000 11. 752 000 12 3 681 000 13 12 180 000
14 1 000 000 15. 9 476 000 16 817 200 17 738 270
18 25 480 000 19. 5 408 100 20 615 720 21. 3 888 000

55

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 11-23 on page 329.
- Provide oral exercises that are similar to Ex. 4-11 in the *Try This* feature and ask the students to write only the products.
- The number strips prepared for addition practice in *Related Activities* on page T19 may be used now to generate multiplication exercises for several one-digit factors. Encourage students to use different methods to find a product and compare the results. For example, for the number strip shown below, it is possible to multiply either from left to right or from right to left to group pairs of factors for products of 10 or multiples of 10.

2	8	5	3	9	6	5	4	7

Ask how the product is affected if one of the factors in the number strip is zero.

- Challenge students to complete multiplication chains similar to the following.

$32 \times 10 \times 10 \times 10 \times 10$
$43 \times 10 \times 20 \times 30$
$24 \times 20 \times 20 \times 20 \times 20$
$12 \times 30 \times 30 \times 30 \times 30 \times 30$

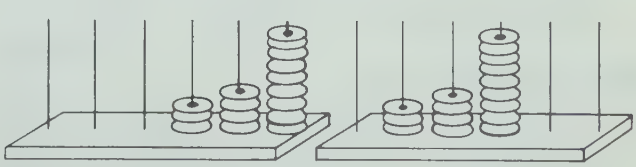
Encourage them to find ways to shorten their work. For example, they may suggest multiplying by 900 once instead of by 30 twice in the last chain.

Ex. 8 and 9 suggest that zeros may be written in the appropriate places of the product first, and then multiplication can continue as if by a one-digit number. Have the students write Ex. 13 in vertical form so that the multiplier is 3000.

Exercises: Remind the students to leave spaces in the products between hundreds and thousands and between hundred thousands and millions.

Try This: The skill of multiplying a number by 10, 100, and 1000 is of particular importance in the work that follows, for example, in relating metric units of measurement. These exercises encourage students to find a pattern and to suggest a rule for multiplying by these numbers. Students who discover patterns on their own and can describe those patterns in their own words are more likely to recall the rules and to apply them. It is recommended that these exercises be discussed with the students when they have completed them. Pay careful attention to exercises such as Ex. 5 and 7 to ensure that the students consider the zeros of both factors, for example, $1\underline{000} \times 32\underline{0} = 32\underline{0}\ \underline{000}$. Ask if it is necessary to copy the factors in order to write the products.

It is helpful for students to see products such as these on an abacus. For Ex. 13, for example, have them show the factor 349 on an abacus and then move the rings to show the product 34 900. In this way, the same number of rings can be seen on corresponding pegs, two places to the left when multiplying by 100.



100 × 349 = 34 900

Assessment

Multiply.

1. 28 × 30 = 840
2. 9120 × 400 = 3 648 000
3. 5000 × 96 480 000
4. 250 × 70 17 500

Pages 56-57

LESSON OUTCOME

Multiply by a two-digit number, multiplicands with up to six digits; solve related word problems; round two factors and multiply to estimate the product, then compare the estimate of the product with the exact product

Vocabulary
day, d

Prerequisite Skills
Multiply by a one-digit number, multiplicands with up to five digits; multiply a number to 9999 by a multiple of ten from 10 to 90; round to the nearest ten, hundred, thousand, or ten thousand

Checking Prerequisite Skills
Multiply.

1. $\begin{array}{r} 648 \\ \underline{8} \\ 5184 \end{array}$ **2.** $\begin{array}{r} 92\,075 \\ \underline{6} \\ 552\,450 \end{array}$

3. $\begin{array}{r} 501 \\ \underline{70} \\ 35\,070 \end{array}$ **4.** $\begin{array}{r} 9380 \\ \underline{90} \\ 844\,200 \end{array}$

Round to the nearest

5. ten. 73 70
6. thousand. 8600 9000
7. ten thousand. 25 314 30 000
8. hundred. 608 600

Multiplying by a Two-Digit Number

Angus delivers 34 newspapers in 1 d (day). He delivered newspapers on 26 d in November. How many newspapers did he deliver in November?

Multiply 26 and 34.

For the product

$\begin{array}{r} 34 \\ \underline{26} \end{array}$ 2 tens 6 ones

you need to know how to multiply 6 and 34.

$\begin{array}{r} {}^{2} \\ 3\ 4 \\ \underline{2\ 6} \\ 2\ 0\ 4 \end{array}$

and how to multiply 2 and 34.

$\begin{array}{r} 3\ 4 \\ \underline{2\ 6} \\ 2\ 0\ 4 \\ 6\ 8\ 0 \end{array}$ 2 tens × 34 = 68 tens or 680.

Then add.

$\begin{array}{r} 3\ 4 \\ \underline{2\ 6} \\ 2\ 0\ 4 \\ \underline{6\ 8\ 0} \\ 8\ 8\ 4 \end{array}$

Angus delivered 884 newspapers in November.

To estimate the product, round each factor and then multiply.

$\begin{array}{rcr} 34 & \longrightarrow & 30 \\ 26 & \longrightarrow & \underline{30} \\ & & 900 \end{array}$

The product is about 900.

Angus delivered about 900 newspapers in November.

56

LESSON ACTIVITY

Before Using the Pages

- State the multiplications 7 × 42 and 30 × 42 and have students write them in vertical form on the board. Have the students find the product for each exercise. Then have them find the sum of the two products. Explain that in this lesson they will review how to combine the three steps into one multiplication exercise to find products such as 37 × 42.

Using the Pages

- Draw attention to the photograph on page 56. Discuss experiences that students have had delivering newspapers. Ask a student to read the word problem. Note the symbol d for days. Discuss the use of multiplication in the solution.

  The worked example demonstrates that multiplication by a two-digit number can be performed by multiplying by the ones as usual, then multiplying by the tens (with care in placing 0 ones in the partial product), and then adding the partial products. Guide the students in a discussion of the steps shown. After completing the last step of the multiplication, develop the solution as shown.

$\begin{array}{r} 34 \\ \underline{\times\ \ 6} \\ 204 \end{array}$ $\begin{array}{r} 34 \\ \underline{\times\ \ 2 \text{ tens}} \\ 68 \text{ tens or } 680 \end{array}$ $\begin{array}{r} 204 \\ \underline{+\ 680} \\ 884 \end{array}$

Compare the work on the board with the multiplication on page 56. Ask a student to read the concluding statement.

- Draw attention to the example below the photograph. Ask students to explain how each number has been rounded. When the students rounded to estimate sums or differences, all the numbers in an exercise were usually rounded to the same place. In multiplication, this is not the case. Tell the students that each factor is rounded so that all but one of the digits are zero. Point out that by rounding factors in this way, basic multiplication facts may be used to estimate products. Emphasize the word ''about'' in the concluding statement.

Working Together: Have the students follow the steps for Ex. 1. Emphasize the need to write 0 in the ones' place of the

Working Together

Multiply by following the steps.

1.

	679
	46
Multiply 6 and 679.	4 074
Write 0 in the ones place. Multiply 4 (tens) and 679.	27 160
Add.	31 234

Estimate each product. Estimates may vary for Ex. 2-7.

2. 389 × 71 28 000
3. 612 × 28 18 000

Estimate each product. Then find the exact product.

4. 825 × 23 = 18 975 (16 000)
5. $1378 × 59 = $81 302 ($60 000)
6. 63 × 7808 491 904 (480 000)
7. 98 × $40 093 $3 929 114 ($4 000 000)

Exercises

Estimate each product. Then find the exact product. Estimates may vary for Ex. 1-26

1. 23 × 17 (400) 391
2. 61 × 49 (3000) 2989
3. 74 × 87 (6300) 6438
4. 98 × 34 (3000) 3332
5. $88 × 66 ($6300) $5808
6. 312 × 24 (6000) 7488
7. 604 × 41 (24 000) 24 764
8. 470 × 72 (35 000) 33 840
9. 587 × 93 (54 000) 54 591
10. $309 × 57 ($18 000) $17 613
11. 2516 × 32 (90 000) 80 512
12. 3709 × 83 (320 000) 307 847
13. 9518 × 54 (500 000) 513 972
14. 2005 × 97 (200 000) 194 485
15. $6917 × 61 ($420 000) $421 937
16. 42 386 × 64 (2 400 000) 2 712 704
17. 90 807 × 71 (6 300 000) 6 447 297
18. 34 006 × 89 (2 700 000) 3 026 534
19. 945 682 × 93 (81 000 000) 87 948 426
20. $408 697 × 56 ($24 000 000) $22 887 032
21. 42 × 608 25 536 (24 000)
22. 54 × 5240 282 960 (250 000)
23. 83 × $3619 $300 377 ($320 000)
24. 68 × 90 554 6 157 672 (6 300 000)
25. 43 × 74 563 3 206 209 (2 800 000)
26. 28 × $480 865 $13 464 220 ($15 000 000)

Use the product in each block as the missing factor in the next.

27.	14 × 18	23 ×	× 19	
	252	5796	110 124	
28.	22 × 17	× 16	× 84	
	374	5984	502 656	

Solve.

29. In October, Angus delivered 34 newspapers each day for 27 d. How many newspapers did he deliver in October? 918

*30. On 6 d each week, Angus delivered 34 newspapers. How many newspapers did he deliver in 52 weeks? 10 608

57

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 24-33 on page 329.
- Have the students estimate the products for Ex. 1-18 on page 53 and then compare the estimates with the exact products obtained earlier for these exercises. Point out that since one factor is a one-digit number, it is necessary to round only the other factor.
- Students having difficulty may benefit from working on squared paper, or lined paper turned sideways, with place-value headings as shown.

hundred thousands	ten thousands	thousands	hundreds	tens	ones
		6	4	0	2
			×	6	3
	1	9	2	0	6
3	8	4	1	2	0
4	0	3	3	2	6

- Have the students find the product of the four numbers in each diagram. This provides an opportunity to review the associative (grouping) property of multiplication, which states that the order in which factors are used in multiplication does not affect the product. Have the students show different groupings on the board.

23	64
36	42

19	51
47	88

second partial product. For Ex. 2-7, have the students round each factor so that all but one of the digits are zero. Explain, however, that a product can be estimated by rounding each factor to any place, and multiplying the rounded numbers. For example, in Ex. 5, 60 × $1400 would give a closer estimate of the exact product than 60 × $1000, but the latter can be completed easily without written work. Compare the exact product and the estimate of the product for each of Ex. 4-7 and review that the estimate can be used as a check for the multiplication.

Exercises: You may wish to have the students use the following format for completing Ex. 1-26. Ensure that an estimate is written before the exact product is found.

	312
	× 24
	1248
	6240
Exact product →	7488
Estimate of product →	$\boxed{6000}$

Have the students compare the estimate and the exact product to check their work. Although an estimate may not appear to be close to the exact product, it really is. In this example, both products show thousands, and the numbers of thousands are close, namely, 6 and 7.

Ensure that the students understand how to complete Ex. 27 and 28. Ex. 30 is starred because the solution requires more than one step. Provide an opportunity for students to show and explain different methods of solution.

Assessment

Estimate each product. Then find the exact product. Estimates may vary.

1. 84 × 66 (5600) 5544
2. 468 × 73 (35 000) 34 164
3. 208 × 95 (20 000) 19 760
4. 14 326 × 38 (400 000) 544 388
5. 91 × 4007 364 637 (360 000)
6. 48 × 715 293 34 334 064 (35 000 000)

Solve.

7. Angus delivered 38 newspapers each day for 23 d in May. How many newspapers did he deliver in May? 874

LESSON OUTCOME

Multiply by a three-digit number, multiplicands with up to four digits

Prerequisite Skills

Multiply by a two-digit number, multiplicands with up to four digits

Checking Prerequisite Skills

Multiply.

1. 98 × 24 = 2352 **2.** 712 × 68 = 48 416 **3.** 5076 × 39 = 197 964

4. 78 × 109 8502

5. 43 × 6574 282 682

Multiplying by a Three-Digit Number

For the product 287 × 365

3 hundreds 6 tens 5 ones

you need to know how to multiply 5 and 287,

```
  4 3
  2 8 7
  3 6 5
1 4 3 5
```

how to multiply 6 and 287,

```
    5 4
    2 8 7
    3 6 5
  1 4 3 5
1 7 2 2 0
```

6 tens × 287 = 1722 tens or 17 220.

and how to multiply 3 and 287.

```
      2 2
      2 8 7
      3 6 5
    1 4 3 5
  1 7 2 2 0
  8 6 1 0 0
```

3 hundreds × 287 = 861 hundreds or 86 100.

Then add.

```
    287
    365
  1 435
 17 220
 86 100
104 755
```

The product of 365 and 287 is 104 755.

For the product 968 × 607

6 hundreds 0 tens 7 ones

you need to know how to multiply 7 and 968.

```
  4 5
  9 6 8
  6 0 7
6 7 7 6
```

how to multiply 0 and 968,

```
9 6 8
6 0 7
6 7 7 6
  0 0
```

0 tens × 968 = 0 tens or 0.

and how to multiply 6 and 968.

```
      4 4
      9 6 8
      6 0 7
    6 7 7 6
5 8 0 8 0 0
```

6 hundreds × 968 = 5808 hundreds or 580 800

Then add.

```
    968
    607
  6 776
580 800
587 576
```

The product of 607 and 968 is 587 576.

58

LESSON ACTIVITY

Before Using the Pages

- Write the following exercises on the board.

842 × 7	842 × 70	842 × 700

Ask how they are alike and how they are different. Then ask how the products are alike and how they are different. Recall that multiplying by a number of tens results in a product with zero ones, and multiplying by a number of hundreds gives a product with zero ones and zero tens. Emphasize that after writing the necessary zeros in the second and third exercises, the multiplication is completed by multiplying 7 and 842. Use other similar examples if necessary.

- Write the first three exercises on the board.

146 × 3	146 × 20	146 × 400	146 × 423

Have the students find the product for each exercise and then find the sum of the three products. Write the fourth exercise on the board and ask the students to suggest what the product would be.

Using the Pages

- The worked example on the left involves a three-digit multiplier without a zero in the tens' place. The worked example on the right involves a three-digit multiplier with zero in the tens' place.

Begin with the multiplication on the left. For each step, read the information above the numerals, focus on the numerals highlighted in red, discuss the sentences in the "thought clouds", and have the students explain the steps in their own words. Review that 365 means 3 hundreds 6 tens 5 ones, or 300 + 60 + 5. Emphasize the need for the zeros in the partial products: when multiplying by ones, the first digit of the product is recorded in the ones' place; when multiplying by tens, zero is recorded in the ones' place and the first digit of the partial product is recorded in

Working Together

Multiply by following the steps.

1.

	809
	814
Multiply 4 and 809.	3 236
Write 0 in the ones place.	8 090
Multiply 1 (ten) and 809.	647 200
Write 0 in the ones place and in the tens place.	658 526
Multiply 8 (hundreds) and 809.	
Add.	

2.

	734
	250
Multiply 0 and 734.	36 700
Multiply 5 (tens) and 734.	146 800
Write 0 in the ones place and in the tens place.	183 500
Multiply 2 (hundreds) and 734.	
Add.	

Multiply.

3. 915 × 902 = 825 330

4. $208 × 670 = $ 139 360

5. 564 × $371 $209 244

Exercises

Multiply. You can check your multiplication by estimating.

1. 298 × 457 = 136 186
2. 601 × 421 = 253 021
3. 765 × 605 = 462 825
4. 192 × 580 = 111 360
5. $309 × 803 = $248 127
6. 894 × 824 = 736 656
7. 713 × 504 = 359 352
8. 641 × 370 = 237 170
9. 739 × 769 = 568 291
10. $165 × 458 = $75 570
11. 625 × 423 264 375
12. 170 × 387 65 790
13. 806 × 690 556 140
14. 245 × $740 $181 300
15. 621 × $961 $596 781
16. 389 × $908 $353 212
17. 476 × 9287 4 420 612
18. 365 × 4071 1 485 915
19. 6234 × 7819 48 743 646

Multiply to find the hidden bingo. A bingo is three products in a row, column, or diagonal.

283 845	282 705	260 664
260 604	261 778	252 456
253 044	252 054	264 024

20. 705 × 401
21. 942 × 268
22. 852 × 297
23. 684 × 386
24. 513 × 508
25. 803 × 326

20 282 705 21 252 456
22 253 044 23 264 024
24 260 604 25 261 778

59

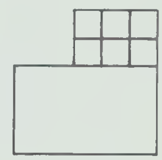
RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 34-38 on page 329.
- Have students find the hidden bingo for the following information.

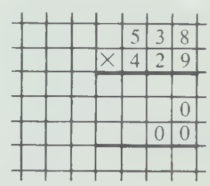
303 136	201 696	204 040
554 400	204 060	303 096
313 130	544 400	244 241

692 × 438 346 × 905
382 × 528 825 × 672
467 × 523 716 × 285

- Provide an opportunity for students to practice basic multiplication facts by playing the game "Product Search" on page T 380.
- Adapt the game "Greatest Sum" on page T 379 for a multiplication game called "Greatest Product". Have students use charts similar to the following.



- Students having difficulty may find it helpful to work multiplication exercises provided on squared paper. Show the necessary zeros in the partial products for a few exercises.



the tens' place; when multiplying by hundreds, zero is recorded in the ones' place and in the tens' place, and then the first digit of the partial product is recorded in the hundreds' place. Use a similar procedure for the second worked example.

To help students understand this example, you may wish to explain it another way. Write the following exercise on the board. Ask which partial product does not affect the sum.

968	
× 607	
6 776	← 7 × 968
0 000	← 0 tens × 968
580 800	← 6 hundreds × 968
587 576	← Add.

Establish that the product for 0 tens and 968 need not be written at all. Multiplication by 6 hundreds can be performed immediately after 7 × 968. To multiply by 6 hundreds, zeros are needed in the ones' place and the tens' place of the second partial product.

Working Together: Ex. 1 and 2 identify the steps for completing the multiplication. Have students give similar statements in an oral explanation of their work for Ex. 3-5. Note that there is a zero in the ones' place of the multiplier in Ex. 2. Point out that the double arrow emphasizes that the product for 5 tens and 734 is written on the same line as the product for 0 ones and 734.

Exercises: Encourage the students to check the products for several exercises by estimating. Ensure that they understand what is required to find the hidden bingo. By matching the six products for Ex. 20-25 with the corresponding numbers in the chart, they will find the three numbers that form the bingo. They may find the hidden bingo without having to complete all six exercises, if they choose to complete the exercises in a different order.

Assessment

Multiply.

1. 149 × 825 = 122 925
2. 673 × 906 = 609 738
3. 4032 × 750 = 3 024 000
4. 501 × 438 219 438
5. 689 × 407 280 423

Pages 60-61

OBJECTIVE

Demonstrate competence in multiplying and in estimating products; solve related word problems

Practice

First, estimate the product without doing any work on paper. Then multiply and compare the product with your estimate. Estimates may vary

1. 732 (4200) 6 4392	2. 8054 (800 000) 99 797 346	3. 63 745 (480 000) 8 509 960	4. 403 (20 000) 50 20 150	5. 46 (4500) 94 4324
6. 128 (6000) 58 7424	7. 607 (540 000) 935 567 545	8. 861 (540 000) 600 516 600	9. 293 (270 000) 903 264 579	10. 485 (250 000) 520 252 200

11. 70 × 9217	12. 43 × 1834	13. 6 × 600 056
14. 400 × 6127	15. 907 × 290	16. 5200 × 60
17. 4932 × 6410	18. 500 × 500	19. 729 × 182
20. 18 × 908	21. 917 × 658	22. 400 × 80
23. 43 × 47 286	24. 89 × 741 664	25. 624 × 2637
26. 248 × 9172	27. 3872 × 5796	28. 7 × 7 030 463

Write only the products.

29. 10 × 6018 60 180	30. 726 × 100 72 600	31. 1000 × 2840 2 840 000
32. 5569 × 1 5569	33. 1000 × 879 879 000	34. 4330 × 100 433 000
35. 0 × 7642 0	36. 6800 × 10 68 000	37. 1 × 2870 2870
38. 2392 × 100 239 200	39. 3761 × 0 0	40. 4200 × 1000 4 200 000

You can change the order of the factors to check your multiplication.

```
   429        87
    87       429
 3 003       783
34 320     1 740
37 323    34 800
          37 323
```

If this result does not match the first result, there is a mistake in your work.

try this

Check your multiplication by changing the order of the factors for Exercises 6, 7, 12, 21 above.

For Exercise 12, which order involved less work?

```
1834
  43
```

11. 645 190 (630 000)	12. 78 862 (80 000)	13. 3 600 336 (3 600 000)
14. 2 450 800 (2 400 000)	15. 263 030 (270 000)	16. 312 000 (300 000)
17. 31 614 120 (30 000 000)	18. 250 000 (250 000)	19. 132 678 (140 000)
20. 16 344 (18 000)	21. 603 386 (630 000)	22. 32 000 (32 000)
23. 2 033 298 (2 000 000)	24. 66 008 096 (63 000 000)	25. 1 645 488 (1 800 000)
26. 2 274 656 (1 800 000)	27. 22 442 112 (24 000 000)	28. 49 213 241 (49 000 000)

60

LESSON ACTIVITY

Using the Pages

- Ex. 1-28 provide practice in estimating products and in multiplying. Remind the students to check this work by comparing the estimates and the exact products. Ex. 29-40 provide practice in multiplication when one of two factors is 1, 10, 100, or 1000.

  Before the students begin Ex. 41-48, discuss the pictographs on page 61. Review the meaning of each symbol and half of a symbol. Ask questions such as the following.

  "What numbers would you multiply to find how many flyers Nicole delivered each week?" (4 and 16)

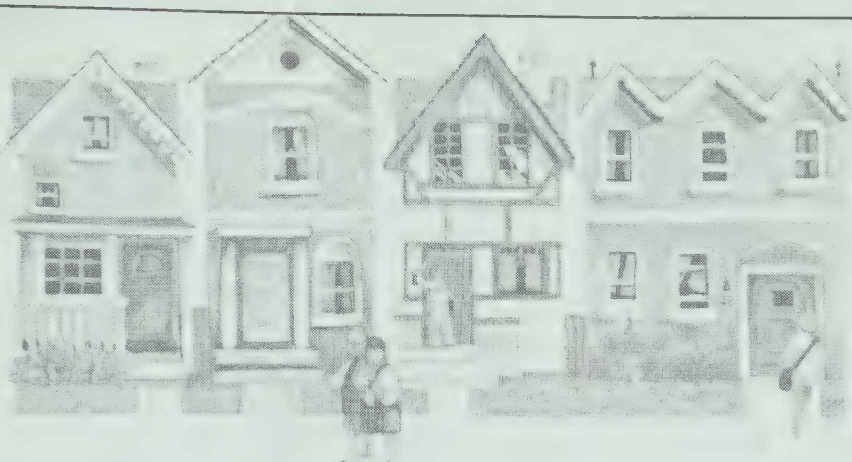
  "How would you find the number of weeks that Ina delivered flyers?" (3 × 2 + 1)

  "How would you find how many flyers Lewis delivered altogether?" (2 × 16 + 8) × (5 × 2)

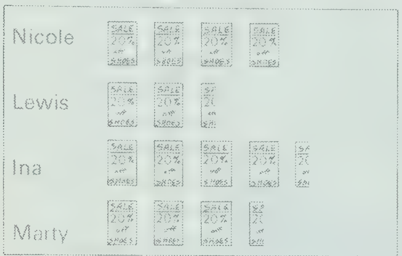
  The students may enjoy sharing the problems that they make up for Ex. 48.

Try This: The example reviews the procedure of checking multiplication by reversing the order of the factors and multiplying. This is possible because multiplication is commutative; that is, the order of multiplying two factors does not affect the product. However, the order can affect the number of partial products, and thus the amount of written work required to complete a multiplication. The students will encounter this when they show two methods for Ex. 12.

```
   1834        43
 ×   43    × 1834
  5 502       172
 73 360     1 290
 78 862    34 400
           43 000
           78 862
```

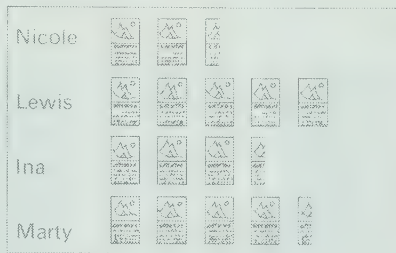


This pictograph shows how many flyers each student delivered in one week.



Each stands for 16 flyers.

This pictograph shows how many weeks each student delivered flyers.



Each stands for 2 weeks.

41. How many flyers did Lewis deliver in 1 week? 40

42. How many flyers did Marty deliver in 2 weeks? 112

43. How many flyers did Nicole deliver in all? 320

44. How many more flyers did Ina deliver than Lewis? 104

45. How many fewer flyers did Nicole deliver than Marty? 184

46. Which 2 students delivered the same number of flyers? Ina and Marty

47. How many flyers did the four students deliver altogether? 1728

48. Make up and solve a word problem for multiplication using information in the pictographs. Answers will vary.

61

RELATED ACTIVITIES

- Have the students prepare two pictographs or two bar graphs related to an activity similar to the one on page 61. Display the graphs and have the students write and solve word problems about them. You may wish to provide the word problems for the students to solve.
- You may wish to demonstrate the procedure for checking multiplication known as "casting out nines". An example is given below.

$$\begin{array}{r} 6342 \\ \times \quad 92 \\ \hline 12\,684 \\ 570\,780 \\ \hline 583\,464 \end{array}$$

Step 1. For the factor 6342,
$6 + 3 + 4 + 2 = 15$,
and $1 + 5 = 6$.

Step 2. For the factor 92,
$9 + 2 = 11$,
and $1 + 1 = 2$.

Step 3. For the results obtained in steps 1 and 2,
$2 \times 6 = 12$,
and $1 + 2 = 3$.

Step 4. For the product 583 464,
$5 + 8 + 3 + 4 + 6 + 4 = 30$, and $3 + 0 = 3$.

Step 5. Compare the results of steps 3 and 4. If they are equal, the multiplication exercise is likely correct.

Students may use this procedure to check exercises on this page and on other pages of this unit.

Pages 62-63

LESSON OUTCOME

Use multiplication to divide, divisors and quotients to 9, remainders; solve related word problems

Materials

counters (optional)

Vocabulary

divide, division, ÷, $\overline{)}$, divisor, dividend, quotient, remainder

Prerequisite Skills

Complete the basic multiplication facts; write the missing factor in a multiplication fact

Checking Prerequisite Skills

Multiply.

1. $\begin{array}{r} 5 \\ \underline{5} \end{array}$ 25
2. $\begin{array}{r} 8 \\ \underline{4} \end{array}$ 32
3. $\begin{array}{r} 7 \\ \underline{6} \end{array}$ 42
4. $\begin{array}{r} 2 \\ \underline{9} \end{array}$ 18
5. 9×8 72
6. 7×3 21

Complete.

7. $3 \times \square = 12$ 4
8. $8 \times \square = 56$ 7
9. $4 \times \square = 36$ 9
10. $7 \times \square = 7$ 1

Using Multiplication to Divide

Kim has 20 crests for 4 hats.
She wants the same number on each.
How many are there for each hat?

Divide 20 by 4.

For $4\overline{)20}$

think $4 \times 5 = 20$.

Write
$$\begin{array}{r} 5 \\ 4\overline{)20} \\ \underline{20} \\ 0 \end{array}$$
5 — crests for each hat
20 — crests used
0 — crests remaining

There are 5 crests for each hat.

Mark has 23 crests for 3 hats.
He wants the same number on each.
How many are there for each hat?

Divide 23 by 3.

For $3\overline{)23}$

think $3 \times 7 = 21$
$3 \times 8 = 24$ . . . too great!

Write
$$\begin{array}{r} 7 \\ 3\overline{)23} \\ \underline{21} \\ 2 \end{array}$$
7 — crests for each hat
21 — crests used
2 — crests remaining

There are 7 crests for each hat and 2 crests left over.

62

LESSON ACTIVITY

Before Using the Pages

- Present a few oral exercises to suggest the concept of sharing equally and to enable students to recall that it is associated with the operation of division. For example, tell the students that 30 dots are to be drawn in 5 rows so that each row shows the same number of dots. Ask how many dots there will be in each row. Some students may draw diagrams or use counters to find the answer. Others may think of a multiplication fact to find the answer. Still others may recall the use of division and write a division exercise to express the information. Have students demonstrate these ways of answering the question. Discuss the inefficiency of methods that involve diagrams or counters. Lead the students to suggest that thinking of multiplication is a more efficient approach; for instance, $5 \times \square = 30$ (5 times what number equals 30) is a related multiplication sentence. Write the sentence "30 divided equally by 5 is 6" on the board. Have students help to translate it into a number sentence by replacing the underlined words with the appropriate mathematical symbols ($30 \div 5 = 6$). Then ask for another way to show the same division to obtain the format $\begin{array}{r} 6 \\ 5\overline{)30} \end{array}$. Elicit the words *division* and *divide* from the students. Note that each form of the division is read "thirty divided by five equals six." Emphasize that the concept of sharing equally is associated with division. Ask whether 26 dots can be drawn in 4 rows with the same number of dots in each row.

Using the Pages

- The worked examples review the partitive (sharing) aspect of division, division terminology, and the concept of remainders. The use of multiplication to divide is emphasized and the written format of the divisions suggests the standard form for division.

  Briefly discuss the photograph to introduce the word problems. Review that division is used because the problems involve sharing equally. Question the students as

To check Mark's division,

think 3 groups of 7 crests or 3×7, and 2 left over.

quotient → 7 R2 ← **remainder**

divisor → $3\overline{)23}$ ← **dividend**

$$\begin{array}{r} 7 \text{ R}2 \\ 3\overline{)23} \\ \underline{21} \\ 2 \end{array}$$

2 ← **remainder**

Write $3 \times 7 = 21$

$21 + 2 = 23$

To check a division, multiply the divisor and the quotient. Then add the remainder.

If the result does not match the dividend, there is a mistake in your work.

Working Together

Give the multiplication fact that you would use to find the quotient.

Example: For $6\overline{)52}$, think

$6 \times 8 = 48$

$6 \times 9 = 54$ . . . too great!

Use $6 \times 8 = 48$.

1. $4\overline{)31}$ 4×7=28
2. $8\overline{)75}$ 8×9=72
3. $49 \div 7$ 7×7=49

Divide. Show the quotient and the remainder. Then check.

4. $4\overline{)25}$ 6 R1
5. $10 \div 7$ 1R3
6. $2\overline{)19}$ 9 R1
7. $42 \div 6$ 7
8. $9\overline{)83}$ 9 R2
9. $4 \div 1$ 4

Exercises

Divide. Write the remainder beside the quotient when the remainder is not 0. Check the ones that you found difficult.

1. $6\overline{)36}$ 6
2. $2\overline{)17}$ 8R1
3. $5\overline{)49}$ 9R4
4. $7\overline{)9}$ 1R2
5. $3\overline{)29}$ 9R2
6. $9\overline{)72}$ 8
7. $8\overline{)47}$ 5R7
8. $4\overline{)38}$ 9R2
9. $1\overline{)9}$ 9
10. $1\overline{)1}$ 1
11. $13 \div 2$ 6R1
12. $61 \div 7$ 8R5
13. $3 \div 2$ 1R1
14. $40 \div 5$ 8
15. $7 \div 4$ 1R3
16. $64 \div 8$ 8
17. $25 \div 6$ 4R1
18. $85 \div 9$ 9R4

Solve.

19. Max has 15 badges to give to 8 friends. If each receives the same number, how many does each receive? 1 How many are left over? 7

20. Sue had 36 crests. She sewed the same number on each of 7 jackets. How many are on each? 5 How many are left over? 1

63

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 39-48 on page 329.
- Because of the relationship between multiplication and division, review basic multiplication facts as well as basic division facts with games, written drill, and oral drill.
- Prepare number wheels and tables from copies of page T391 for practice in writing missing factors and completing division facts.

÷	7
21	3
28	
63	

- Students having difficulty in understanding the concept of division will benefit from using counters to demonstrate division exercises.
- Provide the students with division exercises similar to each of the following. Have them complete each exercise and then write a word problem that would require the exercise for the solution. Students can check their results by multiplying and adding.

$7\overline{)7}$ (a quotient of 1)
$1\overline{)6}$ (a divisor of 1)
$4\overline{)3}$ (a dividend less than the divisor)
$6\overline{)0}$ (a dividend of 0)

you guide them through the examples. For instance, ask, "How many crests are there in all?" "How many hats are there?" "What multiplication fact helps to find the number of crests for each hat?" "How many crests are used?" "What operation is used to find how many crests are left over?" Note that two multiplication facts are named for the second example. Discuss the reasons for this and point out that the subtraction step reveals that 2 crests are left over.

- Draw attention to the division terms and the method for checking division shown at the top of page 63. Have students explain the meaning of *divisor*, *dividend*, *quotient*, and *remainder*. Then have them identify these in the division exercises on page 62 and describe how to check the divisions.

Working Together: Ex. 1-3 deal with the skill of relating a multiplication fact to each division. In the example provided, ask why $6 \times 8 = 48$ is used instead of $6 \times 9 = 54$. Have the students follow the format shown in the worked examples to complete Ex. 4-9. To do this, some division exercises must be rewritten using the symbol $\overline{)}$.

Exercises: After the students have completed the exercises, you may wish to ask for the multiplication fact used for each division. Discuss the result of dividing by one as in Ex. 9 and 10. (The dividend and the quotient are the same.) For Ex. 19 and 20, remind the students to write a statement to answer each question.

Assessment

Divide.

1. $5\overline{)15}$ 3
2. $3\overline{)14}$ 4R2
3. $6\overline{)46}$ 7R4
4. $8\overline{)48}$ 6
5. $4\overline{)31}$ 7R3
6. $35 \div 7$ 5
7. $17 \div 2$ 8R1
8. $67 \div 9$ 7R4

Solve.

9. Steve earned 36 crests in 4 years. If he earned the same number each year, how many crests did he earn each year? 9

Pages 64-65

LESSON OUTCOME

Divide by a one-digit number, dividends with up to five digits, zero as only the last digit in the quotient; solve related word problems

Materials

models for thousands, hundreds, tens, and ones

Prerequisite Skills

Use multiplication to divide, divisors and quotients to 9, remainders; extend basic multiplication facts to multiples of 10, 100, 1000, and 10 000; regroup with numbers to 99 999

Checking Prerequisite Skills

Divide.

1. 5)20 4
2. 7)52 7 R3
3. 8)62 7 R6
4. 54 ÷ 9 6
5. 35 ÷ 6 5 R5

Complete.

6. 4 × 7 tens = 28 tens
7. 6 × 8 hundreds = 48 hundreds
8. 9 × 5 ten thousands = 450 000
9. 7 × 3 thousands = 21 000
10. 5 hundreds 3 tens = 53 tens
11. 8 thousands 0 hundreds = 80 hundreds
12. 4 tens 5 ones = 45 ones

Dividing by a One-Digit Number

Each team is made of 9 cards.
Michael has 306 cards.
How many teams does he have?

Divide 306 by 9. 9)306

306 shows 3 hundreds. Since 3 is less than 9, think of 3 hundreds 0 tens as 30 tens. Then divide the 30 tens.

9 × 3 = 27
9 × 4 = 36 . . . too great!

Use 9 × 3 tens = 27 tens.

Write
```
    3
9)306
  27
   3
```

Think of the 3 tens 6 ones that remain as 36 ones.

```
    3
9)306
  27↓
   36
```

Then divide the 36 ones.

9 × 4 = 36

Write
```
   34
9)306
  27
   36
   36
    0
```

Michael has 34 pictures.

64

LESSON ACTIVITY

Before Using the Pages

- Write the division 4)96 on the board and ask what name is given to the 4 and what term identifies the 96. Ask for the number of tens and the number of ones in the dividend. Have a student use models to represent 96. Review that division is associated with sharing equally. Have four students demonstrate the division 4)96 by sharing the tens first and then the ones, to obtain the quotient 24. Pay particular attention to the regrouping of 1 ten as 10 more ones. Then develop the corresponding division on the board, asking questions similar to the following.
  "What is divided first?"
  "What multiplication fact gives the number of tens in the quotient?" "How many tens are left?"
  "How can the remaining ten be divided?"
  "How many ones are there to divide in all?"
  "What multiplication fact gives the number of ones in the quotient?" "How many ones are left?"

Use a similar procedure with other division exercises such as 5)160 and 3)1284. Emphasize the use of multiplication to obtain digits of the quotient.

Using the Pages

- Ask a student to read the word problem at the top of page 64. The example involves measurement (quotitive) division because the number in each group is known (9). Explain that the cards referred to in the problem have been prepared by cutting pictures into 9 cards each. Discuss how the solution involves the use of division to find the number of groups of 9 in 306.

  Question the students as you lead them through the steps of the solution. Ask why the first digit of the quotient is written in the tens' place rather than in the hundreds' place. Then ask, "If the first digit of the quotient is written in the tens' place, how many digits will there be in the final quotient?" Emphasize the use of multiplication to obtain digits of the quotient. Point out that arrows are not to be drawn in the solution; they illustrate which digit of the

Working Together

Complete.

1. 6)4376 — 72 R — 42, 17, 12, 56 — 9, 2 — 54, 2
2. 5)1948 — 38 R — 15, 44, 40 — 9, 3 — 48, 45, 3
3. 8)93 921 — 1 R — 8, 1 3 — 1740, 1 — 8, 59, 56, 32, 32, 01

To check the division, multiply the divisor and the quotient. Then add the remainder.

Divide. Then check your work.

4. 3)78 26
5. 2)400 200
6. 2)1381 690R1
7. $61 024 ÷ 8 $7628

Exercises

Divide. Check your work for five difficult exercises.

1. 2)37 18R1
2. 5)89 17R4
3. 6)549 91R3
4. 1)4672 4672
5. 3)803 267R2
6. 8)5600 700
7. 9)3242 360R2
8. 7)7000 1000
9. 5)6752 1350R2
10. 4)9320 2330
11. 7)$434 $62
12. 9)$12 078 $1342
13. 1361 ÷ 2 680R1
14. 4908 ÷ 5 981R3
15. 24 788 ÷ 3 8262R2
16. 37 272 ÷ 4 9318
17. 88 475 ÷ 7 12 639R2
18. 89 765 ÷ 6 14 960 R5
19. 1597 ÷ 5 319R2
20. 6920 ÷ 7 988R4
21. 12 740 ÷ 4 3185
22. 9453 ÷ 2 4726R1
23. $3492 ÷ 9 $388
24. $5460 ÷ 7 $780

Each group of numbers forms a division. Write the division.

Example: For 4, 26, 6, 2, write 4)26 6 R2

25. 7, 6264, 6, 894 7)6264 894R6
26. 3, 4, 1269, 5079 4)5079 1269R3
27. 1788, 5, 4, 8944 5)8944 1788R4

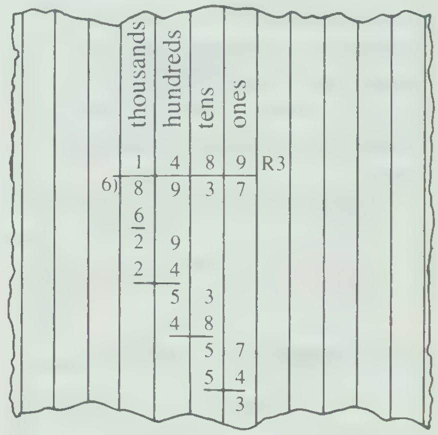
Solve.

28. Adele has 240 cards. Each picture is made of 8 cards. How many pictures does she have? 30
29. There are 132 cards. As many as possible are dealt to 7 players. How many does each get? 18 How many are left over? 6

65

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 49-52 on page 329.
- Students having difficulty with place value in division may benefit from completing division exercises as shown below. Have them turn their lined paper sideways to provide the lines for columns.



- Have the students write and solve word problems that involve interpreting the remainder for some of the exercises on pages 63 and 65. Emphasize the relationship between the question that is asked and the meaning of the remainder. For example, state that there are 27 apples packed in boxes of 6. Ask the following questions.
"How many boxes are full of apples?" (4)
"How many apples are left over after the full boxes are packed?" (3)
"How many boxes are needed to pack all the apples?" (5)

dividend is required at each step. Also, note the importance of aligning the digits in the division process with those of the dividend. Ask a student to read the concluding statement. Then have the students use multiplication to check the division.

Working Together: Ex. 1-3 provide partially completed division exercises. It is important to discuss the completed steps before continuing the division. For Ex. 4-7, ask the students to determine the number of digits there will be in each quotient before they begin the division. For example, in Ex. 6, the first digit of the quotient will be in the hundreds' place, and thus the quotient must have three digits. Similarly, the quotient in Ex. 7 must have four digits. Since Ex. 7 involves an amount of money, tell the students that the symbol $ is recorded at the left of the dividend and at the left of the quotient.

Exercises: Encourage the students to use multiplication (and addition) to check their work. The exercises with amounts of money (Ex. 11, 12, 23, 24) do not have remainders and they involve only dollars, not dollars and cents.

Assessment

Divide.

1. 5)210 42
2. 7)23 903 3 414 R5
3. 9)6191 687 R8
4. 11 674 ÷ 6 1945 R4
5. 5360 ÷ 8 670

Solve.

6. 106 cards are dealt at the beginning of a game. Each of 9 players receives the same number of cards. How many cards does each player receive? 11 How many cards are left over? 7

LESSON OUTCOME

Divide by a one-digit number, dividends with up to five digits, zeros in one or more places in the quotient

Materials

models for hundreds, tens, and ones

Prerequisite Skills

Divide by a one-digit number, dividends with up to five digits

Checking Prerequisite Skills

Divide.

1. 6)318 — 53
2. 8)5143 — 642 R7
3. 2613 ÷ 9 — 290 R3
4. 32 814 ÷ 7 — 4687 R5

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 53-56 on page 329.
- You may wish to teach short division as described below for 4)693.

Divide 6 hundreds.	1
Regroup the remaining 2 hundreds with 9 tens.	4)6₂9 3
Divide 29 tens.	1 7
Regroup the remaining 1 ten with 3 ones.	4)6₂9₁3
Divide 13 ones and write the remainder in the usual way.	1 7 3 R1 4)6₂9₁3

Have the students practice short division with the exercises on page 66 and compare the quotients.

Zeros in the Quotient

Sometimes zeros are needed in the quotient as place holders.

Divide 3042 by 6.

Divide 30 hundreds.	Show the tens that remain.	Divide 4 tens.	Show the ones that remain.	Divide 42 ones.
5 6)3042 30 0	5 6)3042 30↓ 04	50 6)3042 30 04	50 6)3042 30 ↓ 042	507 6)3042 30 042 42 0

3042 ÷ 6 = 507.

The zero is needed in the quotient to show 3042 divided by 6 is 507. 3042 divided by 6 is not 57.

Working Together

Give the next digit for each quotient.

1. 3)1803 — 60 (18, 00)
2. 8)56 621 — 707 (56, 0 62)
3. 7)70 680 — 10 0 (7, 00 6)

Divide.

4. 5)20 150 — 4 030
5. 8)8483 — 1060 R3
6. 41 209 ÷ 2 — 20 604 R1
7. 18 075 ÷ 9 — 2008 R3

Exercises

Divide.

1. 2)10 064 — 5 032
2. 7)42 718 — 6 102 R4
3. 9)72 557 — 8 061 R8
4. 4)93 623 — 23 405 R3
5. 8)40 487 — 5 060 R7
6. 5)102 019 — 20 403 R4
7. 3)211 505 — 70 501 R2
8. 8)72 247 — 9 030 R7
9. 81 010 ÷ 9 — 9001 R1
10. 35 440 ÷ 7 — 5062 R6
11. 40 009 ÷ 4 — 10 002 R1
12. 35 045 ÷ 5 — 7009
13. 60 015 ÷ 3 — 20 005
14. 61 830 ÷ 6 — 10 305
15. 80 600 ÷ 2 — 40 300
16. 72 727 ÷ 8 — 9090 R7

66

LESSON ACTIVITY

Before Using the Pages

- Write the exercises 3)163, 3)715, and 3)615, on the board. For each exercise, have students tell the place value of the first digit of the quotient, explain how this is known, tell the number of digits there will be in the quotient, and explain how this is determined without actually performing the division.

  Ask students to complete the first two division exercises. Then assign the division 3)615. Use models to find the quotient, emphasizing that in each of the 3 equal groups, there are 0 tens. Discuss the importance of writing 0 in the tens' place. Use multiplication to check the quotient 205.

Using the Pages

- The worked example shows and explains the steps in a division for which there are 0 tens in the quotient. Have a student read the statement at the top of the page. Point out that the thousands' digit of the dividend is 3 and the divisor is 6. Ask what the place value will be of the first digit of the quotient and how many digits there will be in the quotient. Lead the students through the example, emphasizing the place-by-place aspect of division. Draw attention to the statements in the "thought cloud". Have students complete the multiplications 6 × 57 and 6 × 507 to check that 507, not 57, is the correct quotient.

Working Together: Have a few students show and explain their work on the board.

Exercises: Emphasize how to determine the number of digits in a quotient prior to the division process, and the use of multiplication and addition to check division.

Assessment

Divide.

1. 3)611 — 203 R2
2. 9)38 708 — 4 300 R8
3. 8)48 424 — 6 053
4. 12 029 ÷ 4 — 3007 R1
5. 60 948 ÷ 7 — 8706 R6

Finding the Average

4 pieces of wood were used for 1 kite, 8 were used for another, and 2 for each of 2 other kites. What was the **average** number of pieces of wood for each kite?

Add 4, 8, 2, and 2. (the numbers in each group)

Then divide the sum by 4. (the number of groups)

$$4 + 8 + 2 + 2 = 16$$

$$16 \div 4 = 4$$

The average number of pieces of wood for each kite was 4.

Working Together

What number would you divide by to find the average?

1. 3, 9 2
2. 7, 4, 6, 0, 8 5

Find the average.

3. 365 in 5 groups 73
4. 1674 in 9 groups 186
5. 7, 7, 7, 7 7
6. 796, 796, 1421, 0, 15, 8 506
7. \$72, \$64, \$137 \$91

Exercises

Copy and complete.

1.

Number in all	2568	5616	7063	? 2715
Number of groups	6	8	7	3
Average	? 428	? 702	? 1009	905

Find the average.

2. 280 in 8 groups 35
3. 6264 in 4 groups 1566
4. 6, 5, 0, 9, 5 5
5. 4, 4, 4, 4, 4, 4, 4, 4, 4 4
6. \$72, \$86, \$137, \$582, \$638 \$303
7. 759 km, 973 km 866 km
8. 1728 kg, 4936 kg, 3725 kg 3463 kg
9. 7 mm, 8 mm, 9 mm, 1 mm, 5 mm 6 mm

Solve.

10. 432 m of string were used for 8 kites. What was the average length of string for each kite? 54 m
11. 24 m² (square metres) of material were used for 8 kites. What was the average amount of material for each? 3 m²

67

Page 67

LESSON OUTCOME

Find the average for a set of numbers; solve related word problems

Materials

16 counters, a metre stick

Vocabulary

average, square metres, m²

Prerequisite Skills

Add more than two addends; divide by a one-digit number

Checking Prerequisite Skills

Add.

1. 764 + 325 + 893 + 80 2062
2. 67 + 92 + 86 + 75 + 58 378

Divide.

3. $8\overline{)344}$ 43
4. $6\overline{)4020}$ 670

RELATED ACTIVITIES

- Help the students find the temperature or the rainfall in various areas for a month and then calculate the average temperature or rainfall for that month.
- Have students help one another to measure their heights. Then have them find the average height of a student, the number of students whose heights are above the average, and the number of students whose heights are below the average.

LESSON ACTIVITY

Before Using the Page

- Display a group of 3 counters and a group of 5 counters. Ask the students how the counters can be arranged so that the number of groups remains the same and each group contains the same number of counters. Have students arrange the counters to show the two groups of 4 counters.
  Repeat this procedure with groups of 5, 6, and 4 counters and ask students to place them in three equal groups. Then ask for a way to find the number in each of the three groups without using the counters.

Using the Page

- Read the word problem and discuss the meaning of *average*. The average is the number in each group if the number of groups remains the same and each group contains the same number of items. Discuss the worked example, noting that the average can be found by using addition and then division. Ask how it is known what number is the divisor.

Working Together: Point out that 0 in Ex. 2 represents one of the groups; thus, the sum is divided by 5, not by 4. For Ex. 3 and 4, the sum is given and it is necessary only to divide to find the average. Since the items for Ex. 7 are amounts of money, explain that the average will also be an amount of money and will require the symbol \$.

Exercises: Remind the students to include the units of measurement in the averages for Ex. 6-11. Note that the symbol m² is used to represent "square metres".

Assessment

Find the average.

1. 24, 89, 63, 72 62
2. 807, 928, 463, 272, 800 654
3. 6483, 9212, 5091, 6073, 6849, 2148 5976

Solve.

4. 8 students made 32 kites to sell at the school fair. What was the average number of kites made by each student? 4

LESSON OUTCOME

Divide by a multiple of ten from 10 to 90, dividends with up to five digits

Prerequisite Skills

Divide by a one-digit number, dividends with up to five digits; multiply a one-digit number by a multiple of ten from 10 to 90

Checking Prerequisite Skills

Divide.

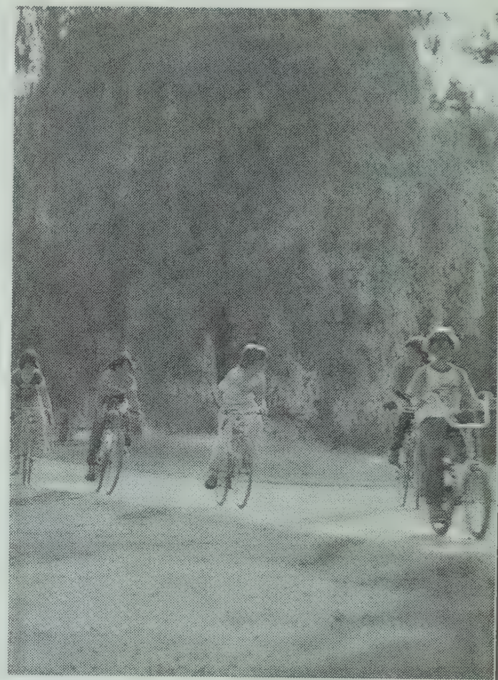
1. $4\overline{)2732}$ 683
2. $8\overline{)33\,612}$ 4 201 R4
3. 23 117 ÷ 6 3852 R5
4. 81 320 ÷ 9 9035 R5

Multiply.

5. 30 × 5 150
6. 60 × 7 420
7. 40 × 9 360
8. 80 × 8 640

Dividing by Multiples of 10

80 students rode 1920 km altogether in a bike-athon. What was the average distance each student rode?



Divide 1920 by 80.

1920 shows 1 thousand.

1920 shows 19 hundreds.

Since 19 is less than 80, think of 19 hundreds 2 tens as 192 tens. Then divide the 192 tens.

80 × 2 = 160
80 × 3 = 240 . . . too great!

Use 80 × 2 tens = 160 tens

Write

$$\begin{array}{r} 2 \\ 80\overline{)1920} \\ \underline{160} \\ 32 \end{array}$$

Think of the 32 tens 0 ones that remain as 320 ones.

$$\begin{array}{r} 2 \\ 80\overline{)1920} \\ \underline{160}\downarrow \\ 320 \end{array}$$

Then divide the 320 ones.

80 × 4 = 320

Write

$$\begin{array}{r} 24 \\ 80\overline{)1920} \\ \underline{160} \\ 320 \\ \underline{320} \\ 0 \end{array}$$

The average distance each student rode was 24 km.

68

LESSON ACTIVITY

Before Using the Pages

- Write examples on the board to review place value in numerals similar to the following.

$\boxed{1}$2 0 0	1 thousand	$\boxed{1}$6 0 0 0	1 ten thousand
$\boxed{1\ 2}$0 0	12 hundreds	$\boxed{1\ 6}$0 0 0	16 thousands
$\boxed{1\ 2\ 0}$0	120 tens	$\boxed{1\ 6\ 0}$0 0	160 hundreds

- Have students suggest numbers to make true sentences for examples similar to the following. Have them focus on the zeros in the factors and the products, and apply basic multiplication facts.

$$\begin{array}{l} 4\ 0 \times 9 = \underline{\qquad} \\ 4\boxed{0} \times \underline{\qquad} = 3\ 6\boxed{0\ 0} \\ 4\boxed{0} \times \underline{\qquad} = 3\ 6\boxed{0\ 0\ 0} \end{array}$$

$$\begin{array}{l} 7\ 0 \times 6 = \underline{\qquad} \\ 7\boxed{0} \times \underline{\qquad} = 4\ 2\boxed{0\ 0} \\ 7\boxed{0} \times \underline{\qquad} = 4\ 2\boxed{0\ 0\ 0} \end{array}$$

- Write the following exercises on the board.

$4\overline{)920}$ $\qquad$ $40\overline{)920}$

Ask how the exercises are alike and how they are different. Have students tell the place value of the first digit in the quotient and the number of digits in the quotient for each exercise. For example, for $40\overline{)920}$, 9 (the number of hundreds in the dividend) is less than 40 (the divisor), but 92 (tens) is greater than 40. Thus, the first digit of the quotient will be in the tens' place and the quotient will have two digits. Have students draw a square frame for each digit there will be in the quotient for each exercise.

$\begin{array}{r} \square\square\square \\ 4\overline{)9\ 2\ 0} \end{array}$ $\qquad$ $\begin{array}{r} \square\square \\ 40\overline{)9\ 2\ 0} \end{array}$

Using the Pages

- Have a student read the word problem at the top of page 68. Point out that the addition step has been completed, as indicated by the total distance of 1920 km for the 80 students. Discuss that only the division step is required to find the average.

Working Together

Round to the nearest ten.

1. 36 40 2. 47 50 3. 32 30 4. 75 80

Complete.

5. 10)43 860 — quotient 4 38 6
40
3 8
3 0
86
80
60
60
0

6. 40)32 597 — quotient 81 4 R 37
32 0
59
40
197
160
37

7. 80)73 924 — quotient 9 24 R 4
72 0
192
160
324
320
4

Divide.

8. 20)3760 188 9. 50)52 360 1 047 R10 10. 27 005 ÷ 60 450 R5

Exercises

Divide. Check the difficult ones.

1. 30)2910 97 2. 40)3880 97
3. 10)8740 874 4. 70)7000 100
5. 80)95 472 1 193 R32 6. 60)85 963 1 432 R43
7. 90)92 006 1 022 R26 8. 50)57 650 1 153
9. 10)6730 673 10. 40)7904 197 R24
11. 36 500 ÷ 50 730 12. 56 002 ÷ 80 700 R2
13. 4356 ÷ 60 72 R36 14. 76 521 ÷ 10 7652 R1
15. 55 000 ÷ 90 611 R10 16. 97 362 ÷ 70 1390 R62

Complete each in three different ways.
Answers will vary

17. 40) R 18. 70) R25

Work inside the parentheses first. Then work from left to right.

1. (23 × 714) ÷ 7 2346
2. 23 × (714 ÷ 7) 2346
3. 48 400 ÷ (8 × 10) 605
4. (48 400 ÷ 8) × 10 60 500
5. (14 760 ÷ 60) ÷ 6 41
6. 14 760 ÷ (60 ÷ 6) 1476
7. 68 × (73 × 84) 416 976
8. (68 × 73) × 84 416 976
9. 794 × (31 900 ÷ 50) 506 572
10. 36 360 ÷ (20 × 2) 909

try this

69

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 57-60 on page 329.
- Have the students find the missing digits in the following exercises.

$$\begin{array}{r} 6\boxed{2}\boxed{5}\ \text{R8} \\ 60\overline{)3\boxed{7}\boxed{5}0\boxed{8}} \\ \underline{\boxed{3}\boxed{6}\boxed{0}} \\ 15\boxed{0} \\ \underline{\boxed{1}\boxed{2}\boxed{0}} \\ \boxed{3}\boxed{0}\boxed{8} \\ \underline{300} \\ \boxed{8} \end{array}$$

$$\begin{array}{r} \boxed{3}0\boxed{1}\boxed{9}\ \text{R}\boxed{5} \\ 20\overline{)603\boxed{8}\boxed{5}} \\ \underline{\boxed{6}\boxed{0}} \\ 0\boxed{3}8 \\ \underline{\boxed{2}\boxed{0}} \\ \boxed{1}\boxed{8}5 \\ \underline{\boxed{1}\boxed{8}\boxed{0}} \\ \boxed{5} \end{array}$$

$$\begin{array}{r} 2\boxed{8}\boxed{6}\ \text{R}\boxed{2}8 \\ 90\overline{)\boxed{2}576\boxed{8}} \\ \underline{\boxed{1}\boxed{8}\boxed{0}} \\ \boxed{7}\boxed{7}\boxed{6} \\ \underline{\boxed{7}20} \\ 56\boxed{8} \\ \underline{\boxed{5}\boxed{4}\boxed{0}} \\ \boxed{2}\boxed{8} \end{array}$$

- Prepare tables for students to practice division, using copies of page T391. Division by 10 is especially interesting because of the relationship between the dividend and the quotient.

Lead the students through the steps of the solution, paying particular attention to the statements that explain why the division starts with 192 tens, not with 1 thousand or 19 hundreds. This explains why the first digit of the quotient is in the tens' place. Have the students work the same division on their own and check their work with the worked example. They may also use multiplication to check their work. Then point out that all the division exercises on page 69 have divisors that are multiples of ten.

Working Together: Ex. 1-4 prepare students for rounding two-digit divisors to the nearest ten for estimating a quotient in the following lesson. For Ex. 5-7, discuss the steps given and then have the students complete the divisions. The grey shapes guide the students by suggesting the number of digits in each numeral. Note that the remainder in Ex. 5 is zero. Pay particular attention to Ex. 8-10: Ex. 8 and 9 involve zero ones in the dividend; Ex. 9 and 10 involve zeros in the quotient.

Exercises: After they have completed the exercises, you may wish to have the students compare the dividend and the quotient for Ex. 3, 9, and 14, to note the result of dividing by 10. Students may enjoy comparing their answers for Ex. 17 and 18.

Try This: You may wish to have the students turn to page 31 for the *Try This* feature involving addition, subtraction, and parentheses. Review that the work inside parentheses is completed first and that the remaining work is completed from left to right. The same procedure will apply in these exercises involving multiplication and division. When the students have completed the exercises, have them compare the answers for the following pairs of exercises: Ex. 1 and 2; Ex. 3 and 4; Ex. 5 and 6; Ex. 7 and 8. Ex. 7 and 8 present an opportunity to review the associative property of multiplication, which states that the way three or more factors are grouped does not affect the product.

Assessment

Divide.

1. 60)4045 67 R25 **2.** 40)17 230 430 R30 **3.** 80)22 809 285 R9
4. 68 342 ÷ 10 6834 R2 **5.** 95 670 ÷ 90 1063 **6.** 21 430 ÷ 70 306 R10

LESSON OUTCOME

Divide by a two-digit number when the trial estimates for digits in the quotient are correct, dividends with up to six digits; solve related word problems

Prerequisite Skills

Divide by a multiple of ten from 10 to 90, dividends with up to five digits; regroup for numbers to 999 999

Checking Prerequisite Skills

Divide.

1. $20\overline{)926}$ 46R6
2. $70\overline{)63\ 150}$ 902 R10
3. 55 187 ÷ 80 689 R67
4. 85 920 ÷ 60 1432
5. 33 358 ÷ 90 370 R58

Complete.

6. 1 hundred 7 tens = 17 tens
7. 8 thousands 2 hundreds = 82 hundreds
8. 5 hundred thousands 6 ten thousands = 56 ten thousands

Dividing by a Two-Digit Number

Divide 4489 by 62.

For $62\overline{)4489}$, think of $60\overline{)4489}$.

62 rounded to the nearest ten is 60.

4489 shows 4 thousands 4 hundreds or 44 hundreds
Since 44 is less than 60, think of
44 hundreds 8 tens as 448 tens.
Then divide the 448 tens.

60 × 7 = 420

60 × 8 = 480 . . . too great!

Use 62 × 7 tens = 434 tens.

Write

$$\begin{array}{r} 7 \\ 62\overline{)4489} \\ 434 \\ \hline 14 \end{array}$$

Think of the 14 tens 9 ones that remain as 149 ones.

$$\begin{array}{r} 7 \\ 62\overline{)4489} \\ 434\!\downarrow \\ \hline 149 \end{array}$$

Then divide the 149 ones.

60 × 2 = 120

60 × 3 = 180 . . . too great!

Use 62 × 2 = 124.

Write

$$\begin{array}{r} 72 \text{ R25} \\ 62\overline{)4489} \\ 434 \\ \hline 149 \\ 124 \\ \hline 25 \end{array}$$

4489 ÷ 62 = 72 R25

70

LESSON ACTIVITY

Before Using the Pages

- Write the division exercise $20\overline{)3658}$ on the board. Have students tell the place value of the first digit of the quotient and the number of digits in the quotient. Have the students complete the division and ask a student to show and explain her/his work on the board.

$$\begin{array}{r} 182 \text{ R18} \\ 20\overline{)3658} \\ 20\!\downarrow \\ \hline 165 \\ 160\!\downarrow \\ \hline 58 \\ 40 \\ \hline 18 \end{array}$$

Write the exercise $21\overline{)3658}$ on the board and have the students note that it is very similar to the exercise $20\overline{)3658}$. Since the dividends are equal and the divisors differ only by one, suggest that the quotients must be close in value. Ask whether the first digit of the quotient for $21\overline{)3658}$ will likely be in the thousands' place, the hundreds' place, or the tens' place. For $21\overline{)3658}$, 3 (thousands) in the dividend is less than 21, the divisor, and thus the division begins with 36 hundreds. Ask the students what the hundreds' digit of the quotient will probably be for $21\overline{)3658}$. They will likely suggest 1, since it was the hundreds' digit of the quotient for $20\overline{)3658}$. Complete the exercise on the board with the students. However, to obtain each digit in the quotient, think of the divisor 21 as 20.

A

$$\begin{array}{r} 17 \\ 21\overline{)3658} \\ 21\!\downarrow \\ \hline 155 \end{array}$$

(Think of the divisor as 20 to obtain 7 in the quotient.)

B

$$\begin{array}{r} 174 \\ 21\overline{)3658} \\ 21 \\ \hline 155 \\ 147\!\downarrow \\ \hline 88 \end{array}$$

(Think of the divisor as 20 to obtain 4 in the quotient.)

C

$$\begin{array}{r} 174 \text{ R4} \\ 21\overline{)3658} \\ 21 \\ \hline 155 \\ 147\!\downarrow \\ \hline 88 \\ 84 \\ \hline 4 \end{array}$$

Working Together

Complete.

1.
```
    3 16 3 R 22
31)98 075
   93
    5 0
    3 1
    1 97
    1 86
      115
       93
       22
```

2.
```
     1 2 7 R 70
75)92 095
   75
   17 0
   150
    209
    150
     595
     525
      70
```

Divide.

3. 13)143 11
4. 89)28 876 324 R40
5. 8917 ÷ 22 405 R7
6. 30 220 ÷ 56 539 R36

Exercises

Divide. Check your work for five difficult exercises.

1. 23)69 3	2. 78)93 1 R15	3. 61)85 1 R24	4. 18)60 3 R6
5. 11)583 53	6. 42)438 10 R18	7. 29)700 24 R4	8. 68)372 5 R32
9. 53)4289 80 R49	10. 87)6498 74 R60	11. 67)4245 63 R24	12. 33)1462 44 R10
13. 34)23 970 705	14. 51)25 908 508	15. 48)79 662 1 659 R30	16. 78)98 092 1 257 R46

17. 54 064 ÷ 19 2845 R9 18. 38 186 ÷ 72 530 R26 19. 63 320 ÷ 89 711 R41

20. 268 630 ÷ 44 6105 R10 21. 129 300 ÷ 27 4788 R24 22. 642 137 ÷ 79 8128 R25

Solve.

23. When 21 scouts cleaned a conservation area, they found 525 cans. What was the average number of cans for each scout? 25

24. 24 scouts collected 264 kg of garbage. What was the average amount of garbage that each scout collected? 11 kg

25. 31 scouts gathered 124 bags of garbage. What was the average number of bags of garbage gathered by each scout? 4

71

RELATED ACTIVITIES

• You may wish to demonstrate the procedure for checking division known as "casting out nines". Have students use the procedure to check division exercises on this page and on other pages of this unit. The steps are shown below for the division given in the worked example of page 70.

$$62\overline{)4489}\quad 72 \text{ R}25$$

Step 1. For the divisor 62, 6 + 2 = 8.

Step 2. For the quotient 72, 7 + 2 = 9.

Step 3. For the remainder 25, 2 + 5 = 7.

Step 4. For the above results, divisor × quotient + remainder
= (8 × 9) + 7
= 72 + 7
= 79.
For 79,
7 + 9 = 16.
For 16,
1 + 6 = 7.

Step 5. For the dividend 4489,
4 + 4 + 8 + 9 = 25
and 2 + 5 = 7.

If this result equals the result obtained in step 4, the division exercise is correct.

Summarize that 20 is a multiple of ten, and that it is helpful in finding the quotient when dividing by 21, which is not a multiple of ten.

Using the Pages

• Rounding the divisor to the nearest ten helps to find a trial estimate for each digit of the quotient. Emphasize that the rounded divisor is used only to find the trial estimate for the quotient; the exact divisor is used to complete each step in the division. Ask why 62 is rounded down to 60 rather than up to 70. Have students explain why the first digit of the quotient is in the tens' place. Review that this implies that the quotient will have two digits. Point out that the rounded divisor 60 is used twice in the solution: once to determine the digit for dividing 434 tens, and again for dividing 124 ones.

Working Together: Before the students begin, have them name the rounded divisor for each exercise, tell the place value of the first digit of the quotient, and tell the number of digits in the quotient.

Exercises: By applying the procedure outlined on page 70, students can find the correct digits for the quotient. Exercises for which the first estimate may not be correct appear in the next lesson. Remind the students to show their work for Ex. 23-25 and write a concluding statement for each.

Assessment

Divide.

1. 18)827 45 R17 **2.** 47)3289 69 R46 **3.** 81)1459 18 R1

4. 276 934 ÷ 92 3010 R14 **5.** 70 352 ÷ 69 1019 R41 **6.** 684 960 ÷ 32 21 405

Solve.

7. 23 scouts collected 483 bottles. What was the average number of bottles collected by each scout? 21

Pages 72-73

LESSON OUTCOME

Divide by a two-digit number when trial estimates for digits in the quotient are incorrect, dividends with up to six digits; round and multiply to estimate a quotient, then compare the estimate of the quotient with the quotient obtained by division

Prerequisite Skills

Divide by a two-digit number when the trial estimates for digits in the quotient are correct, dividends with up to six digits; round to the nearest ten, hundred, thousand, or ten thousand

Checking Prerequisite Skills

Divide.

1. 43)965 — 22 R19
2. 87)13 506 — 155 R21
3. 78)7200 — 92 R24
4. 22)27 390 — 1 245
5. 603 971 ÷ 91 — 6637 R4
6. 284 247 ÷ 59 — 4817 R44

Round to the nearest

7. ten. 74 — 70
8. hundred. 6893 — 6900
9. thousand. 21 106 — 21 000
10. ten thousand. 337 407 — 340 000

Dividing by a Two-Digit Number

Divide 2528 by 32.

For 32)2528, think of 30)2528.

For 30)2528, think of 2528 as 252 tens 8 ones. Then divide the 252 tens.

30 × 8 = 240
30 × 9 = 270 . . . too great!

Try using 32 × 8 tens.

32 × 8 tens = 256 tens

```
     8
32)2528
   256
```

Cannot subtract 256 from 252.

Use 32 × 7 tens.

32 × 7 tens = 224 tens

```
     7
32)2528
   224
    28
```

Then complete the division.

```
    79
32)2528
   224
    288
    288
      0
```

2528 ÷ 32 = 79

Divide 5504 by 67.

For 67)5504, think of 70)5504.

For 70)5504, think of 5504 as 550 tens 4 ones. Then divide the 550 tens.

70 × 7 = 490
70 × 8 = 560 . . . too great!

Try using 67 × 7 tens.

67 × 7 tens = 469 tens

```
     7
67)5504
   469
    81
```

greater than the divisor

Use 67 × 8 tens.

67 × 8 tens = 536

```
     8
67)5504
   536
    14
```

Then complete the division.

```
    82 R10
67)5504
   536
    144
    134
     10
```

5504 ÷ 67 = 82 R10

72

LESSON ACTIVITY

Before Using the Pages

- Write the division 43)3342 on the board. Ask for the place value of the first digit of the quotient and then ask what that digit would be. Students using the method presented in the previous lesson would round 43 to 40 and use 40 × 8 = 320 (40 × 9 = 360 is too great.) Thus, the response likely would be 8. Continue with the division by multiplying 43 (the divisor) and 8 (the first digit of the quotient) to allow the students to discover the difficulty that arises in the subtraction step.

```
     8
43)3342
   344
     ?
```

Although the divisor was rounded correctly, discuss that the estimate obtained by thinking of 40 × 8 = 320 led to a subtraction that is not possible. Ask what can be done to overcome this difficulty.

Using the Pages

- The worked examples illustrate the two kinds of situations that can arise in estimating digits for the quotient, using the procedure described on page 70. In each case, the first trial digit is not correct but is close to the correct digit for the quotient.

  The example on the left shows the case for which the trial estimate (8) multiplied by the divisor (32) results in a number that is too great for the necessary subtraction (252 − 256). The next step is to try a lesser trial estimate for the quotient.

  The example on the right shows the case for which the trial estimate (7) multiplied by the divisor (67) gives a number that results in a difference (81) greater than the divisor after the subtraction. The next step is to try a greater trial estimate for the quotient.
- Begin with the worked example on the left. Have a student explain why the first digit of the quotient is in the tens' place. Ask questions similar to the following.
  "How is the trial divisor obtained?"

Rounding the divisor and the dividend can help you estimate the quotient.

For 67)5504, round 67)5504 → 70)5500

For 70)5500, think

7 or 8? . . . 8 is closer!
7)55

Use 8 in the quotient.

7 × 8 = 56

the rounded divisor — 70 × 8 = 560

70 × 80 = 5600 — 5600 is close to 5500.

80 is an estimate for 5504 ÷ 67.

Working Together

Complete.

1. 72)2447 — 3▢ R▢ — 216, 287, ▢ — 3, 71; 216, 71
2. 57)2570 — 4▢ R▢ — 228, ▢, ▢ — 5, 5; 290, 285, 5

Give the rounded divisor.
Then give the rounded dividend.

3. 44)9720 — 40; 9700
4. 23)43 170 — 20; 43 000

Estimate each quotient. Estimates may vary
Then divide to find the quotient.

5. 84)5620 — 66 R76 (70)
6. 94)$75 012 — $798 ($800)

Exercises

Estimate each quotient.
Then divide to find the quotient. Estimates may vary

1. 63)542 — 8 R38 (9)
2. 38)278 — 7 R12 (7)
3. 76)2300 — 30 R20 (30)
4. 51)3632 — 71 R11 (70)
5. 28)9356 — 334 R4 (300)
6. 12)4789 — 399 R1 (400)
7. 89)38 270 — 430 (400)
8. 42)25 153 — 598 R37 (600)
9. 47)19 923 — 423 R42 (400)
10. 33)25 986 — 787 R15 (800)
11. 94)63 700 — 677 R62 (700)
12. 72)31 639 — 439 R31 (400)
13. 84 010 ÷ 23 — 3652 R14 (4000)
14. 79 102 ÷ 92 — 859 R74 (900)
15. $24 087 ÷ 37 — $651 ($600)
16. 478 057 ÷ 68 — 7030 R17 (7000)
17. 39 285 ÷ 46 — 854 R1 (800)
18. $346 995 ÷ 55 — $6309 ($6000)

73

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 61-68 on page 329.
- The procedure for estimating a quotient described on page 73 may be adapted for division with one-digit divisors and for divisors that are multiples of 10. Have the students estimate the quotients for exercises on pages 65, 66, 69, and 71, and compare the estimates with the quotients for the corresponding exercises.
- For more practice, have the students label each of these estimates as too large, too small, or correct. Then have them complete the division.

8 64)493	5 73)361
6 45)312	1 16)39
9 52)473	5 85)523
4 94)366	3 58)235
8 37)337	7 24)153
3 65)270	6 53)309
8 44)347	4 76)338
6 83)546	2 31)60
5 27)164	9 92)825
4 66)341	5 14)58

"How can you tell that 8 is not the correct tens' digit of the quotient?"
"What digit would you try next as the tens' digit?"
"How can you tell that 7 is the correct tens' digit?"
"How can the rounded divisor help to find the ones' digit of the quotient?"

Discuss the second worked example in a similar way. If necessary, develop the exercise on the board to draw particular attention to the difference, 81, which is greater than the divisor, 67. Demonstrate that continuing the division would necessitate writing 12 in the ones' place of the quotient. Thus, the trial digit 7 (tens) is increased to 8 (tens) to avoid this.

- For the method of estimating quotients shown at the top of page 73, the divisor is rounded to the nearest ten and the dividend is rounded so that there are two digits other than zero at the left of the numeral. This suggests a basic division fact, $7\overline{)55}$ with quotient $\square$ in this case. The one-digit quotient that is closest to the missing factor for the divisor and the dividend is found. The multiplication fact 7 × 8 = 56 is closer than 7 × 7 = 49 for the division 7)55. Multiplying the first factor of 7 × 8 = 56 by 10 gives 70 × 8 = 560, and thus the rounded divisor, 70, is obtained. Multiplying the second factor of 70 × 8 = 560 by 10 gives 70 × 80 = 5600. Multiplying the second factor (the estimate of the quotient) by 10 continues until the product is close to the rounded dividend.

Working Together: Discuss the steps given for Ex. 1 and 2, and then have the students complete the exercises. Point out that estimates can be used to check division exercises.

Exercises: Have the students check their work by comparing their estimates and the quotients obtained by division.

Assessment

Estimate each quotient. Then divide to find the quotient. Estimates may vary.

1. 56)2398 — (40) 42 R46
2. 34)29 706 — (1000) 873 R24
3. 83)240 943 — (3000) 2 902 R77
4. 48 270 ÷ 65 — 742 R40 (700)
5. 145 890 ÷ 47 — 3104 R2 (3000)
6. 63 352 ÷ 94 — 673 R90 (700)

LESSON OUTCOME

Divide by a three-digit number, dividends with up to five digits; solve related word problems

Prerequisite Skills

Divide by a two-digit number, dividends with up to five digits; round a three-digit number to the nearest hundred; multiply by multiples of one hundred from 100 to 900

Checking Prerequisite Skills

Divide.

1. 76)4786 62 R74
2. 54)26 463 490 R3
3. 55 330 ÷ 93 594 R88
4. 22 496 ÷ 37 608

Round to the nearest hundred.

5. 387 400
6. 912 900

Multiply.

7. 400 × 3 1200
8. 800 × 7 5600
9. 600 × 1 600
10. 300 × 9 2700

Dividing by a Three-Digit Number

On "Trees for Canada" Day, 205 scouts planted 12 710 trees. What was the average number of trees planted by each scout?

For 205)12 710, think of 200)12 710.

For 200)12 710, think of 12 710 as 1271 tens 0 ones. Then divide the 1271 tens.

200 × 6 = 1200

200 × 7 = 1400 . . . too great!

Use 205 × 6 tens.

```
      6
205)12 710
    12 30
       41
```

Think of the 41 tens that remain as 410 ones.

```
      6
205)12 710
    12 30↓
      410
```

Then divide the 410 ones.

```
     62
205)12 710
    12 30
      410
      410
        0
```

The average number of trees planted by each scout was 62.

LESSON ACTIVITY

Before Using the Pages

- Write the division 395)2463 on the board. Ask how the divisor in this exercise differs from those in previous division exercises. Review that rounding a two-digit divisor to the nearest ten helped to estimate digits of the quotient. Ask what can be done to help estimate digits of the quotient when the divisor has three digits. Students will likely suggest that rounding the divisor to the nearest hundred would be helpful (400)2463). Have a student explain how to determine the place value of the first digit of the quotient. You may wish to show the following on the board.

400)2463: 2 is less than 400
400)2463: 24 is less than 400
400)2463: 246 is less than 400
400)2463: 2463 is greater than 400

The first digit of the quotient will be in the ones' place. Develop the solution on the board with the students. Point out that thinking of the basic multiplication fact 4 × 6 = 24 for 400 × 6 = 2400 led to 6 as the trial estimate for the quotient. Note that the trial estimate is correct because the subtraction step is possible and the remainder, 93, is less than the divisor, 395.

```
     6 R93
395)2463
    2370
      93
```

Using the Pages

- Begin with a brief discussion of the photograph. Some of the boys in your class may have participated in a tree-planting activity as scouts. Ask a student to read the word problem. Because the problem requires finding an average, note that division is used in the solution.

Working Together

Complete.

1. $395\overline{)24\,687}$ quotient: 6 2 R 197
   23 70
   987
   790
   197

2. $823\overline{)73\,684}$ quotient: 8 9 R 437
   6584
   7844
   7407
   437

Divide.

3. $200\overline{)19\,886}$ 99 R86
4. $104\overline{)10\,812}$ 103 R100
5. $34\,270 \div 745$ 46
6. $82\,893 \div 634$ 130 R473

Exercises

Complete three exercises in each box. Write the result for the fourth one by using the pattern.

1. $122\overline{)860}$ 7 R6	2. $452\overline{)6340}$ 14 R12	3. $642\overline{)13\,500}$ 21 R18	4. $897\overline{)25\,140}$ 28 R24
5. $300\overline{)19\,225}$ 64 R25	6. $610\overline{)45\,170}$ 74 R30	7. $493\overline{)41\,447}$ 84 R35	8. $212\overline{)19\,968}$ 94 R40
9. $102\overline{)10\,710}$ 105	10. $499\overline{)53\,493}$ 107 R100	11. $385\overline{)42\,165}$ 109 R200	12. $648\overline{)72\,228}$ 111 R300
13. $251\overline{)12\,560}$ 50 R10	14. $782\overline{)78\,300}$ 100 R100	15. $523\overline{)78\,460}$ 150 R10	16. $453\overline{)90\,700}$ 200 R100

17. Make up a fifth exercise to fit each of the patterns above. Answers will vary

18. Each of 345 scouts planned to plant the same number of trees. They planned to plant 17 250 trees in all. How many did each scout plan to plant? 50

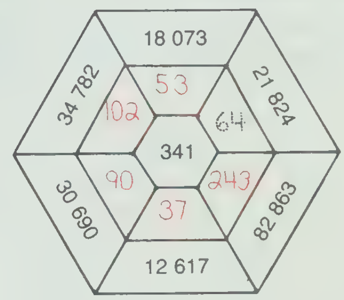
19. Four scout troops have 31 members each. They planted 8308 trees in all. What was the average number of trees planted by each scout? 268

75

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 69-76 on page 329.
- Challenge students to create three division exercises that form a pattern similar to Ex. 1-16 on page 75. Have them exchange papers, divide to find the pattern, and use the pattern to write a fourth and fifth division.
- Use copies of page T 391 to prepare hexagons similar to the following for students to complete. Note that each division gives a remainder of zero. Thus, the inverse relationship between multiplication and division can be emphasized. For example,

$$21\,824 \div 64 = 341$$
and
$$64 \times 341 = 21\,824.$$



Ask why the divisor 205, is rounded down to 200. Review that a rounded divisor helps to obtain trial estimates of the quotient. Have a student explain why the first digit of the quotient is in the tens' place, to emphasize that the dividend, 12 710, is thought of as 1271 tens 0 ones. Discuss each step of the worked example and summarize that dividing by a three-digit number is similar to dividing by a two-digit number.

Working Together: Have the students explain the steps given for Ex. 1 and 2 and then complete the divisions. Note that Ex. 4 involves 0 tens in the quotient and it is necessary to correct the trial estimate for the ones' digit.

Exercises: Remind the students that it may be necessary to adjust the trial estimates for a quotient. You may wish to have them check the results for the fourth exercise in each pattern by dividing or by multiplying and adding. After the students have completed the exercises, encourage them to discuss how they answered Ex. 17.

Assessment

Divide.

1. $684\overline{)15\,732}$ 23
2. $208\overline{)13\,370}$ 64 R58
3. $587\overline{)64\,483}$ 109 R500
4. $99\,345 \div 476$ 208 R337
5. $58\,830 \div 830$ 70 R730
6. $42\,891 \div 923$ 46 R433

Solve.

7. 295 scouts will plant 17 110 trees. Each scout will plant the same number of trees. How many trees will each scout plant? 58

OBJECTIVE

Demonstrate competence in dividing and in estimating quotients; solve related word problems

Vocabulary

hectare, ha

Practice

First, estimate the quotient without doing any work on paper. Then divide and compare the quotient with your estimate.

Estimates may vary

1. 10)4382 438 R2 (440)
2. 63)5150 81 R47 (80)
3. 39)9506 243 R29 (200)
4. 21)$840 $ 40 ($40)
5. 640)78 932 123 R212 (100)
6. 40)956 23 R36 (20)
7. 457)531 1 R74 (1)
8. 100)$52 300 $ 523 ($520)
9. 300)47 286 157 R186 (150)
10. 76)76 152 1002 (1000)
11. 90)3600 40 (40)
12. 510)$40 800 $ 80 ($80)
13. 671)107 360 160 (100)
14. 71)71 071 1001 (1000)
15. 84)480 040 5714 R64 (6000)
16. 70)$28 420 $ 406 ($400)
17. 13 635 ÷ 65 209 R50 (200)
18. 82 606 ÷ 59 1400 R6 (1400)
19. $7000 ÷ 14 $500 ($500)
20. 106 827 ÷ 98 1090 R7 (1100)
21. 728 009 ÷ 208 3500 R9 (3600)
22. $46 556 ÷ 452 $103 ($90)

Five students are playing a game. Each player rolls a die. The number on the die shows how many blocks a player can move. Then the player gives the quotient for the exercise in the block reached. If the quotient is correct, the player places a marker in that block.

Locate each player's marker.

	Player	Last correct quotient
23.	Alison	46
24.	Bert	20
25.	Carol	2001
26.	Dennis	210
27.	Earl	104

Start	47 24)1128	14 62)868
140 92)12 880	2001 15)30 015 Carol	870 40)34 800
450 35)15 750	89 26)2314	46 24)1104 Alison
2 100 22)46 200	2 340)680	2 010 19)38 190
210 73)15 330 Dennis	103 54)5562	121 95)11 495
112 70)7840	44 24)1056	67 92)6164
405 64)25 920	20 34)680 Bert	12 304)3648
Finish	130 84)10 920	104 58)6032 Earl

76

LESSON ACTIVITY

Before Using the Pages

- Write a few division exercises on the board and complete them with the students to review division with two-digit and three-digit divisors.

  35)7210 = 206   64)24 924 = 389 R28   485)27 780 = 57 R135

  Pay particular attention to predicting the number of digits in a quotient, adjusting trial estimates for a quotient, and the occurence of the digit 0 in a quotient.

Using the Pages

- For each of Ex. 1-22, ensure that the students write the estimate first and then find the exact quotient. Encourage them to use the estimates to check their divisions.

  Discuss the rules for the game described above Ex. 23-27. When the students have completed the divisions on the game board, have them indicate the division exercise that corresponds to the last correct quotient for each player.

  Note that the word *hectare* and the symbol ha are introduced at the top of page 77. Ex. 30 and 31 are starred because their solutions require more than one step.

Try This: When the students have finished the exercises, you may wish to have them describe the patterns in an oral discussion. For example, Ex. 1 shows multiples of 4 from 4 to 32, and Ex. 6 shows multiples of 11 from 99 to 22. The numbers in Ex. 7 begin with 52 and increase by 13 each time (multiples of 13 from 52 to 143). Note that the grey shapes guide the students by suggesting the number of digits in each numeral.

- You may wish to have the students investigate one or more of the patterns in Ex. 1 to 12. For example, ask the students to write the first ten numbers of the pattern in Ex. 1. Then have them find the sum of the digits for each number and write the pattern (4, 8, 3, 7, 2, 6, 1, 5, 9, 4). Use copies of

Solve.

28. From 19 ha of land, Bruce's family had 62 415 kg of straw. What was the average amount of straw from each hectare? 3285 kg

29. They grew 98 604 kg of hay on 18 ha. What was the average amount of hay on each hectare? 5478 kg

30. If each of their 4 horses eats 6 kg of hay in one day, in how many weeks would they eat 8736 kg of hay? 52

31. If each of their 43 cattle is fed 2 kg of hay each day, how long would 62 608 kg of hay last? 728 days or 104 weeks

The symbol ha stands for **hectare**.

1 ha = 10 000 m²

Copy and complete the patterns.

1. 4, 8, 12, 16, 20, 24, 28, 32
2. 0, 5, 10, 15, 20, 25, 30, 35
3. 200, 400, 600, 800, 1000, 1200
4. 81, 72, 63, 54, 45, 36, 27
5. 132, 144, 156, 168, 180, 192, 204
6. 99, 88, 77, 66, 55, 44, 33, 22
7. 52, 65, 78, 91, 104, 117, 130, 143
8. 910, 920, 930, 940, 950, 960, 970
9. 168, 152, 136, 120, 104, 88, 72
10. 119, 102, 85, 68, 51, 34, 17, 0
11. 26, 32, 30, 36, 34, 40, 38, 44, 42
12. 15, 21, 26, 30, 33, 35, 36
13. Write three other patterns of numbers. Answers will vary

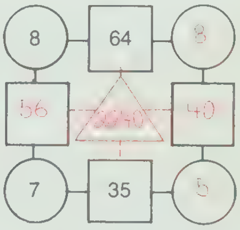
try this

77

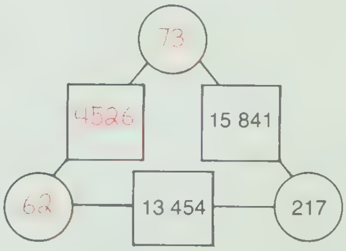
RELATED ACTIVITIES

- Students may enjoy playing the game on page 76. As the player who lands on a block divides to find the quotient, the other players must also divide so that they can check whether the player's answer is correct. To vary the game, have them prepare other game boards with different division exercises.
- For practice in multiplication and division, use copies of page T391 to prepare diagrams similar to the following. Factors are shown in the circles and products are shown in the squares. Note that diagram A suggests a procedure for extending the use of the diagram, namely, multiplying the numbers in opposite squares. If the work is correct, the products are equal. The number may be written in a triangle drawn in the diagram as shown.

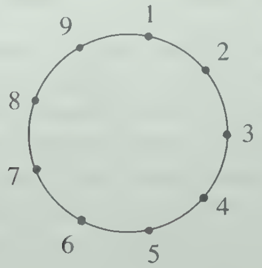
A



B



page T385 to provide each student with a diagram as shown below. Have the students use their straight edges to draw line segments to connect points, following the sequence derived from the sums of the digits. The same procedure can be applied to other number patterns to produce different designs.



Pages 78-79

OBJECTIVE

Prepare a keychart to show the order of pressing two or more of the $\boxed{+}$, $\boxed{-}$, $\boxed{\times}$, $\boxed{\div}$, and $\boxed{=}$ keys on a calculator to solve a problem

Materials

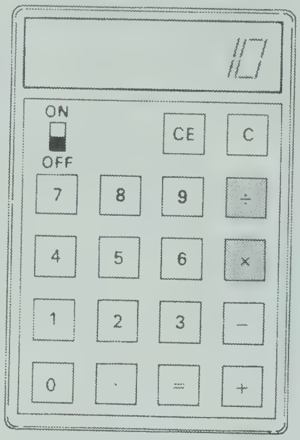
calculators (optional)

Vocabulary

keychart

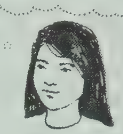
Keycharts and the $\boxed{\times}$ and $\boxed{\div}$ Keys

To use a calculator you need to know the order in which keys should be pressed.



Each of the 10 teams in the softball league plays each of the other 9 teams twice. Then there are 2 play-off games and 1 final game. How many games are there altogether?

Multiply, then multiply again, then add, then add again. . . . $\boxed{\times}$, then $\boxed{\times}$, then $\boxed{+}$, then $\boxed{+}$.



To show which keys to use to solve this problem, Jean made a **keychart**.

10 $\boxed{\times}$ 9 $\boxed{\times}$ 2 $\boxed{+}$ 2 $\boxed{+}$ 1 $\boxed{=}$ ____

10
$\boxed{\times}$
9
$\boxed{\times}$
2
$\boxed{+}$
2
$\boxed{+}$
1
$\boxed{=}$
____

Keycharts may be written up or down.

Keycharts can be different for different calculators. For the calculator you use, are these keycharts correct for finding the result?

78

LESSON ACTIVITY

Using the Pages

- The lesson on pages 32 and 33 emphasizes knowing whether to press the $\boxed{+}$ key or the $\boxed{-}$ key on a calculator. This aspect is extended now to include the $\boxed{\times}$ and $\boxed{\div}$ keys, and a new concept is introduced — knowing the order in which to press the keys.

  Begin with a brief discussion to review the usefulness of a calculator in completing the sequence of operations required to solve a problem. However, emphasize that a calculator can give the correct answer to a problem only if the necessary steps are provided in the correct sequence. Ask a student to read the title of the lesson and the introductory statement at the top of page 78. Draw attention to the two *keycharts* (horizontal and vertical) and note that squares are drawn around the symbols for the operations and the symbol =. Point out that both keycharts indicate that the $\boxed{=}$ key is the last one to be pressed.

Write the two keycharts indicated below on the board and ask whether the calculator would show the same result for each.

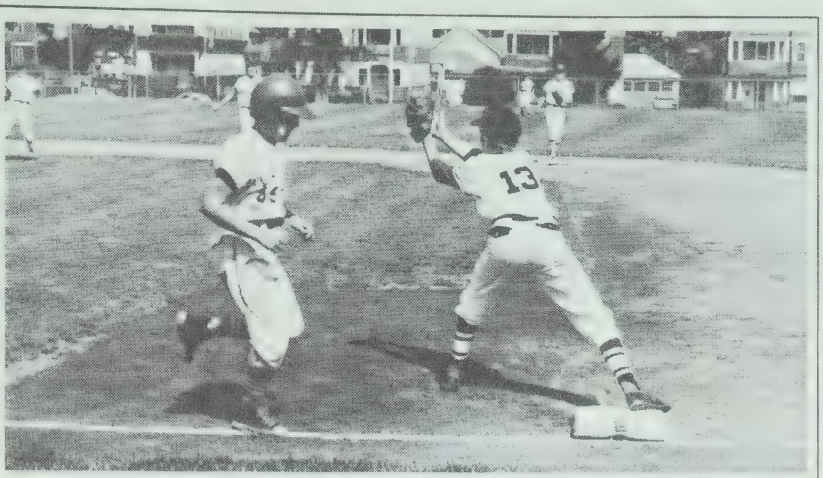
2 $\boxed{\times}$ 8 $\boxed{+}$ 10 $\boxed{=}$ ____

10 $\boxed{+}$ 2 $\boxed{\times}$ 8 $\boxed{=}$ ____

Emphasize that it is necessary to press the keys on a calculator in the correct sequence because the order in which the keys are pressed influences the result.

Write the following problem on the board and ask which of the two keycharts shows how to solve the problem. "There are 8 crayons in a new box. Andy has 2 new boxes of crayons and 10 old crayons. How many crayons does Andy have?"

Return to the example on page 78. Ask a student to read the word problem. Relate the information in the problem to the sequence of steps indicated in the keycharts. Then read and discuss the statements at the bottom of page 78.



Make a keychart that shows how to solve each of these. Keycharts may vary
Answers are also provided for those who complete the solutions

1. Each of the 10 teams in the softball league has 16 players this year. There were 148 players altogether last year. 10 $\boxed{\times}$ 16 $\boxed{-}$ 148 $\boxed{=}$ How many more players are there this year than last? 12

2. Last year the average number of fans for each game was 55. This year the total number of fans for the 183 games was 11 346 $\boxed{\div}$ 183 $\boxed{-}$ 55 $\boxed{=}$ 11 346. How much greater was the average number of fans for each game this year than last? 7

3. The league sold 30 ice cream bars at each of 180 games, 45 at the 2 play-off games, 30 $\boxed{\times}$ 180 $\boxed{+}$ 45 $\boxed{+}$ 45 $\boxed{+}$ 75 $\boxed{=}$ and 75 at the final game. How many were sold in all? 5565

4. 2928 runs were scored in the 183 games this year and 2769 runs last year. 2928 $\boxed{\div}$ 183 $\boxed{=}$ What was the average number of runs per game this year? 16

5. There were 531 home runs this year. Last year there were 594 home runs. How many more home runs were there last year than this? 594 $\boxed{-}$ 531 $\boxed{=}$ 63

6. In 18 games, a team played for 27 h 54 min. What was the average number of minutes for each game? 27 $\boxed{\times}$ 60 $\boxed{+}$ 54 $\boxed{\div}$ 18 $\boxed{=}$ 93

7. The team that won the final game played for 31 h 20 min in 20 games. What was the average 31 $\boxed{\times}$ 60 $\boxed{+}$ 20 $\boxed{\div}$ 20 $\boxed{=}$ number of minutes for each game? 94

Calculator

79

• In Ex. 1-7, the students are required to prepare keycharts that show how to solve the word problems. They are not required to find the solutions. Note that it will be necessary to express in minutes the times given in Ex. 6 and 7.

RELATED ACTIVITIES

• If calculators are available, have the students use them to solve the problems on page 79 or problems and exercises on other pages in this unit.

• Students may enjoy writing keycharts for some of the word problems in Unit 2 or Unit 4.

Page 80

OBJECTIVE

Identify situations that affect answers

RELATED ACTIVITIES

- Ask students to write word problems similar to those on page 80. Then have them discuss the word problems in small groups.
- Students may enjoy dramatizing some of the situations that affect the answers for the word problems on page 80.

Situations That Affect Answers

An answer sometimes depends upon the situation.

Melanie is deciding when to have her party.

If I have the party on Tuesday, it would be on the same day as my birthday.

On Friday or Saturday evening, my friends could stay later.

On the weekend, the party could be during the day.

If I have the party this Saturday, it would be on the same day as the soccer game.

What other situations could affect Melanie's decision?

Different answers are possible for each of these.
Tell how different situations would affect the answers. Answers will vary.

1. At what time will you have the party?
2. Which cake recipe will you use?
3. How high should we make the fence?
4. How many gates should we build?
5. How will we get to the third floor of the building?
6. How many steps would you take in crossing the room?
7. How many stamps will you put on the letter?
8. How many letters will you write to your friends this month?
9. Which colored pencils will you use for the map?
10. How many cans of paint should we buy for this room?
11. Which record will you buy?

PROBLEM SOLVING

80

LESSON ACTIVITY

Before Using the Page

- Begin by asking a question such as "If you were going on a camping trip, what equipment would you take?" Students will likely begin by naming various items, and eventually give a statement such as "It depends." Direct the discussion to describing situations that affect the equipment that would be taken.

Using the Page

- Ask students to read the statements at the top of the page. Then ask them to read the statements describing situations that affect Melanie's decision. Ask students to suggest other situations that might have influenced her decision.
- Because the exercises on this page lend themselves to discussion, it is suggested that answers be considered orally rather than in writing. For large groups of students, guide the discussion and consider the problems in an order determined by the interests of the students. For smaller groups, however, each student has more opportunity to participate in the discussion. Members of a group may take turns reading a problem aloud and leading the discussion. Later, some of the ideas presented in discussions of the small groups may be shared with the rest of the class.

Checking Up

Multiply.

1. 29 × 4 = 116	2. 602 × 8 = 4816	3. 856 × 50 = 42 800	4. 5007 × 81 = 405 567	5. $69 × 96 = $6624
6. 317 × 74 = 23 458	7. 7061 × 29 = 204 769	8. 395 × 205 = 80 975	9. 815 × 840 = 684 600	10. $276 × 563 = $155 388

11. 10 · 2895 28 950
12. 6 · 7403 44 418
13. 400 · $736 $294 400
14. 598 · 937 560 326
15. 361 × 100 36 100
16. 408 · $504 $205 632

Divide.

17. 8)63 7 R7
18. 7)4200 600
19. 20)5683 284 R3
20. 49)$74 480 $1 520
21. 17)39 866 2345 R1
22. 34)61 524 1 809 R18
23. 423)17 018 40 R98
24. 659)$40 858 $62
25. 94 962 ÷ 73 1300 R62
26. 2894 ÷ 30 96 R14
27. $79 870 ÷ 98 $815
28. 65 000 ÷ 500 130
29. 58 292 ÷ 67 870 R2
30. $18 848 ÷ 248 $76

Find the average.

31. 86, 264, 745, 620, 598, 412, 861, 570, 45, 9 421
32. $440, $72, $1561, $838, $500, $60, $9, $732, $258, $192, $13 $425

Solve.

33. Carmen helped her parents plant tomatoes on their farm. They planted 175 rows with 115 plants in each. How many tomato plants were there? 20 125

34. Last summer Sam spent 84 h painting a fence with 336 boards. How many minutes did he spend on each board, if it took him the same amount of time for each? 15

35. During a year, 235 students collected 28 670 kg of paper for recycling. What was the average mass of paper collected by each student? 122 kg

36. Rosemary plans to knit a scarf that has 36 stitches in each row and 375 rows. How many stitches will there be in her scarf? 13 500

81

Page 81

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- For practice with basic multiplication and division facts, prepare a chart similar to the following. Copies of page T 382 can be used.

×			6				7	
							42	
8	72				0			16
			30					
0								
		36				27		
			6			3		
	63			56				

- Have students perform the operations indicated in chains similar to the following.

Start 635 → × 28 (17 780) → ÷ 35 (508) → × 240 (121 920) → ÷ 160 (762) → × 203 (154 686) → ÷ 14 (11 049) → Stop

Skills	Exercises	Related Pages
Multiply by a one-digit number	1, 2, 12	T 58-T 59
Multiply by multiples of 10, 100, 1000	3, 11, 13, 15	T 60-T 61
Multiply by a two-digit number	4-7	T 62-T 63
Multiply by a three-digit number	8-10, 14, 16	T 64-T 65
Divide by a one-digit number	17, 18	T 70-T 72
Divide by a multiple of 10	19, 26	T 74-T 75
Divide by a two-digit number	20-22, 25, 27, 29	T 76-T 79
Divide by a three-digit number	23, 24, 28, 30	T 80-T 81
Find the average	31, 32	T 73
Solve multiplication problems	33, 36	
Solve division problems	34, 35	

Comments

Use of the multiplication and division algorithms involves several skills, some of which are identified as follows.

Multiplication
1. Recall basic multiplication facts
2. Show 0 ones in multiplying by the number of tens
3. Show 0 ones and 0 tens in multiplying by the number of hundreds
4. Regroup in multiplying
5. Add partial products

Division
1. Relate multiplication and division
2. Remember the sequence of steps in long division
3. Subtract with regrouping
4. Round the divisor
5. Obtain trial estimates for digits of the quotient and correct the estimates when necessary

It is important to determine whether most errors in using algorithms are a result of weakness in one particular skill. Appropriate remedial assistance may then be provided.

Encourage the students to check multiplication exercises by changing the order of the factors or by estimating, and to check division exercises by estimating or by multiplying and adding.

Unit 5 Overview

Decimals

A review of one-place and two-place decimals leads to a study of three-place and four-place decimals with emphasis on the names of the place values, especially of the last digit on the right. Forms of decimals with zeros at the right are related to equivalent decimals with fewer places so that comparisons of decimals with different numbers of places may be made. Ordering of decimals is then based upon such comparisons. Addition and subtraction with decimals, and with decimals and whole numbers, re-emphasize the processes and regroupings which also occur with whole numbers. Rounding decimals to any one of several places is presented and is used in estimating sums and differences. Although decimal notation is taken to four decimal places, the majority of the exercises involve practice with up to three decimal places. The *Problem Solving* lesson focuses on two items in measurements: the suitability of the unit of measurement, and the correct position of a decimal point to make the measurement reasonable.

Prerequisite Skills

- compare and order whole numbers
- add and subtract whole numbers
- round whole numbers

Unit Outcomes

- read and write numerals and words for one-place decimals and for two-place decimals
- read and write numerals for three-place decimals and for four-place decimals; interpret place value in three-place decimals and in four-place decimals
- write equivalent decimals using up to four decimal places
- compare two decimals (to ten-thousandths)
- order decimals (to ten-thousandths)
- add from two to five decimals (to ten-thousandths), addends with different numbers of decimal places
- add decimals (to ten-thousandths) and whole numbers, from two to five addends
- subtract decimals (to ten-thousandths) with different numbers of decimal places; use addition to check subtraction
- subtract decimals (to ten-thousandths) and whole numbers
- round decimals with up to four places to the nearest one, to the nearest tenth, to the nearest hundredth, and to the nearest thousandth
- round and add to estimate sums for decimals to ten-thousandths; compare the estimate of the sum with the exact sum; round and subtract to estimate differences for decimals to ten-thousandths; compare the estimate of the difference with the exact difference
- solve word problems involving decimals
- decide whether measurements are reasonable; express an unreasonable measurement as a reasonable one by changing the position of the decimal point and/or the unit

Background

Decimals are used in a wide variety of situations from quantities and prices of gasoline to positions on the FM radio dial. Students will encounter decimals frequently in their studies, particularly in science. A thorough understanding of decimal notation and a healthy respect for the position of the decimal point are essential.

The place-value structure of our numeration system uses the digits 1 to 9 for counting numbers and the digit 0 to serve as a place holder. The place values are in terms of powers of ten, as shown in the chart below for whole numbers. Each position has a value ten times the value of the place on its right, and one-tenth the value of the place on its left.

hundred thousands	ten thousands	thousands	hundreds	tens	ones
10 × 10 000	10 × 1000	10 × 100	10 × 10	10 × 1	1
10^5	10^4	10^3	10^2	10^1	10^0
100 000	10 000	1000	100	10	1

This structure may be extended to the right for powers of ten less than one.

ones	tenths	hundredths	thousandths	ten-thousandths
10^0	10^{-1}	10^{-2}	10^{-3}	10^{-4}
1	0.1	0.01	0.001	0.0001

In any numeral having two or more digits, each digit represents a value equivalent to the product of its own value and the place value of its position. The value of a numeral, or the number represented by it, is the sum of these products. These features are shown in the expanded notation for the numeral 3076.2594: (3 × 1000) + (7 × 10) + (6 × 1) + (2 × 0.1) + (5 × 0.01) + (9 × 0.001) + (4 × 0.0001). Note that the product (0 × 100) is not included in the expanded notation, because it has no value itself. However, the 0 in the numeral has great significance as a place holder to keep the digits 3 and 7 in the thousands' place and the tens' place respectively.

In decimal notation a decimal point separates the whole-number part from the decimal part and the word "and" is read for it. Decimal digits are grouped and named in the same manner as whole-number digits to show thousands, hundreds, tens, and ones, and the place name of the last digit on the right is used. Thus, 3076.2594 is read "three thousand seventy-six and two thousand five hundred ninety-four ten-thousandths". Note that a 0 is written in the ones' place of a decimal which represents a number less than one. The zero is usually not read, but it does ensure that the decimal point is not overlooked and that the digits to the right of it are given the proper place values. The decimal 0.375, for instance, is read "three hundred seventy-five thousandths".

Equivalent decimals are relatively easy to write and interpret. For instance, 3.45 is equivalent to 3.450 and to 3.4500. Because a decimal is named by the place value of the last digit on the right, each zero included on the right changes the name. Thus, 45 hundredths in 3.45 becomes 450 thousandths in 3.450, and 4500 ten-thousandths in 3.4500. Similarly, 2.6900 may be rewritten as 2.690 and as 2.69, the last of which names the same number in simplest form. It should be pointed out that the four-place decimal 2.6900 is presumed to be precise to ten-thousandths, since it was the form given first. In the case of the other number, 3.45 may be presumed to be precise only to hundredths. This brings up the matter of rounding decimals. The same principles apply to rounding decimals as to whole numbers; namely, that the digit to the right of the desired place determines whether to round up or to round down. (See page T 1.) For example, 0.2094 rounded to the nearest thousandth is 0.209, rounded to the nearest hundredth is 0.21, and rounded to

the nearest tenth is 0.2. The farther to the left a number is rounded (to greater place values), the less precise it becomes. For instance, in the case of 3.45, if it is rounded to the nearest tenth as 3.5, it is less precise, since 3.5 is equivalent to 3.50, not 3.45; and if it is rounded to the nearest whole number, 3, it is even less precise, since 45 hundredths are ignored.

When decimals are being compared, it is desirable that they have the same number of decimal places. For instance, 12.7268 and 12.725 may be compared either as four-place decimals by writing 12.7250 for 12.725 (A), or as three-place decimals by rounding 12.7268 to 12.727 (B).

	A	B
12.725 →	12.7250	12.725
12.7268	12.7268 →	12.727

Addition and subtraction of decimals involve the same skills and principles of regrouping as for whole numbers. While it is possible to add whole numbers and decimals to different numbers of decimal places, it is usually more convenient in subtraction to extend whole numbers and decimals to the same number of places.

		$3.5 - 1.705$	$2 - 0.2587$
2.43	2.4300		
3.752	3.7520	3.500	2.0000
+ 12.6245	+ 12.6245	− 1.705	− 0.2587
18.8065	18.8065	1.795	1.7413

The lesson on problem solving looks at types of everyday errors that occur when unreasonable units of measurement are stated or decimal points are positioned in the wrong places. For instance, an advertisement which states that the price of potatoes is .98¢ a bag is obviously wrong. Either the decimal point should not be used with the symbol ¢, or the symbol $ and the decimal point should be used instead of the symbol ¢.

Teaching Strategies

The teaching suggestions for the lessons at the beginning of the unit recommend the use of models to represent ones, tenths, and hundredths. At the same time, place-value charts for these values should be used and later extended to show thousandths and ten-thousandths. Pocket charts which were used for lessons with whole numbers in Unit 1 may be adapted for use with decimals by changing the names and by including a decimal point on a strip between ones and tenths. Numeral cards may be placed in the pockets and extra cards for zero should be available to extend decimals. The following two-level pocket chart is being used to compare 205.76 and 205.7598.

100	10	1		0.1	0.01	0.001	0.0001
2	0	5	•	7	6	0	0
2	0	5	•	7	5	9	8

Students who experience any difficulties adding and subtracting decimals with different numbers of decimal places should be instructed to make them similar by writing zeros in places to the right of the digits given, wherever required. They may also be directed to write the numerals in place-value columns with the decimal points aligned vertically.

Since this unit on decimals is actually an extension of the numeration system and an application of skills which have been used with whole numbers, students should not encounter many difficulties. However, if reteaching, review, and additional practice are needed by some students, they should be grouped according to their needs. Other students should use their time for some of the activities suggested for each lesson, especially those which provide enrichment and extra challenge.

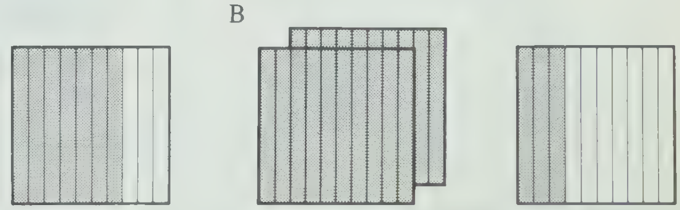
In connection with the lesson on pages 102 and 103, it is suggested in the *Related Activities* that students create Magic Squares. They may use some of the examples on page 103 to guide them. A few hints for preparing Magic Squares are offered here. A 3-by-3 Magic Square may be made using the basic pattern shown. From it other 3-by-3 squares can be made by adding, subtracting, multiplying, or dividing each number by the same number. For example, if 1.2 is added to each number, the new square will also be "magic". Rotating a Magic Square also presents a different aspect. In every case, the sum of any three numbers in a row, a column, or a diagonal of a 3-by-3 Magic Square is three times the number in the center of the square. The more blanks left in a Magic Square, the more challenging it is.

4	3	8
9	5	1
2	7	6

Models for Ones, Tenths, and Hundredths

To present one-place decimals such as 0.7 and 2.3, make white models and blue models using copies of page T 393. To show 0.7, use a white model and color 7 of the 10 strips blue (A). To show 2.3, display 2 blue whole models and color 3 of the 10 strips blue on a white model (B). Cut a few blue models into tenth strips. These may be placed on white models to show regrouping of 10 tenths as 1 whole (or vice versa). Also, students can place tenth strips over hundredths to illustrate equivalent decimals such as 0.2 and 0.20.

A B



0.7 2.3

To represent two-place decimals such as 0.34 and 1.76, make white models and blue models using copies of page T 394. To show 0.34, use a white model and color 34 of the 100 squares blue.

Materials

models for ones, tenths, and hundredths made from copies of pages T 392 - T 394 (or enlarged diagrams for the models) as described above

a ruler marked in centimetres

Vocabulary

one-place decimal
two-place decimal
three-place decimal
four-place decimal
ten-thousandths
gram, g
litre, L
centimetres, cm
helium
carbon dioxide
oxygen
hydrogen
nitrogen
clearance

LESSON OUTCOME

Read and write numerals and words for one-place decimals and for two-place decimals

Materials

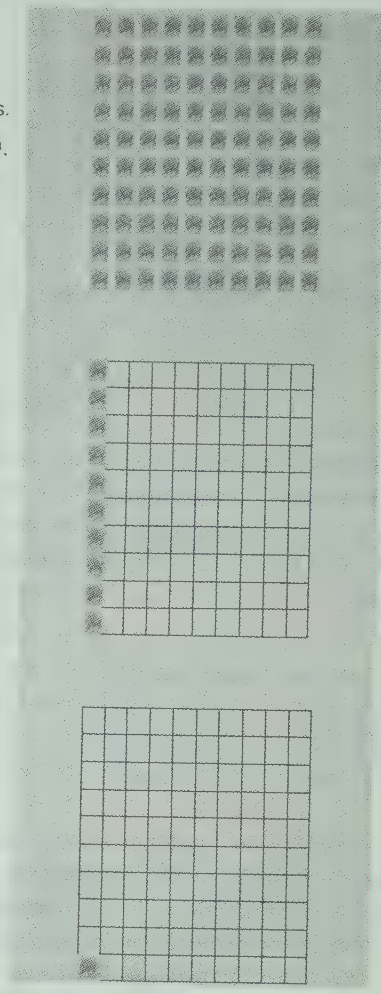
models for ones, tenths, and hundredths made from copies of pages T 392 - T 394 (or enlarged diagrams for the models) as described on page T 89

Vocabulary

one-place decimal, two-place decimal

5 DECIMALS

Tenths and Hundredths



100 one-cent stamps are 1 sheet of stamps.

The value of 100 one-cent stamps is $1.00.

ones	tenths	hundredths
1	0	0

10 one-cent stamps are one-tenth or 0.1 of a sheet of stamps.

A decimal with one digit to the right of the decimal point is a **one-place decimal**.

The value of 10 one-cent stamps is $0.10.

ones	tenths	hundredths
0	1	0

1 one-cent stamp is one-hundredth or 0.01 of a sheet of stamps.

A decimal with two digits to the right of the decimal point is a **two-place decimal**.

The value of 1 one-cent stamp is $0.01.

ones	tenths	hundredths
0	0	1

243 one-cent stamps are two and forty-three hundredths or 2.43 sheets of stamps.

The value of 243 one-cent stamps is $2.43.

ones	tenths	hundredths
2	4	3

Remember that the word "and" is used for the decimal point.

82

LESSON ACTIVITY

Before Using the Pages

- The concept of decimals was introduced earlier on pages 12 and 13 of Unit 1. You may wish to adapt some of the suggestions on pages T 12 and T 13 at this time.
- Use models of ones and tenths to review that a *one-place decimal* names a number of wholes and a number of tenths of another whole. For example, display models for 0.6 and 1.4 and have students name each number and write the corresponding numeral. Review that 0 is written to the left of the decimal point when the number is less than one. Remind the students that 1.4, for example, is read "one *and* four-tenths".

  In a similar manner, use models of ones and hundredths to review that a *two-place decimal* names a number of wholes and a number of hundredths of another whole.
- Place tenth strips on the model of hundredths to show that one-tenth is the same as ten-hundredths, two-tenths is the same as twenty-hundredths, and so on. Place tenth strips on a model for 27 hundredths, for example, to show that it is the same as 2 tenths 7 hundredths. Use other examples.

Using the Pages

- Draw attention to the photograph on page 82. Discuss that postage stamps are often printed in sheets of 100 stamps in a 10-by-10 array. Compare the photograph of the sheet of 100 stamps with the model for hundredths. Point out that one sheet of 100 stamps represents one whole. Because each stamp has a value of one cent, the value of the sheet of stamps is one dollar ($1.00). Point out that $1.00 is a decimal numeral: the ones' place shows the number of dollars, the tenths' place shows the number of tenths of a dollar, and the hundredths' place shows the number of hundredths of a dollar. This concept is shown in the place-value chart.

  The second part of the photograph shows one strip of 10 stamps. Discuss that this represents one-tenth of the sheet of stamps and relate this to the value $0.10 and to the

Working Together

Write the decimals.

1. ones | tenths | hundredths 5.06
   5 0 6

2. 4.1 4.0 4.2 4.6 5.0

3. nine-tenths 0.9

Write the words.

4. tens | ones | tenths forty-two and three-tenths
   4 2 3

5. 1.11 1.10 one and twelve-hundredths 1.20

6. 17.1 seventeen and one-tenth

Exercises

For each of these, write the decimal and the word name.

1. tens | ones | tenths 50.2 fifty and two-tenths
   5 0 2

2. ones | tenths | hundredths 6.39 six and thirty-nine hundredths
   6 3 9

3. 9.00 9.07 9.10 nine and seven-hundredths

4. 10.0 10.5 11.0 ten and five-tenths

Copy and complete the chart.

	ones	tenths	hundredths	Words	Decimals
5.	0	8	8	eighty-eight ? hundredths	0.88 ?
6.	? 7	? 4	? 0	seven and four-tenths	7.4 ?
7.	? 3	? 1	? 5	three and fifteen ? hundredths	3.15
8.	? 9	? 0	? 6	nine and six-hundredths	9.06 ?

In expanded form, 12.75 = 10 + 2 + 0.7 + 0.05

Write each in expanded form.

1. 3.5 3+0.5 2. 6.37 6+0.3+0.07 3. 0.02 0.02 4. 0.85 0.8+0.05 5. 100.25 100+0.2+0.05

Write each in standard form.

6. 1 + 0.6 + 0.06 1.66 7. 7 + 0.9 7.9

8. 30 + 5 + 0.08 35.08 9. 0.2 + 0.04 0.24

try this

83

RELATED ACTIVITIES

• Begin a display board for decimals and continue it as the students are working on this unit. Encourage students to search in newspapers and magazines for examples of decimals to include in the display.

• Some of the activities described in *Related Activities* on page T 13 may be used at this time.

• For practice in writing and interpreting decimals for amounts of money, have the students complete a work sheet similar to the following.

3¢	$0.03
94¢	
	$2.57

• For reinforcement with expanded form, prepare cards similar to the following for multiples of 1, 0.1, and 0.01.

3	0.8	0.06

Ask one student to name a two-place decimal. Have another student select the appropriate cards and show the decimal in expanded form, and then overlap the cards to show the decimal in standard form.

• Have students complete charts similar to the following to show different names for the same number.

1	1.0	1.00
▨	0.1	
▨		4.20
	6.0	

numeral in the place-value chart. Ask what is meant by a one-place decimal.

The third part of the photograph shows a single stamp. Ask what decimal describes this portion of a sheet of 100 stamps. Relate this to the value $0.01 and to the numeral in the place-value chart. Ask what is meant by a two-place decimal.

The example at the bottom of page 82 presents a decimal greater than 1 for wholes and hundredths. Discuss that 243 one-cent stamps would likely be received in the form of 2 whole sheets, 4 tenths of a sheet, and 3 hundredths of a sheet. Relate this to the value $2.43 and to the numeral in the place-value chart.

Working Together: Remind the students to show the decimal point between the ones' place and the tenths' place. Ex. 2 and 5 show decimals marked on a number line. Have students point to each mark on the number lines and name the decimal that is represented.

Exercises: Point out that the chart for Ex. 5-8 shows different ways of representing each decimal.

Try This: You may wish to review expanded form for whole numbers (see pages T 4-T 7). Have the students recall that expanded form uses addition between the values of all digits in a numeral, except 0. If a numeral such as the one in Ex. 3 has only one non-zero digit, the expanded form and the standard form are the same. Review that the standard form for a decimal is 12.75, for example. Emphasize that a numeral in standard form and the numeral in expanded form represent the same number.

Assessment

Write the decimals.

1. ones | tenths | hundredths 7.68
   7 6 8

2. seven-tenths 0.7

3. two and thirty-four hundredths 2.34

4. six and nine-hundredths 6.09

Write the words.

5. 1.0 2.0 one and five-tenths

6. 3.8 three and eight-tenths

7. 4.07 four and seven-hundredths

8. 0.95 ninety-five hundredths

LESSON OUTCOME

Read and write numerals for three-place decimals and for four-place decimals; interpret place value in three-place decimals and in four-place decimals

Vocabulary

ten-thousandths, three-place decimal, four-place decimal

Prerequisite Skills

Read and write numerals and words for one-place decimals and for two-place decimals

Checking Prerequisite Skills

Write the decimals.

1. four and one-tenth 4.1

2. nineteen-hundredths 0.19

3. one and three-hundredths 1.03

Write the words.

4. 0.9 nine-tenths

5. 5.04 five and four-hundredths

6. 2.16 two and sixteen-hundredths

Ex. 5 and 6 of Try This

5. three million six hundred thousand eight hundred ninety and five hundred three thousand six hundred eighty-four millionths

6. nine million three hundred four thousand seven hundred and forty thousand ninety-five millionths

Thousandths and Ten-Thousandths

This stamp is 0.0889 mm thick.

0.0889 means 889 of 10 000 equal parts.

0.0889 is a **four-place decimal.** It has four digits to the right of the decimal point.

A decimal with three digits to the right of the decimal point is a **three-place decimal.**

ones	tenths	hundredths	thousandths	ten-thousandths
0	0	8	8	9

To read any decimal, use the place name of its last digit.

0.08 is "8 *hundredths*".

0.088 is "88 *thousandths*".

0.0889 is "889 *ten-thousandths*".

Working Together

Write the decimals.

1.

ones	tenths	hundredths	thousandths
1	2	0	3

1.203

2. 4.2830 — 4.2840 (4.2831)

Write the decimals.

3. thirty-eight thousandths 0.038

4. four ten-thousandths 0.0004

Write the words.

5. 6.7192 six and seven thousand one hundred ninety-two ten-thousandths

6. 0.301 three hundred one-thousandths

84

LESSON ACTIVITY

Before Using the Pages

- Draw a place-value chart for ones, tenths, and hundredths on the board. Show numbers such as 0.5 and 1.69 in the chart and have students read the decimals. Name a decimal such as one and four-hundredths or twenty-hundredths and have students show the numeral in the chart. Then extend the lines of the chart to include six more columns as shown.

				ones	tenths	hundredths		

Beginning with the tens' place, ask for and write the names of the places to the left of the ones' place. Point out the pattern suggested in the chart on either side of the ones' place (tens ⟶ tenths, hundreds ⟶ hundredths). Ask what names would likely be given to the two places to the right of the hundredths' place. When the nine columns have been named, review the relationships among the places from left to right (1 ten thousand = 10 thousands, and so on) and from right to left (1 ten-thousandth is one-tenth of 1 thousandth, and so on).

Using the Pages

- Draw attention to the numeral 0.0889. Ask why it is described as a four-place decimal. Ask a student to describe a three-place decimal. Explain that for a three-place decimal, the whole has been divided into one thousand equal parts, and for a four-place decimal, the whole has been divided into ten thousand equal parts. Thus, 0.0889 means 889 of 10 000 equal parts. Draw attention to the place-value chart and discuss the procedure described for reading any decimal. Emphasize that the place name of the last digit is used. Then refer to the photograph and discuss that the thickness of the stamp is eight hundred eighty-nine ten-thousandths of a millimetre. This implies that a length of one millimetre is divided into ten thousand equal parts, and 889 of these equal parts represent the thickness of the stamp.

Exercises

Write the decimals.

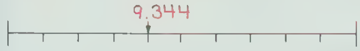
1. | ones | tenths | hundredths | thousandths |
|---|---|---|---|
| 2 | 7 | 9 | 8 |

2.798

2. | ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|
| 3 | 3 | 6 | 0 | 5 |

3.3605

3. 

4. 

5. eight hundred ninety-two thousandths 0.892
6. two thousand four hundred eighty-three ten-thousandths 0.2483
7. six-thousandths 0.006
8. five hundred five-thousandths 0.505
9. one hundred thirty-seven ten-thousandths 0.0137
10. one hundred and thirty-seven ten-thousandths 100.0037

Write the words.

11. 4.235	12. 1.5425	13. 0.248	14. 0.1666	15. 16.809
16. 5.4807	17. 0.0001	18. 1.003	19. 3.1008	20. 2.4036

11. four and two hundred thirty-five thousandths
12. one and five thousand four hundred twenty-five ten-thousandths
13. two hundred forty-eight thousandths
14. one thousand six hundred sixty-six ten-thousandths
15. sixteen and eight hundred nine-thousandths
16. five and four thousand eight hundred seven ten-thousandths

What does each 4 mean?

21. 5.284	22. 0.1824	23. 17.4289	24. 3.347	25. 4.2051
4 thousandths	4 ten-thousandths	4 tenths	4 hundredths	4 ones

try this

Study this place-value chart.

	millions	hundred-thousands	ten-thousands	thousands	hundreds	?	tens	ones	tenths	hundredths	thousandths	ten-thousandths	hundred-thousandths	millionths
	2	7	7	9	1		3	4	1	8	3	2	7	8
5.	3	6	0	0	8		9	0	5	0	3	6	8	4
6.	9	3	0	4	7		0	0	0	4	0	0	9	5

1. Copy and complete the place-value chart.
2. What patterns can you find in the place-value names? Answers will vary.
3. Write the decimal. 2 779 134.183 278
4. Write the word. two million seven hundred seventy-nine thousand one hundred thirty-four and one hundred eighty-three thousand two hundred seventy-eight millionths

For each of these, draw a place-value chart. Then write the word name.

5. 3 600 890.503 684
6. 9 304 700.040 095

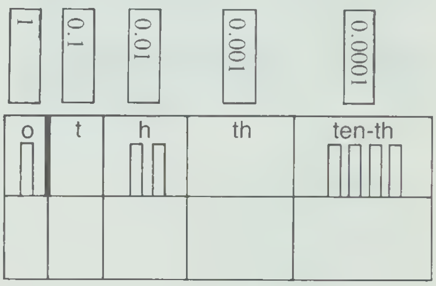
Answers for Ex. 5 and 6 are given on page T 92.

17. one ten-thousandth
18. one and three thousandths
19. three and one thousand eight ten-thousandths
20. two and four thousand thirty-six ten-thousandths

85

RELATED ACTIVITIES

- To provide practice with decimals, prepare work sheets showing place-value charts to ten-thousandths. Have students write numerals and words for one-place, two-place, three-place, and four-place decimals that are given.
- Students who require practice with place value may benefit from using a place-value pocket chart for decimals similar to the one described on page T 378. Strips of paper can be a different color for each place value, or the strips might be marked to show their decimal values.



Students can represent a given decimal or write a decimal represented in the place-value pocket chart.

- Use copies of page T 390. Mark a number line in a way similar to Ex. 3 or 4 on page 85 and have students work in pairs. One student names a decimal and the other locates on the number line the point corresponding to it.
- For more practice with expanded form, select decimals from Ex. 1-25 and have the students write the decimals in expanded form.

Working Together: In Ex. 2, ten-thousandths are related to points on a number line. Have the students read the decimals for the exercises, and emphasize which place name is used for each decimal.

Exercises: In Ex. 9 and 10, it is important to discuss the difference between one hundred thirty-seven ten-thousandths (0.0137) and one hundred *and* thirty-seven ten thousandths (100.0037). Ex. 21-25 provide practice with place value and decimals.

Try This: Completing the place-value chart to the left of the decimal point reviews place-value names presented in Unit 1. The place-value chart can be extended to the right of the decimal point by noting the pattern as suggested in *Before Using the Pages*. Students can compare the place-value names to the left of the ones' place with the corresponding place-value names to the right of the ones' place to help extend the chart. Encourage them to discuss the patterns that they find for Ex. 2. Ex. 3 and 4 refer to the numeral shown in the place-value chart.

Assessment

Write the decimals.

1. | ones | tenths | hundredths | thousandths | ten-thousandths |
|---|---|---|---|---|
| 0 | 3 | 8 | 0 | 9 |

0.3809

2. 

2.172

3. twenty-five thousandths 0.025
4. one and four hundred six ten-thousandths 1.0406

Write the words.

5. 3.7657 6. 0.204

5. three and seven thousand six hundred fifty-seven ten-thousandths
6. two hundred four-thousandths

What does each 3 mean?

7. 0.7836 8. 2.938
3 thousandths 3 hundredths

Pages 86-87

LESSON OUTCOME

Write equivalent decimals using up to four decimal places; compare two decimals (to ten-thousandths)

Materials
models for ones, tens, and hundredths made from copies of pages T 392-T 394 as described on page T 89

Prerequisite Skills
Compare whole numbers; interpret place value in decimals (to ten-thousandths)

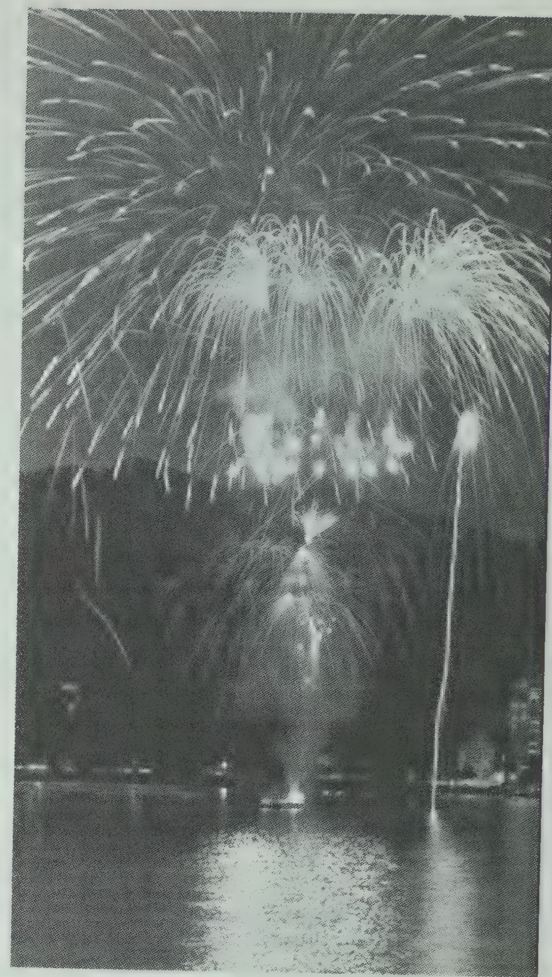
Checking Prerequisite Skills
Use >, <, or = to make true statements.

1. 2936 ⬤ 963 >
2. 44 086 ⬤ 42 086 >
3. 14 144 ⬤ 14 414 <
4. 73 367 ⬤ 73 667 <

What does each 6 mean?

5. 2.9467 6 thousandths
6. 1.6 6 tenths
7. 3.36 6 hundredths
8. 0.5486 6 ten-thousandths

Comparing Decimals



At 15°C, the sound of fireworks travels 341.109 m in 1 s. At 16°C, it travels 341.700 m in 1 s. At which temperature does the sound of fireworks travel faster?

Compare the digits from the left.

341 .109 } show the same
341 .700 } whole number.

341. 1 09 shows 1 tenth.
341. 7 00 shows 7 tenths.

341.700 > 341.109

The sound of fireworks travels faster at 16°C than at 15°C.

Working Together

Write each as a four-place decimal.

1. 4.237 4.2370 2. 1.6 1.6000

Write each as a three-place decimal.

3. 0.56 0.560 4. 7 7.000

Write each as a two-place decimal.

5. 3.1400 3.14 6. 2 2.00

For each of these, show the same number of digits to the right of the decimal point. Then tell which is greater.

7. 4.3 4.30 8. 19 19.0000
   4.13   19.1436

Use >, <, or = to make true statements.

9. 3.313 ⬤ 3.13 >
10. 5 ⬤ 5.00 =

86

LESSON ACTIVITY

Before Using the Pages

- Display a model for four-tenths and ask what decimal is represented. Write the numeral on the board and also show the numeral in a place-value chart. Repeat the procedure using a model for forty-hundredths. Have the students compare the models. Lead them to realize that 0.4 and 0.40 are names for the same number because they represent the same portion of a whole. Ask for other decimals that also name the same number, extending the places as shown.

	ones	tenths	hundredths	thousandths	ten-thousandths
0.4	0	4			
0.40	0	4	0		
0.400	0	4	0	0	
0.4000	0	4	0	0	0

Develop that the value of a number is not changed by writing zeros after the last non-zero digit to the right of the decimal point.

Use a similar procedure with other numbers such as 3.1 (ones and tenths) and 2 (a whole number).

3.1	2	4.25
3.10	2.0	4.250
3.100	2.00	4.2500
3.1000	2.000	
	2.0000	

Using the Pages

- Begin with a discussion of the photograph. Ask if the sound of fireworks is heard at exactly the same instant as the display is seen in the sky. The fact that sound must travel from its source to the listener before it is heard may be new to some students. They may also not realize that temperature affects the speed at which sound travels. Read the introductory statements and review the symbol °C.
- To compare two decimals, the whole number portions are examined first. Discuss that the decimal portions need not be examined for pairs of numerals such as 43.158 and 6.2, and 15.142 and 19.7. Then point out that each of the

Exercises

Write each as a four-place decimal.

1. 7.285 7.2850 2. 4.26 4.2600 3. 1.4 1.4000 4. 0.2 0.2000 5. 9 9.0000

Write each as a three-place decimal.

6. 6.93 6.930 7. 8.7 8.700 8. 2 2.000 9. 0.2430 0.243 10. 15 15.000

Write each as a two-place decimal.

11. 1.2 1.20 12. 5.4000 5.40 13. 6 6.00 14. 0.3 0.30 15. 432 432.00

Write each as a one-place decimal.

16. 3 3.0 17. 10.00 10.0 18. 628 628.0 19. 8.100 8.1 20. 1072 1072.0

Use >, <, or = to make true statements.

21. 8.3246 ◯ 8.2346 >
22. 5.001 ◯ 5.011 <
23. 7.1 ◯ 7.100 =
24. 3.733 ◯ 37.33 <
25. 14 ◯ 14.0 =
26. 9.758 ◯ 9.570 >
27. 6.4320 ◯ 6.432 =
28. 0.25 ◯ 0.225 >
29. 3.000 ◯ 3.0 =
30. 7.36 ◯ 7 >
31. 12.666 ◯ 126.66 <
32. 6.0011 ◯ 6.0001 >

Solve.

33. At 28°C, the sound of fireworks travels 348.715 m in 1 s. At 27°C, it travels 348.132 m in 1 s. At which temperature does it travel faster? 28°C

34. At 17°C, the sound of fireworks travels 342.291 m in 1 s. At 18°C, it travels 342.881 m in 1 s. At which temperature does it travel slower? 17°C

KEEPING SHARP

Add.

1.	2.	3.	4.	5.
119	881	784	1108	747
636	381	976	509	13
95	183	880	3807	1313
850	1445	2640	5424	2073

6. 19 + 654 + 973 1646 7. 1078 + 121 + 32 1231

8. 555 + 27 + 808 1390 9. 202 + 708 + 190 1100

10. 9909 + 1191 + 5 11105 11. 16 + 161 + 1161 1338

87

RELATED ACTIVITIES

• Students may benefit from marking decimals that are being compared, such as the decimals for Ex. 21-34, on a number line and then comparing them. Copies of page T 390 can be used.

• Provide a work sheet with exercises similar to the following and have students ring the different names for the same number.

3.7 : 3.70, 37, 3.07, 3.007, 3.7000
4 : 4.00, 4.004, 40.0, 4.000, 4.0000
0.0200 : 0.2, 0.02, 0.020, 0.002

• Adapt the second of the *Related Activities* on page T 9 for decimals.

• Prepare a numeral card for each digit from 0 to 9 and place-value cards as shown below.

__ __ . __ __

__ . __ __ __ __

Have a student select one place-value card and enough numeral cards for the spaces on the place-value card. As each numeral card is selected, it is placed on the place-value card from left to right. After all the places are filled, the student interchanges two numeral cards so that a greater number is represented. This is continued until the greatest number possible is shown.

• Discuss the results of Ex. 33 and 34 and the worked example on page 86 to suggest how temperature affects the speed at which sound travels.

numerals, 341.109 and 341.700, in the worked example shows the same whole number, 341. For this reason it is necessary to examine the digits from left to right, beginning with the tenths' place. Note that the number of tenths in 341.700 is greater than the number of tenths in 341.109. Lead the students to realize that it is not necessary to compare the digits in the hundredth's place or in the thousandths' place because 7 tenths is greater than 1 tenth. Have a student read the concluding statement.

Working Together: Draw attention to the fact that each numeral in the worked example showed the same number of digits to the right of the decimal point. Explain that in order to compare decimals, each decimal should be thought of as having the same number of digits to the right of the decimal point. Ex. 1-6 prepare students for this skill. Remind the students to write the decimal point for Ex. 4 and 6. Point out that fewer zeros, not more zeros, are needed to complete Ex. 5.

Exercises: Remind the students to examine the digits from left to right in Ex. 21-32.

Keeping Sharp: These exercises review addition with whole numbers and prepare for addition with decimals on pages 90-93. Emphasize that care must be taken to align digits having the same place value when writing the numerals.

Assessment

Write each as a four-place decimal.

1. 1.89 1.8900 **2.** 3.6 3.6000

Write each as a three-place decimal.

3. 0.1550 0.155 **4.** 7.2 7.200

Write each as a two-place decimal.

5. 9.7 9.70 **6.** 4.6100 4.61

Write each as a one-place decimal.

7. 8 8.0 **8.** 1.9000 1.9

Use >, <, or = to make true statements.

9. 4.3011 ◯ 4.3011 = **10.** 6.767 ◯ 6.776 <

11. 2.555 ◯ 2.55 > **12.** 9.0909 ◯ 9.0991 <

Page 88

LESSON OUTCOME

Order decimals (to ten-thousandths)

Vocabulary

gram, g, litre, L, helium, carbon dioxide, oxygen, hydrogen, nitrogen

Prerequisite Skills

Order whole numbers

Checking Prerequisite Skills

List from least to greatest.

1. 3076, 3066, 3776, 3676
2. 2498, 2489, 2448, 2488, 2499

1. 3066, 3076, 3676, 3776
2. 2448, 2488, 2489, 2498, 2499

RELATED ACTIVITIES

- A number line can be used for ordering decimals as suggested in the first of the *Related Activities* on page T 95.
- Have students "comparison shop" by finding the prices for an item at different places or for different brands of the same item and then listing the prices from least expensive to most expensive. Newspaper advertisements and catalogs are sources of prices for items.
- Have each student write a decimal on an index card. Tell some students to write a four-place decimal, others to write a three-place decimal, and so on. Have students draw one card at a time from a box and arrange the cards to order the decimals.

Ordering Decimals

This table gives the mass in grams (g) of 1 L (litre) of each of several different gases.

Gas	Mass of 1 L
Air	1.2928 g
Helium	0.1794 g
Carbon dioxide	1.978 g
Oxygen	1.4286 g
Hydrogen	0.0896 g
Nitrogen	1.2505 g

To list these gases from heaviest to lightest, first write each decimal to show the same number of digits to the right of the decimal point. Then start from the left and look at the digits place by place.

1.2928	1.2928
0.1794	0.1794
1.9780	1.9780
1.4286	1.4286
0.0896	0.0896
1.2505	1.2505

same

1.9780 is the greatest.

0.0896 is the least.

1.9780 > 1.4286 > 1.2928 > 1.2505 > 0.1794 > 0.0896

The gases listed from heaviest to lightest are carbon dioxide, oxygen, air, nitrogen, helium, and hydrogen.

Working Together

Write the numerals to show the same number of decimal places.

1. 1.23, 4, 6.9825, 3.1
   1.2300, 4.0000, 6.9825, 3.1000

List from least to greatest.

2. 2.4, 24.2, 24, 2.44, 0.24
   0.24, 2.4, 2.44, 24, 24.2

Exercises

List from greatest to least.

1. 6.55, 6.555, 6.5, 6.556, 6.565
2. 1.21, 1.222, 1, 1.2122, 0.21
3. 8.88, 8.008, 80.888, 80, 8.0008
4. 93.3, 93.33, 93, 93.39, 93.399
5. 7.218, 7.281, 7.2188, 7.1288, 7
6. 0.85, 0.855, 0.8, 0.555, 0.5

List from least to greatest.

7. 4.44, 44.44, 44.4, 4.444, 4.4444
8. 0.5, 0.05, 5.05, 0.5005, 0.005
9. 2.22, 0.422, 1.92, 1.922, 1.902
10. 1.11, 1.011, 1.191, 2.001, 0.91
11. 0.0235, 0.02355, 0.352, 2.30
12. 0.0001, 0.009, 0.001, 0.0019

1. 6.565, 6.556, 6.555, 6.55, 6.5
2. 1.222, 1.2122, 1.21, 1, 0.21
3. 80.888, 80, 8.88, 8.008, 8.0008
4. 93.399, 93.39, 93.33, 93 3, 93
5. 7.281, 7.2188, 7.218, 7.1288, 7
6. 0.855, 0.85, 0.8, 0.555, 0.5
7. 4.44, 4.444, 4.4444, 44.4, 44.44
8. 0.005, 0.05, 0.5, 0.5005, 5.05
9. 0.422, 1.902, 1.92, 1.922, 2 22
10. 0.91, 1.011, 1.11, 1.191, 2.001
11. 0.0235, 0.02355, 0.352, 2 30
12. 0.0001, 0.001, 0.0019, 0.009

88

LESSON ACTIVITY

Before Using the Page

- Write a few exercises on the board to review that two decimals are thought of as having the same number of digits to the right of the decimal point when they are being compared. Review that the digits are compared from left to right.

Using the Page

- Draw attention to the photograph on page 89. Then refer to the chart at the top of page 88. Read the statement above the chart and introduce the units *gram* and *litre* and the symbols g and L.
- Read the statements that describe the procedure for ordering the masses from heaviest to lightest. Point out that lining up the numerals vertically helps to compare digits having the same place value. Note that the digits are compared from left to right. Because 9 is the greatest digit in the tenths' place for the four decimals highlighted in blue, 1.9780 is the greatest number. Emphasize that after the four decimals having the greatest digit in the ones' place are ordered, it is necessary to return to the ones' place to compare the digits from left to right for the two remaining decimals. Have students explain how the gases are ordered by matching each number with the corresponding name in the chart.

Working Together: For Ex. 1, ask students how they will determine the number of decimal places required.

Exercises: Some students may need to write the numerals in vertical form with the decimal points aligned prior to ordering the numbers.

Assessment

List from greatest to least.

1. 1.007, 0.0777, 1.07, 1.7, 1.077
   1.7, 1.077, 1.07, 1.007, 0.0777
2. 3.66, 3.636, 3, 3.333, 3.336
   3.66, 3.636, 3.336, 3.333, 3

List from least to greatest.

3. 0.955, 0.9555, 0.595, 0.9, 0.0999
   0.0999, 0.595, 0.9, 0.955, 0.9555
4. 6.222, 6.22, 6.6622, 6, 6.6266
   6, 6.22, 6.222, 6.6266, 6.6622

Practice

Write the decimals.

1.

ones	tenths	hundredths	thousandths
3	5	0	9

3.509

2. 4.0600 4.0608 4.0610

3. five-tenths 0.5
4. eighty-seven hundredths 0.87
5. seven and two-thousandths 7.002
6. nine hundred ninety-five ten-thousandths 0.0995
7. nine hundred and ninety-five ten-thousandths 900.0095

Write the words.

8. 6.2 9. 7.009 10. 0.1052
11. 15.015 12. 8.38 13. 1.0204

What does each 2 mean?

14. 0.4372 15. 1.6925 16. 7.2

Copy and complete.

17. 1.28, 1.29, 1.30, 1.31, 1.32, 1.33, 1.34
18. 6.786, 6.787, 6.788, 6.789, 6.790, 6.791, 6.792
19. 0.9997, 0.9998, 0.9999, 1.0000, 1.0001, 1.0002

Use >, <, or = to make true statements.

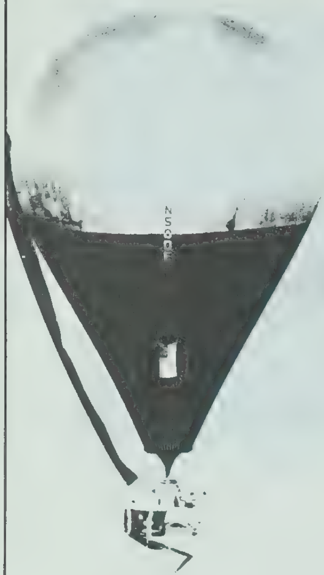
20. 2.988 < 2.998 21. 6 = 6.000
22. 7.3214 > 7.321 23. 5.989 < 5.99
24. 0.520 = 0.52 25. 3.919 < 3.99

List from greatest to least.

26. 3.18, 31.8, 0.318, 318.18, 3.108
318.18, 31.8, 3.18, 3.108, 0.318
27. 0.024, 2.4, 0.24, 24, 2.404
24, 2.404, 2.4, 0.24, 0.024

List from least to greatest.

28. 6.078, 7.860, 78.6, 60.78, 6.0078
6.0078, 6.078, 7.860, 60.78, 78.6
29. 5.009, 5.0099, 5, 5.09, 5.99
5, 5.009, 5.0099, 5.09, 5.99



Helium, hydrogen, or hot air are the gases usually used in balloons.

8. six and two-tenths
9. seven and nine-thousandths
10. one thousand fifty-two ten-thousandths
11. fifteen and fifteen-thousandths
12. eight and thirty-eight hundredths
13. one and two hundred four ten-thousandths
14. 2 ten-thousandths
15. 2 thousandths
16. 2 tenths

89

Page 89

OBJECTIVE

Demonstrate competence in reading and writing numerals and words for decimals to ten-thousandths, in interpreting place value in decimals to ten-thousandths, and in comparing and ordering decimals

RELATED ACTIVITIES

- Write decimals that show the same sequence of digits with different place values. Have students read the decimals or write the word names.

  6.0, 0.6, 0.06, 0.006, 0.0006
  32.8, 3.28, 0.328, 0.0328

- Ask students to write three decimals that form a pattern similar to those in Ex. 17-19. Have other students write the next three decimals for each pattern.

LESSON ACTIVITY

Before Using the Page

- Have the students complete the following patterns with whole numbers.

  6, 7, 8, ____, ____, ____
  13, 14, 15, ____, ____, ____

  Then extend the skill to completing patterns for decimals.

  0.6, 0.7, 0.8, ____, ____, ____
  1.3, 1.4, 1.5, ____, ____, ____
  2.17, 2.18, 2.19, ____, ____, ____

Using the Page

- If students have difficulty comparing or ordering decimals, have them write the decimals in a place-value chart and then compare the digits from the left of the place-value chart to the right. After the students have completed the exercises, you may wish to have them read some of their answers for practice in reading decimals.

Pages 90-91

LESSON OUTCOME

Add from two to five decimals (to ten-thousandths), addends with different numbers of decimal places; solve related word problems

Prerequisite Skills

Add whole numbers; interpret place value in decimals (to ten-thousandths)

Checking Prerequisite Skills

Add.

1.	2.	3.
984	2 478	34 728
367	38 612	180
1351	5 094	79 684
	46 184	3 412
		118 004

4. 728 + 3948 + 71 + 2590 7337

5. 384 + 59 + 2013 + 6754 + 38 9248

What does each 2 mean?

6. 7.248 2 tenths 7. 0.1125 2 thousandths

Adding Decimals

When Greg made soup, he used 1.15 L of soup stock, 0.4 L of condensed tomato soup, 0.078 L of condensed vegetable soup, and 0.125 L of condensed mushroom soup. How much soup did he make?

Add 1.15, 0.4, 0.078, and 0.125.

Use the decimal points to line up the decimals. Add thousandths and regroup.

```
    1
1 . 1 5
0 . 4
0 . 0 7 8
0 . 1 2 5
---------
        3
```

Add hundredths and regroup.

```
  1 1
1 . 1 5
0 . 4
0 . 0 7 8
0 . 1 2 5
---------
      5 3
```

Add tenths.

```
  1 1
1 . 1 5
0 . 4
0 . 0 7 8
0 . 1 2 5
---------
    7 5 3
```

Add ones.

```
    1 1
1 . 1 5
0 . 4
0 . 0 7 8
0 . 1 2 5
---------
1 . 7 5 3
```

Place the decimal point in the sum in line with those in the addends.

Greg made 1.753 L of soup.

Working Together

Line up the decimals in vertical form.

1. 0.6435 + 0.92 0.6435 / 0.92

2. 7.75 + 10.5 + 0.3333 7.75 / 10.5 / 0.3333

Add by following the steps.

3.
```
   111
  2.4679
  1.2957
  ------
  37636
```
Add and regroup.
Add and regroup.
Add and regroup.
Add.
Add.

Add.

4. 10.325 + 4.905 = 15.230 5. 611.68 + 0.3479 = 612.0279

6. 0.7 + 12.32 + 1.0057 14.0257

90

LESSON ACTIVITY

Before Using the Pages

- Ask the students to think of a three-place decimal. Have two students show their decimals in a place-value chart on the board. Ask how the sum of the two numbers can be found. Lead the students to suggest that the addition can be performed using the same procedure as for whole numbers: add place by place from right to left, regrouping as needed. Have students perform the addition on the board and explain the regrouping that is encountered. For instance, in the example below, addition of the thousandths does not require regrouping, but addition of the hundredths requires 11 hundredths to be regrouped as 1 tenth 1 hundredth.

tens	ones	tenths	hundredths	thousandths
	1	1		
	0	8	7	1
+	9	6	4	1
1	0	5	1	2

Using the Pages

- Have a student read the word problem at the top of page 90. Discuss why addition is used to find the solution. Point out that the addition involves more than two addends and the number of decimal places shown varies. Emphasize the importance of lining up the decimals in vertical form so that the digits having the same place value appear in the same column. Have students note that this will occur if the decimal points in the numerals are aligned. Explain that some columns have more digits than others because the addends have different numbers of decimal places. Tell the students that 1.15 and 1.150 name the same number and a 0 may be written in the thousandths' place for the first addend, but it is not necessary. Similarly, it is not necessary to show 0 hundredths and 0 thousandths for the second addend. (Addition of zero in any column does not affect the sum.) Ask students to explain each step of the addition. Emphasize that the decimal point must be shown in the sum and discuss where it is placed. Have a student read the concluding statement.

Exercises

Add.

1. 81.67
   95.84
   177.51

2. 1.379
   1.805
   3.184

3. 32.784
   7.759
   40.543

4. 0.9734
   0.9876
   1.9610

5. $2.73
   1.24
   3.36
   $7.33

6. 0.129
   0.824
   5.2706
   6.2236

7. 2.7
   5.003
   0.5003
   8.2033

8. 0.5436
   0.0604
   0.27
   0.8740

9. $12.47
   4.00
   1.23
   3.90
   $21.60

10. 7.01
    3.47
    10.2761
    6.5
    27.2561

11. 0.72
    0.612
    3.6
    2.0036
    6.9356

12. 4.3
    22.55
    0.073
    6.25
    15.2
    48.373

13. $6.73 + $5.02 + $7.10 $18.85
14. $6.25 + $17.36 + $11.04 $34.65
15. 4.306 + 0.5 + 1.0539 5.8599
16. 5.2 + 11.0568 + 3.0094 19.2662
17. 0.5815 + 5.813 + 1.5815 7.9760
18. 3.1604 + 2.041 + 0.9 6.1014
19. 17.306 + 21.5 + 8.06 46.866
20. 2.9641 + 17.03 + 0.899 20.8931
21. 0.72 + 0.612 + 3.6 + 5.1 10.032
22. 50.02 + 2.837 + 6.04 + 7.5 66.397

For each of these, show the correct addition.

23. 0.2756 | 0.2756
    0.3085 | 0.3085
    2.315 | 2.315
    2 8991 | 2.8991

24. 46.23 | 46.23
    9.1 | 9.1
    10.46 | 10.46
    657.9 | 65.79

25. 50.062 | 50.062
    19.73 | 19.73
    0.917 | 0.917
    52.952 | 70.709

26. 17.163 | 17.163
    1.77 | 1.77
    153.45 | 153.45
    502.08 | 172.383

Solve.

27. Janine made stew with 1.25 L of soup, 0.3 L of meat, and 0.75 L of vegetables. How much stew did she make? 2.3 L

28. John added 0.125 L of soup to 2.5 L of stew. How many litres of stew does he have? 2.625 L

29. Jerome mailed four recipe books. Their masses were 1.35 kg, 1.1 kg, 0.85 kg, and 0.455 kg. How heavy were all four books? 3.755 kg

30. Janis mailed three packages. The postage was $0.95, $1.80, and $2.05. How much did she pay for postage for all three packages? $4.80

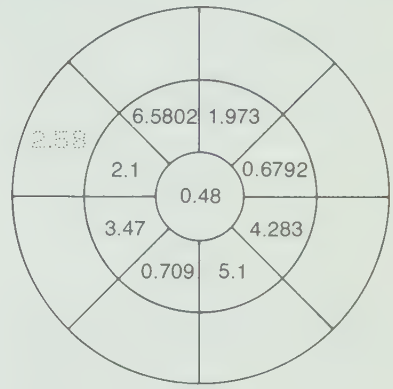
91

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 1-16 on page 330.
- Students having difficulty lining up the addends for the exercises on page 91 may benefit from using a place-value chart as shown below for 5.24 + 0.689 + 2.7.

ones	tenths	hundredths	thousandths
1	1		
5	2	4	
0	6	8	9
+ 2	7		
8	6	2	9

- Have students complete number wheels and tables similar to the following prepared from copies of page T391.



+	2.837
6.9	
4.58	
0.397	

Working Together: Ex. 1 and 2 provide practice in lining up decimals in vertical form. Review that the decimal points will be aligned if place values are aligned. Ex. 3 guides the students by indicating the steps for adding and regrouping. Summarize that adding decimals is similar to adding whole numbers.

Exercises: Review that numerals for dollars and cents are decimals (Ex. 5, 9, 13, 14, 30). Remind the students to record the symbol $ in the sums for those exercises. The additions in Ex. 23-26 are incorrect. For example, the decimal point is not recorded in the sum for Ex. 23. For Ex. 24, the digit in the tenths' place of the sum is not aligned with those of the addends and the decimal point is incorrectly placed in the sum. Have the students rewrite the exercises and find the correct sums. For Ex. 25, the addends were lined up on the right, and for Ex. 26, the addends were lined up on the left, instead of aligning the decimal points or considering place values.

Assessment

Add.

1. 9.387
   6.345
   15.732

2. 7.92
   6.143
   2.5
   16.563

3. 3.0458
   11.9
   4.792
   6.84
   26.5778

4. 7.682 + 9.314 + 5.904 22.900
5. 6.7143 + 17.62 + 9.310 + 4.5 + 6.9875 45.1318
6. 6.238 + 1.94 + 7.5055 + 3.8 19.4835
7. Jack mailed three packages having masses of 1.3 kg, 2.55 kg, and 0.86 kg. How heavy were all three packages? 4.71 kg

LESSON OUTCOME

Add decimals (to ten-thousandths) and whole numbers, from two to five addends

Prerequisite Skills

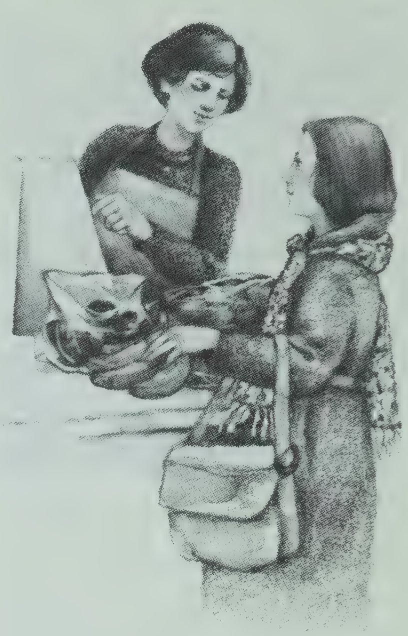
Add from two to five decimals (to ten-thousandths), addends with different numbers of decimal places

Checking Prerequisite Skills

Add.

1. 3.87 + 6.055 + 9.3 19.225
2. 12.682 + 0.83 + 7.9248 + 3.1 24.5368
3. 7.68 + 9.354 + 8.6 + 7.2 32.834

Adding Decimals and Whole Numbers



Rita bought 5 kg of potatoes, 1.3 kg of bananas, and 2.54 kg of tomatoes. How heavy were her groceries?

Add 5, 1.3, and 2.54.

Write 5 as 5.0.

5 is the same as 5.0.

Use the decimal points to line up the decimals. Add hundredths.

```
 5 . 0
 1 . 3
 2 . 5 4
 -------
       4
```

Add tenths.

```
 5 . 0
 1 . 3
 2 . 5 4
 -------
     8 4
```

Add ones.

```
 5 . 0
 1 . 3
 2 . 5 4
 -------
 8 . 8 4
```

Place the decimal point in the sum in line with those in the addends.

The mass of her groceries was 8.84 kg.

Working Together

Write each of these to show tenths.

1. 2 2.0
2. 6 6.0

Line up in vertical form. Show zeros when needed.

3. 5 + 7.2 5.0 / 7.2
4. 2.025 + 4 + 3.25 2.025 / 4.0 / 3.25

Add.

5. 17 + 6.3 23.3
6. 0.425 + 1 + 0.3 1.725
7. $2 + $6.05 + $14 $22.05

92

LESSON ACTIVITY

Before Using the Pages

- Write 1.5 + 7 on the board. Ask a student to read the numerals. Draw a place-value chart for ones and tenths on the board and have students suggest where the numerals should be shown in the chart. Ask the following questions.
  "What is the sum of the tenths?"
  "What is the sum of the ones?"
  Write the numeral for the sum in the chart. Point out that the addend 7 is a whole number and ask what decimal names the same number. Discuss that the place-value chart helps to align the digits of the addends correctly. Ask what helps in aligning decimal addends when no place-value chart is used. Lead the students to suggest the decimal point.

ones	tenths
1	5
+ 7	

Using the Pages

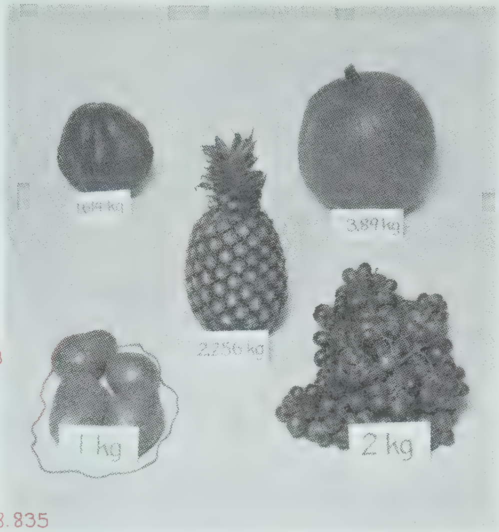
- The worked example demonstrates that writing a whole number addend as a decimal helps in lining up decimal points and thus, digits of the addends. Ask the students how they could use place values to line up the addends. Because 5 and 5.0 represent the same number, 5.0 can be written for the first addend, 5. Have students read the word problem, follow the digits highlighted in blue, and explain the steps in their own words. Review that the decimal point is placed in the sum in line with those in the addends. Have a student read the concluding statement.

Working Together: Ex. 1 and 2 provide practice in writing whole numbers to show tenths. Have students explain where decimal points and zeros are needed when lining up the addends for Ex. 3 and 4 in vertical form. For Ex. 7, point out that $2 and $14 must be written as two-place decimals because two decimal places are shown when dollars and cents are written with the symbol $.

Exercises

Add.

1. 2 + 5.7 7.7
2. 19.3 + 24 43.3
3. 17 + 8.4 25.4
4. 22 + 6.19 28.19
5. 21.45 + 184 205.45
6. 7 + 3.92 + 1.36 12.28
7. 4 + 8.004 + 3 15.004
8. 2 + 1.0055 + 5.619 8.6245
9. 32 + 4.725 + 0.3333 37.0583
10. 6.1 + 7 + 8.28 21.38
11. 5 + 3.8 + 4 12.8
12. 6.9 + 9 + 4.1 20.0
13. 8.153 + 2 + 6.437 16.590
14. 24.16 + 19.3 + 40 + 5.375 88.835
15. 8 + 14.081 + 25.1 + 5 52.181
16. 29 + 0.569 + 2.2 + 4 35.769
17. 6.3044 + 3 + 2 + 0.666 + 1.5 13.4704
18. 3.005 + 6 + 1.3 + 5 + 7.02 22.325
19. 9 + 0.2 + 6.1 + 4.25 + 6.08 25.63
20. \$12 + \$2.85 + \$6.50 + \$4 \$25.35
21. \$7.26 + \$10 + \$6.02 + \$7 \$30.28
22. \$6.14 + \$7.32 + \$1.75 + \$3 \$18.21



Use the items in this photograph to find the mass of each of these.

23. the potatoes and the squash 2.614 kg
24. the pumpkin, the squash, and the grapes 7.504 kg
25. the pineapple, the potatoes, and the pumpkin 7.146 kg
26. the squash, the grapes, and the pineapple 5.870 kg
27. the grapes, the potatoes, the squash, the pineapple, and the pumpkin 10.760 kg

KEEPING SHARP

Subtract.

1. 5555 − 666 = 4889
2. 9876 − 1234 = 8642
3. 1191 − 818 = 373
4. 4321 − 1234 = 3087
5. 60 855 − 40 857 19 998
6. 20 706 − 10 807 9899
7. 1555 − 666 889
8. 9876 − 6789 3087
9. 30 953 − 2095 28 858

93

RELATED ACTIVITIES

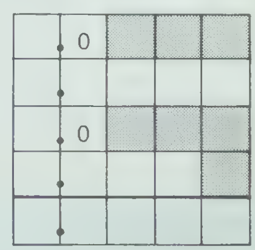
- For further practice, you may wish to have students complete Ex. 17-24 on page 330.
- Prepare number squares similar to the following and have the students find the sum of all the numbers in the square.

7.6	2.389	4.1
0.888	3.42	7
6	0.5	12

Point out that the order of adding the numbers does not influence the sum, and challenge the students to find the sum for squares such as the following without using pencil and paper.

3	0.8	1.5
0.2	1.25	6
2.5	0.1	3.75

- The game "Greatest Sum" described on page T 379 may be adapted for decimals. Prepare charts similar to the following for the game.



Exercises: Remind the students to line up the addends for Ex. 1-22 in vertical form and show zeros when needed. Ex. 23-27 refer to the items in the photograph on page 93.

Keeping Sharp: These exercises review subtraction with whole numbers and prepare for subtraction with decimals on pages 94-97.

Assessment

Add.

1. 6 + 5.2 11.2
2. 9.86 + 5.382 + 7 22.242
3. 4 + 3.941 + 6 13.941
4. 8 + 4.058 + 9.32 + 8.7 30.078
5. 1.6 + 4 + 2.05 + 3 + 7.938 18.588

Pages 94-95

LESSON OUTCOME

Subtract decimals (to ten-thousandths) with different numbers of decimal places; use addition to check subtraction

Vocabulary

clearance

Prerequisite Skills

Subtract whole numbers; add decimals with different numbers of decimal places

Checking Prerequisite Skills

Subtract.

1. 3048
2986
62

2. 4231
1748
2483

3. 60 003 − 19 287 40 716

4. 5208 − 639 4569

Add.

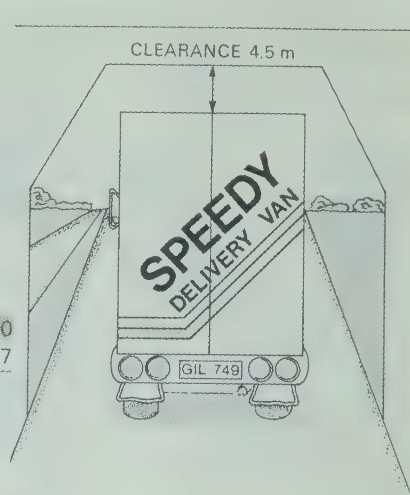
5. 4.28
6.5
10.78

6. 9.37
6.789
16.159

7. 1.327 + 5.0985 6.4255

8. 2.437 + 8.1 10.537

Subtracting Decimals



The height of the truck is 3.97 m. What is the distance between the truck and the bridge?

Subtract 3.97 from 4.5.

Write each decimal to show the same number of digits to the right of the decimal point.

4.5 is the same as 4.50.

Use the decimal points to line up the decimals.

```
  4 . 5 0
  3 . 9 7
```

Regroup 5 tenths 0 hundredths as 4 tenths 10 hundredths. Subtract hundredths.

$$\begin{array}{r} 4\,.\,\overset{4}{\cancel{5}}\,\overset{10}{\cancel{0}} \\ 3\,.\,9\,7 \\ \hline 3 \end{array}$$

Regroup 4 ones 4 tenths as 3 ones 14 tenths. Subtract tenths.

$$\begin{array}{r} \overset{3}{\cancel{4}}\,.\,\overset{14}{\cancel{5}}\,\overset{10}{\cancel{0}} \\ 3\,.\,9\,7 \\ \hline 5\,3 \end{array}$$

Subtract ones.

$$\begin{array}{r} \overset{3}{\cancel{4}}\,.\,\overset{14}{\cancel{5}}\,\overset{10}{\cancel{0}} \\ 3\,.\,9\,7 \\ \hline 0\,.\,5\,3 \end{array}$$

Place the decimal point in line with the others.

The subtraction can be checked by addition.

$$\begin{array}{r} \overset{1}{0}\,.\,\overset{1}{5}\,3 \\ 3\,.\,9\,7 \\ \hline 4\,.\,5\,0 \end{array}$$

If this number does not match the first number used in the subtraction, there is a mistake.

The distance between the truck and the bridge is 0.53 m.

94

LESSON ACTIVITY

Before Using the Pages

- Ask the students to think of a four-place decimal. Have two students write their examples on the board. Ask which number is greater. Ask what operation can be carried out to answer the question "How much greater?" The students will likely suggest the use of subtraction. Have students show the two numbers in a place-value chart on the board. Lead them to suggest that the subtraction can be performed using the same procedure as for whole numbers: subtract place by place from right to left, regrouping as needed. Have students perform the subtraction on the board to show the regrouping that is encountered. For instance in the example given, since 9 ten-thousandths cannot be subtracted from 1 ten-thousandth, 3 thousandths 1 ten-thousandth are regrouped as 2 thousandths 11 ten-thousandths.

ones	tenths	hundredths	thousandths	ten-thousandths
			12	
4	12	7	~~2~~	11
~~5~~	~~2~~	~~8~~	~~3~~	~~1~~
− 2	7	6	5	9
2	5	1	7	2

Point out that both numbers in the subtraction show the same number of decimal places.

Using the Pages

- Draw the students' attention to the illustration at the top of page 94 and ask what the term "CLEARANCE 4.5 m" indicates. Have a student read the word problem. Ask why 3.97 is subtracted from 4.5 to solve the problem. Have the students note that the greater number is a one-place decimal and the lesser number is a two-place decimal. Emphasize that although all the decimals need not be written to show

Here is another example of subtraction with decimals.

Subtract 0.9 from 1.125.

Line up the decimals.	Think of 9 tenths as 900 thousandths or 0.900.	Subtract.
1.125 0.9	1.125 0.900	1.125 0.900 0.225

Working Together

Complete this chart.

1.	7	7.0	7.00	7.000	7.0000
2.	9	? 9.0	? 9.00	? 9.000	? 9.0000
3.		4.7	? 4.70	? 4.700	? 4.7000
4.			6.45	? 6.450	? 6.4500
5.				7.125	? 7.1250

Show zeros when needed for each subtraction.

6. 13.074 − 2.6   13.074 − 2.600
7. 6.2 − 5.875   6.200 − 5.875

Show in vertical form. Show zeros when needed.

8. 5.32 − 4.986   5.320 − 4.986
9. 6.7936 − 2.14   6.7936 − 2.1400

Subtract. Add to check.

10. $4.29 − 0.99 = $3.30
11. 62.75 − 8.9 = 53.85
12. 7.4 − 4.397 = 3.003
13. 25.702 − 9.85 15.852
14. 16.38 − 0.5279 15.8521

Exercises

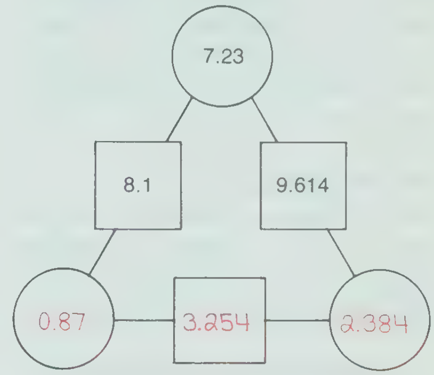
Subtract. Add to check the six most difficult exercises.

1. 0.9876 − 0.5321 = 0.4555
2. 726.4 − 350.8 = 375.6
3. 4.613 − 2.913 = 1.700
4. 16.002 − 5.849 = 10.153
5. $19.05 − 12.56 = $ 6.49
6. 25.702 − 9.85 = 15.852
7. 17.3 − 9.45 = 7.85
8. 57.9 − 8.425 = 49.475
9. 6.526 − 0.9 = 5.626
10. $1.20 − 0.68 = $0.52
11. 40.06 − 17.96 22.10
12. 38.8 − 4.9 33.9
13. 23.54 − 9.6 13.94
14. 64.52 − 19.368 45.152
15. 9.45 − 3.7 5.75
16. 0.8 − 0.309 0.491
17. 3.3 − 2.4008 0.8992
18. 84.067 − 6.4 77.667
19. 17.63 − 9.0666 8.5634
20. $6.92 − $0.85 $6.07
21. $29.50 − $5.90 $23.60
22. $30.00 − $4.65 $25.35

95

RELATED ACTIVITIES

- For further practice you may wish to have students complete Ex. 25-42 on page 330.
- For more practice, use copies of page T391 to prepare diagrams similar to the following. Addends are shown in the circles and sums are shown in the squares.



the same number of digits to the right of the decimal point for addition, they must be shown for subtraction. Explain that 4.50 is written in the subtraction exercise because 4.5 and 4.50 represent the same number. Have students explain the steps of the subtraction in their own words. Review that the decimal point is placed in the difference in line with those of the subtrahend and minuend. Have a student read the concluding statement. Ask what operation can be used to check subtraction. Have students explain the addition for the example on page 94.

- For the worked example on page 94, there are more digits to the right of the decimal point in the subtrahend than there are in the minuend. For the example at the top of page 95, there are more digits to the right of the decimal point in the minuend than there are in the subtrahend. Point out that in each example, both decimals are written to show the same number of decimal places before the subtraction is performed. Summarize that the same procedure is used for subtraction of decimals as for subtraction of whole numbers: the numerals are lined up in vertical form, the subtraction is performed from right to left with regrouping as needed, and the subtraction can be checked by addition.

Working Together: Have students explain when zeros are needed for Ex. 6-14.

Exercises: Encourage the students to use addition to check subtraction whenever they are uncertain about their work.

Assessment

Subtract. Add to check.

1. 4.689 − 3.794 = 0.895
2. 7.1386 − 4.95 = 2.1886
3. 8.2 − 7.305 = 0.895
4. 16.73 − 2.8445 13.8855
5. 9.006 − 3.9 5.106

LESSON OUTCOME

Subtract decimals (to ten-thousandths) and whole numbers; solve related word problems

Materials

a model for ones made from a copy of page T 392, a model for tenths made from a copy of page T 393, and a model for hundredths made from a copy of page T 394 as described on page T 89

Prerequisite Skills

Subtract decimals (to ten-thousandths) with different numbers of decimal places; add decimals (to ten-thousandths) and whole numbers

Checking Prerequisite Skills

Subtract.

1. 6.89 − 3.5476 3.3424
2. 19.046 − 2.75 16.296
3. 5.1 − 0.838 4.262

Add.

4. 2.4 + 8 10.4
5. 7 + 6.9354 13.9354

Subtracting Decimals and Whole Numbers

Debbie cut a piece of wood 1.15 m long from a board 2 m long. How long was the board that remained?

Subtract 1.15 from 2.

Write 1.15 and 2 to show the same number of digits to the right of the decimal point.

2 is the same as 2.00.

Use the decimal points to line up the decimals.

$$\begin{array}{r} 2.00 \\ \underline{1.15} \end{array}$$

2 .00 shows 2 ones.
2.0 0 shows 20 tenths.

Regroup 20 tenths as 19 tenths 10 hundredths.

$$\begin{array}{r} {}^{1}\ \ {}^{9}\ {}^{10} \\ \cancel{2.0}\ \cancel{0} \\ \underline{1\ .\ 1\ 5} \end{array}$$

Then subtract hundredths, tenths, and ones.

$$\begin{array}{r} {}^{1}\ \ {}^{9}\ {}^{10} \\ \cancel{2.0}\ \cancel{0} \\ \underline{1\ .\ 1\ 5} \\ 0\ .\ 8\ 5 \end{array}$$

The board that remained was 0.85 m long.

Here is an example of subtracting whole numbers from decimals.

Subtract 6 from 7.175.

Write 6 and 7.175 to show the same number of digits to the right of the decimal point. Line up the decimals in vertical form.

$$\begin{array}{r} 7.175 \\ \underline{6.000} \end{array}$$

Subtract.

$$\begin{array}{r} 7.175 \\ \underline{6.000} \\ 1.175 \end{array}$$

96

LESSON ACTIVITY

Before Using the Pages

- Review that 1000 can be interpreted in the following ways.

one thousand	$\boxed{1}\,0\,0\,0$	1 thousand
	$\boxed{1\,0}\,0\,0$	10 hundreds
	$\boxed{1\,0\,0}\,0$	100 tens
	$\boxed{1\,0\,0\,0}$	1000 ones

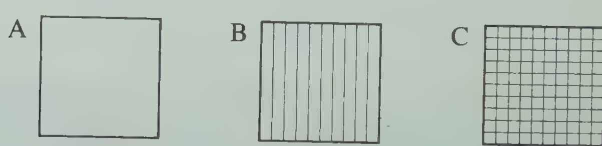
Have the students recall that interpreting 1000 as shown above led to regrouping, for example, 100 tens 0 ones as 99 tens 10 ones, in subtractions similar to the following.

$$\begin{array}{r} 9\ \ 9\ \ 10 \\ \cancel{1\ 0\ 0}\ \cancel{0} \\ \underline{-\quad 2\ \ 6\ \ 7} \\ 7\ \ 3\ \ 3 \end{array}$$

- Review that the whole number 1 has decimal names such as 1.0, 1.00, 1.000, and 1.0000. Then demonstrate that 1.000 can be interpreted in the following ways.

one	$\boxed{1.}\,0\,0\,0$	1 one
	$\boxed{1.\,0}\,0\,0$	10 tenths
	$\boxed{1.\,0\,0}\,0$	100 hundredths
	$\boxed{1.\,0\,0\,0}$	1000 thousandths

The concept can be illustrated using models for decimals, to enable the students to understand that 1 whole (A), 10 tenths (B), and 100 hundredths (C) name the same number.



Use a similar procedure to show different ways of interpreting, for example, 3.0000 for the whole number 3. Then discuss ways of regrouping decimals such as these.

$\overset{99}{\cancel{100}}$ hundredths $\overset{10}{\cancel{0}}$ thousandths $\longrightarrow$ $\overset{9\ \ 9\ \ 10}{\cancel{1.0\ 0}\ \cancel{0}}$

$\overset{2999}{\cancel{3000}}$ thousandths $\overset{10}{\cancel{0}}$ ten-thousandths $\longrightarrow$ $\overset{2\ \ 9\ \ 9\ \ 9\ \ 10}{\cancel{3.0\ 0\ 0}\ \cancel{0}}$

Working Together

Line up in vertical form. Show zeros when needed.

1. 24 − 6.375 24.000 / 6.375
2. 6.9 − 5 6.9 / 5.0
3. $4 − $3.50 $4.00 / 3.50

Subtract.

4. 31.05 − 6 25.05
5. 1 − 0.3333 0.6667
6. $16.30 − $9 $7 30

Exercises

Subtract.

1. 4.236 − 2 2.236
2. 7 − 0.35 6.65
3. 9.6 − 9 0.6
4. 18 − 3.75 14.25
5. 2.6666 − 1 1.6666
6. 4.005 − 4 0.005
7. 1 − 0.0036 0.9964
8. 12 − 6.875 5.125
9. 10 − 3.3333 6.6667
10. $5 − $1.25 $3.75
11. $8.78 − $6 $2.78
12. $1 − $0.63 $0 37
13. $9.45 − $4 $5.45
14. $203 − $8.93 $194 07
15. $18 − $7.36 $10.64

Solve.

16. Brenda's parcel had a mass of 5 kg. Stan's parcel had a mass of 3.855 kg. How much heavier was Brenda's parcel than Stan's? 1.145 kg
17. To qualify for the track team, Enid had to run around the track in 32 s. Her time was 29.88 s. How much time did she have to spare? 2.12 s
18. Olga had $7.38. She spent $6. How much did she have left? $1.38
19. Jerry paid $6.23 with a $10 bill. How much change did he receive? $3.77

Round to the

nearest ten.
1. 364 360
2. 489 490

nearest hundred.
3. 556 600
4. 213 200

nearest ten thousand.
5. 766 000 770 000
6. 521 000 520 000

nearest hundred million.
7. 458 000 000 500 000 000
8. 687 000 000 700 000 000

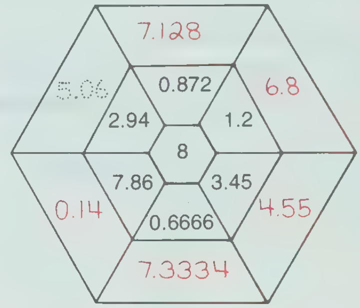
nearest billion.
9. 7 934 000 000 8 000 000 000
10. 9 205 000 000 9 000 000 000

KEEPING SHARP

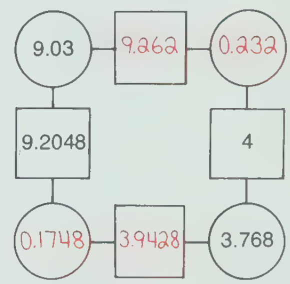
97

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 43-57 on page 330.
- Use copies of page T 391 to prepare hexagons similar to the following for more practice.



- Use copies of page T 391 to prepare diagrams similar to the following. Addends are shown in the circles and sums are shown in the squares.



- Prepare cards similar to the following for the game "Dominoes" described on page T 379.

3.00	4	4.000	16

Explain that this manner of regrouping can be applied in subtracting a decimal from a whole number.

Using the Pages

- Have a student read the word problem. Ask why subtraction is used for the solution. Note that the subtraction involves subtracting a two-place decimal from a whole number. Point out that 2.00 can be written in the subtraction exercise for 2 because 2 and 2.00 represent the same number. Because 5 hundredths cannot be subtracted from 0 hundredths, 2.00 is thought of as 20 tenths 0 hundredths and is regrouped as 19 tenths 10 hundredths.

  The example at the bottom of page 96 involves subtracting a whole number from a decimal. Explain that both decimals are written to show the same number of decimal places and then the subtraction is performed.

Working Together: Remind the students that they must also write the decimal point when they show zeros for the subtraction exercises. For example, Ex. 1 should be written as shown in A, not as in B.

A		B	
	24.000		24 000
	6.375		6.375

Review that two zeros are needed to the right of the decimal point in the minuend for Ex. 3 since the subtrahend involves dollars and cents.

Exercises: Encourage the students to use addition to check their subtraction exercises.

Keeping Sharp: These exercises review rounding whole numbers and prepare for rounding decimals, which is presented on pages 98 and 99.

Assessment

Subtract.

1. 7 − 6.8359 0.1641
2. 12.945 − 8 4.945
3. 5.34 − 5 0.34
4. 9 − 0.05 8.95
5. 15 − 6.9 8.1
6. 1 − 0.333 0.667
7. Louise had $9. She spent $0.86. How much did she have left? $8.14

LESSON OUTCOME

Round decimals with up to four places to the nearest one, to the nearest tenth, to the nearest hundredth, and to the nearest thousandth

Prerequisite Skills

Round whole numbers; interpret place value in decimals (to ten-thousandths)

Checking Prerequisite Skills

Round to the nearest ten.

1. 43 40 **2.** 679 680

Round to the nearest hundred.

3. 850 900 **4.** 2641 2600

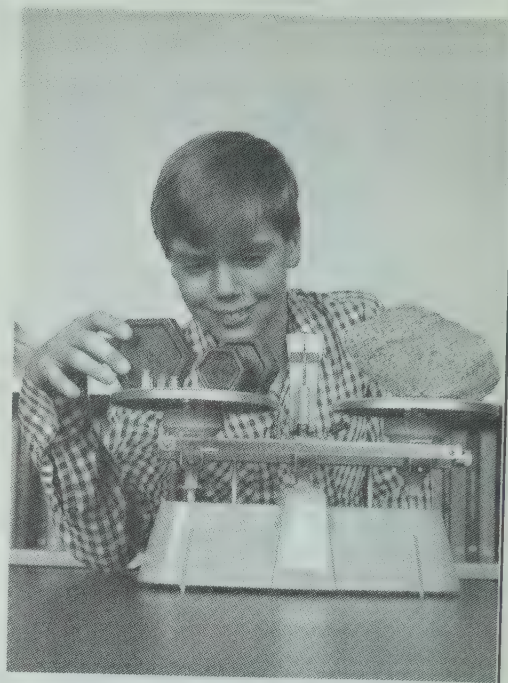
What does each 5 mean?

5. 1.56 **6.** 0.4775

7. 2.865 **8.** 5.03

5. 5 tenths
6. 5 ten-thousandths
7. 5 thousandths
8. 5 ones

Rounding Decimals



During science class, Jim found that the mass of this stone is 1.735 kg. What is the mass of the stone to the nearest kilogram?

To round a decimal to the nearest whole number,

ones	tenths	hundredths	thousandths
this place			

check the digit in the tenths place.

ones	tenths	hundredths	thousandths
	this place		

If the digit in the tenths place is 5, 6, 7, 8, or 9, round up.

If the digit in the tenths place is 0, 1, 2, 3, or 4, round down.

For 1.735, the digit in the tenths place is 7.

Round 1.735 up.

To the nearest kilogram, the mass of the stone is 2 kg.

Working Together

When rounding to

1. the nearest one, check the digit in the __?__ place. tenths
Would you round down or up to the nearest one?

2. the nearest tenth, check the digit in the __?__ place. hundredths
Would you round down or up to the nearest tenth?

3. 4.12 down **4.** 6.948 up **5.** 9.27 up **6.** 0.1432 down

Round to the nearest

7. one. 7.2 7 **8.** tenth. 8.487 8.5 **9.** hundredth. 7.6813 7.68 **10.** thousandth. 0.2107 0.211

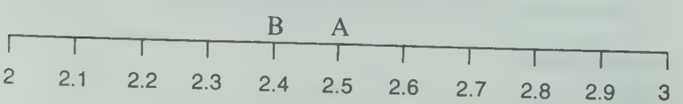
98

LESSON ACTIVITY

Before Using the Pages

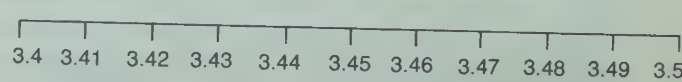
- Draw a number line on the board and mark it into ten equal parts. Label the line as you direct the students and question them as suggested below. Sample answers are given in parentheses.

  "Name a whole number between 0 and 10." (2)
  "Name the next whole number." (3)
  "What decimal names point A? point B?" (2.5, 2.4)



  "Is 2.4 closer to 2 or to 3?" (2)
  "We say that 2.4 rounded to the nearest whole number is 2."
  "Round 2.7 to the nearest whole number." (3)
  "Is 2.5 closer to 2 or to 3?"
  "By agreement, we round 2.5 up to 3 rather than down to 2."
  "Round 14.4, 14.7, and 14.5 to the nearest whole number." (14, 15, 15)

  Use a similar procedure to develop rounding a decimal hundredth to the nearest tenth. For example, review that 3.4 may be written as the two-place decimal 3.40, and that 3.5 may be written as 3.50. Mark the number line in hundredths to show 3 and 40 hundredths to 3 and 50 hundredths. Discuss how 3.42 (3 and 42 hundredths) is rounded down to 3.40 (or 3.4) and how 3.45 is rounded up to 3.50 (or 3.5).



- Review that it is necessary to check only the digit to the right of a given place when rounding a whole number to the given place. The digit 5 or a digit greater than 5 indicates that the number is rounded up. A digit less than 5 indicates that the number is rounded down. Lead the students to

Exercises

Round to the nearest one.

1. 6.7 7 2. 4.37 4 3. 16.059 16 4. 47.9 48 5. 9.201 9
6. 5.7803 6 7. 2.907 3 8. 0.98 1 9. 220.197 220 10. 59.98 60

Round to the nearest tenth.

11. 0.927 0.9 12. 20.189 20.2 13. 5.55 5.6 14. 7.424 7.4 15. 6.39 6.4
16. 4.6385 4.6 17. 108.27 108.3 18. 31.43 31.4 19. 41.301 41.3 *20. 0.999 1.0

Round to the nearest hundredth.

21. 5.283 5.28 22. 9.4358 9.44 23. 1.3586 1.36 24. 3.704 3.70 25. 6.007 6.01
26. 2.893 2.89 27. 0.214 0.21 28. 0.0396 0.04 29. 11.009 11.01 *30. 2.995 3.00

Round to the nearest thousandth.

31. 1.2355 1.236 32. 3.6601 3.660 33. 7.2589 7.259 34. 0.3742 0.374 35. 1.4992 1.499
36. 0.9032 0.903 37. 2.0046 2.005 38. 0.5009 0.501 39. 1.0043 1.004 40. 0.0005 0.001

Solve.

41. What is the mass of Jim's stone to the nearest tenth of a kilogram? 1.7 kg

42. What is the mass of Jim's stone to the nearest hundredth of a kilogram? 1.74 kg

Stacey has three timers. One runs for 3 min, another runs for 5 min, and the other runs for 6 min.

1. How can she time an egg to boil for 4 min?
2. How can she time an egg to boil for 7 min?

Answers are given at the right

PROBLEM SOLVING

RELATED ACTIVITIES

- Have students mark the whole numbers from 0 to 3 on a number line for tenths. (Use copies of page T390.) Then have them write a two-place or a three-place decimal between 0 and 3, locate its position on the number line, and round it to the nearest tenth.
- Challenge students to write word problems similar to those in the *Problem Solving* feature for others to solve.
- Have the students search magazines and newspapers for examples of rounded numbers. These may be reported to the class and placed on the display board.

1. Start the timer that runs for 6 min and one that runs for 5 min at the same time. When the timer that runs for 5 min stops, place the egg in boiling water. When the timer that runs for 6 min stops, start the timer that runs for 3 min. When it stops, take the egg out of the water.
2. Start the timer that runs for 6 min and the one that runs for 5 min at the same time. When the timer that runs for 5 min stops, place the egg in boiling water. When the timer that runs for 6 min stops, start the timer that runs for 3 min. When it stops, start it again. When it stops the second time, take the egg out of the water.

realize that the same procedure can be applied to decimals. Write a few numerals on the board, but do not show all the digits in each numeral. Have students explain how they can round such numbers to the given place.

Round 12.7 — — to the nearest whole number.
Round 0.39 — — to the nearest tenth.
Round 1.426 — to the nearest hundredth.

Using the Pages

- Begin with a brief discussion of the photograph on page 98. Ask students who have rock collections to name the kinds of rocks in their collections and possibly to identify Jim's stone. Lead the students through the worked example. Emphasize that the digit that determines whether 1.735 will be rounded up or rounded down is the digit in the tenths' place. Because the digit in the tenths' place is 7, 1.735 is rounded up to 2, to the nearest whole number.

Working Together: Have the students refer to the place-value chart on page 98 for Ex. 1 and 2. Question the students to learn which digits determined their answers for Ex. 3-6.

Exercises: Ex. 20 is starred because rounding 0.999 to the nearest tenth results in a change in the ones' digit. The 9 in the tenths' place is rounded up so that the number becomes 1.0. Emphasize that 0 tenths must be shown in the numeral. A similar change occurs in Ex. 30. Ex. 41 and 42 refer to the information in the worked example on page 98.

Problem Solving: Students who have difficulty with these word problems can begin with similar ones that are easier. For example, "Stacey has two timers. One runs for 2 min and the other runs for 1 min. How can she time an egg to boil for 3 min?"

Assessment

Round to the nearest one.

1. 8.35 8 2. 16.782 17

Round to the nearest tenth.

3. 17.553 17.6 4. 9.2222 9.2

Round to the nearest hundredth.

5. 0.9308 0.93 6. 6.909 6.91

Round to the nearest thousandth.

7. 5.2818 5.282 8. 0.1396 0.140

LESSON OUTCOME

Round and add to estimate sums for decimals to ten-thousandths; compare the estimate of the sum with the exact sum; round and subtract to estimate differences for decimals to ten-thousandths; compare the estimate of the difference with the exact difference; solve related word problems

Prerequisite Skills

Add from two to four decimals (to ten-thousandths) with different numbers of decimal places; subtract decimals (to ten-thousandths) with different numbers of decimal places

Checking Prerequisite Skills

Add.

1. $\begin{array}{r} 6.35 \\ \underline{4.28} \\ 10.63 \end{array}$ **2.** $\begin{array}{r} 0.8609 \\ 0.98 \\ \underline{0.823} \\ 2.6639 \end{array}$

3. 1.68 + 5.9034 + 2.987 10.5704

4. 9.812 + 2.4 + 3.215 + 6.7983 22.2253

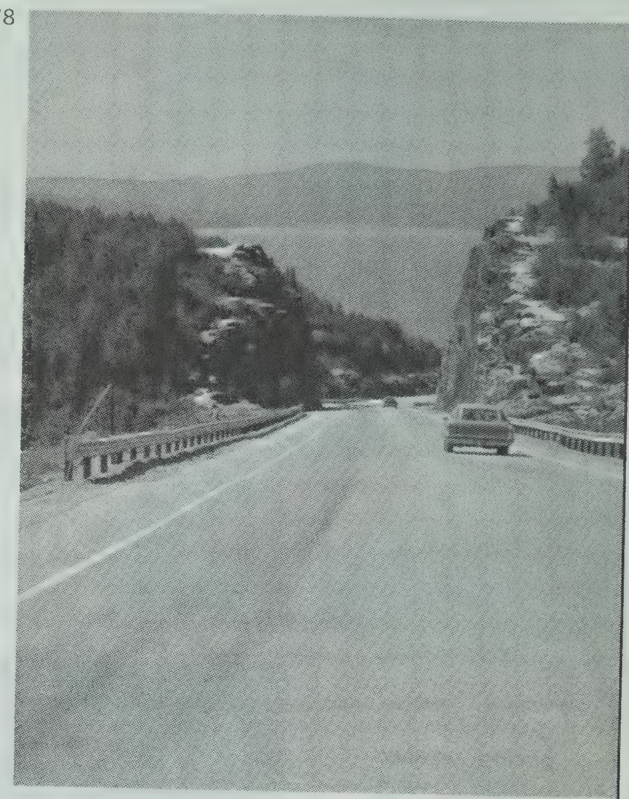
Subtract.

5. $\begin{array}{r} 3.805 \\ \underline{1.296} \\ 2.509 \end{array}$ **6.** $\begin{array}{r} 0.7001 \\ \underline{0.2987} \\ 0.4014 \end{array}$

7. 6.7 − 1.2835 5.4165 **8.** 0.888 − 0.3 0.588

Estimating Sums and Differences

Last week Alice spent $9.78 on gasoline for her car. This week she spent $7.83 on gasoline. About how much more did she spend on gasoline last week?



To estimate the difference of 9.78 and 7.83, round to the nearest one and subtract.

$$\begin{array}{rcr} 9.78 & \longrightarrow & 10 \\ \underline{7.83} & \longrightarrow & \underline{8} \\ & & 2 \end{array}$$

Alice spent about $2 more last week on gasoline.

For the exact difference, subtract in the usual way.

$$\begin{array}{r} {}^{8}\ {}^{17} \\ \cancel{9}.\cancel{7}8 \\ \underline{7.83} \\ 1.95 \end{array}$$

Alice spent exactly $1.95 more last week on gasoline.

Rounding can also help you estimate the sum.

To estimate the sum of 0.38 and 0.794, round to the nearest tenth and add.

$$\begin{array}{rcr} 0.38 & \longrightarrow & 0.4 \\ \underline{0.794} & \longrightarrow & \underline{0.8} \\ & & 1.2 \end{array}$$

For the exact sum, add in the usual way.

$$\begin{array}{r} {}^{1}\ {}^{1}\ \ \ \\ 0.38 \\ \underline{0.794} \\ 1.174 \end{array}$$

The sum of 0.38 and 0.794 is about 1.2.

100

LESSON ACTIVITY

Before Using the Pages

- Review the procedure for estimating the sum of two whole numbers. Write 739 + 482 on the board and ask the students to suggest how to estimate the sum. Review that each addend would likely be rounded to the nearest hundred rather than to the nearest ten. The sum of the rounded addends provides an estimate of the exact sum. Have one student show the addends in vertical form. Have another student write the estimate of the sum. Have a third student find the exact sum. Compare the estimate of the sum with the exact sum.

$$\begin{array}{lr} & 739 \\ & \underline{+\ 482} \\ \text{Exact sum} \longrightarrow & 1221 \\ \text{Estimate of the sum} \longrightarrow & \boxed{1200} \end{array}$$

In a similar manner, review the procedure for estimating the difference of two whole numbers. For an example such as 86 − 29, note that the numbers would be rounded to the nearest ten.

$$\begin{array}{lr} & 86 \\ & \underline{-\ 29} \\ \text{Exact difference} \longrightarrow & 57 \\ \text{Estimate of the difference} \longrightarrow & \boxed{60} \end{array}$$

Using the Pages

- Have a student read the word problem at the top of page 100. Establish that in order to estimate the difference each number is rounded so that it has one digit other than zero. Have students explain the procedure for rounding each number to the nearest one (whole number).

  Ask students to explain the steps for finding the exact difference. Compare the estimate of the difference with the exact difference.

Working Together

Round to the nearest one.

1. 2.1 2
2. 6.56 7

Round to the nearest tenth.

3. 4.51 4.5
4. 0.7834 0.8

Estimate each sum. Estimates may vary.
Then find the exact sum.

5. 0.87 + 0.928 1.798 (1.8)
6. $3.82 + $3.41 + $9.77 $17.00 ($17)

Estimate each difference.
Then find the exact difference.

7. 4.6 − 2.487 2.113 (3)
8. $0.98 − $0.28 $0.70 ($0.70)

Exercises

Estimate each sum. Then find the exact sum. Estimates may vary for Ex 1-31.

1.	2.	3.	4.	5.
1.7	6.48	0.3456	5.26	$8.96
8.2	6.97	0.45	0.798	4.54
9.9 (10)	13.45 (13)	0.7956 (0.8)	6.058 (6)	$13.50 ($14)

6.	7.	8.	9.	10.
0.692	0.2105	4.6	0.4965	$4.37
5.43	0.54	7.7	0.73	6.89
4.178	0.893	2.0	0.2823	0.50
10.300 (10)	1.6435 (1.6)	14.3 (15)	1.5088 (1.5)	$11.76 ($12)

Estimate each difference. Then find the exact difference.

11.	12.	13.	14.	15.
8.9	4.1	0.9286	5.000	$7.25
7.1	1.0	0.3700	2.987	6.35
1.8 (2)	3.1 (3)	0.5586 (0.5)	2.013 (2)	$0.90 ($1)

16.	17.	18.	19.	20.
6.3	0.770	8.98	3.1500	$6.97
4.0	0.395	3.98	2.3333	2.00
2.3 (2)	0.375 (0.4)	5.00 (5)	0.8167 (1)	$4.97 ($5)

Estimate each result. Then find the exact result.

21. 7.669 + 7.38 15.049 (15)
22. 9.3 − 6.3 3.0 (3)
23. 5.34 − 2.6 2.74 (2)
24. 4.9 + 3.5275 + 6 + 5.485 19.9125 (20)
25. 7.643 − 4.8764 2.7666 (3)
26. 5.5 + 7.4 12.9 (13)
27. $8.14 − $2.69 $5.45 ($5)
28. $7.03 + $6.57 + $4 $17.60 ($18)
29. $6.62 − $3.20 $3.42 ($4)

Solve by estimating.

30. A parcel has a mass of 3.7 kg. Another parcel has a mass of 6.5 kg. Find the difference in the masses of the parcels. 3 kg

31. Ed bought bread for $0.67, milk for $0.84, apples for $1.73, and tomatoes for $0.92. About how much did he spend? $5

101

RELATED ACTIVITIES

- For practice with estimating, have students estimate sums for exercises on pages 91 and 93 and differences for exercises on pages 95 and 97. Then have them compare the estimates with the exact results.
- Write a one-digit whole number, such as 3, or a one-place decimal with 0 ones, such as 0.2, on the board. Tell the students that it is an estimate of the sum for an addition exercise. Have them write and complete an addition exercise that would have that number as an estimate of the sum. Discuss their results. This activity may be adapted for subtraction exercises.

Explain that a sum can be estimated in a similar way by rounding the decimal addends. Point out that the addends are rounded to the nearest tenth since they show 0 ones. Have students explain the steps of rounding the addends, estimating the sum, and finding the exact sum. Compare the estimate of the sum with the exact sum.

Working Together: Ex. 1-4 deal with the skill of rounding a decimal. This skill is applied in Ex. 5-8. Note that the students must decide whether to round the numbers to the nearest one or to the nearest tenth. Suggest that they round numbers less than one to the nearest tenth. The numbers greater than one can be rounded to the nearest whole number. Point out that the sums and the differences for exercises involving amounts of money written with decimal points are estimated in the same way as for exercises involving decimals.

Exercises: For Ex. 1-29, have the students write the estimate of the sum (difference) before finding the exact sum (difference). Ex. 30 and 31 require only an estimate.

Assessment

Estimates may vary.
Estimate each sum. Then find the exact sum.

1.	2.
2.93	0.7984
8.265	0.6
11.195 (11)	0.13
	1.5284 (1.5)

3. 0.915 + 0.28 + 0.3179 + 0.5 2.0129 (2.0)
4. 1.89 + 7.642 + 3.08 + 8.1 20.712 (21)

Estimate each difference. Then find the exact difference.

5.	6.
8.53	0.3027
6.68	0.1989
1.85 (2)	0.1038 (0.1)

7. 7.2 − 7.1489 0.0511 (0.1)
8. 6.394 − 2.8 3.594 (3)

Solve by estimating.

9. Doug had a board that was 3.8 m long. He cut off a piece 2.5 m long. How long was the piece that remained? (1 m)

Pages 102-103

OBJECTIVE

Demonstrate competence in rounding decimals, in adding and subtracting decimals, and in estimating sums and differences; solve related word problems

Practice

Round to the nearest one.

1. 2.831 3 2. 4.9 5 3. 1.328 1 4. 12.04 12 5. 0.9467 1

Round to the nearest tenth.

6. 5.624 5.6 7. 2.342 2.3 8. 17.45 17.5 9. 0.6432 0.6 10. 2.95 3.0

Round to the nearest hundredth.

11. 7.473 7.47 12. 1.2498 1.25 13. 0.731 0.73 14. 3.6201 3.62 15. 1.4005 1.40

Round to the nearest thousandth.

16. 4.2857 4.286 17. 0.5860 0.586 18. 1.3702 1.370 19. 4.7485 4.749 20. 0.9999 1.000

Estimate each sum. Then find the exact sum. Estimates may vary

21. 4.607	22. 0.4792	23. 2.7	24. 4.36	25. $5.40
1.69	0.86	4.3	5.0	7.61
4.3	0.9018	1.0	7.932	3.00
10.597 (11)	2.2410 (2.3)	9.5	6.5	1.98
		17.5 (18)	23.792 (24)	$17.99 ($18)

26. 2.972 + 3.18 + 4.035 10.187 (10) 27. $9.50 + $6.47 $15.97 ($16)
28. 1.254 + 3.496 + 6 + 7.08 17.83 (17) 29. $7 + $8.40 + $9.67 $25.07 ($25)

Estimate each difference. Then find the exact difference. Estimates may vary

30. 3.05	31. 5.6	32. 2.4	33. 1.1736	34. $4.74
1.47	3.4	1.387	0.98	3.98
1.58 (2)	2.2 (3)	1.013 (1)	0.1936 (0.2)	$0.76 ($1)

35. 5.932 − 1.781 4.151 (4) 36. 6.75 − 4.1 2.65 (3) 37. $7.52 − $5 $2.52 ($3)
38. 7.38 − 4 3.38 (3) 39. 1 − 0.932 0.068 (0.1) 40. $9 − $2.38 $6.62 ($7)

Add or subtract.

41. 14.73 + 0.964 15.694 42. 4.387 − 2.099 2.288 43. 5.32 + 7 + 6.41 18.73
44. 12.024 − 5.83 6.194 45. 6 + 3.06 + 4.1 13.16 46. 16.8 − 4.298 12.502
47. 23.087 − 3.987 19.100 48. 7.43 + 0.6211 8.0511 49. 9.806 − 5 4.806
50. 1 + 0.538 + 2 3.538 51. 7 − 3.428 3.572 52. 7.38 + 2 + 9.145 + 9 27.525
53. $3.20 − $3 $0.20 54. $9.14 + $7 + $6.32 $22.46 55. $14 − $9.36 $4.64
56. $4.72 + $6 $10.72 57. $8.05 − $2.19 $5.86 58. $7.39 − $4 $3.39

102

LESSON ACTIVITY

Before Using the Pages

- Draw the following Magic Square on the board and have the students begin to find the sum for each column, row, and diagonal. When they notice a pattern for the sums, ask for the sum they expect to find. Then add to check. Explain that in a Magic Square, the sums for the columns, rows, and diagonals are the same. Then erase the numeral in one of the squares and ask the students to suggest how to find the missing number.

7	6	11
12	8	4
5	10	9

Using the Pages

- For Ex. 21-40, ensure that the students write the estimates of the sums and the differences before finding the exact results. Because the Magic Squares for Ex. 62-67 have more than one missing number, suggest that the students find a column, row, or diagonal that has only one missing number and then find that number first. This procedure may be repeated until the Magic Square is completed.

Find the Magic Squares.

In a Magic Square, the sums for the columns, rows, and diagonals are the same.

59. a Magic Square

1.2	1.7	1.6
1.9	1.5	1.1
1.4	1.3	1.8

60. not a Magic Square

1.12	1.21	1.31
1.3	1.21	1.13
1.22	1.22	1.2

61. not a Magic Square

32.5	15	27.25
20	25	30.5
22.5	35.5	17.75

Complete these Magic Squares.

62.

3.25	1.5	2.75
2	? 2.5	3
? 2.25	3.5	? 1.75

63.

? 9.25	? 3.5	6.75
4	6.5	9
6.25	? 9.5	? 3.75

64.

3.1	1.2	? 1.7
? 0.6	2	3.4
2.3	? 2.8	0.9

65.

? 0.25	? 0.875	0.75
? 1.125	0.625	? 0.125
0.5	? 0.375	1

66.

? 0.3	? 0.425	0.4
? 0.475	? 0.375	? 0.275
0.35	0.325	0.45

67.

? 7	8.25	? 8
? 8.75	7.75	? 6.75
7.5	7.25	? 8.5

For each of these, estimate the answer. Then find the exact answer.

68. Mark bought a jacket that was on sale for \$7.98. The cost of the jacket had been \$9.98. How much did Mark save by buying the jacket on sale? \$2.00 (\$2)

69. Sheila bought a dress for \$9.95, a pair of jeans for \$8.80, a sweater for \$6, and a belt for \$2. How much did she spend? \$26.75 (\$27)

103

RELATED ACTIVITIES

- Some students may be able to develop Magic Squares and then write the Magic Squares with missing decimals for others to complete.
- For practice in comparing and rounding decimals, state, "I am thinking of a two-place decimal. When it is rounded to the nearest tenth, it is 4.6. What may the number be?" Have students list the possibilities (4.55, 4.56, 4.57, . . ., 4.64).
- To practice addition and subtraction skills and to reinforce the relationship between addition and subtraction, prepare a work sheet with additions and subtractions that have missing digits. The number of missing digits and the number of regroupings may vary to provide various levels of difficulty. The following are some examples.

1. $\begin{array}{r} 4.\boxed{9}\boxed{5} \\ +\boxed{1}.2 \\ \hline 6.15 \end{array}$

2. $\begin{array}{r} 0.57\boxed{9} \\ -0.\boxed{1}67\boxed{3} \\ \hline 0.4117 \end{array}$

3. $\begin{array}{r} \boxed{2}.6\boxed{9}\boxed{3} \\ +2.\boxed{7}3 \\ \hline 5.423 \end{array}$

4. $\begin{array}{r} 7.0\boxed{8}\boxed{6}\boxed{7} \\ -\boxed{3}.\boxed{8}9 \\ \hline 3.1967 \end{array}$

5. $\begin{array}{r} \boxed{3}.47\boxed{3}\boxed{6} \\ +3.\boxed{7}\boxed{8} \\ \hline 7.2536 \end{array}$

Page 104

OBJECTIVE

Decide whether measurements are reasonable; express an unreasonable measurement as a reasonable one by changing the position of the decimal point and/or the unit of measurement

Materials

a ruler marked in centimetres

Vocabulary

centimetres, cm

RELATED ACTIVITIES

- Ask students to write sentences using an unreasonable measurement which can become a reasonable measurement by changing the unit of measurement or the position of the decimal point. Then have other students write the sentences giving reasonable measurements.

Reasonable Measurements

To decide whether a measurement is reasonable, think about the unit of measurement and the position of the decimal point.

Mary walks 1.65 m to school.

This measurement is unreasonable. If you change the unit, the measurement is reasonable. Mary walks 1.65 km to school.

Jim's height is 13.7 cm.

This measurement is unreasonable. If you change the position of the decimal point, the measurement is reasonable. Jim's height is 137 cm.

Write each sentence giving a reasonable measurement.
Other answers are possible.

Think about the unit of measurement and the position of the decimal point.

1. The width of the river is 155 cm (centimetres). 155 m
2. At the equator, the temperature at noon is about 3.5°C. 35°C
3. Irma ran 1 km in 6 s. 6 min
4. The mass of the necklace is 45 kg. 4.5 g
5. The aquarium holds 45 mL of water. 4.5 L
6. The mass of a quarter is about 500 g (grams). 5 g
7. An average letter has a mass of 1.5 kg. 1.5 g
8. The boy's waist measurement is 58 mm. 58 cm
9. Joel drinks 0.05 L of milk every day. 0.5 L
10. The mass of an electric typewriter is 9.5 g. 9.5 kg
11. The sun is about 149 000 000 cm from Earth. 149 000 000 km

PROBLEM SOLVING

104

LESSON ACTIVITY

Before Using the Page

- Write the following statement on the board.
  "A student in Grade Six is about 1.4 cm tall."
  Ask whether this is a reasonable statement. Display a ruler marked in centimetres and ask a student to indicate a length of 1.4 cm. Ask the students to change either the unit of measurement or the position of the decimal point so that the measurement becomes a reasonable one for the situation. This results in two possibilities: 1.4 m and 140 cm.

Using the Page

- Have a student read the information at the top of the page. Allow the students a few moments to read the first example. Ask if changing the position of the decimal point could also result in a reasonable measurement. Establish that 1650 m is also a reasonable measurement.

  Use a similar procedure with the second example. Some students may suggest changing the position of the decimal point and changing the unit of measurement to give 1.37 m.

- Note that different answers may be possible, but there may be a preferred way to express the measurement. For example, 155 m, 15 500 cm, and 155 000 mm are reasonable measurements for Ex. 1, but 155 m is the preferred way to write the measurement. You may wish to discuss the students' answers after they have completed the exercises, to enable them to name the possible choices and to give reasons for their choices.

Checking Up

Write the decimal for each of these.

1\.

tens	ones	tenths	hundredths
1	2	0	8

12.08

2\. 

2.1243

3\. one and eight-tenths 1.8
4\. seven and nine-hundredths 7.09
5\. eighty-six thousandths 0.086
6\. forty-five ten-thousandths 0.0045

Write each of these using words.

7\. 6.76 8. 55.3 9. 0.405 10. 22.2002 11. 0.2022

Write each as a four-place decimal.

12\. 2.377 2.3770 13. 4.1 4.1000

Write each as a three-place decimal.

14\. 6.2 6.200 15. 7 7.000

Write each as a two-place decimal.

16\. 5.5 5.50 17. 6.7500 6.75

Write each as a one-place decimal.

18\. 36 36.0 19. 4.000 4.0

Use >, <, or = to make true statements.

20\. 2.244 ◯ 2.2444 < 21. 9.6600 ◯ 9.66 = 22. 7.717 ◯ 7.7117 >

List from greatest to least.

23\. 6.65, 6.6555, 6, 6.6, 6.665
6.665, 6.6555, 6.65, 6.6, 6

List from least to greatest.

24\. 1.001, 1.01, 1.0101, 1.101, 1.1
1.001, 1.01, 1.0101, 1.1, 1.101

Round to the nearest

25\. ten. 95.4 95
26\. one. 6.84 7
27\. tenth. 1.1165 1.1
28\. hundredth. 0.7355 0.74
29\. thousandth. 3.1489 3.149

Add.

30\. 4.2331 + 6.0824 = 10.3155
31\. 13.4 + 1.7918 = 15.1918

Subtract.

32\. 4.637 − 1.989 = 2.648
33\. 8.342 − 6.9 = 1.442
34\. 2.9 − 0.2437 = 2.6563

Find the result.

35\. 9.2 + 3.497 + 2.04 14.737 36. 13.17 − 11.208 1.962 37. 3.94 + 6.1238 10.0638
38\. 1.0695 + 11 + 4.28 16.3495 39. 2.3942 − 1 1.3942 40. 1.5 − 0.611 0.889
41\. 5.0008 + 1.99 + 3 9.9908 42. 7.04 − 6.9981 0.0419 43. 100 − 0.732 99.268
44\. \$9 + \$4.95 + \$1.45 \$15.40 45. \$565.72 − \$477 \$88.72 46. \$29 − \$11.45 \$17.55

7 six and seventy-six hundredths
8 fifty-five and three-tenths
9 four hundred five-thousandths
10 twenty-two and two thousand two ten-thousandths
11 two thousand twenty-two ten-thousandths

105

Skills	Exercises	Related Pages
Write decimals	1-6	T 90-T 93
Write word names for decimals	7-11	T 90-T 93
Show a number as a four-place decimal, as a three-place decimal, as a two-place decimal, and as a one-place decimal	12-19	T 94-T 95
Compare decimals	20-22	T 94-T 95
Order decimals	23, 24	T 96
Round decimals	25-29	T 106-T 107
Add decimals	30, 31, 35, 37	T 98-T 99
Add decimals and whole numbers	38, 41, 44	T 100-T 101
Subtract decimals	32-34, 36, 40, 42	T 102-T 103
Subtract decimals and whole numbers	39, 43, 45, 46	T 104-T 105

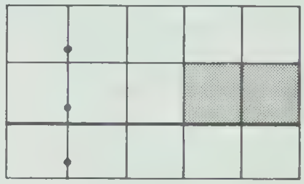
Page 105

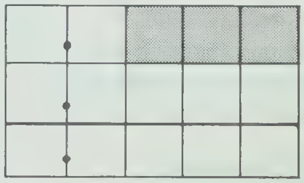
OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- For practice with place value, provide exercises such as the following.
  7 ones 2 tenths 5 thousandths = _____ thousandths
  1 tenth 3 ten-thousandths = _____ ten-thousandths
  802 thousandths = _____ tenths _____ thousandths
- Adapt the third activity in *Related Activities* on page T 7 for decimals.
- Adapt the rules of the game "Greatest Sum" on page T 379 for a game called "Least Difference" involving subtraction with decimals. Use charts similar to each of the following.





Comments

If students have difficulty writing or comparing decimals, provide more practice with models, place-value charts, and number lines. Encourage the students to draw a place-value chart or a number line for assistance when they have difficulties with decimals.

Remind the students that they can estimate the results for Ex. 35-46 and use the estimates as a check. Also they can use addition to check subtraction exercises.

Unit 6 Overview

Measurement

This unit explores three types of measurement — length, area, and volume — in terms of metric units. The first lesson directs attention to selecting appropriate units of length in a variety of situations. Perimeter is first established as the total length of the sides of a polygon, and then the specialized techniques are developed for finding perimeters of rectangles, squares, and regular polygons. The topic of area is approached by counting square units and parts of square units which are contained in polygons. The rule for finding the area of a rectangle is developed and applied, not only to rectangles but also to polygons which may be divided into rectangles. Finding the area of a parallelogram involves a slide of a triangular portion from one side to another to form a rectangle, followed by application of the rule for finding the area of a rectangle. The method for finding the area of a triangle is derived from the one for a parallelogram by noting that a triangle is one-half of a parallelogram with the same base and the same height. The topic of volume is introduced by counting the cubes in various regular and irregular structures. Finding the volume of a rectangular prism is approached first through multiplying the number of cubes in one layer and the number of layers, and then through multiplying the area of the base and the height. The lesson in problem solving introduces the use of a tree diagram to find the number of possibilities when several combinations of items are made.

Prerequisite Skills

- add whole numbers
- add decimal tenths
- measure line segments to the nearest centimetre
- multiply whole numbers
- write a multiplication sentence for an array

Unit Outcomes

- estimate and measure length in millimetres, in centimetres, in decimetres, and in metres
- choose the preferred unit of length from millimetre, centimetre, decimetre, metre, kilometre
- use addition to find the perimeter of a shape
- measure the sides of a shape to the nearest centimetre and find the perimeter
- use multiplication and addition to find the perimeter of a rectangle
- use multiplication to find the perimeter of a regular polygon (square, pentagon, hexagon, octagon, decagon)
- find the area in square centimetres by counting whole square centimetres and parts of square centimetres
- use multiplication to find the area of a rectangle and of a square
- use multiplication and addition to find the area of a shape that can be divided into rectangles and squares
- use multiplication and division to find the area of a triangle
- find volume in cubic centimetres by counting
- find the volume of a rectangular prism in cubic centimetres by multiplying the number of centimetre cubes in one layer and the number of layers
- find the volume of a rectangular prism by multiplying the area of the base and the height
- solve word problems involving measurements
- use a tree diagram to find the number of possibilities

Background

The metric system was introduced in France in the latter part of the eighteenth century by a committee which was formed to develop a rational system of measurement. As a result, the *metre*, *litre*, and *kilogram* were created as standard units. Since then, more and more countries have adopted the system and, in 1960, agreements were reached concerning the various units and their symbols. It has been called the *International System of Units*, or SI (from the French name, Le Système International d'Unités).

Prior to the adoption of the metric system, most measurement systems lacked rational structure. Units of measurement were often developed from parts of the body, such as the hand and the foot, and varied according to local customs and adaptations. Conversions from one unit to another involved such diverse factors as 2, 3, 4, $5\frac{1}{2}$, 12, 16, and 5280. The metric system, on the other hand, is based on the simple and logical decimal system, the same as our base-ten numeration system. Units are related by factors such as 10, 100, and 1000, and conversion is easily accomplished from one unit to another by multiplication and division. Computation is, therefore, much easier with metric measures. Furthermore, the units for length, capacity, and mass are themselves related.

The *metre* is the base unit of length. All other units of length are derived by multiplying one metre by a power of ten. Prefixes added to the word ''metre'' indicate which power of ten is the multiplier. For some types of measurement many different terms are used, while in others only a few are commonly used. The usual prefixes are shown in the chart at the right. For linear measure, the most common units are kilometre (km), metre (m), centimetre (cm), and millimetre (mm). Metric units for area are derived from linear units. A square with sides 1 cm long encloses an area of 1 square centimetre (1 cm²). Other common units for area are the square metre (m²) and the square kilometre (km²). The most common metric units for volume are the cubic metre (m³) and the cubic centimetre (cm³), although the structure of the metric system makes others possible.

kilo	—	1000
hecto	—	100
deca	—	10
———	—	1
deci	—	0.1
centi	—	0.01
milli	—	0.001

In *Starting Points in Mathematics 5* the terms *length* and *width* are used to name the longer and the shorter sides of rectangles, but in this book the terms *base* and *height* are used. The two sets of terms are synonymous, but the latter is better at this stage because the areas of parallelograms and triangles are to be found. The height in these cases is measured perpendicular to the base, and this permits the rule for finding the area of a rectangle to be used and adapted since the sides of a rectangle are perpendicular to the base.

There is no direct relationship between perimeter and area. This can be discovered by students using a geoboard and geopaper. For example, in forming different rectangular shapes that have a perimeter of 16 cm, students can discover that the square has the greatest area (A). But for rectangular shapes having a given area, the square has the least perimeter (B).

A

Length	Width	Perimeter	Area
7 cm	1 cm	16 cm	7 cm^2
6 cm	2 cm	16 cm	12 cm^2
5 cm	3 cm	16 cm	15 cm^2
4 cm	4 cm	16 cm	16 cm^2

B

Length	Width	Perimeter	Area
64 cm	1 cm	130 cm	64 cm^2
32 cm	2 cm	68 cm	64 cm^2
16 cm	4 cm	40 cm	64 cm^2
8 cm	8 cm	32 cm	64 cm^2

Teaching Strategies

For the first lesson in this unit, it may be advisable to organize the class into small groups because some of the exercises require the students to move about the classroom. It may also be desirable for each student to have a partner for this lesson and for some of the activities suggested for other lessons. Later lessons in the unit do not involve moving about the classroom to complete the exercises.

In connection with the topic of perimeter it is important that students associate a literal meaning to the term: *peri* means around, *meter* means to measure. The perimeter of any polygon may be found by measuring around it, or, in other words, by adding the measures of all the sides. For special kinds of polygons, such as rectangles, squares, and regular polygons, shorter methods may be used. The perimeters of squares and other regular polygons may be found by multiplying the length of one side by the number of sides. The following rule applies only to rectangles.

$$\text{Perimeter} = (2 \times \text{base}) + (2 \times \text{height})$$

In this unit, measurement is applied to one-dimensional, two-dimensional, and three-dimensional shapes, and students should be helped to associate with them the appropriate units of measurement and the ways of writing the symbols. For one-dimensional shapes (line segments), the most common units of length are represented by the symbols km, m, cm, and mm; for two-dimensional shapes (plane figures), the most common square units are represented by the symbols km^2, m^2, and cm^2; and for three-dimensional shapes (solids), the most common cubic units are represented by the symbols m^3 and cm^3. It may be helpful to point out to students that the exponent 2, as in m^2 and cm^2, relates to the two dimensions used in finding area, and that the exponent 3, as in m^3 and cm^3, relates to the three dimensions used in finding volume.

On page 115 there are three starred exercises dealing with area in which the students are challenged to find one of the two dimensions. Before assigning these exercises it may be desirable to review briefly the inverse relationship between multiplication and division, as shown.

factor		factor		product		product		factor		factor
7	×	▦	=	63		63	÷	7	=	9

In the lesson on page 118, the formula for finding the area of a triangle is developed.

$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$

This may be the first occasion where students see a denominator used in an expression to denote division. Prior to this lesson it would be wise to spend a few minutes discussing this notation. Students should have no difficulty explaining, for instance, that $\frac{1}{2}$ represents 1 object divided into 2 equal parts, and that $\frac{1}{4}$ represents 1 object divided into 4 equal parts. Then they may be led to see that $\frac{24}{2}$ can represent 24 objects divided into 2 equal groups, for which $24 \div 2$ has the same meaning. With this background, they may be asked to interpret such expressions as $\frac{6 \times 4}{2}$ and $\frac{15 \times 20}{2}$, which are similar to the type encountered in the formula for finding the area of a triangle.

For the exercises on pages 122 and 123, the students must visualize the cubes that are hidden in the structures. Most students will respond well to this feature and will probably enjoy the *Related Activities* suggested for this lesson.

Results of the *Checking Up* lesson for this unit should be examined to detect any errors, misconceptions, and misuses of the various units of measurement. Appropriate reteaching and additional work should be arranged. The exercises of *Checking Skills* on page 133 should be used to assess the students' strengths in the four basic operations and to diagnose specific weaknesses that they may have. Selected review and practice should be provided before the students begin multiplying and dividing decimals in Unit 7.

Materials

- metre sticks and objects for measuring length; a ruler marked in centimetres and millimetres and an unmarked straight edge for each student
- a trundle wheel (optional)
- a ruler marked in centimetres and decimetres
- plastic drinking straws and string or pipe cleaners, or cardboard strips and pronged paper fasteners
- copies of page T 395
- a regular pentagon and a regular hexagon prepared from straws or cardboard strips
- copies of page T 396, crayons, and scissors for each student
- a large sheet of square grid paper (optional)
- tracing paper (optional)
- a rectangular card such as a birthday card
- cutouts of parallelograms, a display grid (optional)
- centimetre cubes or other cubes; a few prisms (copies of the patterns of prisms on pages T 386 - T 388 can be used to make models)
- a rectangular prism or a small box
- a rectangular shape cut from a blank sheet of paper

Vocabulary

kilometre, km	area
metre, m	square millimetre, mm^2
decimetre, dm	square centimetre, cm^2
centimetre, cm	square decimetre, dm^2
millimetre, mm	square metre, m^2
to the nearest unit	square kilometre, km^2
line segment	diagonal
perimeter	parallelogram
polygon	dimensions
triangle	triangular
quadrilateral	volume
pentagon	cubic centimetre, cm^3
hexagon	cubic decimetre, dm^3
height, base	cubic metre, m^3
regular polygon	surface area
rectangle	face
square	solid
octagon	rectangular prism
decagon	tree diagram

Pages 106-107

LESSON OUTCOME

Estimate and measure length in millimetres, in centimetres, in decimetres, and in metres; choose the preferred unit of length from millimetre, centimetre, decimetre, metre, kilometre

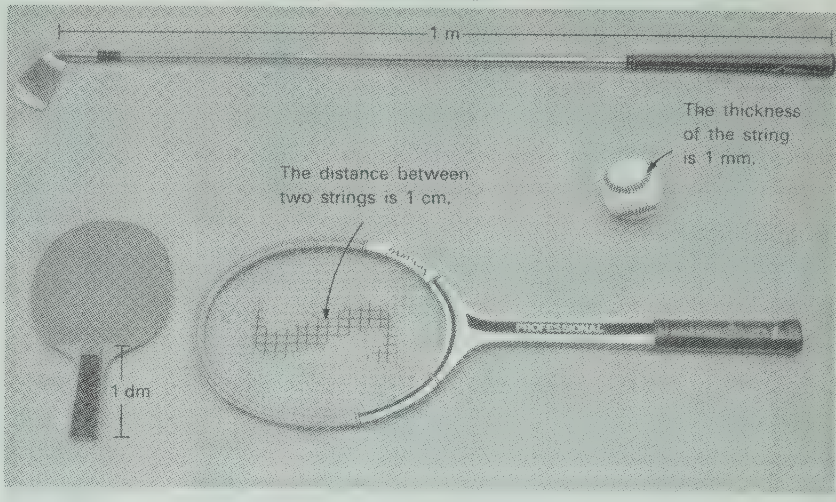
Materials
metre sticks and objects for measuring length; a ruler marked in centimetres and millimetres and an unmarked straight edge for each student; a trundle wheel (optional)

Vocabulary
kilometre, km, metre, m, decimetre, dm, centimetre, cm, millimetre, mm, to the nearest unit, line segment

Background
At this time, the students are involved in choosing appropriate units of length and in measuring objects. The relationships among the units of length, for example, 1 m = 100 cm, are developed in Unit 8.

6 MEASUREMENT

Measuring and Estimating Length



To measure greater lengths, you can use the kilometre.
9 times around a baseball diamond in a stadium is about 1 km.

Working Together

Tell whether you would use millimetres, centimetres, decimetres, metres, or kilometres to measure each of these.

1. the distance you ride on a bicycle kilometres
2. the thickness of a spoke in a bicycle wheel millimetres
3. the height of a basketball hoop metres
4. the length of a badminton racket decimetres or centimetres

Complete the chart. Choose a unit of measurement. Estimate to the nearest unit. Then measure to the nearest unit. Answers will vary.

	Object	Unit	Estimate	Measurement
5.	the length of your classroom	?	?	?
6.	the height of your desk	?	?	?

106

LESSON ACTIVITY

Before Using the Pages

- Display a metre stick. Have students identify it and describe its use. Ask them to suggest lengths that are about 1 m, for example, the distance from a doorknob to the floor. Have students measure the suggested lengths. Ask them to estimate a length to the nearest metre and then to measure the length, for example, the length of the chalkboard or the width of the hall.

  Discuss that the metre is not a suitable unit for measuring lengths such as the height of a shoe box and the length of a shoelace. Lead the students to suggest the centimetre as an appropriate unit of length. Have them note the centimetre marks on their own centimetre rulers and state the length of their rulers in centimetres. Point out that the ruler must be aligned carefully with an object for which the length is being measured. (Some rulers show a mark for 0 cm; others do not, but the end of the scale is clearly marked.)

  Ask students to suggest lengths that are about 1 cm, for example, the width of a paper clip, and then to check by measuring.

  If possible, have students name other units of length such as the kilometre, the decimetre, and the millimetre. They will likely recall encountering such units in word problems of previous units.

Using the Pages

- The photograph enables students to associate lengths of 1 m, 1 dm, 1 cm, and 1 mm with familiar objects. Ask students to identify the objects shown and to describe the indicated lengths. For example, the handle of the table tennis paddle has a length of 1 dm. Have them note the millimetre marks on their rulers and the decimetre marks on a metre stick. Draw attention to the symbols for the units of measurement. Write each symbol on the board, have students write the words, and correct the spelling as required. Have a student read the statements below the photograph. Ask

Exercises

Choose a unit of measurement for each.

1. the distance you would travel to a nearby city kilometres
2. the width of your hand centimetres
3. the distance across a pond metres

Complete the chart. Choose a unit of measurement. Estimate. Then measure to the nearest unit. Answers will vary

	Object	Unit	Estimate	Measurement
4.	your height	?	?	?
5.	the length of your arm	?	?	?
6.	the length of your shortest finger	?	?	?
7.	the length of your pencil	?	?	?
8.	the width of your classroom	?	?	?
9.	the height of your chair	?	?	?
10.	the distance from your desk to the door	?	?	?
11.	the width of your ruler	?	?	?
12.	the height of the chalkboard	?	?	?
13.	the length of a page in this book	?	?	?
14.	the thickness of this book	?	?	?

15. Choose five items and complete a similar chart for these five items. Answers will vary

For each of these lengths, use a straight edge to draw a line segment that you estimate has that length. Then measure the line segment to check. Answers will vary

16. 5 cm 17. 1 dm 18. 7 mm 19. 12 cm 20. 1 cm

107

RELATED ACTIVITIES

- Have students work together to list items having lengths of 1 mm, 1 cm, 1 dm, 1 m, and 1 km. Ask them to illustrate some of the items on the list. Display the pictures for reference.
- Have students list familiar lengths such as the height of a bottle of paint or the length of the hall. Ask them to write an estimate for each length and then to measure the length. A trundle wheel is useful, for example, for measuring the length of the hall. Display the list to enable the students to refer to it frequently.
- For practice in estimating and measuring lengths, have students play the following game in pairs. The first player names a length such as the thickness of a book or the height of a chair. The other player names a unit of measurement and estimates the length to the nearest unit. Both players measure the length to the nearest unit and find the difference between the estimate and the measurement. The first player scores one point for each unit in the difference. For the next round and each successive round, players reverse their roles. Thus, each player has several turns as "first player". The player who first scores 25 points is the winner.

students to suggest other lengths that would be measured in kilometres. They may recall from the lesson on pages 2 and 3 that the distance from the earth to the sun is given in kilometres.

Working Together: Ask students to explain their answers for Ex. 1-4. For Ex. 5 and 6, emphasize that an estimate is a carefully considered opinion, not a haphazard guess. However, explain that the estimate and the measurement will probably not be the same, but will be close. It may be necessary to review the meaning of the phrase "to the nearest unit". Remind the students to include the unit of measurement with the estimate and with the measurement.

Exercises: Provide the students with equipment for measuring in metres, in decimetres, in centimetres, and in millimetres. Before they begin, discuss the directions for each group of exercises, particularly for Ex. 16-20. Remind the students of the meaning of the term *line segment*. Draw a line segment on the board and have students note the end points.

Discuss that an *unmarked* straight edge, such as the edge of a card, is used to draw line segments. Then, a ruler is used to measure their lengths.

Assessment

For each of these, choose a unit of measurement. Estimate. Then measure to the nearest unit.

1. the width of your desk Answers will vary.

2. the length of your eraser

For each of these lengths, use a straight edge to draw a line segment that you estimate has that length. Then measure the line segment to check.

3. 6 cm **4.** 9 mm 9 mm

6 cm

Pages 108-109

LESSON OUTCOME

Use addition to find the perimeter of a shape; measure the sides of a shape to the nearest centimetre and find the perimeter; solve related word problems

Materials

a ruler marked in centimetres and decimetres and an unmarked straight edge for each student; plastic drinking straws and string, or cardboard strips and pronged paper fasteners

Vocabulary

perimeter, polygon, triangle, quadrilateral, pentagon, hexagon

Prerequisite Skills

Add whole numbers; add decimal tenths; measure line segments to the nearest centimetre

Checking Prerequisite Skills

Add.

1. 17 + 29 + 38 + 147 231

2. 6.2 + 4.3 + 8.9 19.4

3. 9.8 + 5.5 + 2.7 + 7.1 + 1.6 26.7

Use an unmarked straight edge to draw a line segment.

4. Measure the length of the line segment to the nearest centimetre. Answers will vary.

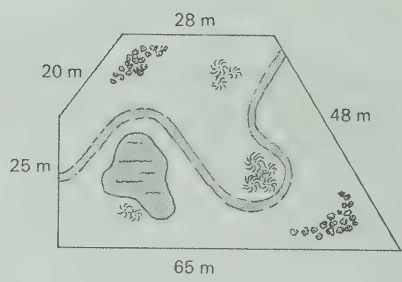
Finding Perimeter

Walter ran around the park.
How far did he run?

Add the lengths of the five sides.

$$\begin{array}{r} 65 \\ 48 \\ 28 \\ 20 \\ \underline{25} \\ 186 \end{array}$$

Walter ran 186 m around the park.

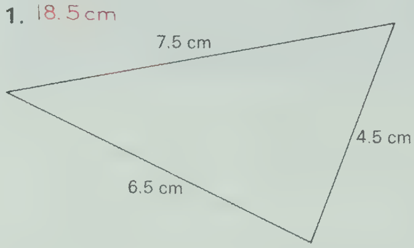


The distance around the park is the **perimeter**.

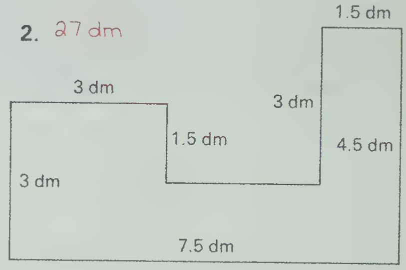
Working Together

Find the perimeter of each shape.

1. 18.5 cm

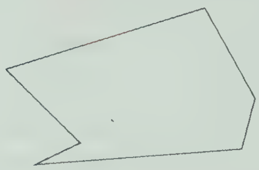


2. 27 dm

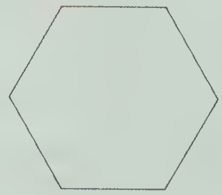


Measure to the nearest centimetre.
Then find the perimeter.

3. 14 cm



4. 12 cm



Answers will vary for Ex. 5 and 6

5. the top of your desk

6. the cover of this book

108

LESSON ACTIVITY

Before Using the Pages

- Have the students help to prepare one or both of the following materials.
  1. plastic drinking straws cut to various lengths and string or pipe cleaners for joining the straws
  2. cardboard strips cut to various lengths with a hole punched in both ends of each strip and pronged paper fasteners for joining the strips

  Have the students work individually or in small groups, joining from three to six straws (strips) to form a closed shape. Introduce the term *polygon* to describe shapes having three or more sides, noting that the sides suggest line segments. (The prefix "poly" is from the Greek word for "many", and "gon" means "angles".) Elicit the following names from the students to describe shapes that they have made: triangle (3 sides); quadrilateral (4 sides); pentagon (5 sides); and hexagon (6 sides). Ask the students to find the total length of straws (strips) used to form each shape, that is, to find the distance around the shape. Ask several students to explain the procedure they used.

Using the Pages

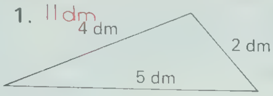
- Have the students read the word problem at the top of page 108 silently. Ask what word describes the distance around a shape. Note the spelling of the word *perimeter*. Have them trace the perimeter of the park in the illustration with one finger, discussing that they must end at the point where they started. Note that the shape is a polygon because the sides are represented by line segments. Ask what name describes the polygon (pentagon). Point out that the length of each side is expressed in the same unit of length (metres). Ask a student to explain how the perimeter of a shape is found if the length of each side is known. Review that the order of adding several numbers does not affect the sum.

Working Together: Some students may not realize that the length of one of the sides in Ex. 2 is not provided, and will suggest that the perimeter is 24 cm, not 27 cm. Lead the students to discover why 24 cm is not the correct perimeter.

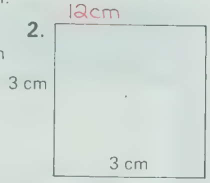
Exercises

Find the perimeter of each.

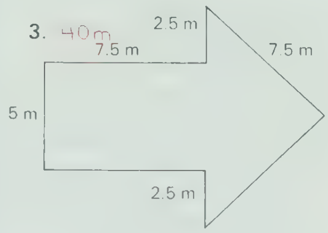
1. 11 dm



2. 12 cm

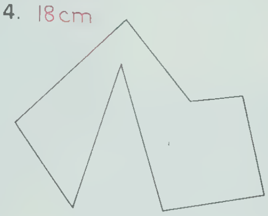


3. 40 m

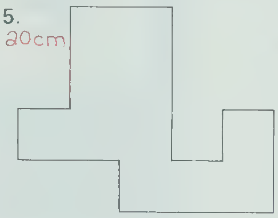


Measure to the nearest centimetre.
Then find the perimeter.

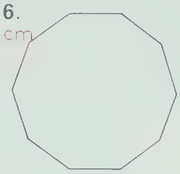
4. 18 cm



5. 20 cm



6. 10 cm



Find the perimeter of each of these.

7. a hexagon with sides 3 cm, 3 cm, 4 cm, 2 cm, 2 cm, 4 cm 18 cm
8. a polygon with sides 14 m, 15 m, 26 m, 21 m, 12 m 88 m
9. a polygon with sides 12 m, 5 m, 10 m, 5 m 32 m
10. a triangle with sides 15 mm, 15 mm, 15 mm 45 mm
11. a quadrilateral with sides 5 cm, 9 cm, 8 cm, 2 cm 24 cm
12. a triangle with sides 7.1 dm, 9.3 dm, 5.2 dm 21.6 dm
13. a pentagon with sides 5.4 m, 8.2 m, 6.1 m, 7.3 m, 8.9 m 35.9 m
14. a polygon with sides 6.3 m, 7.1 m, 6.5 m, 9.2 m 29.1 m

Solve.

15. The sides of the yard at Lakeview School are 243 m, 118 m, 137 m, and 96 m. What is the perimeter of the yard? 594 m
16. The sides of a park are 107.3 m, 92.1 m, 46.5 m, and 42.0 m. How long is the fence around the park? 287.9 m

Use a straight edge. Draw polygons you estimate to have these perimeters. Then measure to check. Answers will vary

17. 8 cm
18. 15 cm
19. 22 cm
20. 4 dm

109

RELATED ACTIVITIES

- Prepare cards showing irregular polygons. Have students measure the sides to the nearest centimetre or to the nearest millimetre and find the perimeter of each.
- For several items such as the top of a table or the schoolyard, have the students choose a unit of length, measure, and find the perimeter.
- Ask each student to cut a piece of string that is 1 m long and to write her/his name on a piece of tape attached to the string. This string may be used to find the perimeter to the nearest metre of rooms at home.
- Students can use string to find the perimeter of a curved object, for example, a jar, by marking the length of string required to go around the object. Then the length of string can be measured, using a metre stick or a centimetre ruler.
- Provide students with copies of page T 395. Have them use a straight edge to join dots and form a polygon. Have them exchange sheets, measure the sides, and find the perimeter of the polygon.
- Ask students to prepare shapes as described in *Before Using the Pages*. Display the shapes as mobiles and attach a card to each shape to indicate its name and its perimeter.

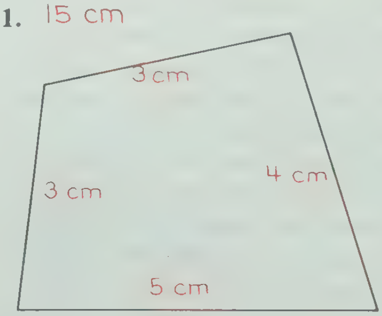
Discuss different ways for finding the length of the unmarked side. They cannot measure the side in decimetres, but they can use a ruler to discover that this side matches a side marked 3 dm. Other students may suggest the use of addition and subtraction: 7.5 − (3 + 1.5). Still others may realize that the unknown length need not be found at all, because the lengths of the three sides that are parallel to the longest side must have a sum of 7.5 (dm). Have the students use their centimetre rulers for Ex. 3-6.

Exercises: Remind the students to show the unit of length in stating the perimeter for each exercise. Note that the lengths for some of the sides in Ex. 2 and 3 are not provided. For Ex. 17-20, point out that an unmarked straight edge is to be used to draw the polygons. Then, a ruler is to be used to measure the sides. Explain that it is not likely that the perimeters of their polygons will be identical to the perimeters given, but they should be close.

Assessment

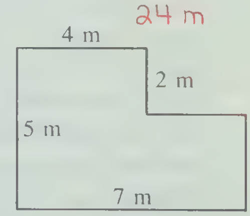
Measure to the nearest centimetre.
Then find the perimeter.

1. 15 cm



Find the perimeter.

2. 24 m



3. a triangle with sides 15 mm, 27 mm, and 34 mm 76 mm

Solve.

4. The sides of a plot of land are 70 m, 125 m, 70 m, 45 m, 35 m, and 45 m long. What is the perimeter of the plot? 390 m

Pages 110-111

LESSON OUTCOME

Use multiplication and addition to find the perimeter of a rectangle; use multiplication to find the perimeter of a regular polygon (square, pentagon, hexagon, octagon, decagon); solve related word problems

Materials

an unmarked straight edge and a ruler marked in centimetres for each student, copies of page T395, metre sticks (optional), a regular pentagon and a regular hexagon prepared from straws or cardboard strips

Vocabulary

height, base, regular polygon, rectangle, square, octagon, decagon

Prerequisite Skills

Add whole numbers; multiply whole numbers

Checking Prerequisite Skills

Add.

1. 98 + 46 144

2. 34 + 22 56

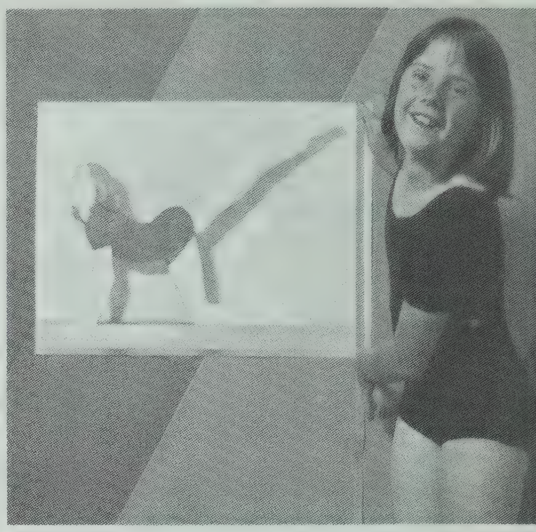
Multiply.

3. 4 × 48 192

4. 8 × 19 152

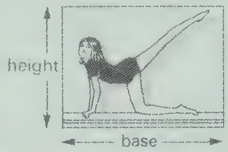
Using a Rule to Find Perimeter

Stacey's poster is 61 cm by 46 cm. What length of wood does she need to frame her poster?



Add the lengths of the four sides.

61
61
46
46
214



For a rectangle:

Perimeter = (2 × base) + (2 × height)
= (2 × 61) + (2 × 46)
= 122 + 92
= 214

Stacey needs a piece of wood that is 214 cm long.

Working Together

Complete the chart for **regular polygons**.

For a regular polygon, all sides have the same length and all angles have the same measurement.

	Regular polygon	Number of sides	Rule for perimeter	Length of each side	Perimeter
1.	square	4	4 × side	3 cm	12cm ?
2.	pentagon	5	5 × side	2 m	10m ?
3.	hexagon	6	6 x side ?	7 cm	42cm ?
4.	octagon	8	8 x side ?	9 m	72m ?

For each, give the rule for finding the perimeter. Then find the perimeter.

5. a rectangle having a base of 4 dm and a height of 2 dm
Perimeter = (2 x base) + (2 x height)
12 dm

6. a square having a side of 5 cm
Perimeter = 4 x side
20 cm

7. a triangle having sides of 9 m, 7 m, and 3 m
Perimeter = side + side + side
19 m

110

LESSON ACTIVITY

Before Using the Pages

- Ask students to name objects in the classroom for which the shapes suggest rectangles and squares, for example, the chalkboard, a book, the door, a ceiling tile, and a piece of paper. Have them identify sides that are the same and note that the angles suggest a "square corner". Emphasize that the four sides of a square have the same length, but for a rectangle, pairs of opposite sides have the same length. You may wish to have students use metre sticks to measure some of the objects named and find the perimeter. Discuss whether it is necessary to measure all the sides of a shape.

Using the Pages

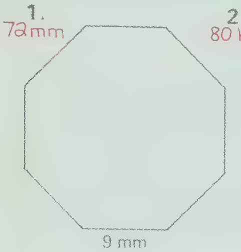
- Direct the students' attention to the photograph at the top of page 110 and ask what name describes the shape of the poster. Have a student read the word problem. Discuss that it is necessary to find the perimeter of the poster to solve the problem. Refer to the photograph again and ask whether Stacey must measure the four sides to find the perimeter. Refer to the diagram of the poster to emphasize that only two measurements are required: the *height* and the *base*. Discuss that the first method for finding the perimeter shows the sum of the four lengths. Ask a student to explain the second method which shows the use of multiplication before the use of addition. Then ask whether addition is necessary for finding the perimeter of a square.

Working Together: Review that a polygon may have three or more sides. Then draw attention to the term *regular polygon* and discuss its meaning. To emphasize the concept, display a regular pentagon and a regular hexagon prepared from one of the materials described on page T118 in *Before Using the Pages*. Ask how the perimeter of a regular polygon can be found without using addition. As the students complete these exercises, remind them to write the unit of length when stating the perimeter. Note that the term *octagon* is introduced in Ex. 4. Ask why multiplication cannot be used for completing Ex. 7.

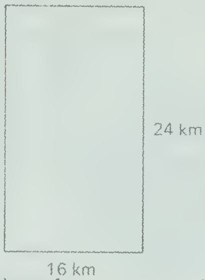
Exercises

Find the perimeter.

1. 72 mm



2. 80 km



	Regular polygon	Length of each side	
3.	hexagon	12 cm	72 cm
4.	octagon	1 m	8 m
5.	decagon	3 dm	30 dm
6.	pentagon	61 mm	305 mm
7.	octagon	92 cm	736 cm

8. a triangle having sides of 64 mm, 27 mm, and 61 mm 152 mm
9. a square, sides 15 m 60 m
10. a rectangle, 11 dm by 35 dm 92 dm
11. a square, sides 432 m 1728 m
12. a rectangle, base 1 km, height 5 km 12 km
13. a regular pentagon, sides 9 cm 45 cm

Use a straight edge. Draw polygons you estimate to have these perimeters. Then measure to check. Answers will vary

14. a square with a perimeter of 25 cm
15. 3 different rectangles, each with a perimeter of 24 cm

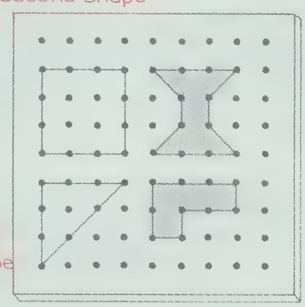
Solve.

16. A gym floor is 32 m long and 18 m wide. What is its perimeter? 100 m
17. A gymnastics mat is 61 cm wide and 152 cm long. What is its perimeter? 426 cm

Which has a greater perimeter, the first shape or the second shape?
the second shape

1.

2. the first shape



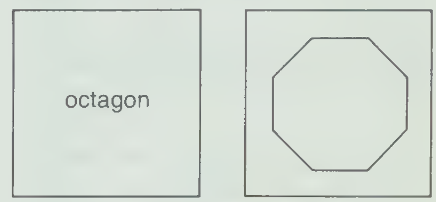
try this

111

RELATED ACTIVITIES

• Provide each student with a copy of pages T383-T385. Ask the students to find the perimeter of each shape by measuring as few sides as possible. Ask them to record the perimeter and to indicate the number of sides that were measured. Note that one of the shapes on page T385 is a circle. Challenge the students to find the perimeter of the circle. Some may suggest matching a string with the perimeter of the circle and then measuring the length of the string.

• To reinforce the names of polygons, prepare cards similar to the following for the game "Concentration" described on page T379. The shapes on pages T383-T385 may be used to prepare the cards.



• Have students use rubber bands and geoboards, or copies of page T395. Ask them to show several rectangles having a given perimeter, for example, 16 units.

• Have the students work in pairs using rubber bands and geoboards, or copies of page T395. One student shows two polygons and the other determines which polygon has the greater perimeter.

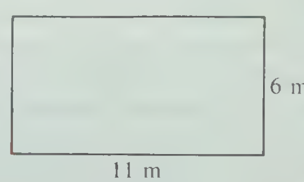
Exercises: Draw attention to Ex. 5. Lead the students to suggest the number of sides in a decagon by relating the name to the word "decimal". Provide each student with a straight edge and a ruler marked in centimetres for Ex. 14 and 15. Encourage them to attempt to draw shapes having perimeters close to the given perimeters, but point out that they are not expected to draw shapes with exactly those perimeters. For Ex. 15, point out that many rectangles can have a perimeter of 24 cm. Remind the students to answer Ex. 16 and 17 with concluding statements and to include the units of measurement.

Try This: To help in solving these exercises, have the students wrap string around corresponding nails on geoboards or draw the shapes on copies of page T395. Encourage them to explain their answers in terms of the distance between adjacent dots. The diagonal line segment between two adjacent dots is longer than the horizontal line segment or the vertical line segment between two adjacent dots. Ex. 2 is more challenging than Ex. 1 because the perimeters are nearly equal.

Assessment

Find the perimeter.

1. 34 m



2. a square, sides 8 cm 32 cm
3. a rectangle, 3 km by 5 km 16 km
4. a regular octagon, sides 8 m 64 m
5. a regular pentagon, sides 15 mm 75 mm

Solve.

6. A photograph is 45 cm by 29 cm. What length of wood is needed to frame the photograph? 148 cm

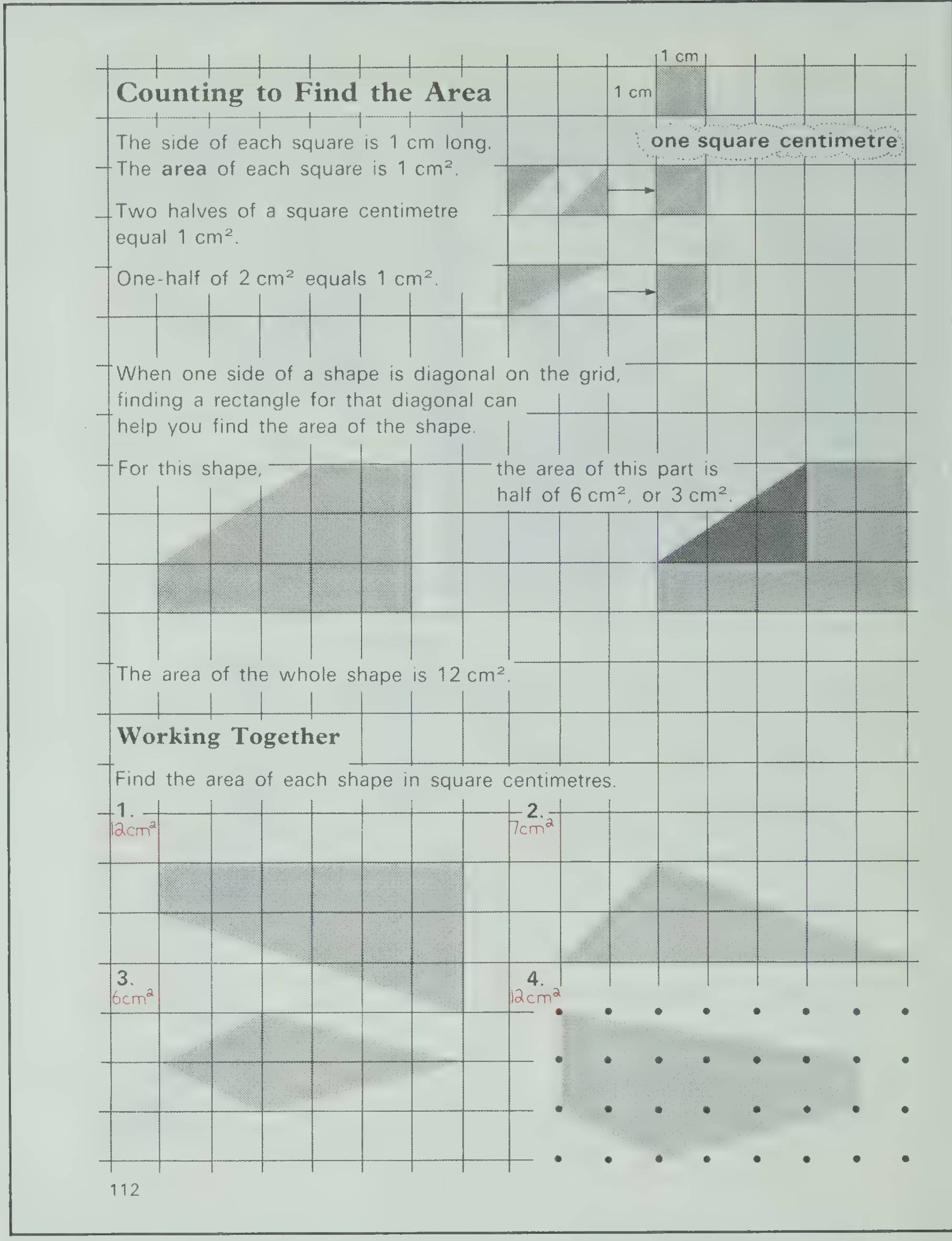
Pages 112-113

LESSON OUTCOME

Find the area in square centimetres by counting whole square centimetres and parts of square centimetres

Materials
a copy of page T 396, a straight edge, and a crayon for each student; a large sheet of square grid paper (optional); tracing paper (optional)

Vocabulary
area, square centimetre, cm², square metre, m², diagonal


Counting to Find the Area

The side of each square is 1 cm long.
The **area** of each square is 1 cm².

Two halves of a square centimetre equal 1 cm².

One-half of 2 cm² equals 1 cm².

When one side of a shape is diagonal on the grid, finding a rectangle for that diagonal can help you find the area of the shape.

For this shape, the area of this part is half of 6 cm², or 3 cm².

The area of the whole shape is 12 cm².

Working Together

Find the area of each shape in square centimetres.

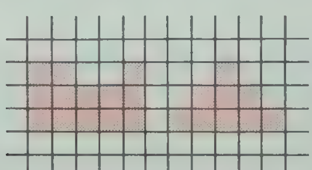
1. 12cm²
2. 7cm²
3. 6cm²
4. 12cm²

LESSON ACTIVITY

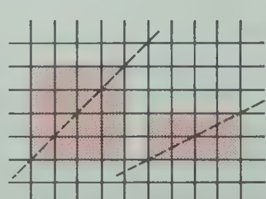
Before Using the Pages

- Give the students copies of page T 396. Have them color parts of the grid to show polygons having boundaries that follow the grid lines (A). Encourage them to draw polygons other than rectangles and squares. For each polygon, have them state the number of squares colored. Have several students show and explain their work to the other students. You may wish to have them copy their shapes on a larger sheet of grid paper for discussion.

A



B



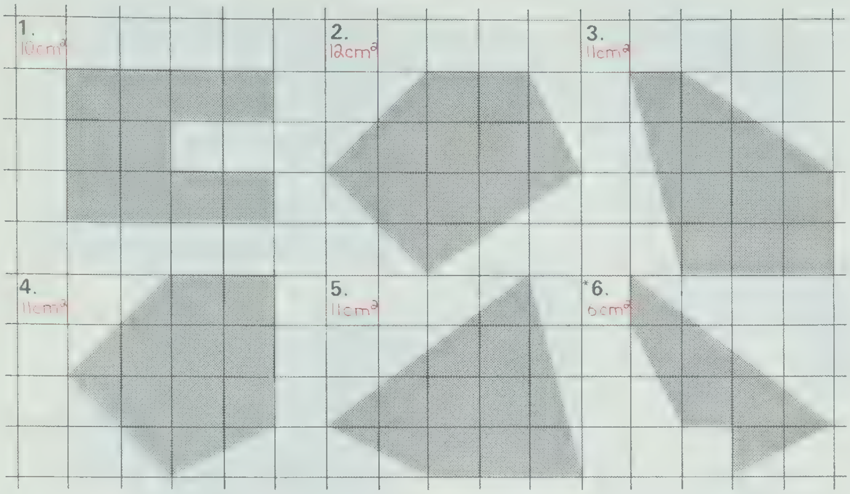
Have the students color inside 16 squares to form a large square (B). Ask them to fold the square in half to obtain two triangles and to determine the number of squares in each half. Ask how this number is determined. Repeat this procedure for a rectangle having dimensions of 2 cm and 4 cm.

Using the Pages

- Have a student read the title at the top of page 112 and the first two statements below the title. Ask what is meant by the term *area*. Note that the grid lines on the page form centimetre squares. Discuss that a square 1 cm by 1 cm has an area of *one square centimetre* which is written 1 cm². Relate the numeral 2 in cm² to the fact that a square has two dimensions: length and width. Discuss the diagrams showing two halves of a square centimetre and one-half of 2 cm². Review the term *diagonal* to describe the sides of a shape that do not follow grid lines. For the polygon shown, ask how many sides there are (5), how many sides follow grid lines (4), and how many sides are diagonal on the grid

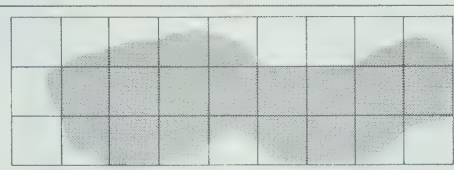
Exercises

Find the area of each shape in square centimetres.



Use grid paper.
Answers will vary

9. Draw 3 shapes that have an area of 4 cm².



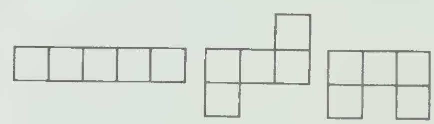
This shows a small pond.
Each square represents 1 m².

1. How many squares are completely covered by the pond? 9
2. How many squares are covered or partly covered by the pond? 23
3. The area of the pond is between __ m² and __ m². 9m² and 23m²
4. How would you estimate the area of the pond? Answers will vary

PROBLEM SOLVING

RELATED ACTIVITIES

- Have the students use decimetre squares prepared from copies of page T 392 to find the areas of surfaces in the classroom. Ask them to cover a surface with the squares and then to count the squares to find the area in square decimetres.
- Have students trace around various objects, such as their hands, on copies of page T 396 and then find the approximate area that is covered.
- Provide copies of page T 395 or T 396 for students to outline and color different shapes having the same area, for example, 16 cm².
- To explore the concept presented in the *Problem Solving* feature, name an area, such as 10 cm², as a lower limit and name another area, such as 24 cm², as an upper limit. Provide copies of page T 396 and ask the students to draw three shapes having an area that is between these two limits.
- Have the students draw as many different shapes as possible, formed by joining five squares so that every square shares at least one side with another square. Three examples are shown.



(1). Have the students focus on the polygon for which a triangular region is grey, and ask them to picture the rectangle for which this triangular region represents one-half. This will help them to understand why the grey region has an area of 3 cm². Once this is found, students can count the centimetre squares that remain, to arrive at an area of 12 cm². Summarize that perimeter refers to the distance around a shape and involves a unit of length such as centimetres. In contrast, area refers to the number of identical squares covered by a shape, and thus involves a unit such as square centimetres.

Working Together: Some students will develop a method of their own for counting squares and parts of squares. Others will need to be advised to concentrate first on each side that is diagonal on the grid, picturing the related rectangles for each, determining the area for each of those regions, and adding the whole squares that remain. Because there are different ways for finding the areas, it is important to discuss the procedures used. Point out that the dots in Ex. 4 suggest squares that have an area of 1 cm².

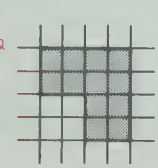
Exercises: Ex. 6 and 8 are starred because they involve subtraction as well as addition to find the area. Provide each student with a copy of page T 396 and a straight edge for Ex. 9. Encourage them to draw shapes with sides diagonal on the grid. Later, have them compare their results to note different shapes that have an area of 4 cm².

Problem Solving: The answer to Ex. 1 provides the lower limit for the area of the pond. The answer to Ex. 2 provides the upper limit for the area of the pond. Ex. 3 establishes that the exact area of the pond is between the lower limit and the upper limit. Ask the students to explain their answers for Ex. 4.

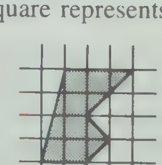
Assessment

Find the area of each shape. Each square represents 1 cm².

1. 12 cm²



2. 9 cm²



Pages 114-115

LESSON OUTCOME

Use multiplication to find the area of a rectangle and of a square; use multiplication and addition to find the area of a shape that can be divided into rectangles and squares

Materials

a copy of page T396, a rectangular card such as a birthday card

Vocabulary

square millimetre, mm^2, square decimetre, dm^2, square kilometre, km^2

Prerequisite Skills

Write a multiplication sentence for an array; multiply whole numbers

Checking Prerequisite Skills

Write a multiplication sentence to match each array.

1.

● ● ●
● ● ●
● ● ●
● ● ●

4 × 3 = 12

2.

● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●
● ● ● ● ● ●

5 × 6 = 30

Multiply.

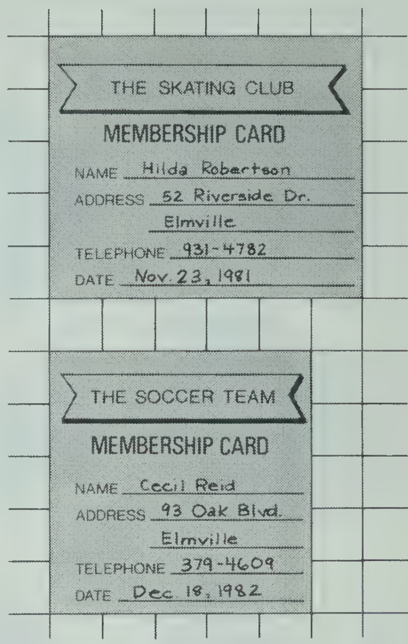
3. 28 × 34 — 952

4. 61 × 73 — 4453

Using a Rule to Find Area

What is the area of each card?

The centimetre squares are in rows and columns.



For Hilda's card: 5 rows, 6 columns

5 × 6 = 30

Area = 30 cm^2

For a rectangle:

Area = base × height
= 6 × 5
= 30

The area of Hilda's card is 30 cm^2.

For Cecil's card: 5 rows, 5 columns

5 × 5 = 25

Area = 25 cm^2

For a square:

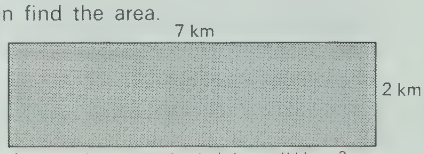
Area = side × side
= 5 × 5
= 25

The area of Cecil's card is 25 cm^2.

Working Together

Give the rule for finding the area.
Then find the area.

1.



Area = base × height 14 km^2

2. a rectangle 9 cm by 7 cm

Area = base × height
63 cm^2

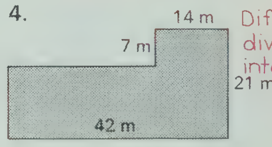
3. a square, sides 27 dm

Area = side × side
729 dm^2

Divide the region into rectangles.
Then find the area.

A square is also a rectangle.

4.



Different ways of dividing the region into rectangles are possible.
686 m^2

114

LESSON ACTIVITY

Before Using the Pages

- Review that the area of a shape depends on the number of square units in the shape. Display a copy of page T396 and review that the grid lines form centimetre squares. Trace the boundary of the grid and review that the shape is a rectangle. Ask how to find the area of the rectangle without counting all the squares one by one and lead the students to suggest the use of multiplication. Establish that this is possible since a rectangle can show rows of squares with the same number of squares in each row, in this case, 24 rows of 18 squares (or 18 rows of 24 squares). The product of 24 and 18 is 432, thus the area of the rectangle is 432 cm^2.
- Display a rectangular card such as a birthday card. Develop that it is necessary to know the number of rows of squares (square units) and the number of squares in each row to find the area of the card. Discuss different ways of finding the information required.

Using the Pages

- The worked example demonstrates that a rectangular shape can be placed on a centimetre grid to determine the number of rows and the number of columns of centimetre squares. Point out that although the grid lines do not appear on the cards, the number of rows and the number of columns of centimetre squares for a card can be determined in the following way:

1. note the number of centimetres in the base (columns);
2. note the number of centimetres in the height (rows).

The number of square centimetres may be found by multiplying the numbers for the base and the height.

Have students read the information aloud and explain the procedure. Ask how the shape of Cecil's card differs from that of Hilda's card. Note that Cecil's card involves the same number of rows and columns. This helps to explain the rule "Area = side × side". Because the base and the height of a square are of equal measure, each can be referred to as a side.

Exercises

Find the area.

Complete the chart for each rectangle.

	Base	Height	Area
10.	36 cm	24 cm	864cm²?
11.	19 m	47 m	893m²?
12.	1 dm	102 dm	102dm²?
*13.	5km?	9 km	45 km²
*14.	16 m	24m ?	384 m²

Complete the chart for each square.

	Base	Area
15.	87 m	7569 ?m²
16.	1 mm	1mm²?
17.	38 cm	1444?cm²
18.	163 m	26569?m²
*19.	1km ?	1 km²

115

RELATED ACTIVITIES

- Have students use a straight edge to draw a rectangle or a square having dimensions in centimetres and find the area. Then have them place the paper showing the shape on a copy of page T 396 and count to check the area.
- Have the students measure such items as envelopes, sheets of paper, and cards, and use multiplication to find the area of each. Similarly, have them measure the schoolyard and find the area. A trundle wheel is a useful device for this purpose. Measurements should be taken to the nearest whole unit.
- Students who are having difficulty dividing a region into rectangles may benefit from drawing a shape on a copy of page T 397 by following the grid lines and then dividing the shape into rectangles.
- For practice in dividing a shape into rectangles, have students make puzzles by cutting two or three rectangles from construction paper. Then have them place the rectangles on a piece of paper and trace the outline. Ask the students to exchange the puzzles and to fit the rectangles inside the outline.

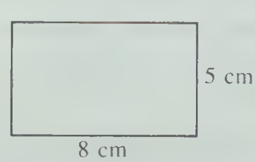
Working Together: Have the students note that for all the shapes on these pages, no sides are diagonal. This implies that the rules involving multiplication will be useful for finding the areas. For Ex. 1, point out that the unit of length is the kilometre, and thus the area will be named in square kilometres. Similarly, discuss the units of measurement for Ex. 3 and 4. Draw attention to the statements to the left of Ex. 4. Ask what name identifies the polygon for Ex. 4 and point out that two of the six sides are not marked to show their length. Ask students to explain how to determine each length. (See *Working Together* on page T 119.) Then have them show different methods for finding the area. Three ways to consider the area are indicated below, each of which involves multiplication and then addition.

Exercises: For the starred exercises, Ex. 13, 14, and 19, the area is given. From this, the unknown dimension is found by division.

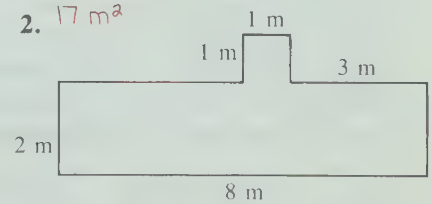
Assessment

Find the area.

1. 40 cm²



2. 17 m²



3. a rectangle having a base of 9 dm and a height of 8 dm
72 dm²

4. a square having a side of 29 mm
841 mm²

Pages 116-117

LESSON OUTCOME

Use multiplication to find the area of a parallelogram

Materials

one or both of the materials described on page T118 in *Before Using the Pages*, a 4 cm-by-8 cm section of centimetre grid paper and scissors for each student

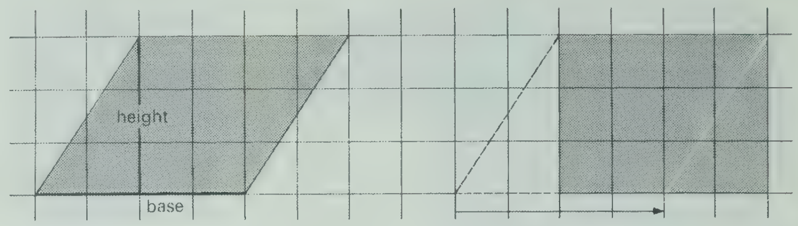
Vocabulary

parallelogram, dimensions

Finding the Area of a Parallelogram

Slide part
of the parallelogram . . .

. . . to make a rectangle.



The area of the rectangle is 12 cm², so the area of the parallelogram is 12 cm².

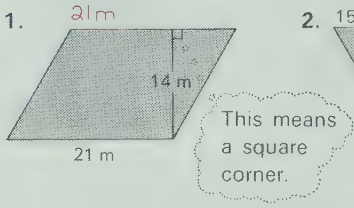
For a parallelogram:
Area = base × height
= 4 × 3
= 12

The area of the parallelogram is 12 cm².

Working Together

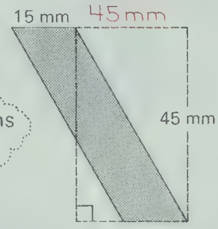
Give the base.

1. 21m

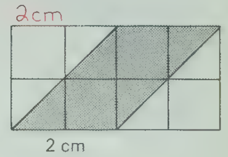


Give the height.

2. 15 mm 45mm



3. 2cm



Give the rule for finding the area.

4. a parallelogram having a base of 7 m and a height of 9 m Area = base x height

Find the area.

5. ← 18 cm → 108 cm²

6. a parallelogram having a base of 17 dm and a height of 24 dm 408 dm²

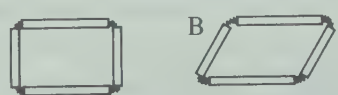
116

LESSON ACTIVITY

Before Using the Pages

- Have the students use one or both of the materials described on page T118 in *Before Using the Pages*. Have them prepare several rectangular shapes. Ask what must be true about the lengths of the straws (sticks) used to prepare the shapes. (The straws used for opposite sides of a rectangle must be the same length.) Demonstrate how the model of a rectangular shape (A) can be altered (B).

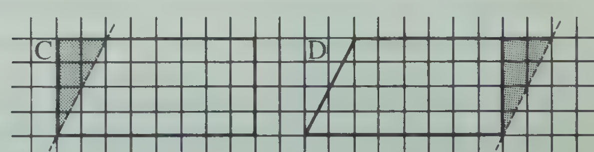
A B



Ask questions about the altered shape (B) such as "Are the opposite sides the same length?" and "Why is the shape not a rectangle?" Introduce the term *parallelogram* to identify the new shape.

- Give each student a section of centimetre grid paper that measures 4 cm by 8 cm from copies of page T396.

Establish that the shape is a rectangle. Ask a student to explain the rule for finding the area of a rectangle and ask another student to find the area. Demonstrate how the rectangular shape can be cut into two parts, beginning at a vertex, so that one part is a triangular shape (C). Ask the students to copy the procedure and then to arrange the two parts to form a parallelogram (D). Ask what the area of the parallelogram must be, and have students explain their answers.



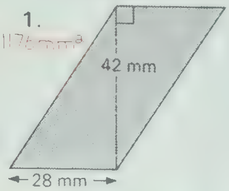
Using the Pages

- The worked example demonstrates that a rule involving multiplication can be applied for finding the area of a parallelogram, even though two of its sides are diagonal on a grid. In other words, it is not necessary to note the squares

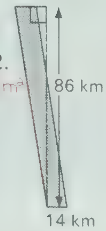
Exercises

Find the area.

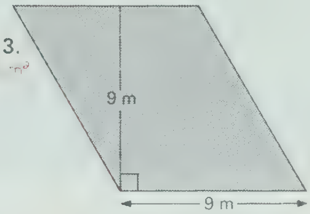
1. 1176 mm²



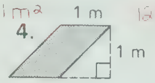
2. 1204 km²



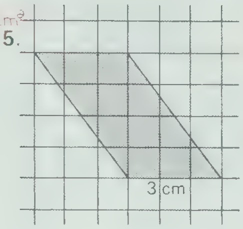
3. 81 m²



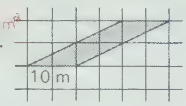
4. 1 m²



5. 12 cm²



6. 100 m²



Complete the chart for each parallelogram.

	Base	Height	Area
7.	94 m	73 m	6862 ? m²
8.	16 mm	9 mm	144 ? mm²
9.	1 dm	8 dm	8 ? dm²
10.	107 m	58 m	6206 ? m²
11.	4 km	5 km	20 ? km²
12.	7 cm	8 cm	56 ? cm²
13.	85 m	30 m	2550 ? m²
14.	4 km	1 km	4 ? km²
15.	62 cm	49 cm	3038 ? cm²
16.	1 mm	1 mm	1 ? mm²
*17.	25 cm	25 cm ?	625 cm²
*18.	12 km ?	8 km	96 km²
*19.	8 mm ?	19 mm	152 mm²
*20.	45 m	3 m ?	135 m²

Complete the table beginning with a square 1 m by 1 m, and then doubling the dimensions for each square.

	Base	Height	Perimeter	Area	
1.	1 m	1 m	4 m ?	?	1 m²
2.	2 m	2 m	8 m ?	?	4 m²
3.	4 m	4 m	16 m ?	?	16 m²
4.	8 m	8 m	32 m ?	?	64 m²
5.	16 m	16 m	64 m ?	?	256 m²
6.	32 ? m	32 ? m	128 m ?	?	1024 m²
7.	64 ? m	64 ? m	256 m ?	?	4096 m²
8.	128 ? m	128 ? m	512 m ?	?	16 384 m²

9. What patterns can you find?
Answers will vary

Make a similar table beginning with a rectangle 3 cm by 1 cm.
A table is given on page T370.

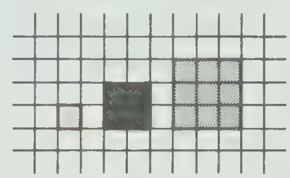
10. What patterns can you find in this table?
Answers will vary

try this

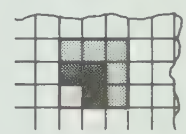
RELATED ACTIVITIES

- Have the students cut two identical parallelograms from copies of page T396. Have them paste one of the parallelograms on a large sheet of paper and cut the other parallelogram along a grid line to obtain two parts, one of which is triangular. Have them paste the parts beside the first parallelogram, to show the corresponding rectangle having the same area. Below the shapes, ask the students to write "Area = base × height".
- Some students may enjoy investigating patterns for "how squares grow". Have them cut square shapes from centimetre graph paper, starting with a square having a side of 1 cm, then 2 cm, and so on. Ask them to color inside the shapes for each exercise, using a different color (A). Then have them overlap the shapes, matching lower left corners (B). The shapes may be pasted to retain the arrangement.

A



B



The areas of the squares will lead to the pattern 1, 4, 9, 16, … . The overlapping squares (B) suggest the pattern 1, 3, 5, 7, … .

that are partly covered by the parallelogram and to associate these with the corresponding rectangles as on pages 112 and 113. Emphasize that the height of a parallelogram is not associated with a side. The height is the distance along a grid line between the base and the side opposite the base.

Working Together: Draw attention to the symbol for a square corner in Ex. 1, 2, and 5. Also, remind the students that the height is the distance indicated from the base to the opposite side. For each of Ex. 1-3, ask what unit of length is indicated for the base and for the height. Discuss why the height for Ex. 2 is 45 mm. The height for Ex. 3 is not shown, but the students can recognize that the grid lines form centimetre squares because the base is given as 2 cm.

Exercises: To find the height for the parallelogram in Ex. 5, the students must recognize that the length of one side of each square represents 1 cm. For Ex. 6, they must determine that the length of one side of each square represents 5 m. For Ex. 17-20, the students must recognize that the height (base) of a parallelogram can be found by dividing the area of the parallelogram by the base (height).

Try This: Explain what is meant by *dimensions*. Have the students note that the dimensions named in one exercise are double those named in the preceding exercise for Ex. 2-5. Ask how to find the dimensions of the squares for Ex. 6-8. Provide an opportunity to discuss the patterns discovered. Some students may have completed Ex. 6-8 by extending the patterns discovered in Ex. 1-5. For example, the area of a square is four times the area of the preceding square. The perimeter is twice that of the preceding square. In general, if the dimensions of a square are doubled, the perimeter is also doubled but the area is four times as great.

Assessment

Find the area.

1. 84 m²

2. a parallelogram with a base of 22 cm and a height of 35 cm 770 cm²

3. a parallelogram with a base of 3 km and a height of 8 km 24 km²

LESSON OUTCOME

Use multiplication and division to find the area of a triangle; solve related word problems

Materials

a parallelogram having a base of 12 cm and a height of 5 cm; a copy of page T 396 and scissors for each student

Vocabulary

triangular

Prerequisite Skills

Use multiplication to find the area of a parallelogram

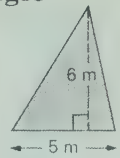
Checking Prerequisite Skills

Find the area.

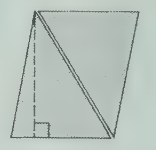
1. a parallelogram with a base of 5 m and a height of 3 m 15 m²
2. a parallelogram with a base of 25 cm and a height of 17 cm 425 cm²

Finding the Area of a Triangle

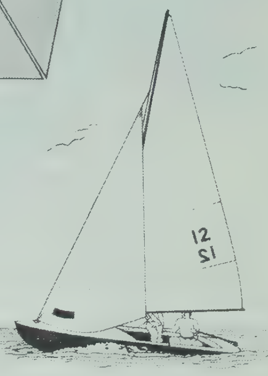
This sail has the shape of a triangle.
The base of the triangle is 5 m.
The height of the triangle is 6 m.
What is the area of the sail?



Two triangles fit together to make a parallelogram.



The area of the triangle is half the area of the parallelogram, or half the product of the base and the height.



For a triangle:

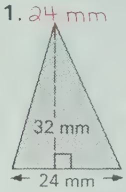
$$\text{Area} = \frac{\text{base} \times \text{height}}{2}$$
$$= \frac{5 \times 6}{2}$$
$$= \frac{30}{2} \qquad \frac{30}{2} \text{ means } 30 \div 2$$
$$= 15$$

The area of the sail is 15 m².

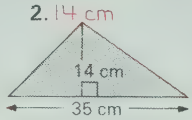
Working Together

Give the base. Give the height.

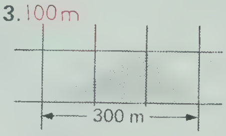
1. 24 mm



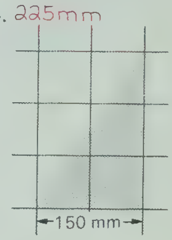
2. 14 cm



3. 100 m



4. 225 mm

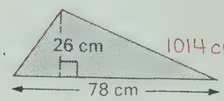


Give the rule

5. for finding the area of a triangle. $\text{Area} = \frac{\text{base} \times \text{height}}{2}$

Find the area.

6. a triangle, base 15 m, height 10 m 75 m²
7. a triangle, base 10 dm, height 15 dm 75 dm²
8. 1014 cm²



118

LESSON ACTIVITY

Before Using the Pages

- Display a parallelogram (cut from a copy of page T 396) for which the base is 12 cm and the height is 5 cm. Have students help to show on the board the method for finding the area of the parallelogram.

$$A = \text{base} \times \text{height}$$
$$= 12 \times 5$$
$$= 60$$

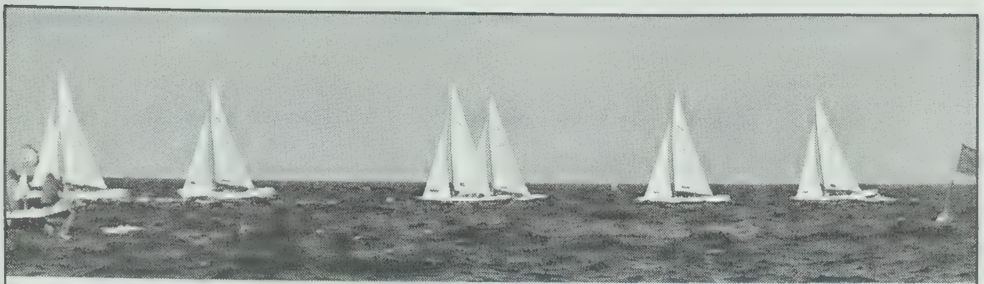
The area of the parallelogram is 60 cm².

Use a ruler to draw a line segment that joins one pair of opposite vertices. Cut the parallelogram along the line segment drawn. Have a student identify the shape of each part (triangle). Match the two parts to show that they are not only the same shape but also the same size. Ask what the area of each triangle must be. Develop that the area of each triangle is one-half the area of the parallelogram, namely, 30 cm².

- Have each student trace and cut a parallelogram from a copy of page T 396. Have the students cut their parallelograms into two triangular shapes as described in the preceding activity. Then have them exchange shapes, reassemble the triangles to form a parallelogram, and find the area of one triangle by first finding the area of the parallelogram.

Using the Pages

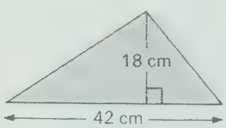
- Draw attention to the photograph at the top of page 119. Note that the sails on a sailboat suggest triangles. Have the students refer to the illustration at the top of page 118 to recognize what is meant by the base and the height of a triangle. Emphasize that the height is the distance between the base and the vertex of the triangle opposite the base. Point out the symbol for a square corner, to emphasize that



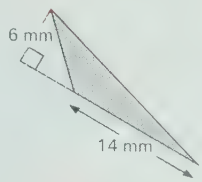
Exercises

Use the rule to find the area.

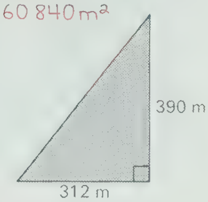
1\. 378 cm²



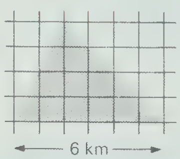
2\. 42 mm²



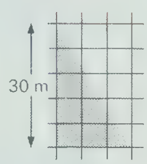
3\. 60 840 m²



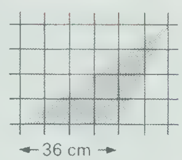
4\. 12 km²



5\. 270 m²



6\. 648 cm²



Copy and complete the chart for each triangle.

	Base	Height	Area
7.	19 m	56 m	532? m²
8.	63 cm	184 cm	5796? cm²
9.	23 mm	4 mm	46? mm²
10.	1 km	10 km	5? km²
11.	43 cm	106 cm	2279? cm²
˙12.	3 m	16m ?	24 m²
˙13.	16dm?	8 dm	64 dm²
˙14.	16 mm	24mm?	192 mm²

Solve.

15\. A triangle has a height of 8 cm and a base of 12 cm. What is its area? 48 cm²

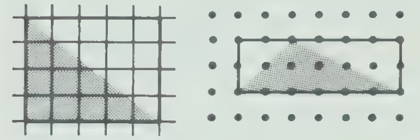
16\. What is the area of a triangular sail with a base of 12 m and a height of 16 m? 96 m²

˙17. A triangular sail has a base of 15 m and an area of 300 m². What is the height of the sail? 40 m

119

RELATED ACTIVITIES

• Have the students draw triangles on copies of page T 395 or T 396 and then draw a rectangle for each triangle as shown.



Have the students find the area of each of the triangles by using the rule $\frac{\text{base} \times \text{height}}{2}$, by multiplying to find the area of the rectangle and dividing by two, and by counting the centimetre squares in the region covered by the triangle.

• Have students help to prepare a chart to summarize the rules presented in this unit for finding the areas of specific polygons. Display the chart for several days to enable students to refer to it as required.

Diagram	Shape	Rule for finding area
height, base	rectangle	A = base × height

the height suggests a vertical line on a grid. Discuss that the two triangles fit together to form a parallelogram. Ask a student to give the rule for finding the area of a parallelogram. Have students explain how this rule is adapted for finding the area of a triangle. Point out that $\frac{30}{2}$ means 30 ÷ 2.

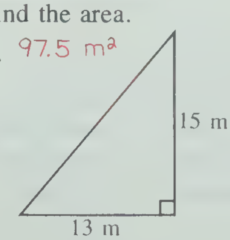
Working Together: For Ex. 3 and 4, the students must determine the length represented by one side of a square, and then find the height of each triangle.

Exercises: Discuss why 6 mm is the height of the triangle for Ex. 2, even though it is not shown within the triangle. For Ex. 4-6, the students must determine the length represented by one side of a square and use this information to find the height of each triangle. Ex. 12, 14, and 17 require the students to use the fact that the area of a triangle is found by multiplying the height by the base and dividing by two. That is, they are to find the height of a triangle by dividing the area by the base and then multiplying by two. For Ex. 13, they must divide the area by the height and multiply by two to find the base. Have the students check their answers for Ex. 12, 13, 14, and 17 by using the rule for finding the area of a triangle and comparing each answer with the area that is given.

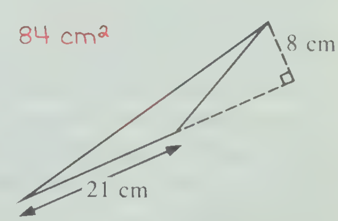
Assessment

Find the area.

1. 97.5 m²



2. 84 cm²



3. a triangle with a base of 4 km and a height of 1 km 2 km²

4. a triangle with a base of 28 mm and a height of 35 mm 490 mm²

Solve.

5. The height of a sail is 7 m and the base is 4 m. What is the area of the sail? 14 m²

OBJECTIVE

Demonstrate competence in finding perimeter and area; solve related word problems

Practice

Choose the possible units of measurement.

1. Area of a triangle: cm, cm², m², km, mm²
2. Perimeter of a square: m, km², mm, m²

Complete the charts.

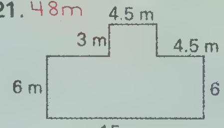
	Regular polygon	Length of each side	Perimeter
3.	octagon	7 m	56m ?
4.	hexagon	23 cm	138cm ?
5.	pentagon	36 m	180 m ?
6.	decagon	15 dm	150 dm ?
7.	hexagon	46 mm	276mm ?
*8.	pentagon	20cm ?	100 cm

	Polygon	Base	Height	Perimeter
9.	rectangle	15 mm	17 mm	64mm ?
10.	rectangle	107 m	59 m	332 m ?
11.	square	48 cm	48 ?cm	192cm ?
12.	square	12 ?m	12 m	48m ?
*13.	rectangle	54 ?dm	3 dm	114 dm

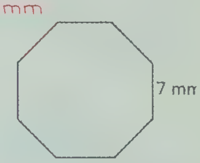
	Polygon	Base	Height	Area
14.	rectangle	15 m	74 m	1110 ?m²
15.	parallelogram	5 mm	7 mm	35 ?mm²
16.	triangle	8 cm	11 cm	44 ?cm²
17.	square	68 cm	68 ?cm	4624 ?cm²
*18.	parallelogram	103 m	82 ?m	8446 m²
*19.	rectangle	23 ?cm	19 cm	437 cm²
*20.	triangle	6 dm	8 ?dm	24 dm²

Find the perimeter.

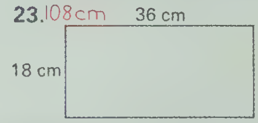
21. 48m



22. 56mm



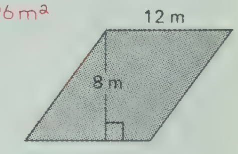
23. 108cm



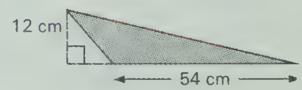
Find the area.

24. 2420dm²

25. 96m²



26. 324cm²



120

LESSON ACTIVITY

Before Using the Pages

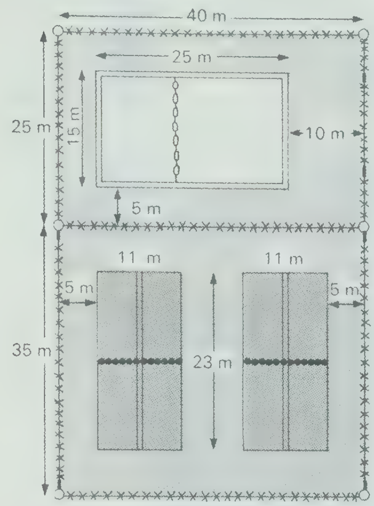
- Review the terms *triangle*, *pentagon*, *hexagon*, *octagon*, and *decagon*. Name a shape and ask for the number of sides. Name the number of sides and ask for the name of the shape. Ask what word describes a polygon for which all the sides and all the angles are equal (regular). Ask how a square differs from a rectangle and how a rectangle differs from a parallelogram. Review that squares, rectangles, and parallelograms may all be described as quadrilaterals. Briefly review how to find the perimeter and the area of various shapes.

Using the Pages

- Ex. 1 and 2 review the symbols for different units of measurement and emphasize that square units are required for area, whereas linear units are required for perimeter. Ex. 3-7 provide practice in finding the perimeter of a regular polygon. They require that the students recall the number of sides for each polygon. For Ex. 8, the students must realize that since a pentagon has five sides, the length of each side of a regular pentagon is found by dividing the perimeter by five. Ex. 11, 12, and 17 review that all the sides of a square have the same length. The students are required to use the lengths that are given to find the lengths of the remaining sides of the polygons for Ex. 21-23.

You may wish to refer to the photograph on page 121 to motivate a discussion about a neighborhood swimming pool or tennis court. Encourage the students to think about their sizes to help them relate to the diagram above the photograph. Discuss each item in the legend and have the students relate the legend to the diagram. To help them interpret the diagram, ask questions such as the following.
''What is the length of the swimming pool?''
''How wide is the swimming pool?''
''How long is each tennis court?''
Remind the students to show their work for Ex. 27-34 and to write concluding statements that include the units of measurement.

The park has a swimming pool and two tennis courts.



Legend

swimming pool

tennis courts

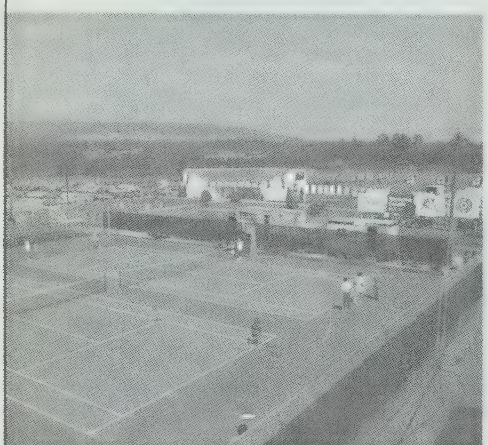
concrete

clay

fence

gate
(Each gate is 1 m wide.)

tennis net



27. What is the perimeter of the swimming pool? 80 m
28. What is the area of the pool? 375 m^2
29. What area is covered by the tennis courts? 506 m^2
30. What is the perimeter of each tennis court? 68 m
31. How much greater is the area of the tennis courts than the area of the pool? 131 m^2
32. Which has the smaller area, the clay or the concrete? concrete
33. Which has the greater area, the pool or the concrete? concrete
34. How long is the fencing in and around the park? 234 m

121

RELATED ACTIVITIES

- Have the students change the base of a parallelogram to three times its original length and find the area. Have them change the height of the original parallelogram to three times its original length and find the area. Then have them change both the base and the height to three times their original lengths and find the area. Compare the results obtained. Ask them to use a similar procedure with triangles and with rectangles.
- To encourage the students to think of applications of perimeter, have them play the game ''Inventory'' described on page T 380 and list reasons for finding the perimeter. For applications of area and comparisons of perimeter and area, have the students play the same game and list reasons for finding the area.
- Have the students list the dimensions of all the rectangles for which the perimeter is 16 cm and the length of each side is a whole number of centimetres. (Remind them that a square is also a rectangle.) Then have them find the area of each rectangle and decide which has the greatest area. Have them repeat this for rectangles having perimeters of 20 cm and 36 cm. Discuss the results. (For each perimeter, the square has the greatest area.) They may draw the rectangles on copies of page T 396 to help reinforce the fact that rectangles having the same perimeter can have different areas.

Pages 122-123

LESSON OUTCOME

Find volume in cubic centimetres by counting

Materials

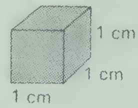
centimetre cubes or other cubes; a few prisms similar to those illustrated on page 123 (copies of the patterns of prisms on pages T386-T388 can be used to make models); a ruler marked in centimetres for each student

Vocabulary

volume, cubic centimetre, cm³, surface area, face, solid

Finding Volume by Counting Cubes

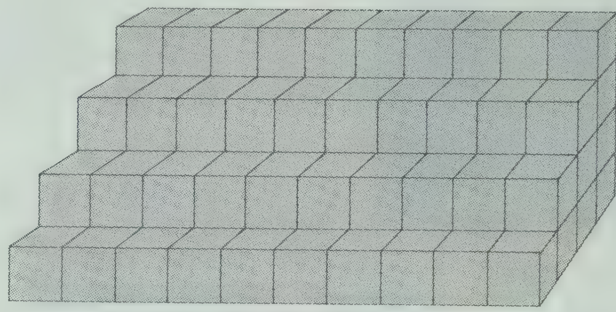
Each edge of the cube is 1 cm long.



The **volume** of the cube is **one cubic centimetre**, or **1 cm³**.

You can find volume by counting centimetre cubes.

Remember the hidden cubes.

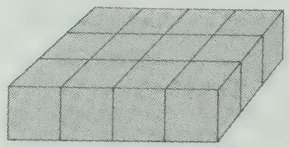


100 cm³

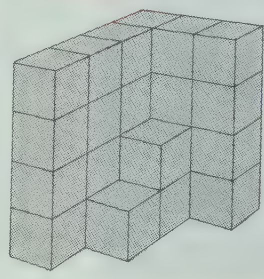
Exercises

Find the volume in cubic centimetres.

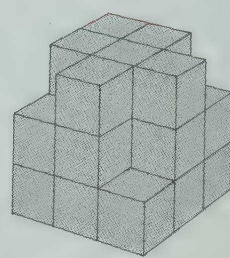
1. 12 cm³



2. 27cm³



3. 23 cm³



122

LESSON ACTIVITY

Before Using the Pages

- Review that a cube is a three-dimensional solid in which all the edges have the same length. Note that each *face* of a cube is square. Have the students work in small groups to build shapes using cubes such as centimetre cubes. Have members of a group count the cubes used to build each of the shapes, without taking the shapes apart. Discuss that some of the cubes in a shape are hidden from view. Ask for ways to determine the number of hidden cubes without turning the shapes or viewing them from a different side. Ask if they know the word that describes the number of identical cubes contained in a three-dimensional shape.

Using the Pages

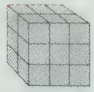
- Introduce the word *volume*. Ask students to name the length, width, and height of the cube illustrated. Explain that the cube has a volume of one cubic centimetre, or 1 cm³, since each edge of the cube is 1 cm long. Relate the word "cubic" to "cube" and the numeral 3 to the three equal dimensions of a cube.

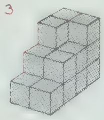
  Have the students count the cubes to check that the volume of the shape illustrated is 100 cm³. Remind them to consider the cubes that are hidden from view. Point out that the volume is given in cubic centimetres because the shape is made of centimetre cubes.

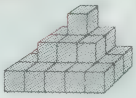
Exercises: Students who have difficulty with Ex. 1-9 may benefit from writing the volume that they think is correct for the shape, building the shape with cubes, and then counting the cubes without taking the shape apart. They can take the shape apart and count the cubes again to check. Provide each student with a ruler marked in centimetres for Ex. 10. You may wish to have the students draw the shapes for Ex. 10 on a separate sheet of paper for display. This would enable them to see many different shapes which have the same volume. If students have difficulty with Ex. 10, have them build a shape with 24 cubes and then draw the shape.

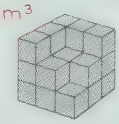
Find the volume.
Each cube represents a cubic centimetre.

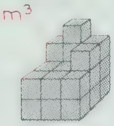
4. 10 cm³


5. 18 cm³


6. 17 cm³


7. 22 cm³


8. 22 cm³


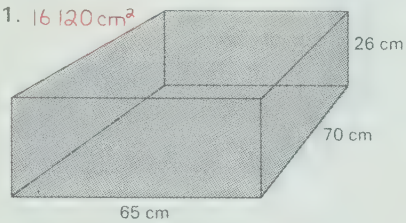
9. 23 cm³


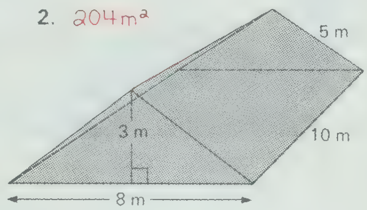
Draw

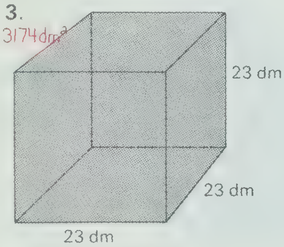
10. three different shapes with a volume of 24 cm³. Answers will vary.

The **surface area** of a solid
is the area of its whole surface.
To find the surface area,
find the area of each face.
Then add the areas of all the faces.

Find the surface area of each.

1. 16 120 cm²


2. 204 m²


3. 3174 dm²


4. a book
5. an object in your classroom

Answers will vary.

try this

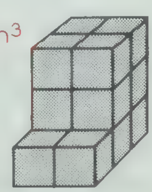
RELATED ACTIVITIES

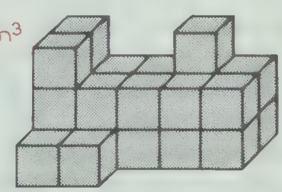
- Students can work in pairs so that each student builds a shape using centimetre cubes and determines the volume of the shape built by the other student. They may take the shape apart and count the cubes to check the volume.
- Have students use centimetre cubes to build different shapes having the same volume, for example, 20 cm².
- Have each student make as many different shapes as possible with three cubes so that each cube shares at least one face with another cube. You may wish to have the students find which shape has the least surface area and which has the greatest surface area. Have students repeat the activity using four cubes.
- Ask students to bring boxes of various sizes to school. Have them find the surface area of each box. The concept of surface area can be clarified for some students if they cover each face of a box with centimetre graph paper.

Try This: Have students read the introductory statements. Ask students to explain the term *surface area* in their own words. Display a few prisms similar to those illustrated and ask students to point to each face that must be considered for finding the surface area. Repeat the procedure for various objects in the classroom. If students have difficulty with Ex. 1-3, provide a model of the prism shown and compare it with the drawing. Some students may realize that some of the faces of a solid have the same area, and use multiplication to find the total surface area. For Ex. 4 and 5, have the students select a reasonable unit of measurement and measure to the nearest unit.

Assessment

Find the volume.
Each cube represents a cubic centimetre.

1. 14 cm³


2. 25 cm³


Pages 124-125

LESSON OUTCOME

Find the volume of a rectangular prism in cubic centimetres by multiplying the number of centimetre cubes in one layer and the number of layers

Materials

centimetre cubes or other cubes, a rectangular prism or a small box

Vocabulary

rectangular prism

Finding Volume from Cubic Units in Layers

Find the volume of the **rectangular prism**.

Each surface of a rectangular prism is a rectangle.

Each layer contains the same number of centimetre cubes.

For the bottom layer,

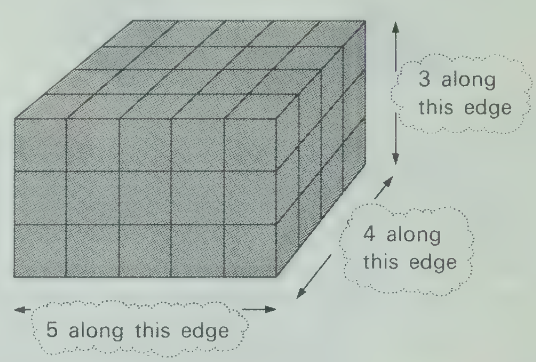
$5 \times 4 = 20$

there are 20 cubes.

There are 3 layers

$3 \times 20 = 60$

or 60 cubes in all.



Volume = number of cubes in each layer × number of layers

$= 20 \times 3$

$= 60$

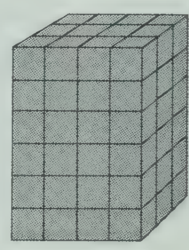
The volume of the rectangular prism is 60 cm^3.

Working Together

Complete the chart for this rectangular prism.
Each cube represents a cubic centimetre.

1.

Does each layer contain the same number of cubes?	? yes
Number of cubes in each layer	12?
Number of layers	6?
Volume	72?



124

LESSON ACTIVITY

Before Using the Pages

- Display a rectangular prism or a small box having a rectangular shape. Ask the students to build shapes like the one shown, using centimetre cubes. The shapes they build need not be the same size as the one shown. Have them exchange shapes, find the volume, and explain the procedure used. If no student suggests it, ask for a way to find the volume that is faster than counting individual cubes.

Using the Pages

- Introduce the term *rectangular prism* and tell the students that the shapes for which they found the volume in the preceding activity were rectangular prisms. Ask them to name objects that suggest rectangular prisms. They may suggest boxes or books. Ask what is known about the layers of cubes that form a rectangular prism. (Each layer contains the same number of cubes.) You may wish to show this with a rectangular prism composed of cubes. Have students explain the procedure for finding the number of cubes in the bottom layer and the number of cubes in all. Summarize that the use of multiplication provides a quick method to obtain the volume. Explain that the volume is given in cubic centimetres because the shape is made of centimetre cubes.

Working Together: Discuss the significance of the answer to the first question. Remind the students to use the symbol cm^3 for the volume of the rectangular prism.

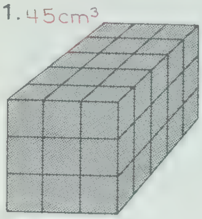
Exercises: Ex. 4 and 5 show only some of the cubes. Have students explain how they found the number of cubes that would fill each rectangular prism.

To complete Ex. 10 and 11, students must recognize that since the volume of a rectangular prism is found by multiplying the number of cubes in each layer and the number of layers, then the number of layers can be found by dividing the volume by the number of cubes in each layer. Similarly, the number of cubes in each layer can be found by dividing the volume by the number of layers.

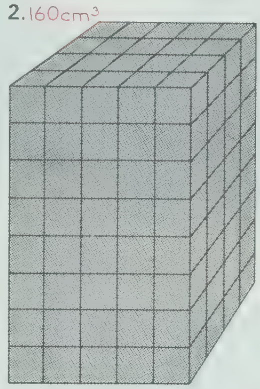
Exercises

Find the volume of each rectangular prism.
Each cube represents a cubic centimetre.

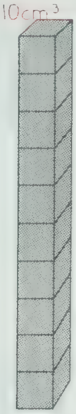
1. 45cm³



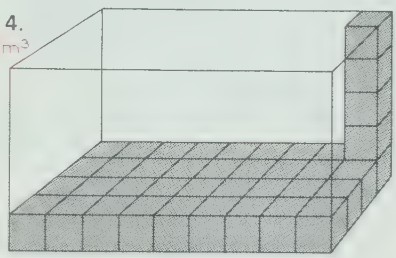
2. 160cm³



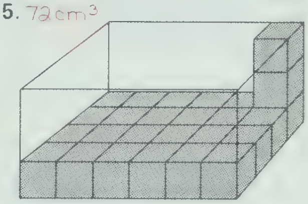
3. 10cm³



4. 180cm³



5. 72cm³



Copy and complete the chart for each rectangular prism made of centimetre cubes.

	Cubes in each layer	Number of layers	Volume
6.	8	7	56? cm³
7.	12	9	108? cm³
8.	25	73	1825? cm³
9.	182	98	17 836? cm³
*10.	6	14?	84 cm³
*11.	22 ?	15	330 cm³

125

RELATED ACTIVITIES

- Have the students place centimetre cubes in boxes as shown for Ex. 4 and 5 on page 125 and find the volume of each box.
- For practice in estimating volume, place one centimetre cube in a box and ask students to estimate the volume of the box. Then have them fill the box with cubes to check the estimate.
- Display a box that has the shape of a rectangular prism, for example, a box that contained crackers or cookies. Place a card showing the following question in front of the box.

How many cubic centimetres?

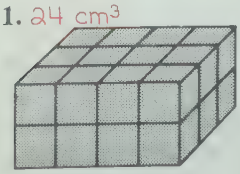
Have students write their estimates on slips of paper showing their names and place the papers in the box. At the end of one week, have some students determine the volume of the box by filling it with centimetre cubes. Announce the names of the three students whose estimates were closest to the exact number of cubic centimetres.

Students who are unsure of their answers for Ex. 10 and 11 can multiply the number of layers and the number of cubes in each layer and compare the product with the volume that is given.

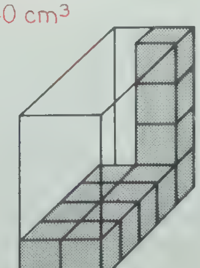
Assessment

Find the volume.
Each cube represents a cubic centimetre.

1. 24 cm³



2. 40 cm³



Copy and complete the chart for each rectangular prism made of centimetre cubes.

	Cubes in each layer	Number of layers	Volume
3.	24	6	144 cm³
4.	18	14	252 cm³

Pages 126-127

LESSON OUTCOME

Find the volume of a rectangular prism by multiplying the area of the base and the height; solve related word problems

Materials

centimetre cubes or other cubes, a box (optional), rectangular sections of graph paper cut from copies of page T 396, a rectangular shape cut from a blank sheet of paper

Vocabulary

cubic metre, m^3, cubic decimetre, dm^3

Prerequisite Skills

Use multiplication to find the area of rectangles and squares; find the volume of a rectangular prism by multiplying the number of cubic units in one layer and the number of layers

Checking Prerequisite Skills

Find the area.

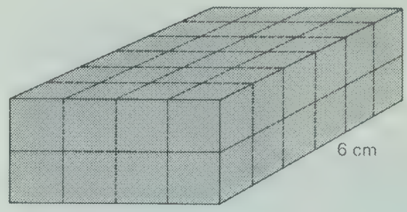
1. a rectangle with a base of 7 cm and a height of 12 cm 84 cm^2
2. a square with sides of 15 m 225 m^2

Find the volume of each rectangular prism made of centimetre cubes.

	Cubes in each layer	Number of layers	
3.	36	8	288 cm^3
4.	20	12	240 cm^3

Finding the Volume of a Rectangular Prism by Multiplying the Area of the Base and the Height

Find the volume of this rectangular prism.



For the bottom layer,
$4 \times 6 = 24$
there are 24 cubes.

There are 2 layers
2×24
or 48 cubes in all.

Volume
= cubes in each layer × number of layers
= 24 × 2
= 48

The volume of the prism is 48 cm^3.

The area of the base
$4 \times 6 = 24$
is 24 cm^2.

The height is 2 cm.

There are 2 × 24,
or 48 cm^3 in all.

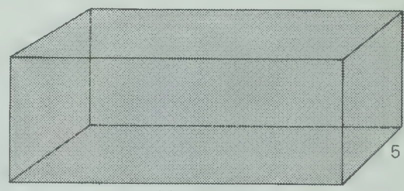
Volume
= area of base × height
= 24 × 2
= 48

The volume of the prism is 48 cm^3.

Working Together

Complete the chart for this rectangular prism.

1.	Area of base	Height	Volume
	? 70 m^2	? 6m	? 420 m^3



Find the volume of each rectangular prism.

2. Base is 8 cm by 4 cm. Height is 6 cm. 192 cm^3
3. Base is 2 dm by 1 dm. Height is 9 dm. 18 dm^3

126

LESSON ACTIVITY

Before Using the Pages

- Cut various rectangular sections of centimetre graph paper from copies of page T 396. Display one section, noting that the shape is a rectangle. Tell the students to imagine that a rectangular prism is to be built on that section as a base. Then ask how many centimetre cubes there would be in the bottom layer and thus how many there would be in each layer of the prism.

  Display a rectangular shape, for example, 6 cm by 9 cm, cut from a blank sheet of paper. Tell the students that the length of the rectangular shape is 9 cm and the width is 6 cm. Ask how to find the number of centimetre cubes there would be in each layer of the prism built on this base. They will likely suggest multiplying 9 and 6. Review that the product 9 × 6, or 54, determines the area of the rectangular shape, that is, the area of the base of the prism.

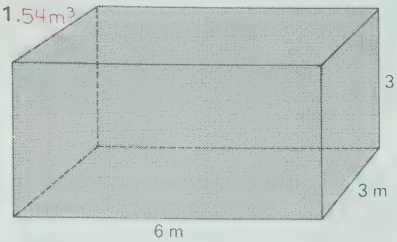
Using the Pages

- The worked example presents two methods for finding the volume of the rectangular prism illustrated. On the left side of the page, the procedure that was presented on page 124 is reviewed: multiply the number of cubes in each layer and the number of layers. On the right side of the page, the volume is found by multiplying the area of the base and the height. Compare corresponding steps of the two methods. For example, the number for the area of the base corresponds to the number of cubes in the bottom layer; the number for the height of the prism corresponds to the number of layers. Multiplying the area of the base and the height of the prism corresponds to multiplying the number of cubes in each layer and the number of layers. Because area is measured in square units, the symbol cm^2 is required in naming the area. Similarly, the symbol cm is required in naming the height. Point out that the volume is given in cubic centimetres because each edge of the rectangular prism is named in centimetres.

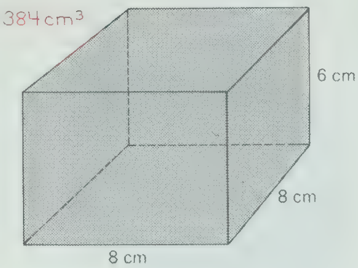
Exercises

Find the volume of each rectangular prism.

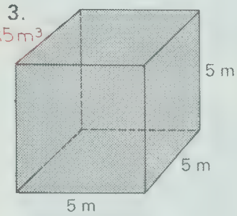
1. 54 m³



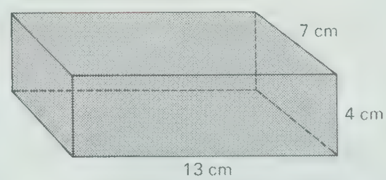
2. 384 cm³



3. 125 m³



4. 364 cm³



Find the volume of each rectangular prism.

	Base	Height	
5.	5 cm by 2 cm	3 cm	30 cm³
6.	25 m by 25 m	25 m	15 625 m³
7.	10 m by 7 m	20 m	1400 m³
8.	82 cm by 6 cm	100 cm	49 200 cm³
9.	100 dm by 100 dm	100 dm	1 000 000 dm³
10.	22 cm by 15 cm	6 cm	1980 cm³
11.	15 m by 15 m	15 m	3375 m³
12.	20 dm by 61 dm	43 dm	52 460 dm³

Solve.

13. A soccer team has a storage box 49 cm wide, 105 cm long, and 50 cm high. What is its volume? 257 250 cm³

14. A carton has a base 80 cm by 96 cm and is 55 cm high. How many boxes with a base 8 cm by 8 cm and a height of 11 cm would fit in the carton? 600

*15. A carton has a base 112 cm by 72 cm and a height of 84 cm. How many boxes with a base 9 cm by 16 cm and a height of 15 cm would fit in the carton? 280

127

RELATED ACTIVITIES

• Have the students measure the dimensions of various boxes or items having the shape of rectangular prisms and find the volume.

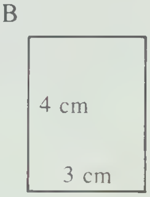
• To explore the concept suggested in Ex. 15 on page 127, have students measure the dimensions of small boxes and calculate the number of centimetre cubes that would fill the boxes. Then have them fill the boxes with centimetre cubes to demonstrate and check their calculations.

• Have students tape together six copies of the diagram on page T 392 to form a decimetre cube.

• Students having difficulty with the exercises on these pages would benefit from using cubes to build rectangular prisms on given bases. If centimetre cubes are to be used, cut rectangular sections of page T 396 as bases (A). When the students have mastered this process, have them build rectangular prisms on unmarked rectangular bases (B).

A

B



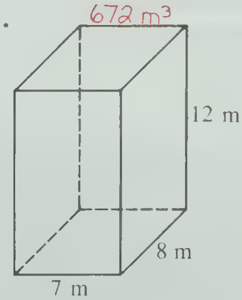
Working Together: Discuss that the volume for Ex. 1 will involve *cubic metres* (m³) and that Ex. 3 will involve *cubic decimetres* (dm³). Emphasize that the unit of length shown for the dimensions of a prism determines the *square unit* for the area of the base and the *cubic unit* for the volume of the prism. The chart for Ex. 1 can reinforce this concept.

Exercises: In Ex. 14, the number of boxes 8 cm by 8 cm by 11 cm that would fit a carton 80 cm by 96 cm by 55 cm is found by dividing the dimensions of the carton by the corresponding dimensions of one box. Because each dimension of the carton is a multiple of one dimension of the box, the boxes can fit in the carton and there will be no empty spaces. Ex. 15, however, is starred because the dimensions of the carton are not multiples of the dimensions of the box. The shape of the boxes and the arrangement of the boxes in the carton can affect the answer. Even when the boxes are arranged for a maximum number to fit in the carton, there is some empty space in the carton. You may wish to demonstrate this concept with a small box and several cubes that cannot be arranged to fill the box completely.

Assessment

Find the volume of each rectangular prism.

1. 672 m³



	Base	Height
2.	3 cm by 6 cm	14 cm
3.	10 dm by 9 dm	2 dm
4.	5 m by 15 m	4 m

2. 252 cm³
3. 180 dm³
4. 300 m³

Solve.

5. A storage locker is 95 cm long, 100 cm wide, and 225 cm high. What is the volume of the locker?
2 137 500 cm³

Pages 128-129

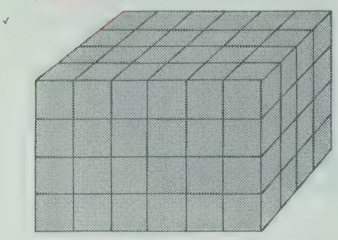
OBJECTIVE

Demonstrate competence in finding volume; solve related word problems

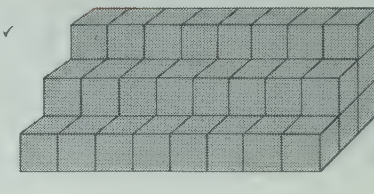
Practice

Find the volume.
Each cube represents a cubic centimetre.

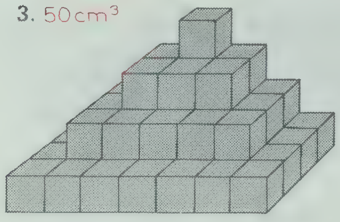
1. 96 cm³



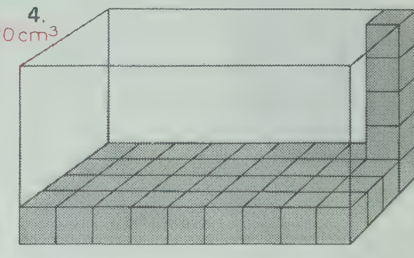
2. 48 cm³



3. 50 cm³

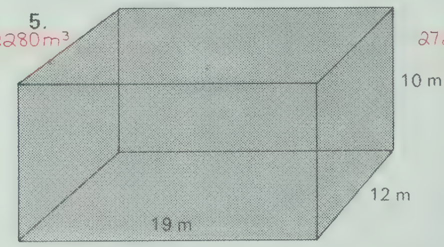


4. 180 cm³

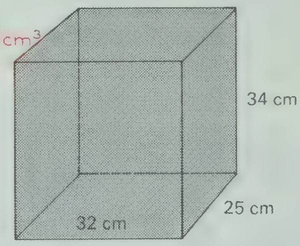


Find the volume of each rectangular prism.

5. 2280 m³



6. 27 200 cm³



7. Base is 9 cm by 6 cm.
   Height is 8 cm. 432 cm³

8. Base is 183 cm by 35 cm.
   Height is 2 cm. 12 810 cm³

9. Base is 20 m by 91 m.
   Height is 40 m. 72 800 m³

10. Base is 4 dm by 860 dm.
    Height is 34 dm. 116 960 dm³

11. Base is 100 m by 100 m.
    Height is 10 m. 100 000 m³

12. Base is 73 m by 60 m.
    Height is 85 m. 372 300 m³

128

LESSON ACTIVITY

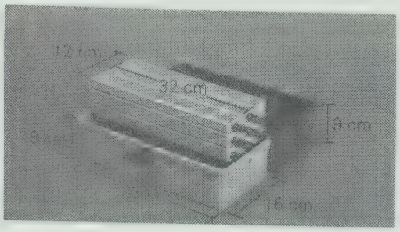
Before Using the Pages

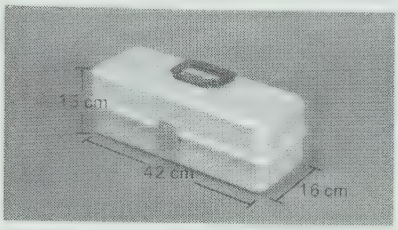
- You may wish to review finding the volume of an object by asking students to explain how to find the volume of an irregular shape composed of centimetre cubes and how to find the volume of a rectangular prism.

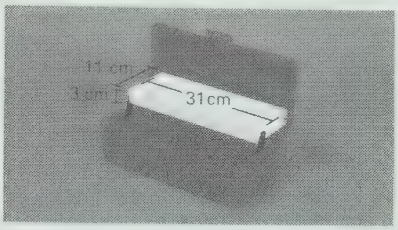
Using the Pages

- For shapes that are not rectangular prisms, such as those in Ex. 2 and 3, the students can use multiplication and addition to find the volume. Ex. 4-6 show rectangular prisms for which multiplication is used to find the volume. Ex. 13-17 refer to the fishing tackle boxes in the photographs on page 129. For Ex. 13, the students must consider the volume of the top and of the bottom of the brown fishing tackle box. Ex. 14 involves comparing volumes after they have been calculated. Ex. 15 and 17 review the concept of area. After the students have completed the exercises, ask why the units of measurement in the answers for Ex. 15 and 17 are different from those for Ex. 13, 14, and 16.

Keeping Sharp: Ex. 1-8 review addition and subtraction with whole numbers and with decimals. Ex. 9-16 review multiplication and division with whole numbers. For Ex. 17-21, the students must work inside the parentheses first and then work from left to right.







Solve.

13. What is the volume of the brown fishing tackle box? 9504 cm³

14. How much greater is the volume of the silver tackle box than the volume of the brown tackle box? 576 cm³

15. What is the area of the tray in the red tackle box? 341 cm²

16. What is the volume of the tray in the red tackle box? 1023 cm³

17. What is the area of one tray in the brown tackle box? 384 cm²

Add.

1. 38 764 + 53 829 92 593

2. 72 941 + 25 308 + 921 99 170

3. 10.58 + 9.56 = 20.14

4. 0.467 + 5.296 = 5.763

Subtract.

5. 31 108 − 2 347 28 761

6. 70 039 − 52 986 17 053

7. 42.78 − 33.14 = 9.64

8. 7.001 − 0.439 = 6.562

Multiply.

9. 305 × 68 = 20 740

10. 438 × 507 = 222 066

11. 600 × 679 407 400

12. 871 × 583 507 793

Divide.

13. 40)287 7 R7

14. 71)382 5 R27

15. 68 287 ÷ 64 1066 R63

16. 40 023 ÷ 631 63 R270

Find the result.

17. 785 + (9207 − 4608) 5384

18. 230 − (90.46 + 55.76) 83.78

19. (42 416 ÷ 482) ÷ 44 2

20. 609 × (4725 ÷ 63) 45 675

21. 34 960 ÷ (23 × 38) 40

KEEPING SHARP

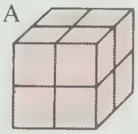
129

RELATED ACTIVITIES

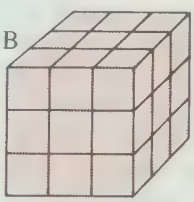
• For the game "Inventory" described on page T 380, have the students list reasons for finding the volume, such as finding the amount of storage space in a trunk. Also, have them list examples, such as fishing tackle boxes, for which they would be able to find the volume.

• Display a cube composed of eight smaller cubes (A). Have students imagine that the faces of the large cube are to be painted red and ask, "How many of the small cubes will have more than three red faces?" "How many cubes will have three red faces?" "How many cubes will have only two red faces?" "How many cubes will have only one red face?" "How many cubes will have no red faces?" Challenge the students with the same questions for a cube made of 27 smaller cubes (B).

A



B



OBJECTIVE

Use a tree diagram to find the number of possibilities

Vocabulary
tree diagram

RELATED ACTIVITIES

- Challenge the students to use a tree diagram to plan a school activity, for example, the organizing of teams in different ways for sports events.
- Provide headings for four menu categories.

Soup	Sandwich	Drink	Dessert

Ask students to suggest from one to three items for each category. Ask them to find the number of different ways to select a meal, if one item and only one item must be selected from each category. They may draw tree diagrams to help solve the problem.

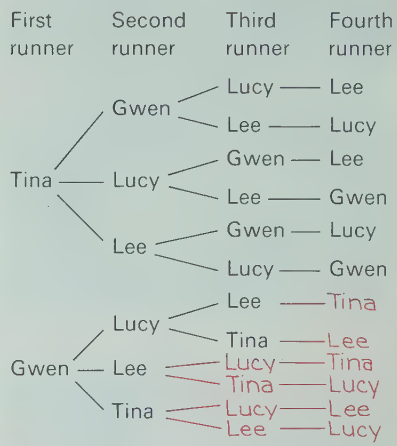
Increase the number of categories from four to six to provide a greater challenge.

Finding the Number of Possibilities

Tina, Gwen, Lucy, and Lee are on a relay team. How many possible choices are there for the order of the runners on the team?

Any of the 4 girls can be first. Any of the remaining 3 girls can be second. Either of the remaining 2 girls can be third. The remaining girl would be fourth.

To find the possible choices, you can draw a **tree diagram**.



The tree diagram shows that there are 6 possible choices, if Tina is the first runner.

1. Complete the diagram to show the possible choices for the runners if Gwen is first.
2. Complete the other diagrams to show all the possibilities for the order of the runners. Diagrams are shown below.

Draw a diagram that shows the possible choices for each. Then give the number of possible choices.

3. If 3 girls are running on a relay team, how many possible choices are there for the order of the runners? 6
4. If 5 girls are swimming on a relay team, how many possible choices are there for the order of the swimmers? 120
5. If 15 boys are racing, how many different possibilities are there for first place? 15
6. If 15 boys are racing, how many possibilities are there for second place after one has crossed the finish line? 14
7. If 4 boys are racing, how many different ways can they finish the race? 24

PROBLEM SOLVING

130

LESSON ACTIVITY

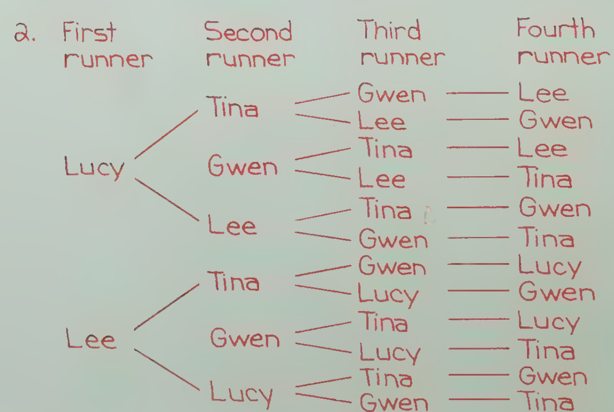
Before Using the Page

- Ask two students to stand side by side. Ask how many different possibilities there are for the order in which these two students can stand side by side. Write the students' names on the board to show each order and have the two students demonstrate the orders. Write the number of possibilities on the board.

  Repeat this procedure with three students. Then ask students to estimate the number of possibilities that there would be for four students.

Using the Page

- Have a student read the word problem at the top of the page. Then discuss the information in the "thought cloud". Introduce the *tree diagram* as a method for finding the number of possibilities and discuss the part of the tree diagram that is shown. You may wish to have four students demonstrate possible choices for the order of the runners if Tina is the first runner, using the tree diagram as a guide.
- Have the students explain the tree diagrams that they draw for Ex. 1-7. Some students may realize that multiplication can be used to find the number of possibilities, particularly after a tree diagram has been drawn for one exercise.



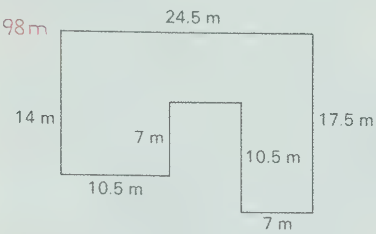
Checking Up

Choose the best unit to measure each.

1. the distance between cities kilometres
2. the length of a tennis racket decimetres or centimetres

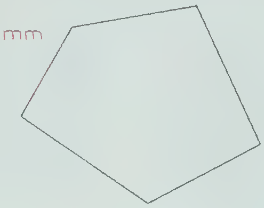
Find the perimeter.

3. 98 m



Measure to the nearest millimetre.
Then find the perimeter.

4. 130 mm

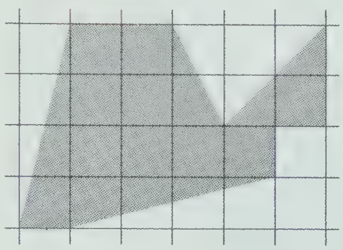


Find the perimeter of each.

5. a rectangle 14 m by 35 m 98 m
6. a square with sides 93 mm 372 mm

Find the area in square centimetres.

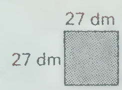
7. 15 cm^2



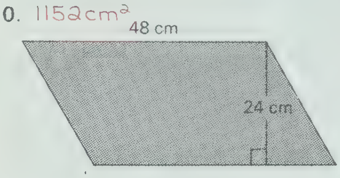
Find the area.

8. 1792 m^2

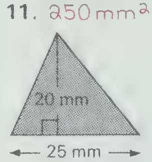
9. 729 dm^2



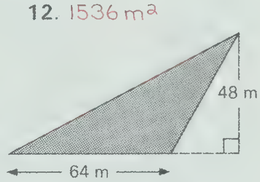
10. 1152 cm^2



11. 250 mm^2



12. 1536 m^2



Turn the page for more exercises.

Pages 131-132

OBJECTIVE

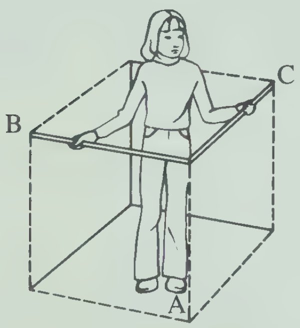
Demonstrate an understanding of the concepts and skills presented in this unit

Materials

a ruler marked in millimetres for each student

RELATED ACTIVITIES

- To reinforce the concepts of perimeter, area, and volume, have the students write word problems similar to Ex. 24-27 on page 132. Then have other students decide whether a problem involves perimeter, area, or volume and state the unit of measurement that would be used for the answer.
- Have students use copies of page T396 and draw a horizontal number line and a vertical number line starting from the same point as described on page 42. Have them draw a rectangle or a square on the grid, find the perimeter and the area of the shape, and identify the ordered pair for each vertex. Have them write the ordered pairs for other students to plot on a grid and to find the perimeter and the area of the shape.
- Help students to visualize a metre cube by marking one corner of the classroom as indicated below. A student standing at Point A may hold two metre sticks to touch the walls at points B and C.



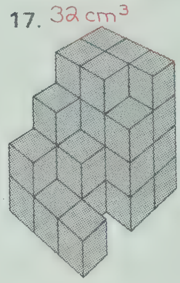
Find the area of each.

13. a rectangle 17 cm by 14 cm 238 cm²
14. a square with sides 32 m 1024 m²

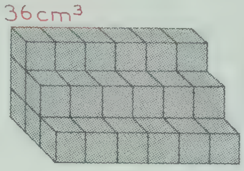
15. a parallelogram with height 64 mm and base 93 mm 5952 mm²
16. a triangle with height 14 m and base 95 m 665 m²

Find the volume.
Each cube represents one cubic centimetre.

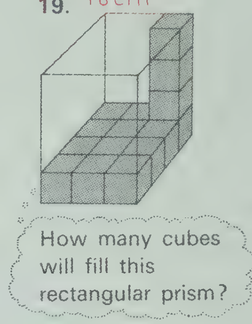
17. 32 cm³



18. 36 cm³

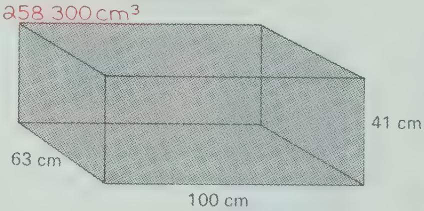


19. 48 cm³

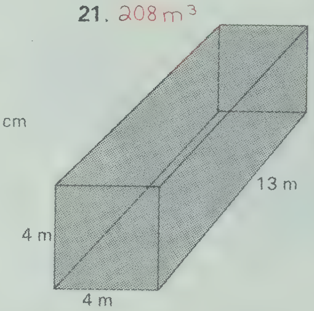


Find the volume.

20. 258 300 cm³



21. 208 m³



22. Base is 17 m by 16 m. Height is 22 m. 5984 m³

23. Base is 1 dm by 52 dm. Height is 13 dm. 676 dm³

Solve.

24. How long is a fence around a skating rink 21 m by 15 m? 72 m

25. What is the area of a gymnastics mat 205 cm by 180 cm? 36 900 cm²

26. A box of golf tees is 44 cm long, 41 cm wide, and 46 cm high. What is its volume? 82 984 cm³

27. A triangular sign with a base of 122 cm is 92 cm high. What is its area? 5612 cm²

132

Skills	Exercises	Related Pages
Choose the preferred unit of length	1, 2	T116-T117
Use addition to find the perimeter of a shape	3, 4	T118-T119
Find the perimeter of a rectangle and of a square	5, 6	T120-T121
Find the area by counting	7	T122-T123
Find the area of a rectangle and of a square	8, 9, 13, 14	T124-T125
Find the area of a parallelogram	10, 15	T126-T127
Find the area of a triangle	11, 12, 16	T128-T129
Find the volume by counting	17, 18	T132-T133
Find the volume of a rectangular prism	19-23	T134-T137
Solve word problems	24-27	

Comments

Remind the students that there are two pages for *Checking Up*.

Provide each student with a ruler marked in millimetres for Ex. 4.

Difficulties with Ex. 3 may be due to forgetting the side for which the measurement is not given. Determine whether mistakes with perimeter, area, or volume are caused by forgetting the procedure used to determine each, by confusing the meaning of perimeter, area, and volume, or by misusing linear units, square units, and cubic units. Provide practice with measuring the distance around shapes to find the perimeter, covering shapes with centimetre squares or decimetre squares to find the area, and filling shapes with centimetre cubes to find the volume.

Checking Skills

Multiply.

1. 6427 9 57 843	2. 85 319 8 682 552
3. 346 30 10 380	4. 975 70 68 250
5. 48 99 4752	6. 306 62 18 972
7. 379 400 151 600	8. 492 800 393 600
9. 276 108 29 808	10. 815 570 464 550
11. 405 937 379 485	12. 683 642 438 486

Divide.

13. 6)427 71 R1	14. 9)8570 952 R2
15. 10)6703 670 R3	16. 30)19 203 640 R3
17. 68)7223 106 R15	18. 43)8869 206 R11
19. 92)50 320 546 R88	20. 79)64 312 814 R6
21. 87)9310 107 R1	22. 64)5545 86 R41
23. 26)80 540 3097 R18	24. 98)69 770 711 R92
25. 200)1387 6 R187	26. 700)91 324 130 R324
27. 570)76 002 133 R192	28. 430)30 100 70
29. 625)73 842 118 R92	30. 751)64 853 86 R267
31. 204)52 287 256 R63	32. 943)99 463 105 R448

Add or subtract.

1. 24.98 + 12.74 + 33.56 71.28
2. 96.502 − 28.431 68.071
3. 6.8 + 9.2 + 3.5 + 6.1 + 4.4 30.0
4. 2.32 + 4.76 + 9.83 + 4.21 21.12
5. 68.3 + 52.1 + 32.6 + 56.2 209.2
6. 0.926 − 0.489 0.437
7. 337.64 − 46.21 291.43
8. 0.005 + 0.007 + 0.008 0.020
9. 630.05 − 0.08 629.97
10. 1.0001 − 0.0009 0.9992
11. 2.4831 + 2.4831 4.9662
12. 52.008 − 51.009 0.999
13. 2.9486 − 0.7285 2.2201
14. 6.03 + 6.03 + 6.03 18.09
15. 9.87 + 9.87 + 9.87 + 9.87 39.48

Find the result.

16. 28 347 + (97 423 − 64 999) 60 771
17. (28 347 + 97 423) − 64 999 60 771
18. 125 × (684 ÷ 18) 4750
19. (125 × 684) ÷ 18 4750
20. 8073 − (429 + 1237) 6407
21. (8073 − 429) + 1237 8881
22. 52 731 ÷ (27 × 31) 63
23. (52 731 ÷ 27) × 31 60 543
24. (90 000 − 11 111) − 1 119 77 770
25. 90 000 − (11 111 − 1 119) 80 008
26. (7744 ÷ 176) ÷ 44 1
27. 7744 ÷ (176 ÷ 44) 1936
28. 81 000 ÷ (30 × 30) 90
29. (196 × 248) ÷ 98 496
30. 7.903 − (2.845 + 5.057) 0.001

133

Page 133

OBJECTIVE

Demonstrate competence in performing addition, subtraction, multiplication, and division, and in simplifying a number phrase involving parentheses

RELATED ACTIVITIES

- For practice in interpreting place value in decimals, prepare cards similar to the following. Have students use the cards to play the game "Concentration" described on page T 379.

3.78	3 ones 7 tenths 8 hundredths

- For extra practice and review, students may be asked to complete one or more of the following.
  1. Use multiplication and addition to check the answers for several division exercises.
  2. Write the word name for each number in a selection of exercises.
  3. Write the expanded form for each number in a selection of exercises.

LESSON ACTIVITY

Using the Page

- These exercises help to review and maintain skills in addition and subtraction with whole numbers and with decimals, and in multiplying and dividing whole numbers. The latter skills prepare students for similar work with decimals in Unit 7. Expressions involving parentheses are also presented on this page. Remind the students to complete the work within parentheses first and then work from left to right for Ex. 16-30 of the second column. They should note the similarities for two exercises such as Ex. 16 and 17, and Ex. 20 and 21, and observe whether different results are obtained.

Unit 7 Overview

Multiplying and Dividing Decimals

The first lesson in this unit reviews multiplying a decimal by a whole number with attention to place values in the decimal factor and in the product. Placing the decimal point is considered from two points of view: by rounding the factors and then estimating the whole-number product, and by counting the decimal places in the factor. Both methods are used in the subsequent lessons in which both factors are decimals. Special attention is then given to products less than 1, particularly those which require the insertion of one or more zeros immediately to the right of the decimal point. In the division of a decimal by a whole number, place values in both the dividend and the quotient are emphasized. Extension of the dividend by adding zeros to the right is also presented. Rounding quotients to various decimal places applies the same principles as rounding whole numbers. The use of the decimal key on the calculator is presented for both multiplication and division. The *Problem Solving* lesson involves solutions requiring two or more steps.

Prerequisite Skills

- multiply a whole number by a number with up to three digits
- interpret place value for decimals with up to three places
- write decimals
- use extra zeros for showing equivalent decimals
- round whole numbers
- round decimals to the nearest one
- use multiplication to find the area of a rectangular region in square metres
- divide a whole number by a number with up to three digits

Unit Outcomes

- multiply a decimal with up to three decimal places by a whole number with up to three digits
- place the decimal point in the product by rounding and multiplying to estimate the product, and by counting decimal places in each factor to determine the number of decimal places in the product
- find the product of two one-place decimals
- multiply a one-place, two-place, or three-place decimal by a one-place decimal or a two-place decimal
- multiply two decimals, products less than one
- divide a decimal by a one-digit number, using extra zeros in the dividend
- divide a decimal by a whole number having two or three digits, using extra zeros in the dividend
- round quotients to the nearest tenth, to the nearest hundredth, and to the nearest thousandth
- solve word problems involving decimals
- use a calculator to multiply and divide with decimals
- solve word problems in two or more steps

Background

The same basic multiplication and division facts and the same algorithms are used with decimals as with whole numbers. Place values are emphasized even more with decimals because the decimal point must be placed correctly in the product and in the quotient.

There are two methods for determining the correct location of the decimal point in a product: one involves making a whole-number estimate of the product and leads to placing the decimal point directly to the right; the other involves considering the place values to be represented by the decimal portion of the product and thus the number of digits that must be shown to the right of the decimal point.

First Method

42 × 7.895

Round and estimate whole-number product.

$$\begin{array}{r} 40 \\ \times\ 8 \\ \hline \underbrace{320} \end{array} \qquad \begin{array}{r} 7.895 \\ \times\ \ 42 \\ \hline 15\ 790 \\ 315\ 800 \\ \hline \underbrace{331}.\underbrace{590} \end{array}$$

Second Method

42 × 7.895

Consider place value.
7.895 names thousandths.
7.895 has 3 decimal places.
Product needs 3 decimal places.

The same methods are equally effective in locating the decimal point when both factors are decimals. For the second method the decimal places in the two factors are counted and the sum is the number of decimal places required in the product. Because this approach often loses its place-value meaning, it is important that a few basic generalizations should be understood in using it; namely, that "tenths × tenths = hundredths" (2 places), that "hundredths × tenths = thousandths" (3 places), and that "hundredths × hundredths = ten-thousandths" (4 places).

Multiplication of a decimal or a whole number by a power of ten, whether it be a whole-number power such as 10, 100, or 1000, or a decimal power such as 0.1, 0.01, or 0.001, causes a shift in the relative positions of the digits and the decimal point. Products may be calculated mentally by moving the digits to the left or to the right. Division of a decimal by a whole-number power of ten may also be calculated mentally by moving the digits to the right. These quotients are identical to the products shown below where 5437.9 is multiplied by decimal powers of ten. Note that there is no attempt to develop this inverse relationship in this unit.

Whole-number power	Digits move to the left
	24.653
Multiply by 10.	246.53
Multiply by 100.	2465.3
Multiply by 1000.	24653.

Decimal power	Digits move to the right	Whole-number power
	5437.9	
Multiply by 0.1.	543.79	Divide by 10.
Multiply by 0.01.	54.379	Divide by 100.
Multiply by 0.001.	5.4379	Divide by 1000.

Calculators that have a built-in feature called a "floating decimal point" move the decimal point, while the digits remain fixed on the digital displays.

In this unit only whole-number divisors are encountered, so the traditional algorithm is used in dividing each place value of the dividend starting from the left. When the division procedure passes from ones to tenths in the dividend, a corresponding decimal point must be placed in the quotient. The regrouping of values represented by a remainder at any step combined with the next digit of a dividend is performed in the same manner as with

whole numbers. The only new feature in dividing decimals is the optional extension of a dividend by writing zeros on the right. This writing of extra zeros should be approached meaningfully by examining equivalent decimals such as 3.4 and 3.40, and 1.656 and 1.6560.

The principles for rounding a whole number are discussed in the Overview for Unit 1, and it is emphasized that attention needs to be focused on two digits — on the one in the place value to which the number is to be rounded and on the one in the next place to the right. The same general principle applies to decimals, but an additional step is involved when a decimal quotient is to be rounded to a specific place. If the division were carried out only to that place, there would be no digit to the right, so it is necessary to continue the division to one more place. The extra digit in the quotient determines whether the quotient should be rounded up or down. For example, in finding the quotient for 7.65 ÷ 12 rounded to the nearest hundredth (2 places), the division must be performed to thousandths (3 places). Since the digit in the thousandths' place is 7, the quotient is rounded up to 0.64.

$$
\begin{array}{r}
0.637 \\
12\overline{)7.650} \\
\underline{7\ 2} \\
45 \\
\underline{36} \\
90 \\
\underline{84} \\
6
\end{array}
$$

One cannot work with numbers without becoming fascinated by the relationships between operations, by the recurrence of certain numbers more often than others, and by the patterns of digits which occur frequently in decimal quotients. In this unit the method of recording the repetition of digits by drawing a horizontal bar above them is introduced. In some instances a single digit is repeated, whereas in others two or more digits establish a pattern which may be extended indefinitely. It is the base-ten structure of our numeration system which produces such interesting patterns, because some numbers are not exactly divisible by certain divisors, even if the dividends are extended by adding zeros.

$$
\begin{aligned}
8 \div 6 &= 1.333\ldots \\
&= 1.\overline{3} \\
15 \div 11 &= 1.3636\ldots \\
&= 1.\overline{36} \\
12 \div 37 &= 0.324324\ldots \\
&= 0.\overline{324}
\end{aligned}
$$

Teaching Strategies

If significant differences in students' abilities become apparent in multiplying and dividing with whole numbers in Unit 4, it may be advisable to structure two or more instructional groups for the work of this unit. Those students who experienced difficulty with the earlier work with whole numbers may need more emphasis on place values as they perform the operations. They may also be helped in their work in multiplication by naming the values represented by the decimal factors, as shown on page 134. In their work with division of decimals, it may be beneficial for them to name the columns represented by the digits in the dividends, as suggested in the *Related Activities* for teaching pages 148 and 149.

Two methods are presented for placing the decimal point in a product, and more capable students should have no difficulty using either of them. Less capable students may find it harder to estimate a whole-number product and will probably prefer the procedure of counting decimal places. Both of the methods for placing the decimal point in a product can be used in a game-like manner. Students can be challenged to predict how many digits will be in either the whole-number part or in the decimal part of a product. For instance, in the case of 7.35 × 45.86, the product will have a three-digit whole-number part, since 7 × 46 has a product of hundreds, and there will be four decimal places, since the product of hundredths (2 digits) and hundredths (2 digits) is ten-thousandths (4 digits).

On page 145 the use of the commutative property of multiplication is shown for checking accuracy in multiplying decimals. No method is shown for checking accuracy in dividing decimals, but the students are familiar with the one for whole numbers in which the quotient and the divisor are multiplied, and then the remainder is added. This method may also be applied to checking accuracy in dividing decimals, but the place values of remainders must be considered. As shown in the example, the remainder 2 means 2 hundredths and must be written as 0.02.

$$
\begin{array}{r}
6.26 \\
6\overline{)37.58} \\
\underline{36} \\
1\ 5 \\
\underline{1\ 2} \\
38 \\
\underline{36} \\
2
\end{array}
\qquad
\begin{array}{r}
\text{Check} \\
6.26 \\
\underline{\times\ \ \ 6} \\
37.56 \\
\underline{+\ 0.02} \\
37.58
\end{array}
$$

The work with repeating digits in quotients can be more challenging if students have access to calculators. Students may divide consecutive numbers beginning at 1 by the divisors 3, 6, 9, 15, and 18 for repetition of single digits, by 11 and 22 for repetition of two digits, and by 37 for repetition of three digits. Examples for each kind are shown (A). For enrichment, students may be challenged to study the relocations of digits when numbers are divided by 7, 14, 19, and 21. Examples are given for 7 as a divisor (B).

A	$19 \div 18 = 1.0\overline{5}$	B	$1 \div 7 = 0.\overline{142857}$
	$9 \div 11 = 0.\overline{81}$		$2 \div 7 = 0.\overline{285714}$
	$39 \div 37 = 1.\overline{054}$		$3 \div 7 = 0.\overline{428571}$

Some supermarkets post "unit prices" for customers to use in comparative shopping. The *Problem Solving* lesson in this unit requires students to use division to establish unit prices which may be compared. Some of the exercises also make comparisons between the quantities obtainable at unit prices. Decimal quotients are encountered in either case, and comparisons are made between the numbers to one, two, or three decimal places. Students may also be challenged to compare unit prices and quantities from newspaper advertisements or by actual research in stores. Besides groceries, comparative pricing may be studied in connection with items from drugstores, hardware stores, and building supply centers.

The skills for multiplying and dividing decimals as presented in this unit are assessed in the *Checking Up* exercises; the results should be studied carefully to note whether any concepts or processes require additional attention. Students with similar needs should be grouped for reteaching, review, and further practice.

Materials

models for ones, tenths, and hundredths made from copies of pages T 393 and T 394 as described on page T 89

models for ones, tenths, and hundredths prepared from copies of pages T 392-T 394 as shown on page T 150

water, a transparent drinking glass, and objects such as spoons and drinking straws (optional)

calculators (optional)

Vocabulary

milligram, mg

Pages 134-135

LESSON OUTCOME

Multiply a decimal with up to three decimal places by a whole number with up to three digits; solve related word problems

Materials

models for tenths made from copies of page T 393 as described on page T 89

Prerequisite Skills

Multiply whole numbers; interpret place value for decimals with up to three decimal places

Checking Prerequisite Skills

Multiply.

1. 718 × 9 = 6462

2. 752 × 374 = 281 248

3. 62 × 48 2976

4. 805 × 3609 2 905 245

What does each 5 mean?

5. 0.57 5 tenths

6. 2.385 5 thousandths

7 MULTIPLYING AND DIVIDING DECIMALS

Multiplying Decimals and Whole Numbers

The camera store has 27 rolls of one type of film. The price of each roll is $3.15. What is the value of the 27 rolls?

Multiply 27 and 3.15.

$3.15 has the same value as 315 cents. Think of 3.15 as 315 hundredths.

For the product,

3 1 5	3 1 5 hundredths
2 7	2 7

you need to know how to multiply 7 and 315,

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5

and how to multiply 2 and 315.

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5
6 3 0 0	6 3 0 0

Then add and place the decimal point.

3 . 1 5	3 1 5 hundredths
2 7	2 7
2 2 0 5	2 2 0 5
6 3 0 0	6 3 0 0
8 5 . 0 5	8 5 0 5 hundredths

The value of the 27 rolls of film is $85.05.

134

LESSON ACTIVITY

Before Using the Pages

- Write several numerals on the board, such as 4235, 617, and 8092. Ask students to mark a decimal point in one of the numerals to show thousandths, in another to show tenths, and so on, to review decimals with up to three decimal places. Include such numerals as 4 and 6500 to obtain such decimals as 0.04 and 6.500. For some numerals, ask students to mark a decimal point and the symbol $ to show an amount of money for dollars-and-cents notation. Point out that such numerals show hundredths (of a dollar). For each example, ask a student to read the decimal.
- Display three models for six-tenths, one at a time. Establish that each model represents the number 0.6. Group the models together and ask how many tenths there are in all. Ask for a simpler way to show 18 tenths (1.8). Regroup the models as 1 whole and 8 tenths to demonstrate this.

  Write the addition $0.6 + 0.6 + 0.6 = 1.8$ on the board and ask for a shorter way to express the same idea. Lead the students to suggest the use of multiplication and the sentence $3 \times 0.6 = 1.8$. Point out the similarity between the multiplication $3 \times 6 = 18$ and the multiplication 3×6 tenths $= 18$ tenths. Develop that the product would be 18 hundredths (thousandths) if 6 tenths were replaced by 6 hundredths (thousandths). Use other examples as needed. Then ask how the number of decimal places in the product can be determined before multiplying.

Using the Pages

- Ask a student to read the problem. Discuss that multiplication, as opposed to addition, is an efficient way to find the solution. Note that interpreting $3.15 as 315 hundredths of dollars helps to demonstrate that multiplying a decimal by a whole number involves the same steps as multiplying two whole numbers. After adding the partial products, a decimal point is placed to show hundredths in the final product. Since the problem involves an amount of money, the position of the decimal point can also be explained in terms of dollars-and-cents notation.

Working Together

Complete each multiplication.

1.

481	481 tenths	48.1
7	7	7
3367	3367 tenths	336.7

2.

145	145 hundredths	1.45
22	22	22
3190	3190 hundredths	31.90

3.

2875	2875 thousandths	2.875
35	35	35
100625	100625 thousandths	100.625

4.

13	13 thousandths	0.013
6	6	6
78	78 thousandths	0.078

Multiply.

5.	6.	7.	8.	9.	10.
3.15	14.3	4.756	0.67	0.004	20.3
8	79	12	15	23	375
25.20	1129.7	57.072	10.05	0.092	7612.5

Exercises

Multiply.

1.	2.	3.	4.	5.	6.
57.7	5.77	2.695	0.48	375.6	$123.45
28	28	9	75	237	67
1615.6	161.56	24.255	36.00	89017.2	$8271.15

7. 13 × 1.07 13.91 **8.** 6 × 58.6 351.6 **9.** 39 × 0.003 0.117 **10.** 10 × 20.72 207.2

11. 40 × 3.7 148.0 **12.** 76 × 0.085 6.460 **13.** 41 × 74.8 3066.8 **14.** 5 × $97.34 $486.70

Solve.

15. The sports store has 7 tennis rackets in stock at $43.75 each. What is the value of the tennis rackets? $306.25

16. The supermarket had a delivery of 144 packages of butter which were to be priced at $1.39 each. What was the value of the butter? $200.16

When a number is multiplied by 10, each digit moves one place to the left.

Example:

	tens	ones	tenths	hundredths		tens	ones	tenths	hundredths
10 ×		3	6	7	=	3	6	7	

Multiply each number by 10. Do these in your head.
Write only the products.

17. 1.14 11.4 **18.** 28.62 286.2 **19.** 3.588 35.88 **20.** 670 6700 **21.** 0.016 0.16 **22.** 7.5 75

135

RELATED ACTIVITIES

- To help students understand the product of a decimal and a whole number, provide them with copies of page T 390 and have them show multiplications such as 3 × 0.4 on a number line.

- Students having difficulty multiplying decimals may find it helpful to write exercises using the following format.

577 tenths	577 hundredths
× 28	× 28
16 156 tenths	16 156 hundredths

- For practice with basic multiplication facts, prepare cards similar to the following for the game "Dominoes" described on page T 379.

3 × 8	36		4 × 9	48

- You may wish to use the first three activities on page T 93 to review place value for decimals.
- For enrichment, have students complete pairs of products similar to the following.

$3 \times 0.7 =$ ___ $18 \times 0.21 =$ ___
$7 \times 0.3 =$ ___ $21 \times 0.18 =$ ___

Then ask the students to write multiplications that have the same products as Ex. 7-13 on page 135. For example, for Ex. 9, one solution would be $3 \times 0.039 = 0.117$.

Working Together: Ex. 1-4 demonstrate that multiplying a decimal by a whole number is similar to multiplying two whole numbers. Because the product names a number of tenths, hundredths, or thousandths, a decimal point is placed in the product in the appropriate place. Pay particular attention to Ex. 2. The product is 3190 hundredths, or 31.90. Point out that since 31 and 90 hundredths is equal to 31 and 9 tenths, the product 31.90 can also be written as 31.9.

Exercises: Ex. 17-22 are of particular importance because they involve multiplication by ten. The example above these exercises shows that each digit of a number moves to a greater place value, specifically, one place to the left when that number is multiplied by ten. This can be pointed out earlier in Ex. 10. Also, because the concept was presented with whole number factors in *Try This* on page 55, you may wish to draw attention to it briefly. Note that the students are to write only the products for Ex. 17-22. Although 00.16 is correct for Ex. 21, point out that this is usually written as 0.16.

Assessment

Multiply.

1.	2.	3.
5.97	10.8	0.964
7	46	285
41.79	496.8	274.740

4. 53 × 2.5 132.5 **5.** 281 × 8.367 2351.127
6. 10 × 34.26 342.6 **7.** 10 × 0.058 0.58

Solve.

8. A store sold 36 watches for $24.95 each. What was the value of the watches? $898.20

Pages 136-137

LESSON OUTCOME

Multiply a decimal with up to three decimal places by a whole number; place the decimal point in the product by rounding and multiplying to estimate the product, and by counting decimal places in each factor to determine the number of decimal places in the product; solve related word problems

Prerequisite Skills

Multiply whole numbers; round whole numbers; round decimals to the nearest one

Checking Prerequisite Skills

Multiply.

1. $\begin{array}{r} 248 \\ \underline{8} \\ 1984 \end{array}$ **2.** $\begin{array}{r} 96 \\ \underline{74} \\ 7104 \end{array}$

3. 325 × 713 231 725

4. 610 × 5364 3 272 040

Round

5. 34 to the nearest ten. 30

6. 78 to the nearest ten. 80

7. 251 to the nearest hundred. 300

8. 3.84 to the nearest one. 4

9. 11.267 to the nearest one. 11

Placing the Decimal Point

The steps for multiplying decimals are the same as the steps for multiplying whole numbers, but you have to place the decimal point in the product.

Here are two ways that help you remember where to place the decimal point in the product.

Estimate the product first, then multiply like whole numbers.

For 31 × 6.832,

31 $\xrightarrow{\text{rounds to}}$ 30

6.832 $\xrightarrow{\text{rounds to}}$ 7

The product will be about 210.

$$\begin{array}{r} 6\,832 \\ \underline{31} \\ 6\,832 \\ \underline{204\,960} \\ 211\,792 \end{array}$$

The product of 31 and 6832 is → 211 792

Count the decimal places in the factors.

For 31 × 6.832,
6.832 has 3 decimal places.
31 has 0 decimal places.
There will be 3 decimal places in the product.

$$\begin{array}{r} 6.832 \\ \underline{31} \\ 6\,832 \\ \underline{204\,960} \\ 211.792 \end{array}$$

one two three

For 31 × 6.832, the decimal point goes here.

Working Together

Round and multiply to estimate each product. Estimates may vary.

1. $\begin{array}{r} 3.04 \\ \underline{19} \\ 60 \end{array}$ **2.** $\begin{array}{r} 27.9 \\ \underline{8} \\ 240 \end{array}$ **3.** $\begin{array}{r} 2.839 \\ \underline{42} \\ 120 \end{array}$

How many decimal places will there be in each product?

4. $\begin{array}{r} 612.5 \\ \underline{1 \quad 71} \end{array}$ **5.** $\begin{array}{r} 6.295 \\ \underline{3 \quad 7} \end{array}$ **6.** $\begin{array}{r} 23.59 \\ \underline{2 \quad 34} \end{array}$

Show the decimal point in each product.

7. $\begin{array}{r} 3.04 \\ \underline{19} \\ 5776 \\ 57.76 \end{array}$ **8.** $\begin{array}{r} 27.9 \\ \underline{8} \\ 2232 \\ 223.2 \end{array}$ **9.** $\begin{array}{r} 2.839 \\ \underline{42} \\ 119\,238 \\ 119.238 \end{array}$ **10.** $\begin{array}{r} 612.5 \\ \underline{71} \\ 434\,875 \\ 43\,487.5 \end{array}$ **11.** $\begin{array}{r} 23.59 \\ \underline{134} \\ 316\,106 \\ 3161.06 \end{array}$

Multiply.

12. $\begin{array}{r} 6.295 \\ \underline{7} \\ 44.065 \end{array}$ **13.** $\begin{array}{r} 6.49 \\ \underline{38} \\ 246.62 \end{array}$ **14.** $\begin{array}{r} 70.4 \\ \underline{6} \\ 422.4 \end{array}$ **15.** $\begin{array}{r} 5.837 \\ \underline{49} \\ 286.013 \end{array}$ **16.** $\begin{array}{r} 34.95 \\ \underline{265} \\ 9261.75 \end{array}$

136

LESSON ACTIVITY

Before Using the Pages

- Write 8 × 1.064 in vertical form on the board and have students help to complete the multiplication. Ask them to explain the steps using statements such as "8 times 4 thousandths is equal to 32 thousandths, or 3 hundredths 2 thousandths".

  Use other exercises on the board to review the lesson on pages 134 and 135. After several exercises, ask a few students to tell how they decided where to place the decimal point in each product. They may suggest one or both of the methods presented on page 136.

Using the Pages

- The worked example shows two ways of determining the position of the decimal point in a product. Some students may prefer to use the first method, while others may prefer to use the second. Also, one method may be used to place the decimal point and the other method used as a check.

For the first method, the whole number is rounded to the nearest ten, the decimal is rounded to the nearest one, and the rounded numbers are multiplied to provide a whole-number estimate of the product. (Note that this involves the basic fact 7 × 3 = 21 applied to 7 × 3 tens = 21 tens, and 7 × 30 = 210.) Then the given factors are multiplied like whole numbers and the estimate is used to determine where to place the decimal point. Because the exact product will be close to the estimate, 210, the decimal point is placed between the 1 and the 7 to give 211.792. It is helpful to write, for example, 21.1792 and 2117.92 on the board to emphasize that 211.792 is closest to 210, the estimate.

For the second method, the factors are multiplied in the same way as for whole numbers. Then the decimal places are counted in each factor and the sum of the decimal places in the two factors is determined. The sum indicates the number of decimal places in the product. Counting the decimal places in each factor at this time prepares the students for applying the same procedure when both factors are decimals.

Exercises

Solve.

1. A transport truck carries 9 cars, each with a mass of 806.8 kg. How heavy is the load on the truck? 7261.2 kg
2. A dentist pulled 23 teeth in a day at \$22.50 per tooth. How much did the dentist charge that day for pulling teeth? \$517.50

Multiply.

3. 2.9 × 83 = 240.7
4. 7.59 × 47 = 356.73
5. 57.83 × 5 = 289.15
6. 0.832 × 52 = 43.264
7. 64.3 × 170 = 10 931.0
8. \$7.24 × 927 = \$6711.48
9. 32 × 3.97 127.04
10. 6 × 5.683 34.098
11. 561 × 16.4 9200.4
12. 8 × 810.24 6481.92
13. 100 × 1.014 101.4
14. 504 × \$6.68 \$3366.72

When a number is multiplied by 100, each digit moves 2 places to the left. When it is multiplied by 1000, each digit moves 3 places to the left.

Examples: 100 × 5.382 = 538.2
1000 × 5.382 = 5382

Multiply each number by 10, 100, and 1000. Write only the products.

Do these in your head.

15. 3.859
16. 12.674
17. 41.82
18. 6.27
19. 3.8
20. 0.016

15 38.59, 385.9, 3859 16 126.74, 1267.4, 12 674
17 418.2, 4182, 41 820 18 62.7, 627, 6270
19 38, 380, 3800 20 0.16, 1.6, 16

Study these. Then complete each of these.

A. 40 × 20 = 800
B. 800 × 70 = 56 000
C. 30 × 6000 = 180 000

1. 4 tens × 2 tens = 8 ? 8 hundreds
2. 8 hundreds × 7 tens = 56 ? 56 thousands
3. 3 tens × 6 thousands = 18 ? 18 ten thousands

For multiplying, in general,

4. tens × tens = ? hundreds
5. hundreds × tens = ? thousands
6. tens × thousands = ? ten thousands
7. hundreds × thousands = ? hundred thousands

Find the product of each without using paper or pencil.

8. 30 × 30 = 900
9. 90 × 60 = 5400
10. 400 × 80 = 32 000
11. 200 × 50 = 10 000
12. 8000 × 40 = 320 000
13. 6000 × 90 = 540 000
14. 5000 × 300 = 1 500 000
15. 7000 × 800 = 5 600 000
16. 300 × 200 = 60 000

KEEPING SHARP

137

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 1-15 on page 331 and Ex. 1-16, Ex. 33-48, and Ex. 65-80 on page 333.
- To provide practice in placing the decimal point, prepare a work sheet similar to the following.

Place the decimal point in each product.
28 × 6.4 = 1792
37 × 8.1 = 2997
59 × 9.02 = 53 218
613 × 47.8 = 293 014
42 × 6.89 = 28 938
157 × 34.7 = 54 479

- For practice in multiplying by 10, 100, and 1000, have the students complete charts similar to the following on copies of page T 382. Discuss patterns shown in the completed chart.

×	4.9	0.38	0.184
10			
100			
1000			

- For the following exercises and others similar to them, discuss why the method of counting the decimal places may be preferable to estimating the product as described on page 136.

0.015 × 23 0.29 × 6

Working Together: Ex. 1-3 provide practice in estimating a product. Ex. 4-6 deal with counting the decimal places in the factors to determine where to place the decimal point in the product. The results of Ex. 1-6 help students to place the decimal point in each product for Ex. 7-11. For Ex. 12-16, the students must multiply the factors and place the decimal points in the products. Rounding and estimating may be used, as well as counting decimal places, for Ex. 12-15, but for Ex. 16 the counting method is probably simpler.

Exercises: After the students have completed Ex. 1-14, ask them to explain how they decided where to place each decimal point. Ex. 15-20 extend the concept of multiplying by ten (see Ex. 17-22 on page 135) to multiplying by 100 and by 1000.

Keeping Sharp: These exercises review multiplications for which the two factors are multiples of 10, 100, or 1000. Discuss how the number of zeros in the two factors together gives the number of zeros in the product in each of examples A, B, and C. Relate these to the place names of the factors and products in Ex. 1-3. After the students have completed the exercises, ask them to explain in their own words the answers for Ex. 4-16. A thorough discussion of these exercises is helpful in preparing the students for similar work when both factors are decimals (see *Try This* on page 143).

Assessment

Multiply. For Ex. 5, tell how you decided where to place the decimal point in the product.

1. 4.9 × 28 = 137.2
2. 0.73 × 936 = 683.28
3. 8.6 × 7 = 60.2
4. 53 × 5.89 312.17
5. 814 × 1.325 1078.550
6. 100 × 0.025 2.5
7. 1000 × 25.47 25 470

Solve.

8. A book has a mass of 1.35 kg. What would be the mass of 24 of these books? 32.40 kg

Pages 138-139

LESSON OUTCOME

Find the product of a whole number and a one-place decimal; find the product of two one-place decimals

Materials

models for ones, tenths, and hundredths prepared from copies of pages T 392-T 394 for diagrams as shown in *Before Using the Pages*

Prerequisite Skills

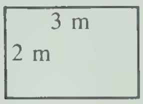
Multiply a three-digit number by a two-digit number; use multiplication to find the area of a rectangular region in square metres; write decimals

Checking Prerequisite Skills

Multiply.

1. 23 × 825 18 975 **2.** 146 × 67 9782

Find the area.

3. 

6 m²

Complete.

	Length	Width	Area
4.	17 m	6 m	102 m²
5.	14 m	12 m	168 m²

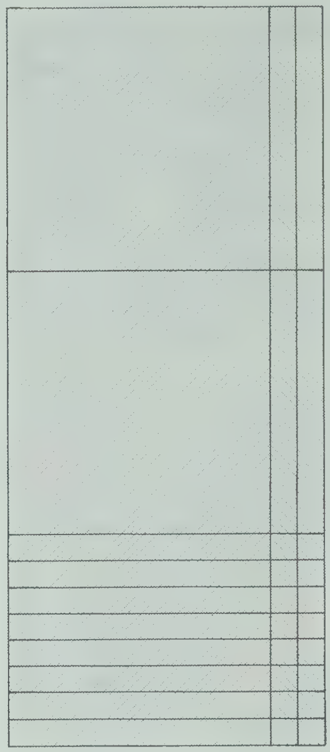
Write the decimal.

6. 3 ones 4 tenths 6 hundredths 3.46

7. 5 ones 0 tenths 8 hundredths 5.08

Multiplying Tenths

The top of the display table is 2.8 m long and 1.2 m wide. What is the area of the table top?



Multiply 1.2 and 2.8.

For the product

```
  2 . 8
  1 . 2
```

you need to know how to multiply 2 and 28,

```
  2 . 8
  1 . 2
  5   6
```

and how to multiply 1 and 28.

```
  2 . 8
  1 . 2
  5   6
2 8   0
```

Then add and place the decimal point.

```
  2 . 8
  1 . 2
  5   6
2 8   0
3 . 3 6
```

Remember, you can use either of these ideas to help you place the decimal point:

2.8 $\xrightarrow{\text{rounds to}}$ 3

1.2 $\xrightarrow{\text{rounds to}}$ 1

The product will be about 3.

2.8 has 1 decimal place.
1.2 has 1 decimal place.
There will be 2 decimal places in the product.

The area of the table top is 3.36 m².

138

LESSON ACTIVITY

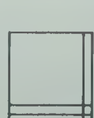
Before Using the Pages

- Prepare the students for the concept of this lesson in the following manner, using models for ones, tenths, and hundredths prepared from copies of pages T 392-T 394. Tell the students that the model for one represents a square having sides 1 m in length, and ask for the area (1 m²). Establish that a tenth strip (cut from a model for tenths) represents a rectangle with dimensions 1 m by 0.1 m and an area of 0.1 m²; similarly, a hundredth square (cut from a model for hundredths) represents a square with sides 0.1 m in length and an area of 0.01 m².

  Before the lesson, prepare diagrams similar to the following by pasting models on construction paper.

A  2 m² 0.3 m²

2.3 m²

B  1 m² 0.3 m² 0.02 m²

1.32 m²

Ask the students to find the area of each diagram by counting the whole square metres, tenths of a square metre, and hundredths of a square metre. Ask for another way to find the area of a rectangular region. (Multiply the length and the width.) Mark the dimensions on each shape and let the students attempt the multiplications. This can lead them to discover that the product of two one-place decimals, such as 1.1 and 1.2 for diagram B, must be a two-place decimal (1.32).

Using the Pages

- Ask a student to read the word problem. Note that the diagram representing the table top shows a length of 2 m and 8 tenths of a metre and a width of 1 m and 2 tenths of a metre. Ask the students to count the whole square metres (2), the tenths of a square metre (12), and the hundredths of a square metre (16) in the diagram. Point out that 10 hundredths can be regrouped as 1 more tenth, and 10 tenths can be regrouped as 1 more whole. Thus, the area is the sum of 3 m², 0.3 m², and 0.06 m², which is 3.36 m².

Working Together

Round the factors and multiply to estimate each product.

1. 9.8 × 2.4 = 20
2. 2.3 × 6.2 = 12
3. 5.7 × 32 = 180

How many decimal places are in each factor? How many will be in each product?

4. 11.2 (1) × 4.7 (1) → 2
5. 1.7 (1) × 0.9 (1) → 2
6. 7.2 (1) × 28 (0) → 1

Show the decimal point in each product.

7. 9.8 × 2.4 = 2352 — 23.52
8. 5.7 × 32 = 1824 — 182.4
9. 1.7 × 0.9 = 153 — 1.53

Multiply.

10. 2.3 × 6.2 = 14.26
11. 7.2 × 28 = 201.6
12. 1.7 × 0.9 = 1.53

Exercises

What is the area of this table top? 4.08 m²

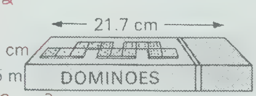
1.

Multiply.

2. 3.7 × 2.9 = 10.73
3. 40.3 × 5.6 = 225.68
4. 72 × 0.6 = 43.2
5. 28.4 × 49 = 1391.6
6. 4.4 × 4.8 21.12
7. 0.7 × 5.4 3.78
8. 1.3 × 7.1 9.23
9. 0.8 × 5.3 4.24
10. 66 × 3.5 231.0
11. 9.2 × 11.7 107.64

For this dominoes box, what is the area of

12. the top? 112.84 cm²
13. the front? 54.25 cm²
14. the end? 13.00 cm²
15. all the faces? 360.18 cm²



When a number is multiplied by 0.1, each digit moves one place to the right.

Example:

	tens	ones	tenths	hundredths		tens	ones	tenths	hundredths
0.1 ×	7	2	5		=		7	2	5

Multiply each number by 0.1. Write only the products.

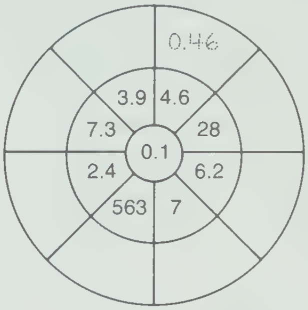
Do these in your head.

16. 39.5 3.95
17. 15.7 1.57
18. 3.8 0.38
19. 75 7.5
20. 0.9 0.09
21. 510 51

139

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 16-30 on page 331.
- For practice in multiplying by 0.1, use copies of page T391 to provide number wheels similar to the following.



- Have the students adapt models of ones, tenths, and hundredths (copies of pages T392-T394) to prepare diagrams similar to the one on page 138 for an appropriate multiplication exercise on page 139.
- Ask students to measure to the nearest tenth of a metre the dimensions of a rectangular shape such as the top of a desk. Then have them use multiplication to find the area of the rectangular shape.

Note that the product of the whole numbers 12 and 28 is 336. Review that the position of the decimal point for the product of 1.2 and 2.8 can be determined two ways as indicated in the "thought clouds" at the bottom of page 138. Summarize the three methods that indicate that the product of two one-place decimals is a two-place decimal: using a diagram; rounding factors to estimate the product; counting the decimal places in the two factors. For the third method, emphasize that the sum of the decimal places in the factors gives the number of decimal places in the product. Emphasize that multiplication is performed as for whole numbers.

Working Together: For Ex. 1-3, the students are asked to round the factors and multiply to estimate each product. Ex. 4-6 relate the number of decimal places in the product to the number of decimal places in the two factors. Ex. 7-9 provide practice in placing the decimal point in a product. Note that some of Ex. 1-12 are repeated, but the instructions are changed so that finding a product is developed in stages.

Exercises: For exercises such as Ex. 10, the products may be written without unnecessary zeros (231 rather than 231.0, for example). Remind the students that the answers for Ex. 12-15 require the symbol cm². Ex. 16-21 are of particular importance because they involve multiplication by 0.1. Note that extra zeros will be required for Ex. 18 and Ex. 20, and the answer for Ex. 21 may be shown simply as 51 rather than 51.0. The procedure for multiplying by 0.1 should be contrasted with the procedure for multiplying by 10 (see Ex. 17-22 on page 135) because they are inverse procedures.

Assessment

Multiply.

1. 9.4 × 3.8 = 35.72
2. 76.2 × 15 = 1143.0
3. 8.5 × 0.7 = 5.95
4. 4.3 × 4.9 21.07
5. 28 × 6.7 187.6
6. 0.1 × 9.3 0.93
7. 0.1 × 120 12

LESSON OUTCOME

Multiply a one-place, two-place, or three-place decimal by a one-place decimal or a two-place decimal

Prerequisite Skills

Multiply a three-digit number by a three-digit number; multiply a four-digit number by a two-digit number

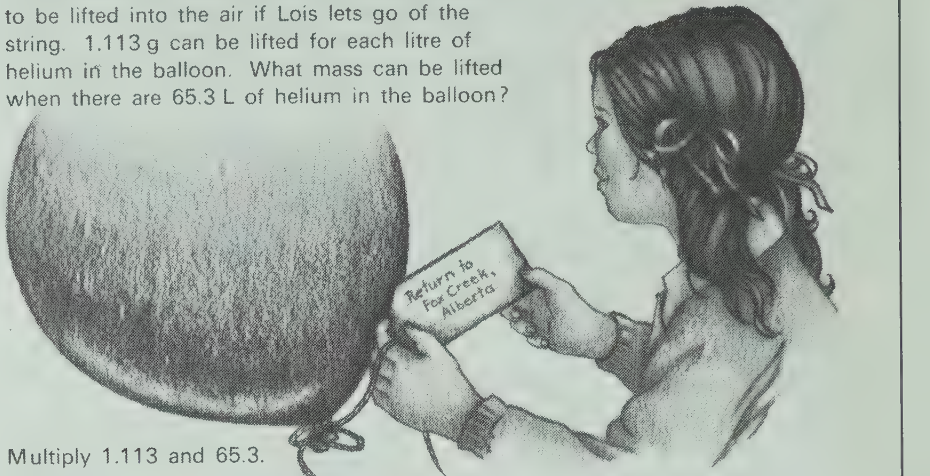
Checking Prerequisite Skills

Multiply.

1. 983 × 346 = 340 118
2. 1950 × 58 = 113 100
3. 27 × 7526 203 202
4. 418 × 907 379 126

Multiplying Hundredths and Thousandths

The helium in the balloon would cause the balloon to be lifted into the air if Lois lets go of the string. 1.113 g can be lifted for each litre of helium in the balloon. What mass can be lifted when there are 65.3 L of helium in the balloon?



Multiply 1.113 and 65.3.

Remember, you can multiply decimals like whole numbers, but then place a decimal point in the product.

For 1.113 × 65.3,

65.3 —rounds to→ 65

1.113 —rounds to→ 1

The product will be about 65.

For 1.113 × 65.3,

65.3 has 1 decimal place.

1.113 has 3 decimal places.

There will be 4 decimal places in the product.

```
    65.3
   1.113
   1 959
   6 530
  65 300
 653 000
 72.6789
```

The decimal point goes here.

A mass of 72.6789 g, or about 73 g, can be lifted.

140

LESSON ACTIVITY

Before Using the Pages

- Write the six digits 412305 on the board. Ask a student to place a decimal point so that a numeral with four decimal places will be shown. Ask another student to read the numeral. Ask students to tell the place value of each digit in the numeral. Ask a student to round the decimal to the nearest whole number (41) and to round that number to the nearest ten (40).
- Complete one or two exercises on the board with the students to review the work of the lesson on pages 138 and 139.

  12.3 × 2.1
  73 × 6.5

Using the Pages

- Begin with a discussion of the illustration. Have the students recall that a balloon filled with helium will rise because helium is lighter than air. This concept was encountered in the lesson on pages 88 and 89. The photograph on page 89 shows that large balloons are still in use. The one shown on page 89 was involved in a crossing of the Atlantic Ocean in 1978. The trip from Maine, U.S.A., to a point west of Paris, France, took six days to complete. Mention the fact that if a balloon is increased in size, more helium is required to inflate it, and thus it can lift a greater mass.

Ask a student to read the word problem. Discuss that a mass of 1.113 g can be lifted for 1 L of helium in the balloon, 2.226 g for 2 L, 3.339 g for 3 L, and so on. Establish that multiplication can be used to find the mass that can be lifted for 65.3 L of helium.

Review that the procedure for multiplying decimals is the same as for multiplying whole numbers, except that a decimal point must be placed in the product. Ask how the position of the decimal point can be determined. (Round each factor to estimate the product or count the decimal places in the two factors.) Have a student explain each method used for the worked example. Point out that the example shows that the product of tenths and thousandths is ten-thousandths.

Working Together

Round the factors and multiply to estimate each product.

1. 1.786 × 9.3 = 18
2. 4.21 × 3.83 = 16
3. 3.956 × 71.6 = 280

How many decimal places are in each factor? How many will there be in each product?

4. 46.09 (2) × 7.43 (2) → 4
5. 2.374 (3) × 2.6 (1) → 4
6. 25.5 (1) × 8.49 (2) → 3

Show the decimal point in each product.

7. 1.78 × 9.3 = 16 554 → 16.554
8. 2.374 × 2.6 = 61 724 → 6.1724

Multiply.

9. 4.21 × 3.83 = 16.1243
10. 25.5 × 8.49 = 216.495
11. 7.08 × 6.4 = 45.312
12. 37.4 × 89.2 = 3336.08

Exercises

Multiply.

1. 2.403 × 1.2 = 2.8836
2. 3.14 × 3.14 = 9.8596
3. 2.78 × 27.8 = 77.284
4. 6.27 × 4.5 = 28.215
5. 253.3 × 0.875 = 221.6375
6. 9.51 × 8.36 = 79.5036
7. 4.968 × 6.9 = 34.2792
8. 61.5 × 6.7 = 412.05
9. 0.98 × 9.05 = 8.8690
10. 5.88 × 4.23 — 24.8724
11. 2.6 × 15.23 — 39.598
12. 5.09 × 8.62 — 43.8758
13. 0.01 × 579.6 — 5.796
14. 22.9 × 0.73 — 16.717
15. 1.396 × 5.6 — 7.8176

When a number is multiplied by 0.01, each digit moves 2 places to the right. When it is multiplied by 0.001, each digit moves 3 places to the right.

Examples: 0.01 × 108.5 = 1.085
0.001 × 108.5 = 0.1085

Multiply each number by 0.1, 0.01, and 0.001. Write only the products. Do these in your head.

16. 348.7 17. 4762 18. 3.67
19. 29 20. 5.1 21. 0.7

Discuss the answers to these questions. Answers will vary.

1. Why does the balloon being blown up become larger fast at first and larger more slowly later?
2. How far does a balloon travel when you blow it up and let it go?
3. How heavy is a balloon filled with helium?
4. How high can a balloon rise?
5. Where do balloons go that escape into the air?

PROBLEM SOLVING

16 34.87, 3.487, 0.3487
17 476.2, 47.62, 4.762
18. 0.367, 0.0367, 0.00367
19 2.9, 0.29, 0.029
20. 0.51, 0.051, 0.0051
21 0.07, 0.007, 0.0007

141

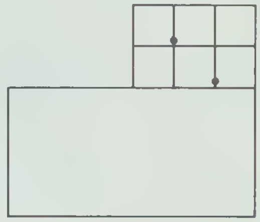
RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 31-61 on page 331 and Ex. 17-32 on page 334.
- For practice in multiplying by 0.1, 0.01, and 0.001, provide tables similar to the following on copies of page T 391.

×	2.38	4.1	9
0.1			

×	301	2.5	16
0.001			

- Adapt the rules of the game "Greatest Sum" on page T 379 for a game "Greatest Product", using charts similar to the following.



- Outline a place-value chart on a flannel board or a magnetic board. Represent a number by placing four digits (felt numerals or magnetic numerals) on the chart. Ask students to move the digits on the chart so that the number is multiplied by 10 (100, 1000, 0.1, 0.01, 0.001). This activity can be used to help develop the concept that if a number is multiplied by 10 (100, 1000) and the answer is multiplied by 0.1 (0.01, 0.001), the result is the original number.

Working Together: Both methods for placing the decimal point in a product are practiced in Ex. 1-6. The students can use either method to complete Ex. 7-12.

Exercises: For Ex. 5 and Ex. 9, it may be necessary to review that a factor such as 0.875 rounded to the nearest whole number is one. For Ex. 13, ask which of the two methods is preferable for determining the position of the decimal point and ask students to explain their choice. It is likely that most students will count the decimal places in the factors because 0.01 rounded to the nearest whole number is zero.

Note that multiplication by 0.01 and by 0.001 is given special attention in Ex. 16-21. Emphasize that the digits of a number move to positions of lesser place value when the number is multiplied by 0.1, 0.01, or 0.001.

Problem Solving: Provide an opportunity for the students to discuss these exercises.

Assessment

Multiply.

1. 1.57 × 5.9 = 9.263
2. 0.802 × 6.4 = 5.1328
3. 5.73 × 4.79 = 27.4467
4. 6.9 × 3.587 — 24.7503
5. 2.6 × 4.18 — 10.868

Pages 142-143

LESSON OUTCOME

Multiply two decimals, products less than one

Vocabulary

milligram, mg

Prerequisite Skills

Multiply a decimal with up to three decimal places by a decimal with up to two decimal places, products greater than one

Checking Prerequisite Skills

Multiply.

1. 1.85 × 6.2 = 11.470

2. 0.738 × 2.94 = 2.16972

3. 0.9 × 6.7 6.03

4. 4.8 × 3.086 14.8128

Two Decimal Factors, Products Less Than 1

The cheese pizza contains 0.32 mg (milligram) of vitamin B_1.

A milligram is one-thousandth of a gram.

Nick will eat three-tenths (0.3) of the pizza. How many milligrams of vitamin B_1 will Nick get from the pizza?

Multiply 0.3 and 0.32.

For 0.3 × 0.32,

0.32 has 2 decimal places.
0.3 has 1 decimal place.

There will be 3 decimal places in the product.

$$\begin{array}{r} 0.32 \\ \underline{0.3} \\ 0.096 \end{array}$$

The decimal point goes here.

Nick will get 0.096 mg of vitamin B_1 from the pizza.

142

LESSON ACTIVITY

Before Using the Pages

- Write five numerals similar to the following on the board.

  14 235 6978 205 34 8

  For each numeral, have students count the digits from right to left, to determine where to place the decimal point so that the numeral shows a one-place decimal. Note that a zero will be required to express 8 as 0.8. Ask students to read the decimals. Repeat the procedure so that each numeral shows a two-place decimal. Continue the procedure to show three and four decimal places. Pay particular attention to the use of more zeros in a numeral; for example, to express 34 as a four-place decimal it is necessary to write 0.0034.

Using the Pages

- Encourage the students to comment on the photograph. Note that the pizza is cut into ten equal parts. Ask students to read the statements above the photograph. Draw attention to the word *milligram* and the symbol mg, and ask why multiplication is used to find the solution. Note that the decimal factors suggest the whole numbers 3 and 32. Ask why rounding the factors to the nearest whole number does not show where the decimal point should be placed in the product. Develop that it is necessary to count the decimal places in the two factors to determine where to place the decimal point in the product.

Working Together: Ex. 1-3 guide the students as they determine the number of decimal places in each product. Ex. 4-6 present the same multiplications with the factors multiplied as whole numbers. The students are required to place the decimal point in each product. The preceding skills are applied in completing Ex. 7-9. Pay particular attention to the use of more zeros to show the required number of decimal places in a product, as in 0.08 × 0.12 = 0.0096 for Ex. 7.

Exercises: Ex. 14-16 provide practice in multiplying by 0.1, 0.01, and 0.001.

Working Together

How many decimal places are in each factor? How many will there be in each product?

1. 0.138 3
   0.7 1
   4
2. 0.18 2
   0.05 2
   4
3. 0.02 2
   0.4 1
   3

Show the decimal point in each product.

4. 0.138
   0.7
   966
   0.0966
5. 0.18
   0.05
   90
   0.0090
6. 0.02
   0.4
   8
   0.008

Multiply.

7. 0.12
   0.08
   0.0096
8. 0.79
   0.8
   0.632
9. 0.2
   0.4
   0.08

Exercises

Multiply.

1. 0.14
   0.7
   0.098
2. 0.64
   0.32
   0.2048
3. 0.075
   0.1
   0.0075
4. 0.794
   0.6
   0.4764
5. 0.3
   0.2
   0.06
6. 0.019
   0.3
   0.0057
7. 0.88
   0.8
   0.704
8. 0.02
   0.03
   0.0006
9. 0.18
   0.09
   0.0162

10. 0.04 × 0.2 0.008
11. 0.5 × 0.7 0.35
12. 0.27 × 0.03 0.0081
13. 0.2 × 0.398 0.0796

Multiply each number by 0.1, 0.01, and 0.001. Write only the products.

Do these in your head.

14. 1.7 0.17, 0.017, 0.0017
15. 0.6 0.06, 0.006, 0.0006
16. 53 5.3, 0.53, 0.053

Study these.	Then complete each of these.
A. 0 . 4 0 . 2 0 . 0 8	1. 4 tenths 2 tenths 8 ? 8 hundredths
B. 0 . 0 8 0 . 7 0 . 0 5 6	2. 8 hundredths 7 tenths 56 ? thousandths
C. 0 . 3 0 . 0 0 6 0 . 0 0 1 8	3. 3 tenths 6 thousandths 18 ? ten-thousandths

For multiplying, in general,

4. tenths
   tenths
   ? hundredths
5. hundredths
   tenths
   ? thousandths
6. tenths
   thousandths
   ? ten-thousandths
7. hundredths
   thousandths
   ? hundred-thousandths

Find the product of each without using paper or pencil.

8. 0.3
   0.3
   0.09
9. 0.9
   0.6
   0.54
10. 0.04
    0.8
    0.032
11. 0.02
    0.5
    0.010
12. 0.008
    0.4
    0.0032
13. 0.006
    0.9
    0.0054
14. 0.005
    0.03
    0.00015
15. 0.007
    0.08
    0.00056

try this

143

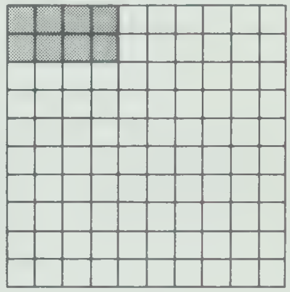
RELATED ACTIVITIES

• For further practice, you may wish to have the students complete related exercises from Ex. 62-73 on page 331 and others from Ex. 49-64 and Ex. 81-96 on page 334.

• To help the students understand multiplications such as 0.2 × 0.4, provide them with copies of the hundredths model on page T 394 to represent metre squares. Review that each side of the square represents a length of 1 m marked into tenths of a metre. Each small square represents an area of 0.01 m². Have them outline a rectangular region having dimensions of 0.2 m and 0.4 m, noting that the area of the region represented is 0.08 m².

0.4 m

0.2 m



• Adapt the game "Product Search" on page T 380 for decimals by marking 0.1, 0.2, 0.6, 0.7, 0.8, and 0.9 on one die, and 0.3, 0.4, 0.5, 0.06, 0.07, and 0.09 on the other die. Write the decimals shown on the first die in the top row of the game board and the decimals shown on the second die in the left column of the game board.

Try This: Before the students begin, have them compare these exercises with those of *Keeping Sharp* on page 137. Similarities between corresponding exercises can be pointed out. For example, Ex. 1 on each page involves the basic multiplication fact 2 × 4 = 8; also, the whole number place value, tens, on page 137, is replaced by the corresponding decimal place value, tenths, on page 143. When the students have completed these exercises, it is important to make similar comparisons again with exercises on page 137. For example, the product for Ex. 2 on each page relates thousands and thousandths, but the numbers of zeros in the products (Ex. B) are different.

Assessment

Multiply.

1. 0.07
   0.6
   0.042
2. 0.3
   0.8
   0.24
3. 0.94
   0.04
   0.0376

4. 0.5 × 0.517 0.2585
5. 0.08 × 0.09 0.0072
6. 0.2 × 0.01 0.002
7. 1.3 × 0.001 0.0013

Pages 144-145

OBJECTIVE

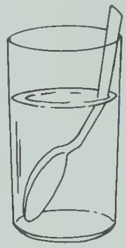
Demonstrate competence in multiplying decimals

Materials

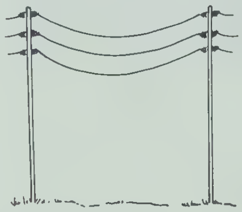
water, a transparent drinking glass, and objects such as spoons and drinking straws (optional)

Practice

Science Fact:
If a volume of gas is lighter than the same volume of air, the gas rises.



Science Fact:
Light rays seem to bend when they pass from water to air.



Science Fact:
Metal expands as the temperature increases.

Solve.

1. 1.113 g can be lifted for each litre of helium in a balloon. What mass can be lifted by 163.4 L of helium? 181.8642g
2. 1.203 g can be lifted for each litre of hydrogen in a balloon. What mass can be lifted by 329.7 L of hydrogen? 396.6291g
3. Can more be lifted with 108 L of helium, or with 100 L of hydrogen? 100 L of hydrogen

In water an object is 1.33 times deeper than it appears to be. How deep is an object that appears to be

4. 0.3 m deep? 0.399m
5. 1.8 m deep? 2.394m

Solve.

6. 1000 m of a steel wire will expand 0.012 m for a 1°C increase in temperature. How much will 1000 m of this wire expand for an increase of 10°C? 0.12m of 0.5°C? 0.006m of 19.7°C? 0.2364m

*7. 1 km of aluminum wire will expand 0.026 m for a 1°C increase in temperature. For a 1°C increase in temperature, what will the length become for 1.5 km of wire? 1500.039m for 700 m of wire? 700.0182m

Each student bought a Science Fact trading card at the store. Each card cost the same amount. The total cost for the group was $2.03.

1. How many students were there? 7 or 29

PROBLEM SOLVING

2. How much did each Science Fact card cost? 29¢ or 7¢

144

LESSON ACTIVITY

Using the Pages

- Begin with a discussion of the Science Fact cards shown on page 144. The first card may remind the students of similar examples in the lessons on pages 88, 89, and 140. (A balloon filled with a gas that is lighter than air rises.) For the second card, students may suggest that they have noticed that an object, for example, a drinking straw, appears to be bent when it is partially immersed in water. It would be helpful to demonstrate the concept for the students by using a transparent drinking glass, water, and such objects as drinking straws and spoons. For the third card, explain that metal expands as the temperature rises, and thus telephone wires increase in length when the weather becomes warmer and decrease in length as the weather becomes cooler. A similar example may be described for the metal rails of a railroad track. Point out that the exercises on page 144 involve the science facts shown on the cards. Ex. 7 is starred because the solution involves more than one step and because different units of length are involved (kilometres and metres).

The example at the top of page 145 demonstrates that some products must be rounded to the nearest cent (hundredth of a dollar) since amounts of money for dollars-and-cents notation are written as two-place decimals. For instance, to buy 1.25 kg of fruit that costs $1.57 for 1 kg, the product $1.9625 is rounded to $1.96. Ask the students what digits in the third decimal place would result in rounding the product to $1.97. Ex. 8-11 involve similar examples.

The product of two whole numbers can be checked by reversing the order of the factors and multiplying. This procedure also applies for decimal factors as shown in the example to the right of Ex. 12-17. You may wish to discuss the example before the students begin the exercises.

Students encountered Magic Squares previously on page 103. Note that diagrams A and B are identified as Magic Squares because the sums of the numbers for any row, column, or diagonal are equal. Thus, there is no need to

When an amount of money and a decimal are multiplied, the product is usually rounded to the nearest cent.

Example:

$$\begin{array}{r} \$1.57 \\ 1.25 \\ \hline \$1.9625 \end{array} \xrightarrow{\text{rounds to}} \$1.96$$

Multiply and round to complete these supermarket price tags.

8. Spareribs

Price per kilogram	Total price	Net mass in kilograms
$3.70	$6.07	1.64

9. Chicken legs

Price per kilogram	Total price	Net mass in kilograms
$3.89	$5.17	1.33

10. Pork chops

Price per kilogram	Total price	Net mass in kilograms
$5.39	$9 54	1.77

11. Lamb roast

Price per kilogram	Total price	Net mass in kilograms
$2.65	$5 59	2.11

You can change the order of the factors to check multiplication.

Multiply and check your work.

Example:

$$\begin{array}{r} 1.38 \\ 21.6 \\ \hline 828 \\ 1\,380 \\ 27\,600 \\ \hline 29.808 \end{array} \qquad \begin{array}{r} 21.6 \\ 1.38 \\ \hline 1\,728 \\ 6\,480 \\ 21\,600 \\ \hline 29.808 \end{array}$$

If these products do not match, there is a mistake.

12. 2.15 × 49.4 = 106.210

13. 7.2 × 9.3 = 66.96

14. 89.5 × 0.64 = 57.280

15. 42.8 × 355 = 15 194.0

16. 0.18 × 0.27 = 0.0486

17. 362 × 3.67 = 1328 54

In a Magic Square, the sums of the numbers for any row, column, or diagonal are equal.

A

1.4	4.9	4.2
6.3	3.5	0.7
2.8	2.1	5.6

B

1.1	1.2	0.7
0.6	1.0	1.4
1.3	0.8	0.9

18. For Magic Square A, multiply each number by 1.06. Do the products form a Magic Square? yes

19. For Magic Square B, multiply each number by 0.27. Then add 2.859. Do the numbers that result form a Magic Square? yes

20. Add the two numbers in the matching positions of your two new squares. Do the numbers that result form a Magic Square? yes

Magic Squares for Ex. 18–20 are shown below

145

RELATED ACTIVITIES

- Provide a work sheet with multiplications such as 64 × 738 = 47.232, showing a decimal point in the product but not in each factor. Have the students rewrite the multiplications showing a decimal point in one or both factors in as many ways as possible to obtain the given products. For example, the previous sentence can be rewritten as follows.

  64 × 0.738 = 47.232
  6.4 × 7.38 = 47.232
  0.64 × 73.8 = 47.232
  0.064 × 738 = 47.232

  If you wish, adapt exercises for whole numbers from pages 56 to 60 to prepare the work sheet.
- Ask the students to bring to school price tags from packages of meat, fruits, vegetables, and so on, used in the home. The tags should indicate the price per kilogram, the mass of the food, and the price paid. These tags may be used for self-checking exercises in multiplication. For example, students can multiply the number of kilograms and the cost per kilogram, round the product to the nearest cent, and check their answer with the price indicated on the tag.
- Have the students form new Magic Squares by the use of multiplication and/or addition for the Magic Squares on page 103. Some students may also wish to test the use of division and/or subtraction.

check these sums, but it is advisable to have students tell the sum related to each Magic Square by performing at least two additions for that square. Ex. 18 and Ex. 19 will enable students to discover that multiplying each number of a Magic Square by the same number, and/or increasing it by the same amount, gives another Magic Square. Also, by adding numbers in corresponding squares of two Magic Squares, another Magic Square is obtained (Ex. 20).

Problem Solving: Students will likely use a "guess and test" strategy to solve this problem. Emphasize that each student in the group bought one Science Fact card; thus, the number of cards bought is the same as the number of students. Also, it is stated that each card cost the same amount; this implies that fractions of a cent are not involved. For example, for a group of 5 students a cost of 40.6¢ for each student is rejected as a possible solution. Note that the two possible answers involve the factors 7 and 29 for the number 203.

18.

1.484	5.194	4.452
6.678	3.710	0.742
2.968	2.226	5.936

19.

3.156	3.183	3.048
3.021	3.129	3.237
3.210	3.075	3.102

20.

4.640	8.377	7.500
9.699	6.839	3.979
6.178	5.301	9.038

Pages 146-147

LESSON OUTCOME

Divide a decimal by a one-digit number, using extra zeros in the dividend; solve related word problems

Materials

models for ones, tenths, and hundredths made from copies of pages T 393 and T 394 as described on page T 89

Prerequisite Skills

Divide a four-digit number by a one-digit number; use extra zeros for showing equivalent decimals

Checking Prerequisite Skills

Divide.

1. 8)2616 327
2. 9)6246 694
3. 4)232 58
4. 6)4230 705

Write each as a four-place decimal.

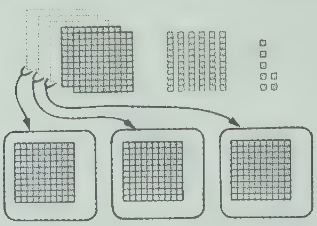
5. 9.768 9.7680
6. 4.13 4.1300

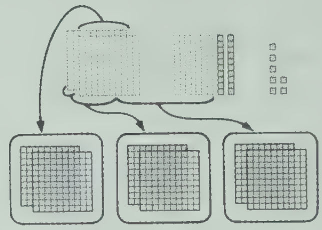
Write each as a three-place decimal.

7. 5.2 5.200
8. 0.47 0.470

Dividing by a One-Digit Number

Divide 5.67 by 3.





For 3)5.67, divide the 5 ones first.

3 × 1 = 3
3 × 2 = 6 . . . too great!

Use 3 × 1 = 3.

```
           1
Write   3 )5.67
           3
           2
```

Think of the 2 ones 6 tenths that remain as 26 tenths.

```
  1
3)5. 6 7
  3 ↓
  2 6
```

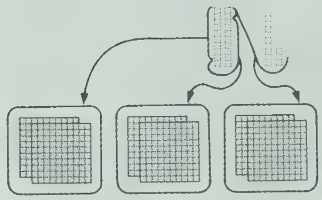
Then divide the 26 tenths.

3 × 8 = 24
3 × 9 = 27 . . . too great!

Use 3 × 8 tenths = 24 tenths.

Place a decimal point in the quotient.

```
          1. 8
Write  3 )5. 6 7
          3
          2 6
          2 4
            2
```



Think of the 2 tenths 7 hundredths that remain as 27 hundredths.

```
   1.8
 3)5.6 7
   3
   2 6
   2 4↓
     2 7
```

Then divide the 27 hundredths.

3 × 9 = 27

Use 3 × 9 hundredths = 27 hundredths.

```
          1.8 9
Write  3 )5.6 7
          3
          2 6
          2 4
            2 7
            2 7
              0
```

5.67 divided by 3 equals 1.89.

146

LESSON ACTIVITY

Before Using the Pages

- Students who can divide a whole number by a whole number less than ten have little difficulty applying the procedure to similar exercises with decimal dividends. However, it is advisable to review that division is related to sharing, in order that students may renew their understanding of the concept as it relates to decimal dividends. For example, have the students recall that 4)84 (quotient 21) can be demonstrated by sharing 8 tens and then 4 ones equally among 4. Ask what interpretation can be given to divisions such as 4)8.4, 4)0.84, and 4)0.084, demonstrating one or more by sharing the appropriate models. Then discuss examples such as 4)5.2 which require regrouping. Emphasize the left-to-right order of dividing (sharing) the dividend place by place.

Using the Pages

- The worked example demonstrates division of decimal hundredths with two regroupings. Have students help to explain the steps of the division, considering carefully the basic multiplication facts that help to derive digits of the quotient. Associate the procedure with the sharing and the regrouping of models as indicated in the diagrams. Emphasize that the decimal point is placed in the quotient above the decimal point in the dividend. Note that decimal points are not shown in the work below the dividend, but place values are aligned as indicated by the arrows. Ask the students to check the accuracy of the division by using multiplication.

Working Together: Ex. 1-3 provide partially completed examples. Grey shapes guide the students by indicating the positions in the algorithm for the digits and the number of digits in each position.

Working Together

Complete.

1. $\begin{array}{r} 0.14 \\ 7\overline{)0.98} \\ \underline{7} \\ 28 \\ \underline{28} \\ 0 \end{array}$

2. $\begin{array}{r} 2.85 \\ 8\overline{)22.80} \\ \underline{16} \\ 68 \\ \underline{64} \\ 40 \\ \underline{40} \\ 0 \end{array}$

3. $\begin{array}{r} 0.307 \\ 9\overline{)2.763} \\ \underline{27} \\ 063 \\ \underline{63} \\ 0 \end{array}$

For this division,

4. what is the place value of each 5?

$\begin{array}{r} \text{ones } 5.375 \text{ thousandths} \\ 4\overline{)21.500} \text{ tenths} \\ \underline{20} \\ 1\,5 \text{ tenths} \\ \underline{1\,2} \\ 30 \\ \underline{28} \\ 20 \\ \underline{20} \\ 0 \end{array}$

An extra zero can be used in the dividend to help divide decimals.

Divide.

5. $\begin{array}{r} 1.75 \\ 5\overline{)8.75} \end{array}$ 6. $\begin{array}{r} 0.92 \\ 7\overline{)6.44} \end{array}$ 7. $\begin{array}{r} 4.062 \\ 4\overline{)16.248} \end{array}$ 8. $\begin{array}{r} 7.35 \\ 6\overline{)44.1} \end{array}$ 9. $\begin{array}{r} 0.305 \\ 8\overline{)2.44} \end{array}$

Exercises

Divide.

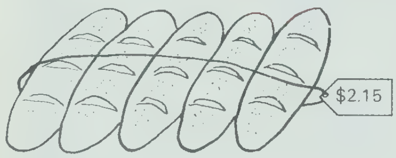
1. $\begin{array}{r} 0.6 \\ 3\overline{)1.8} \end{array}$ 2. $\begin{array}{r} 4.75 \\ 2\overline{)9.5} \end{array}$ 3. $\begin{array}{r} 0.9 \\ 8\overline{)7.2} \end{array}$ 4. $\begin{array}{r} 7.08 \\ 6\overline{)42.48} \end{array}$ 5. $\begin{array}{r} 4.02 \\ 3\overline{)12.06} \end{array}$

6. $\begin{array}{r} 0.11 \\ 7\overline{)0.77} \end{array}$ 7. $\begin{array}{r} 1.48 \\ 8\overline{)11.84} \end{array}$ 8. $\begin{array}{r} 0.025 \\ 4\overline{)0.1} \end{array}$ 9. $\begin{array}{r} 2.92 \\ 9\overline{)26.28} \end{array}$ 10. $\begin{array}{r} 1.02 \\ 5\overline{)5.1} \end{array}$

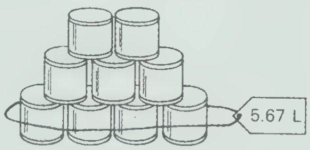
11. $\begin{array}{r} 15.75 \\ 2\overline{)31.5} \end{array}$ 12. $\begin{array}{r} 0.08 \\ 6\overline{)0.48} \end{array}$ 13. $\begin{array}{r} 0.071 \\ 7\overline{)0.497} \end{array}$ 14. $\begin{array}{r} 0.375 \\ 8\overline{)3.0} \end{array}$ 15. $\begin{array}{r} 2.09 \\ 9\overline{)18.81} \end{array}$

Solve.

16. What is the cost of each loaf? $0.43



17. How much syrup is in each can? 0.63 L



147

RELATED ACTIVITIES

- For several exercises on page 147, especially those for which extra zeros were used in the dividend, ask the students to check their work by multiplying the divisor and the quotient.
- Prepare a work sheet with exercises similar to the following to help students relate division of whole numbers and division of decimals.

> Complete the first division. Use your answer to write the quotients for the other divisions.
>
> $3\overline{)46.17}$ $3\overline{)416.7}$
>
> $3\overline{)4167}$ $3\overline{)4.167}$

- Students having difficulty with dividing decimals may benefit from using models to show some of the exercises on page 147.
- Have the students create and solve word problems similar to Ex. 16 and Ex. 17 on page 147.
- Provide a work sheet with exercises similar to the following. Have the students ring the numeral in each row that does not name the same number as the others in the row.

1. 4.7, 4.700, 4.070, 4.7000, 4.70
2. 0.20, 0.02, 0.2, 0.2000, 0.200
3. 6.0, 6.000, 6, 6.00, 0.6
4. 3.1200, 3.12, 3.120, 3.102

Review that zero is recorded in the ones' place of the quotient for Ex. 1 because the quotient is less than one. Ex. 2 and Ex. 4 introduce the need for extra zeros in the dividend. For the exercises in this lesson, the division process should continue with extra zeros in the dividend as needed, until the remainder is zero. Pay particular attention to recording 0 ones in the quotient for Ex. 6 and Ex. 9 and to the use of extra zeros in the dividends for Ex. 8 and Ex. 9. Ask students to use multiplication to check several quotients.

Exercises: Some students may need assistance with exercises that involve zeros in the quotient, as in Ex. 4 and Ex. 5. For example, for Ex. 4, a student may write $\begin{array}{r} 7.8 \\ 6\overline{)42.48} \end{array}$ rather than $\begin{array}{r} 7.08 \\ 6\overline{)42.48} \end{array}$.

Assessment

Divide.

1. $\begin{array}{r} 3.58 \\ 7\overline{)25.06} \end{array}$ 2. $\begin{array}{r} 7.46 \\ 5\overline{)37.3} \end{array}$ 3. $\begin{array}{r} 9.055 \\ 6\overline{)54.33} \end{array}$

4. $\begin{array}{r} 3.315 \\ 4\overline{)13.26} \end{array}$ 5. $\begin{array}{r} 5.68 \\ 9\overline{)51.12} \end{array}$ 6. $\begin{array}{r} 1.1375 \\ 8\overline{)9.1} \end{array}$

Solve.

7. If 8 cans of tomato soup cost $3.04, what is the cost of each can of soup? $0.38

Pages 148-149

LESSON OUTCOME

Divide a decimal by a whole number having two or three digits, using extra zeros in the dividend

Prerequisite Skills

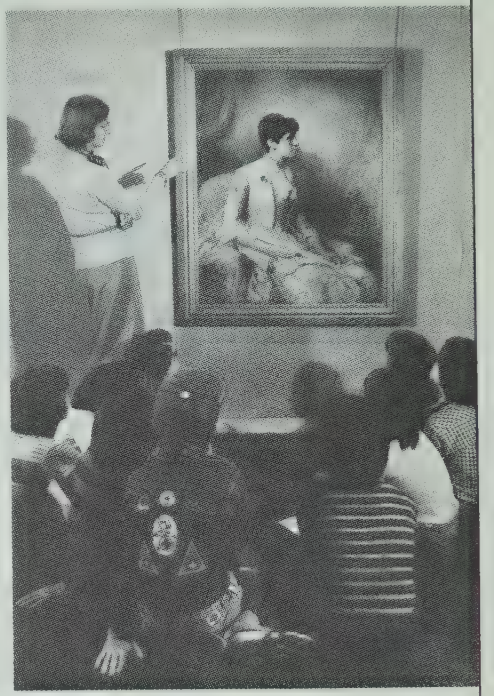
Divide a whole number by a whole number having two or three digits; divide a decimal by a one-digit number, using extra zeros in the dividend

Checking Prerequisite Skills

Divide.

1. 28)18 060 — 645
2. 93)47 151 — 507
3. 678)28 476 — 42
4. 504)32 760 — 65
5. 7)47.67 — 6.81
6. 5)146.64 — 29.328
7. 8)72.9 — 9.1125
8. 9)18.72 — 2.08

Dividing by a Two-Digit or a Three-Digit Number



Museum tickets for 31 students cost \$23.25. How much did each ticket cost?

Divide 23.25 by 31.

Since 31 is greater than 23, the first digit in the quotient is 0.

Write 31)23.25 — 0

Think of the 23 ones 2 tenths as 232 tenths.
Then divide the 232 tenths.

31 rounded to the nearest ten is 30.

Think of 30)23.25 — 0

30 × 7 = 210
30 × 8 = 240 . . . too great!

Use 31 × 7 tenths = 217 tenths.

```
       0. 7
Write  31 )23. 25
           21 7
              1 5
```

Place the decimal point.

Think of the 15 tenths 5 hundredths that remain as 155 hundredths.

```
     0.7
31 )23.2 5
    21 7 ↓
       1 5 5
```

Then divide the 155 hundredths.

30 × 5 = 150
30 × 6 = 180
. . . too great!

Use 31 × 5 hundredths
= 155 hundredths.

Write

```
     0.7 5
31 )23.2 5
    21 7
       1 5 5
       1 5 5
           0
```

Each ticket cost \$0.75.

148

LESSON ACTIVITY

Before Using the Pages

- Write the division 42)6384 on the board. Ask a student to tell the place value of the first digit in the quotient and to explain how this is known. Complete the division on the board with the students, reviewing that digits in the quotient can be estimated by using a divisor of 40 (42 rounded to the nearest ten). Write statements to explain the steps, as indicated below.

```
    152
 42)6384
    42
    218
    210
     84
     84
      0
```

1. Divide 63 hundreds.
   42 × 1 hundred = 42 hundreds.
2. Divide 218 tens.
   42 × 5 tens = 210 tens.
3. Divide 84 ones.
   42 × 2 ones = 84 ones.

Mark a decimal point between the 3 and the 8 of the dividend and ask what number is represented by the dividend as a result of this change. Ask how the quotient will change. The students will likely suggest that a decimal point is required between the 1 and the 5. Indicate that this is acceptable if the steps of the explanation can be altered to justify the quotient for the new dividend. For each underlined word in turn, erase the word and ask a student to write another word to describe the corresponding step of the new division. For example, hundreds would be replaced by ones. The completed division is shown below. Ask the students to use multiplication to check the quotient. The product of 42 and 1.52 should equal the dividend 63.84.

```
    1.52
 42)63.84
    42
    21 8
    21 0
       84
       84
        0
```

1. Divide 63 ones.
   42 × 1 one = 42 ones.
2. Divide 218 tenths.
   42 × 5 tenths = 210 tenths.
3. Divide 84 hundredths.
   42 × 2 hundredths = 84 hundredths.

Working Together

To divide decimals correctly, you should know

1. how to divide whole numbers.
2. what to do with zeros in the quotient.
3. how to use extra zeros in the dividend.

374	3.74 ?	0.62	4.08	1.375
21)7854	21)78.54	39)24.18	62)252.96	108)148.5

The quotient is less than 1.   There is a 0 here.

4. how to work with rounded divisors.

8.6	5.87	4.2
79)679.4	43)252.41	285)1197.0
Try using 80.	Try using 40.	Try using 300.

Divide.

5. 58)190.24 3.28
6. 377)867.477 2.301
7. 319)242.44 0.76
8. 67)34.84 0.52

Exercises

Divide.

1. 41)108.24 2.64
2. 19)4.75 0.25
3. 24)49.44 2.06
4. 94)63.45 0.675
5. 86)262.3 3.05
6. 29)2.349 0.081
7. 36)60.444 1.679
8. 426)391.92 0.92
9. 371)779.1 2.1
10. 10)557.7 55.77
11. 817)849.68 1.04
12. 245)9.065 0.037
13. 456)1909.5 4.1875
14. 517)4239.4 8.2
15. 792)11.088 0.014
16. 475)1463 3.08

When a number is divided by 10,
each digit moves one place to the right.

Example:

tens	ones	tenths	hundredths		tens	ones	tenths	hundredths
5	2	3		÷ 10 =		5	2	3

Divide each number by 10.
Write only the quotients.   Do these in your head.

17. 29.3 2.93
18. 3.64 0.364
19. 275 27.5
20. 0.8 0.08
21. 75.375 7.5375
22. 0.25 0.025

149

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 1-20 on page 332 and Ex. 1-16 on page 334.
- Students having difficulty with place value and division with decimals may benefit from completing division exercises as shown.

	ones	tenths	hundredths
	0.	0	6
42)	2.	5	2
	2	5	2
			0

- Students may enjoy completing the following pairs of division exercises and multiplying the quotients obtained as indicated in the following example.

$$\begin{array}{r}2.5\\2\overline{)5.0}\end{array}\qquad\begin{array}{r}0.4\\5\overline{)2.0}\end{array}\qquad\begin{array}{r}2.5\\\times\ 0.4\\\hline 1.00\end{array}$$

1. $2\overline{)5}$, $5\overline{)2}$
2. $3\overline{)12}$, $12\overline{)3}$
3. $7\overline{)14}$, $14\overline{)7}$
4. $8\overline{)5}$, $5\overline{)8}$
5. $4\overline{)5}$, $5\overline{)4}$
6. $1\overline{)16}$, $16\overline{)1}$

Using the Pages

- Read the word problem and establish that division is used to find the solution.

  Discuss each step for the division and direct the students' attention to the numerals highlighted in blue. Point out that zero is recorded in the ones' place because the quotient is a decimal less than one. Point out that, as for whole numbers, the rounded divisor is used only to estimate the quotient; the actual divisor is used to complete the multiplication steps in the division.

Working Together: Ex. 1-4 emphasize that the procedure for dividing a decimal is the same as for dividing a whole number. However, it is necessary at times to use more zeros in a decimal dividend to terminate the quotient. A thorough discussion of these exercises will help to reveal which sub-skills require more attention. You may wish to complete Ex. 1-4 on the board with the students. Alternatively, you may prefer to have them work individually and then ask a few students to explain their work on the board. Then, as the students complete Ex. 5-8, pay particular attention as each of the situations shown for Ex. 1-4 arises.

Exercises: For Ex. 16, explain that extra zeros can be used in the dividend to complete the division. A decimal point should be placed between the ones' place and the tenths' place of the dividend and the quotient. Ex. 17-22 are of particular importance because they involve dividing by ten. The example above these exercises shows that when a number is divided by ten, each digit of that number moves to a lesser place value, that is, one place to the right. When they have completed these exercises, have the students turn to page 139 and recall the effect of multiplying by 0.1 in Ex. 16-21. They should note that multiplying a number by 0.1 gives the same result as dividing the number by 10.

Assessment

Divide.

1. 48)147.6 3.075
2. 63)81.207 1.289
3. 815)409.13 0.502
4. 392)1117.2 2.85
5. 721)605.64 0.84
6. 165)253.44 1.536

Pages 150-151

LESSON OUTCOME

Round quotients to the nearest tenth, to the nearest hundredth, and to the nearest thousandth; solve related word problems

Prerequisite Skills

Divide a decimal by a whole number having up to three digits, using extra zeros in the dividend

Checking Prerequisite Skills

Divide.

1. 38)93.1 2.45
2. 6)30.756 5.126
3. 75)178.5 2.38
4. 116)14.616 0.126

Rounding Quotients

In the passenger cabin of this airplane, 12 seats, one behind the other, take up a length of 9.75 m. What length, to the nearest hundredth of a metre, does this allow for each seat?

Divide 9.75 by 12. Round the quotient to the nearest hundredth.

To round the quotient to the second decimal place, you should divide to three decimal places.

$$\begin{array}{r} 0.812 \\ 12\overline{)9.750} \\ \underline{9\,6} \\ 15 \\ \underline{12} \\ 30 \\ \underline{24} \\ 6 \end{array}$$

0.812 rounds to 0.81.

There is 0.81 m for each seat.

Working Together

Round to the

1. nearest tenth. 6.32 6.3
2. nearest hundredth. 15.827 15.83
3. nearest thousandth. 8.2785 8.279

To how many decimal places should you divide so that you can round the quotient to the

4. nearest tenth? 2 23)7.45
5. nearest hundredth? 3 7)18.17
6. nearest thousandth? 4 16)33.7

Divide. Round the quotient to the

7. nearest tenth. 23)7.45 0.3
8. nearest hundredth. 7)18.17 2.60
9. nearest thousandth. 16)33.7 2.106

150

LESSON ACTIVITY

Before Using the Pages

- Review the procedure of rounding a number to a given place: it is necessary to consider only the digit to the right of the given place. In other words, to round 0.348☐ to the nearest hundredth, it is not necessary to know what digit appears in the ten-thousandths' place.

Using the Pages

- The photograph can motivate a discussion about air travel. Direct the discussion towards the seating arrangements for certain planes and introduce the word problem. Emphasize that the length is to be determined to the nearest hundredth of a metre. Establish that the division must be performed to the thousandths' place, or to three decimal places, so that the quotient can be rounded to the nearest hundredth. Point out the need for one extra zero in the dividend. In this example, the digit in the thousandths' place of the quotient indicates that the hundredths' digit remains the same. In other words, 0.812 is rounded down to 0.81, and there is 0.81 m for each seat. However, if the digit in the thousandths' place had been 5, 6, 7, 8, or 9, the digit in the hundredths' place would have been rounded up. The result would have been a quotient for which the digit in the hundredths' place was one greater than if the division had been terminated at the hundredths' place.

Working Together: Ex. 1-3 review rounding decimals. You may wish to ask the students to round the numbers in Ex. 2 and Ex. 3 to the nearest tenth, and Ex. 3 to the nearest hundredth. Ex. 4-6 draw the students' attention to the need to continue the division one place past the place to which the quotient is to be rounded. The divisions for Ex. 7-9 are the same as for Ex. 4-6 in order to emphasize the steps involved in rounding quotients.

Exercises: Give special attention to Ex. 32-37 which review division by 10 and present division by 100 and by 1000. When they have completed these exercises, have the students turn to page 141 and recall how to multiply by 0.1, 0.01, and 0.001 in Ex. 16-21. They should note that

Exercises

Divide. Round each quotient to the nearest tenth.

1. 3)21.4 7.1
2. 7)51.6 7.4
3. 8)92.9 11.6
4. 3)0.8 0.3
5. 8)0.7 0.1
6. 12)28.92 2.4
7. 22)60.72 2.8
8. 100)185.3 1.9
9. 212)576 2.7
10. 39)27.2 0.7

Divide. Round each quotient to the nearest hundredth.

11. 7)2.65 0.38
12. 8)16.38 2.05
13. 5)34.08 6.82
14. 7)62.8 8.97
15. 3)82.4 27.47
16. 14)91.9 6.56
17. 19)3.5 0.18
18. 82)347.6 4.24
19. 198)376 1.90
20. 534)100 0.19

Divide. Round each quotient to the nearest thousandth.

21. 3)12.8 4.267
22. 6)27.2 4.533
23. 9)4.25 0.472
24. 7)156.4 22.343
25. 7)0.6 0.086
26. 15)450.22 30.015
27. 32)16.23 0.507
28. 78)289.6 3.713
29. 124)50 0.403
30. 75)200 2.667

Solve.

31. On a DC-9 airplane, passengers are seated 5 abreast. The seating space is 2.29 m wide. What width, to the nearest hundredth of a metre, does this allow each passenger? 0.46m

When a number is divided by 100, each digit moves 2 places to the right. When it is divided by 1000, each digit moves 3 places to the right.

Examples: 73.6 ÷ 100 = 0.736
73.6 ÷ 1000 = 0.0736

Divide each number by 10, 100, and 1000. Write the quotients.
Do these in your head.

32. 4.6
33. 732.8
34. 5.8
35. 0.7
36. 1057
37. 25 000

32 0.46, 0.046, 0.0046
33. 73.28, 7.328, 0.7328
34 0.58, 0.058, 0.0058
35. 0.07, 0.007, 0.0007
36 105.7, 10.57, 1.057
37 2500, 250, 25

For some divisions, you could use extra zeros in the dividend forever.

3)4.000 . . . = 1.333 . . .
11)32.0000 . . . = 2.9090 . . .

When this happens, one or more of the digits in the quotient will follow a repeating pattern.

Write

1.333 . . . = $1.\overline{3}$

2.9090 . . . = $2.\overline{90}$

Use a bar above the repeating digits to show the quotients.

try this

1. 6)10 $1.\overline{6}$
2. 12)50 $4.1\overline{6}$
3. 18)7 $0.3\overline{8}$
4. 9)5 $0.\overline{5}$

151

RELATED ACTIVITIES

- For further practice, you may wish to have the students complete Ex. 21-44 on page 332 and Ex. 33-48 and Ex. 65-80 on page 334.
- Mark a yellow die to show 3.1, 15.6, 4, 7.3, 82, and 9.5 and a blue die to show 10, 100, 1000, 10, 100, and 1000. Students may play in small groups taking turns tossing the dice and dividing the number on the yellow die by the number on the blue die. After several rounds, each player adds the quotients obtained. The player with the greatest sum is the winner.
- Have students write and solve word problems that involve rounding quotients. The topic of airplanes may suggest ideas for the word problems.
- Some students may enjoy working other divisions that result in quotients that are repeating decimals. You may assign the following exercises and also ask the students to create their own division exercises. Students will thus be challenged in the attempt to find divisions for which the quotients do not terminate. For their own divisions, you may suggest that they restrict the divisor to a number less than ten.

1. 3)16 $5.\overline{3}$
2. 33)4 $0.\overline{12}$
3. 111)40 $0.\overline{360}$
4. 7)1 $0.\overline{142857}$
5. 7)3 $0.\overline{428571}$
6. 7)2 $0.\overline{285714}$

multiplying a number by 0.1 (0.01, 0.001) gives the same result as dividing that number by 10 (100, 1000).

Try This: These exercises present the concept of repeating digits in the quotient. Because many divisions have repeating digits in the quotients, a method of showing such quotients is suggested. A bar is drawn over the first group of repeating digits, and care must be taken so that the bar does not include those digits that are not part of the repeating pattern. For example, for 2.9090..., it would be incorrect to show $2.\overline{90}$. Also, $2.\overline{909}$ is incorrect because it implies the pattern 2.909909909... . Students should work the two division examples to understand why the digits follow a repeating pattern. Then they may proceed with Ex. 1-4. Such divisions help to emphasize the importance of the skill of rounding quotients. After the students have completed these exercises, you may wish to have them determine which quotients for Ex. 1-30 on page 151 involve repeating digits.

Assessment

Divide. Round each quotient to the

1. nearest tenth.
   84)18.2 0.2
2. nearest hundredth.
   673)340.2 0.51
3. nearest thousandth.
   55)7.6 0.138

Solve.

4. The mass of three pumpkins is 23.04 kg. What is the average mass, to the nearest tenth of a kilogram, of each pumpkin? 7.7 kg

Page 152

OBJECTIVE

Demonstrate competence in dividing decimals

RELATED ACTIVITIES

- Have the students divide each number in Magic Square A on page 145 by 28, 14, or 70, and then find whether the numbers that result form a Magic Square.
- Have the students cut advertisements from newspapers showing items and prices similar to those in the *Problem Solving* feature. These may be displayed on a bulletin board or mounted on cards. Have the students find the supermarket cost for one of each item.
- Use an overhead projector to demonstrate the product of a number and a power of ten. Write a numeral, for example, 3764, on one piece of acetate. Place it over another piece of acetate showing a place-value chart and a decimal point (A). Move the acetate sheet showing the numeral to the left or to the right over the place-value chart (B).

A

Th	H	T	o	t	h
		3	7	• 6	4

B

Th	H	T	o	t	h
	3	7	6	• 4	

Practice

When the quotient is exact, multiplication can be used to check division.

Example: $7\overline{)4.06}$ = 0.58

If the dividend and the product do not match, there is a mistake.

$$\begin{array}{r} 0.58 \\ \times\ 7 \\ \hline 4.06 \end{array}$$

Divide. Multiply to check.

1. $6\overline{)34.8}$ 5.8
2. $7\overline{)165.34}$ 23.62
3. $8\overline{)36.6}$ 4.575
4. $54\overline{)36.18}$ 0.67
5. $26\overline{)209.3}$ 8.05

Solve.

6. 98 tickets were sold for $171.50. What was the price for one ticket? $1.75
7. The 48 hockey pucks have a mass of 8.16 kg. How heavy is each hockey puck? 0.17 kg or 170g

When division is used with an amount of money and the quotient is not exact, the quotient is often rounded to the nearest cent.

Example: $5\overline{)\$1.780}$ = \$0.356 $\xrightarrow{\text{rounds to}}$ \$0.36

Divide and round the quotient to the nearest cent.

8. $4\overline{)\$5.35}$ $1.34
9. $7\overline{)\$53.80}$ $ 7.69
10. $12\overline{)\$0.88}$ $0.07
11. $6\overline{)\$1}$ $0.17
12. $144\overline{)\$10}$ $ 0.07

Solve. Give the answer to the nearest cent.

13. A dozen eggs cost $0.97. How much is this for each egg? $0.08
14. 15 apples cost $1.98. How much is this for each apple? $0.13
15. 6 L of detergent cost $5.14. How much is this for 1 L? $0.86
16. 10 kg of potatoes cost $2.39. How much is this for 1 kg? $0.24

Supermarket items are often priced at 3 for $1.00.

In the supermarket,

$3\overline{)\$1.000}$ = \$0.333 $\xrightarrow{\text{rounds to}}$ \$0.34.

1. Why is this type of rounding used in a supermarket?
2. Find out how a calculating machine rounds the quotient 1 ÷ 3.

PROBLEM SOLVING

152

1. The price of one item is raised to the next cent. The saving in cost occurs only in buying the whole set
2. For the quotient 1÷3, the display on a calculating machine will show 0.333333... .

LESSON ACTIVITY

Using the Page

- The use of multiplication to check division of a decimal is shown in the example at the top of page 152. You may wish to discuss the example with the students. Review that two-place decimals are used to write amounts of money as dollars and cents. Draw attention to the example below Ex. 6 and Ex. 7. Ask students to explain why the division is continued to the thousandths' place. Lead them to suggest that it must be determined whether the digit in the thousandths' place is 5, 6, 7, 8, or 9, which would result in the digit in the hundredths' place being rounded up. This is necessary to express the amount of money to the nearest cent. If the division stopped at the hundredths' place in similar examples, the hundredths' digit would be one less than it should be.

Problem Solving: For the division in this example, the students will note that the quotient $0.333 is rounded up to $0.34 rather than down to $0.33. At 33¢ each, the price of 3 apples would be 99¢ and the offer of 3 for $1.00 would not be reasonable because there would be no savings involved. At 34¢ each, the cost of 3 apples would be $1.02, and 2¢ can be saved when buying three apples priced at 3 for $1.00. For Ex. 2, students can discover that most calculators do not round quotients for divisions such as 1 ÷ 3; they show, for example, 0.333333… .

The Decimal Point and the $\boxed{\cdot}$ Key on a Calculator

A calculator displays decimals positioned from the right. This means that place value is not shown by the position of a digit in the display. Instead place value is found according to the position of the decimal point.

A. Enter 456. | 456. | 4 hundreds

B. Press $\boxed{\div}$ 10 $\boxed{=}$ | 45.6 | 4 tens

C. Press $\boxed{\div}$ 10 $\boxed{=}$ | 4.56 | 4 ones

D. Press $\boxed{\div}$ 10 $\boxed{=}$ | 0.456 | 4 tenths

E. Press $\boxed{\div}$ 10 $\boxed{=}$ | 0.0456 | 4 hundredths

The digits stay in the same positions. The decimal point moves. Place values change.

In each display above, what is the place value

1. of the 5? 2. of the 6?

1 A tens
B ones
C tenths
D hundredths
E thousandths

2 A ones
B tenths
C hundredths
D thousandths
E ten-thousandths

What will each display show?

3. Press 2.54 $\boxed{\times}$ | 2.54

4. Then press 3.2 $\boxed{\times}$ | 8.128

5. Then press 7.65 $\boxed{=}$ | 62.1792

Start over. What will the display show after each $\boxed{=}$ is pressed?

6. 2.45 $\boxed{\times}$ 2.5 $\boxed{\times}$ 4.8 $\boxed{=}$ 7. 1239 $\boxed{\div}$ 6 $\boxed{\div}$ 28 $\boxed{=}$

29.4 7.375

Calculator

153

Page 153

OBJECTIVE

Use the calculator to multiply and divide with decimals

Materials

calculators (optional)

RELATED ACTIVITIES

- The concept of a floating decimal point can be demonstrated on the chalkboard if magnets will adhere to it. Write a five-digit numeral on the board and place a round magnet between two of the digits as a decimal point. Have students change the position of the magnet as you indicate multipliers such as 0.01 and 10.

  1234.5 ⟶ 12.345 ⟶ 123.45

- Students can prepare a numeral card showing a sequence of five digits such as 24036. A clear acetate sleeve showing a decimal point can be made to slide along the card. The sleeve can be moved to show the product when multiplying by 10, 100, 1000, 0.1, 0.01, or 0.001, or to show the quotient when dividing by 10, 100, or 1000. The device can be used to encourage mental computation.

240.36

$240.36 \div 10 = 24.036$

24.036

LESSON ACTIVITY

Before Using the Page

- Write 78 on the board and ask a student to tell the place value of each digit. Ask for the quotient of 78 ÷ 10. Write 7.8 on the board and ask for the place value of each digit. Repeat this procedure for 7.8 ÷ 10 and for 0.78 ÷ 10. You may wish to show each quotient in a place-value chart to note the different positions taken by the digits. This may be contrasted later with the way a calculator displays such quotients.

tens	ones	.	tenths	hundredths	thousandths
7	8				
	7		8		
	0		7	8	
	0		0	7	8

Using the Page

- Discuss the information at the top of the page, emphasizing the importance of the decimal point in determining the place value of each digit. Draw the students' attention to the $\boxed{\cdot}$ key on the calculator.

  For A, explain that the decimal point is shown on the calculator to the right of the ones' place, although it is not used when writing whole numbers. For each display for A to E, have students read the numeral and explain how the place value of the digit 4 can be determined. Emphasize that although the digits remain in the same position in each display, the place values change because the decimal point moves. Tell the students that for calculators, the moving decimal point is referred to as "the floating decimal point".

  If calculators are available, have the students use them to check their answers for Ex. 1-7.

Page 154

OBJECTIVE

Solve word problems in two or more steps

RELATED ACTIVITIES

- Have the students "comparison shop" by finding the prices of different sizes of packages or of different brands for an item and comparing the prices in one or both of the ways presented on page 154. Newspaper advertisements or clippings from catalogs can provide prices for comparison.

  For grocery items, students can discover that the smallest package is usually the most expensive way to purchase an item. Perhaps they can suggest reasons for this and also suggest why some customers may prefer to buy the smallest package.

Solving Problems in Two or More Steps

Which is the best buy?

For each bag, find the cost of each kilogram.

Divide.

$5\overline{)\$2.920}$ = $0.584 $3\overline{)\$1.750}$ = $0.583 . . . $2\overline{)\$1.18}$ = $0.59

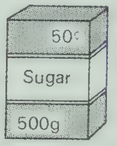
Since 0.583 is less than both 0.584 and 0.59, the 3 kg bag is the best buy.

Which is the better or best buy,

1. 12 peaches for $2.40 or 24 peaches for $4.40?
2. 32 m of paper towels for 77¢ or 41 m of paper towels for 99¢?
3. 2 L of cider for 98¢, 4 L of cider for $1.89, or 10 L of cider for $4.75?
4. 4 kg of potatoes for 89¢, 10 kg of potatoes for $2.10, or 25 kg of potatoes for $4.99?
5. 50 g of nutmeg for $1.05, 500 g of nutmeg for $9.75, or 1000 g of nutmeg for $18.75?
6. What is the best buy for the nutmeg in Exercise 5 with a "15¢ off" coupon for any size? 50g

Sometimes it is easier to find how much you get at each price for a certain amount of money.

Example:



$50\overline{)500}$ = 10
50
00

You get 10 g for each cent.

Which is the better buy,

7. 500 g of salmon for $3 or 750 g of salmon for $4?
8. 400 mL of vinegar for 34¢ or 900 mL of vinegar for 76¢?
9. 125 serviettes for 63¢ or 200 serviettes for 98¢?

PROBLEM SOLVING

154

LESSON ACTIVITY

Before Using the Page

- Tell the students that a bag of 8 apples cost $1.20 and ask how to find the cost of one apple. Develop that division would be used and have the students complete the division to find that each apple cost $0.15.

Using the Page

- Read the title of the lesson and explain that solving some problems requires more than one step. For the first example and for Ex. 1-6, the prices are compared by finding the cost of each unit and then comparing the costs. The package for which the cost of each unit is the least is the best buy. Ask how the best buy can be found for the flour. If the cost of 1 kg of flour is found for each bag, these costs can be compared to find the bag for which each kilogram is cheapest. Have students help to complete the three divisions on the board. Point out that extra zeros are needed in the dividends for the first two divisions. For $3\overline{)\$1.75}$, lead the students to realize that the division can continue beyond thousandths but that only three decimal places are needed for comparing the quotients. Read the concluding statement.
- For the second example and for Ex. 7-9, the prices are compared by finding how much of each item can be bought for a certain amount of money. The package in which more can be bought for a certain amount of money is the better buy. Explain that instead of finding the cost of one gram, for example, the procedure used is to find how many grams can be bought for each cent (dollar). Develop that the amount of sugar that can be bought for each cent is found by dividing the amount of sugar in the box by the cost of the box of sugar.

Checking Up

Multiply.

1. 2.8 × 6 = 16.8
2. 4.245 × 4 = 16.980
3. 3.2 × 47 = 150.4
4. $8.87 × 9 = $79.83
5. 3.86 × 19 = 73.34
6. 5.291 × 43 = 227.513
7. 57.1 × 314 = 17 929.4
8. $0.93 × 192 = $178.56
9. 7.5 × 5.4 = 40.50
10. 9.29 × 0.7 = 6.503
11. 5.227 × 3.6 = 18.8172
12. 1.69 × 9.05 = 15.2945
13. 0.4 × 0.2 = 0.08
14. 1.47 × 0.4 = 0.588
15. 0.897 × 0.8 = 0.7176
16. 0.25 × 0.07 = 0.0175

Round the product to the nearest tenth.

17. 4.1 × 8.6 = 35.3
18. 0.64 × 5.6 = 3.6
19. 18.6 × 5.55 = 103.2

Round the product to the nearest hundredth.

20. 7.162 × 7.9 = 56.58
21. $8.33 × 0.74 = $6.16
22. 9.672 × 19 = 183.77

Divide.

23. 3)14.4 — 4.8
24. 7)37.268 — 5.324
25. 26)94.9 — 3.65
26. 5)$10.75 — $2.15
27. 66)174.24 — 2.64
28. 43)84.194 — 1.958
29. 842)6272.9 — 7.45
30. 565)$2090.50 — $3.70
31. 56)77 — 1.375
32. 38)7.942 — 0.209
33. 45)63.27 — 1.406
34. 16)$104 — $6.50

Round the quotient to the nearest tenth.

35. 7)36 — 5.1
36. 128)48.6 — 0.4

Round the quotient to the nearest hundredth.

37. 8)39 — 4.88
38. 26)$4.40 — $0.17

Solve.

39. 47 L of gasoline cost $10.81. What was the cost of each litre? $0.23
40. The average wage for each of 27 workers was $243.17. How much did the workers earn in all? $6565.59
41. How many square metres are in a wall that is 4.84 m long and 3.25 m high? 15.73
42. Ed cut a rope 14.6 m long into 4 pieces that are the same length. How long is each piece? 3.65m

155

Page 155

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Have the students find the product of all the numbers in the square. Ask the students to multiply the factors in various orders to review that the order of the factors does not affect the product.

3.1	0.5
6	7.23

- Have the students divide 99.5904 by any number in the square, then divide the quotient by another number in the square, and continue the procedure until all of the numbers in the square have been used as divisors. Ask the students to repeat the activity using a different order for the divisors, to enable them to discover that the order of choosing the divisors does not affect the final result.

3	76
24	91

Skills	Exercises	Related Pages
Multiply a decimal by a whole number	1-8	T 146-T 149
Multiply tenths	9	T 150-T 151
Multiply hundredths and thousandths	10-12	T 152-T 153
Multiply decimals, products less than one	13-16	T 154-T 155
Multiply and round the product	17-22	T 156-T 157
Divide by a one-digit number	23, 24, 26	T 158-T 159
Divide by a two-digit number or by a three-digit number	25, 27-34	T 160-T 161
Divide and round the quotient	35-38	T 162-T 163
Solve multiplication word problems	40, 41	
Solve division word problems	39, 42	

Comments

Determine whether errors in multiplication with decimals are caused by poor recall of basic multiplication facts, by difficulty with the algorithm for multiplication, or by difficulty with the concept of decimals. Note whether errors in division with decimals are caused by poor recall of basic multiplication facts, by difficulty in relating multiplication and division, by difficulty with the division algorithm, or by difficulty with the concept of decimals. Lesson suggestions and related activities for Unit 4 can be used to reinforce multiplication and division skills. Activities described in Unit 5 can be used to review decimal concepts.

Unit 8 Overview

Measurement

This unit reviews and extends some of the metric measurements which students have used in previous work. The first two lessons review linear units and conversions from one unit to another by multiplying or dividing by powers of ten. The seven linear metric units are presented; these include the less familiar units (hectometre, decametre, and decimetre), as well as the more familiar kilometre, metre, centimetre, and millimetre. Studies of capacity and volume feature the litre and the millilitre, the cubic centimetre and the cubic metre. Mass is measured in grams and in kilograms and conversions are made between them. Then volume, capacity, and mass of water are considered together to emphasize the relationships which exist in these three kinds of metric measurement. One lesson associates Celsius temperatures with a variety of seasonal activities. Time on a 12-hour clock, on a 24-hour dial clock, and on a 24-hour digital clock is read to the second and recorded in the standard ways. Time zones in Canada are introduced and conversions are made for times in one zone to times in another zone. Problem solving often involves finding information and the lesson in this unit provides experiences in reading a railroad timetable as an example of this skill. Two of the four *Try This* features provide brief experiences with some of the less familiar measures of capacity and mass, another suggests a method for finding the volume of an irregularly-shaped object, and the fourth one presents the format used in numeric dating.

Prerequisite Skills

- multiply by 10, 100, 1000, 0.1, 0.01, and 0.001
- divide by 10, 100, and 1000
- find the volume of a rectangular prism

Unit Outcomes

- express a length given in one unit in terms of another unit for metres, decimetres, centimetres, and millimetres
- compare measurements expressed in different units of length for metres, decimetres, centimetres, and millimetres
- express a length given in one unit in terms of another, for the seven units from kilometres to millimetres; compare measurements expressed in different units of length; choose the best unit for measuring length
- estimate capacity in millilitres and in litres; express millilitres as litres and litres as millilitres; choose the millilitre or the litre as the better unit for measuring capacity
- estimate mass in grams and in kilograms; express grams as kilograms and kilograms as grams; choose the gram or the kilogram as the better unit for measuring mass
- convert between measurements for volume and measurements for capacity; find the capacity in litres of containers for which the dimensions are given in centimetres
- convert among measurements for volume, capacity, and mass of water; find the mass in grams and in kilograms of water held by containers for which the dimensions are given in centimetres
- relate temperatures in degrees Celsius to activities; read a thermometer
- write numerals for times to the second using a.m. or p.m. and as shown on 24-hour dial and digital clocks; read times from 24-hour dial and digital clocks; show a given time on dial and digital clock faces; express the time in one time zone in Canada as the time in another time zone in Canada
- solve word problems involving measurements
- find the information needed to solve a word problem

Background

The origin and structure of the metric system of linear measurement is presented in the Overview for Unit 6. Reference to that *Background* will provide a foundation for the lessons in this unit. It is pointed out there how the units for linear measurement are used in the naming of units for area; for example, a square with sides 1 cm long has an area of 1 cm^2. The same consistent style also appears in units for measuring volume. A cube with edges 1 cm long has a volume of 1 cm^3, and a cube with edges 1 m long has a volume of 1 m^3. Powers of ten are used to convert from one unit to another in all three types of measurement. For conversion from one unit of length to the next, the factor 10 is used; for conversion from one unit of area to the next, 100 (10^2) is used; and for conversion from one unit of volume to the next, 1000 (10^3) is used.

$$10 \text{ cm} = 1 \text{ dm}$$
$$100 \text{ cm}^2 = 1 \text{ dm}^2$$
$$1000 \text{ cm}^3 = 1 \text{ dm}^3$$

A change from a larger unit of measure to a smaller unit is made by multiplication, and a change from a smaller unit to a larger unit is made by division (or by multiplication by a corresponding decimal). Changes between units which are not in order of size also involve powers of ten. For example, between centimetres and metres the factor is 100 (10×10, or 10^2), and between cubic centimetres and cubic metres the factor is 1 000 000 (1000×1000, or 10^6).

The concepts of volume and capacity are almost the same. Briefly, volume refers to how much space an object occupies, and capacity to how much space is inside a container. For instance, a closed empty carton occupies a certain amount of space (volume), but it can hold almost the same amount of material (capacity). Volume is measured in cubic units which derive their names from their linear dimensions — cubic centimetre from centimetre, cubic decimetre from decimetre, and cubic metre from metre. Capacity may be measured in the same units as volume, but it is usually measured in millilitres, litres, and kilolitres. The unit of 1 L is the same size as 1000 cm^3, and, therefore, 1 mL is the same size as 1 cm^3.

Another type of measurement deals with the mass of an object. Mass refers to the quantity of matter in an object and may be measured by balancing an object and standard units of mass on balance scales. Mass is not the same as weight. Weight is the measure of the force required to lift an object. Since the force of gravity can vary, so can weight — even to the point of weightlessness — but mass cannot vary.

Temperature is measured in degrees Celsius, named after the Swedish astronomer, Anders Celsius, who designed a scale of 100 degrees between the freezing point (0°C) and the boiling point (100°C) of water. Besides the boiling point and the freezing point of water, it is important to know other temperatures in degrees Celsius for making comparisons.

Human body	37°C
Very warm day	30°C
Pleasant day	20°C
Cool day	10°C
Cold day	– 10°C
Very cold day	– 20°C

Time is not measured in metric units, but the second (s) is recognized as an SI unit. The other units of time — hour (h), minute (min), and day (d) — although non-metric, are accepted universally. There seems to be no international symbol for week, month, or year, although "a", for *annum*, is sometimes used for year. Since there are 24 hours in a day, if a 12-hour basis is used to record time, it is necessary to add the abbreviations a.m. (ante meridiem) for times before noon and p.m. (post meridiem) for times after noon. Time on a 24-hour basis may be indicated by using two digits for each hour, minute, and second. Spaces may separate the pairs of digits, or colons may be placed between them. Thus, 15:35:40 indicates 15 h, 35 min, and 40 s. Since each day begins and ends at midnight, that time may be, for example, either Thursday at 24:00:00 or Friday at 00:00:00. On a 24-hour basis, hours after noon are greater than 12 (13 to 24). It should be noted that if the number for the hours, the minutes, or the seconds is less than ten, a zero is written to the left of the single digit so that each unit of time is represented by two digits. For instance, 09:05:01 indicates 9 h, 5 min, and 1 s after midnight. It should be pointed out that expressing 15:00 as "fifteen hundred hours" is incorrect because the two zeros do not indicate hundreds. Because time is based on a non-metric system of relationships, conversions and regrouping in operations cannot be made in terms of ten, but rather in terms of 24 and 60 (1 d = 24 h, 1 h = 60 min, 1 min = 60 s).

Numeric dating, that is, writing dates without the names of months in words, is becoming standard practice. The year, the month, and the day, (units in descending order of size), are indicated with eight digits: four for the year, two for the month, and two for the day. Again, if the number for the month or the day is less than ten, a zero is written to the left of the single digit. Therefore, July 1, 1867, the day of Confederation for Canada, is indicated by 1867 07 01. In numeric dating, spaces are left between the pairs of digits. If the century is obvious, the first two digits of the date may be omitted, as in 69 07 20 (July 20, 1969), the date when man first landed on the moon.

Successful solving of word problems requires a number of skills including an ability to read about situations and circumstances, to select only the pertinent data, and to determine the types of relationships which exist between the numbers chosen. A different kind of reading skill is required when the information is provided in charts, such as in timetables, schedules, and tables of numerical data. The selection process is much more complex because so much more data is provided. In every form of tabular presentation there are rows and columns of numbers and other information for which meanings are obtained from the headings above or beside them, and a quick scanning of these features is required before the details are examined. Many programs designed to develop reading comprehension do not include these special skills which are so critical in solving many everyday problems involving quantities, time, and money. It is important, therefore, to devote time in the mathematics program to develop the particular skills of reading which are used to interpret tables of numerical data.

Teaching Strategies

Activities suggested in the lesson outlines, especially those in the *Related Activities*, require a variety of materials. These should be available, not only during the lessons, but also before and after the lessons so that students may benefit from both informal and planned experiences. It may be advisable to form a number of instructional groups and to schedule opportunities for the students to carry out the activities. To prevent spills in measuring capacity, a sink or a water table is desirable if liquids are used, and a sand table if dry substances are used. Even a large dishpan or a tub placed on a plastic sheet can minimize spillage of the materials.

The students may be helped to understand the structure of the metric system and the meanings of the prefixes if a chart showing the units is developed, as shown below for metres. Similar charts may be prepared for showing litres and grams.

kilo-metre	hecto-metre	deca-metre	metre	deci-metre	centi-metre	milli-metre
thousands	hundreds	tens	ones	tenths	hundredths	thousandths
1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Converting metric measures from one unit to another involves multiplication and division by powers of ten, with 10 and 1000 used most frequently. Students may be directed to use multiplication by a corresponding decimal in lieu of division by a whole number. If so, this alternative method should be developed in a brief lesson using examples such as $2480 \div 100$ and 2480×0.01, $32 \div 10$ and 32×0.1, $3625 \div 1000$ and 3625×0.001.

In connection with the lesson on time it is suggested that there be two clocks in the classroom, a 12-hour dial clock and a 24-hour digital clock. If desired, gummed tabs for 13 to 24 may be fastened to the dial of a 12-hour clock to help students to associate times after noon with 24-hour notation. In the *Related Activities* for this lesson, one of the suggestions involves addition and subtraction of times. Students should be reminded that regrouping for times is not based on tens, but rather on 60 between seconds and minutes and between minutes and hours, and on 24 between hours and days.

Materials

metre sticks and rulers marked in decimetres, in centimetres, and in millimetres
a string longer than one decametre
a variety of containers that hold 1 L, more than 1 L, and less than 1 L; measuring cups marked in litres and millilitres
several masses of 1 g and several of 1 kg, other masses from 1 g to 1 kg, balance scales and such objects as pencils, books, nails, corks, and stones
a decimetre cube, materials for the experiment on page 164
balance scales, masses from 1 g to 1 kg, two identical boxes, an object for which the mass is to be measured, two identical one-litre containers
a demonstration thermometer (optional), a large piece of paper (optional), a copy of page T 397 for each student
a dictionary for each student

Vocabulary

hectometre, hm
decametre, dam
capacity
kilolitre, kL
hectolitre, hL
decalitre, daL
decilitre, dL
centilitre, cL
mass
hectogram, hg
decagram, dag
decigram, dg
centigram, cg
time zone
Pacific, Mountain, Central, Eastern, Atlantic, Newfoundland standard time
daylight saving time
decade, century
millennium, eon
fortnight, twinkling
digital clock

Pages 156-157

LESSON OUTCOME

Express a length given in one unit in terms of another unit for metres, decimetres, centimetres, and millimetres; compare measurements expressed in different units of length for metres, decimetres, centimetres, and millimetres

Materials

metre sticks and rulers marked in decimetres, in centimetres, and in millimetres

Prerequisite Skills

Multiply by 10, 100, and 1000; divide by 10, 100, and 1000

Checking Prerequisite Skills

Multiply each number by 10, 100, and 1000.

1. 16 160 1600 16 000 **2.** 0.3 3 30 300 **3.** 0.85 8.5 85 850

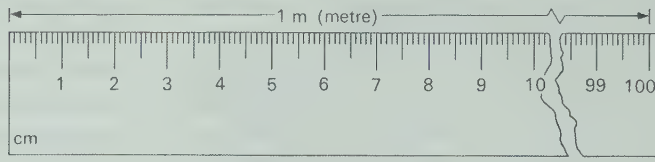
Divide each number by 10, 100, and 1000.

4. 1400 140 14 1.4 **5.** 920 92 9.2 0.92 **6.** 14.6 1.46 0.146 0.0146

8 MEASUREMENT

Metres, Decimetres, Centimetres, and Millimetres

Units of measure are related by multiples of 10.



1 dm (decimetre)

1 cm (centimetre)

1 mm (millimetre)

1 m = 10 dm	1 dm = 0.1 m
1 m = 100 cm	1 cm = 0.01 m
1 m = 1000 mm	1 mm = 0.001 m

To change between units, multiply or divide by 10.

$$25\text{ m} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} 250\text{ dm} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} 2500\text{ cm} \underset{\div 10}{\overset{\times 10}{\rightleftarrows}} 25\,000\text{ mm}$$

When a larger unit is changed to a smaller unit, there will be more units.

Multiply by 10, 100, or 1000.

When a number is multiplied by 10, 100, or 1000, the digits move one, two, or three places to the left.

25 × 10 = 250
25 × 100 = 2500
25 × 1000 = 25 000

When a smaller unit is changed to a larger unit, there will be fewer units.

Divide by 10, 100, or 1000.

When a number is divided by 10, 100, or 1000, the digits move one, two, or three places to the right.

25 000 ÷ 10 = 2500
25 000 ÷ 100 = 250
25 000 ÷ 1000 = 25

156

LESSON ACTIVITY

Before Using the Pages

- Reacquaint the students with the metre, the decimetre, the centimetre, and the millimetre as units of length. They may work in small groups, using metre sticks marked to show decimetres, centimetres, and millimetres. Ask them to place one hand a distance of 1 m from the floor, to place their two hands 1 m apart, to place two fingers 1 dm apart, and so on. Ask them to check each estimate with the appropriate unit on a metre stick. Have them note the number of decimetres in 1 m, the number of centimetres in 1 m and in 1 dm, and the number of millimetres in 1 cm, in 1 dm, and in 1 m. Develop the following statements on the board, pointing out that the relationships between the units involve the numbers 10, 100, and 1000.

10 mm = 1 cm	100 cm = 1 m
10 cm = 1 dm	1000 mm = 1 m
10 dm = 1 m	

- Ask the students to identify a length of 0.8 m on metre sticks. Then ask them to express this length as decimetres, as centimetres, and as millimetres, referring to a metre stick if necessary. Write the results on the board. Use a similar activity, beginning with a length of 750 mm. Ask how the number 10 relates 8 dm and 80 cm, how the number 100 relates 75 cm and 0.75 m, and other similar examples.

$\boxed{0.8\text{ m}} \rightarrow 8\text{ dm} \rightarrow 80\text{ cm} \rightarrow 800\text{ mm}$

$0.75\text{ m} \leftarrow 7.5\text{ dm} \leftarrow 75\text{ cm} \leftarrow \boxed{750\text{ mm}}$

Using the Pages

- For the diagram, ask which is the largest unit and which is the smallest unit indicated. Ask students to name the units from least to greatest and from greatest to least. Discuss that 10, 100, and 1000 are the multiples of ten that relate these units. Note that each relationship is expressed in two ways, for example, 1 m = 10 dm and 1 dm = 0.1 m.

  Lead the students in a discussion of the multiplications and divisions shown for changing between the units. Emphasize that there will be more smaller units than larger

Working Together

Multiply by 10 to complete.

1. 4 m = 40 dm
2. 2.3 cm = 23 mm

Divide by 10 to complete.

3. 23 dm = 2.3 m
4. 48.2 cm = 4.82 dm

Multiply by 10 two times, or by 100.

5. 3.6 m = 360 cm
6. 0.9 dm = 90 mm

Divide by 10 two times, or by 100.

7. 435 cm = 4.35 m
8. 27 mm = 0.27 dm

Multiply by 10 three times, or by 1000.

9. 2.5 m = 2500 mm
10. 0.75 m = 750 mm

Divide by 10 three times, or by 1000.

11. 4000 mm = 4 m
12. 375 mm = 0.375 m

Complete.

13.

1.3 m	13 dm	130 cm	1300 mm

Give each length in metres.

14. 120 cm 1.2 m 15. 5.7 dm 0.57 m 16. 8600 mm 8.6 m

Give each length in centimetres.

17. 3.6 m 360 cm 18. 7000 mm 700 cm 19. 0.42 dm 4.2 cm

Exercises

Complete.

1.	6.3 m	63 dm	630 cm	6300 mm
2.	8.5 m	85 dm	850 cm	8500 mm
3.	0.39 m	3.9 dm	39 cm	390 mm
4.	28 m	280 dm	2800 cm	28 000 mm
5.	70 m	700 dm	7000 cm	70 000 mm
6.	0.15 m	1.5 dm	15 cm	150 mm

Give each length in metres.

7. 380 cm 3.8 m
8. 14 dm 1.4 m
9. 920 mm 0.92 m
10. 4 cm 0.04 m
11. 200 mm 0.2 m
12. 6.8 dm 0.68 m

Give each length in centimetres.

13. 1.6 m 160 cm 14. 55 dm 550 cm 15. 400 mm 40 cm

Give each length in millimetres.

16. 2.8 m 2800 mm 17. 49 cm 490 mm 18. 0.4 dm 40 mm

Which length is greater,

19. 500 cm or 2 m? 20. 80 dm or 0.9 m? 21. 4000 mm or 5 m?
22. 7.3 m or 800 mm? 23. 380 cm or 4 m? 24. 12 dm or 1.6 m?
25. 920 cm or 8300 mm? 26. 4.8 cm or 0.05 m? 27. 58 cm or 680 mm?

157

RELATED ACTIVITIES

- Ask students to suggest objects having lengths of 1 m, 1 dm, 1 cm, and 1 mm respectively. Prepare a list of the objects named and display the list for several days.

1 m	1 dm	1 cm	1 mm
distance from the floor to the door-knob	height of a soup can	width of a thumb-tack	thick-ness of a dime

- Ask students to estimate and then to measure the length of an object using one of these units: metre, decimetre, centimetre, or millimetre. Ask them to write the measurement using a red pencil. Then ask them to write that length in each of the other three units, using a blue pencil.
- For practice in multiplying and dividing by 10, 100, and 1000, have the students prepare cards similar to the following and play the game "Concentration" described on page T 379.

8 ÷ 100	0.08

- You may wish to review that multiplication by 0.1 (0.01, 0.001) is equivalent to division by 10 (100, 1000).

units; for example, more decimetres than metres are needed to express the length of a corridor. Explain that multiplication is used to change from a larger unit to a smaller unit of length, since there will be more units. Similarly, to change from a smaller unit of length to a larger unit, division is used, since there will be fewer units. Ask students to explain when the multiplier (divisor) used is 10, 100, or 1000.

Working Together: Ex. 1-12 emphasize that multiplication is used to change from a larger unit to a smaller unit, and that division is used to change from a smaller unit to a larger unit. Ask students to explain why 10 (100, 1000) is suggested as the multiplier or the divisor. Ask students to describe patterns that they notice in Ex. 13, which presents units in sequence from greatest to least. For each of Ex. 14-19, ask students to explain their answers. You may wish to have students use metre sticks or rulers to show some of the lengths.

Exercises: For assistance with these exercises, students may refer to exercises of *Working Together* and to examples on page 156. For each of Ex. 19-27, have the students express one of the lengths in the unit of measurement given for the other length. You may wish to provide metre sticks and rulers to enable students to check some of their answers.

Assessment

Complete the chart.

1.	4.38 m	43.8 dm	438 cm	4380 mm
2.	0.6 m	6 dm	60 cm	600 mm
3.	0.075 m	0.75 dm	7.5 cm	75 mm
4.	0.92 m	9.2 dm	92 cm	920 mm

Give each length in metres.

5. 630 cm 6.3 m 6. 90 mm 0.09 m

Give each length in centimetres.

7. 3.5 dm 35 cm 8. 724 mm 72.4 cm

Which length is greater,

9. 8.7 m or 89 cm? 10. 436 mm or 52 dm?

LESSON OUTCOME

Express a length given in one unit in terms of another, for the seven units from kilometres to millimetres; compare measurements expressed in different units of length; choose the best unit for measuring length; solve related word problems

Materials

a string longer than one decametre, a metre stick

Vocabulary

hectometre, hm, decametre, dam

Prerequisite Skills

Multiply by 1000, 100, 10, 0.1, 0.01, and 0.001; divide by 10, 100, and 1000

Checking Prerequisite Skills

Multiply each number by 10, 100, and 1000.

1. 6 60 600 6000 **2.** 0.15 1.5 15 150 **3.** 2.1 21 210 2100

Multiply each number by 0.1, 0.01, and 0.001.

4. 230 23 2.3 0.23 **5.** 4.2 0.42 0.042 0.0042 **6.** 25 2.5 0.25 0.025

Divide each number by 10, 100, and 1000.

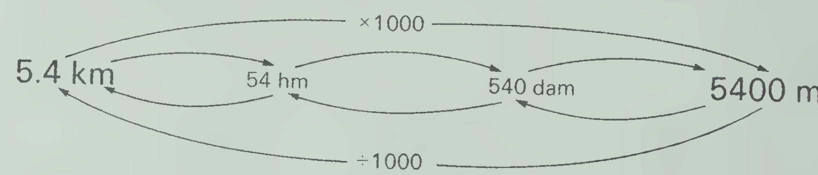
7. 230 23 2.3 0.23 **8.** 34 3.4 0.34 0.034 **9.** 2600 260 26 2.6

Units of Length, Kilometre to Millimetre

The names for different units of length all use "metre". The most common units are kilometre, metre, centimetre, and millimetre.

Unit	Symbol	Relation to metre
kilometre	km	1000 m
hectometre	hm	100 m
decametre	dam	10 m
metre	m	1 m
decimetre	dm	0.1 m
centimetre	cm	0.01 m
millimetre	mm	0.001 m

When a larger unit is changed to a smaller unit, there will be more units. To change kilometres to metres, multiply by 10 three times, or by 1000.



When a smaller unit is changed to a larger unit, there will be fewer units. To change metres to kilometres, divide by 10 three times, or by 1000.

Multiplying by 0.1 three times, or by 0.001, gives the same result as dividing by 1000.

When a number is divided by 10, 100, or 1000, each digit moves one, two, or three places to the right.

5400 ÷ 10 = 540
5400 ÷ 100 = 54
5400 ÷ 1000 = 5.4

When a number is multiplied by 0.1, 0.01, or 0.001, each digit moves one, two, or three places to the right.

5400 × 0.1 = 540
5400 × 0.01 = 54
5400 × 0.001 = 5.4

158

LESSON ACTIVITY

Before Using the Pages

- Briefly review the work of the previous lesson. Have students complete the following chart on the board. Discuss the use of multiplication and division to complete the chart.

1.	1.2 m	dm	cm	mm
2.	m	dm	371 cm	mm
3.	m	dm	cm	48 mm

For the sequence mm ⟶ cm ⟶ dm ⟶ m, emphasize that ten of one unit can be renamed as one of the next unit, for example, 10 cm = 1 dm. For continuing the sequence to the right, develop that a length of 10 m can be renamed as a new unit. Have students help to cut a piece of string 10 m long. Introduce the term *decametre* (dam) for the new unit. Discuss that 10 dam can be renamed as 1 hm and 10 hm as 1 km. Write the complete sequence on the board.

mm ⟶ cm ⟶ dm ⟶ m ⟶ dam ⟶ hm ⟶ km

Using the Pages

- Begin with a discussion of the chart at the top of page 158. Ask how many units of length are shown, which is the largest unit, which is the smallest unit, and which word appears in the name of each unit. Have students identify the symbol for each unit, noting the difference in meaning for dm and dam. Ask students to name the units with which they are most familiar. They will likely suggest the four units named in the statement above the chart.

  To help students understand how the six units are related to the metre, develop the following chart on the board. It can be pointed out, for example, that the prefix "kilo" means "thousand" and thus 1 km = 1000 m. Through examples of this kind, students can see that metric units of length are related in the same way as place values of the base-ten numeration system.

kilometre	hectometre	decametre	metre	decimetre	centimetre	millimetre
thousands	hundreds	tens	ones	tenths	hundredths	thousandths
1000 m	100 m	10 m	1 m	0.1 m	0.01 m	0.001 m

Working Together

Multiply by 10, 100, or 1000 to complete.

1. 7 km = 70 hm 2. 0.95 km = 950 m 3. 3.62 m = 3620 mm

Divide by 10, 100, or 1000 to complete.

4. 76 hm = 7.6 km 5. 400 m = 0.4 km 6. 270 cm = 2.7 m

Multiply by 0.1, 0.01, or 0.001 to complete.

7. 76 hm = 7.6 km 8. 400 m = 0.4 km 9. 270 cm = 2.7 m

Complete.

10.

	25	250	2500
2.5 km	hm	dam	m

11.

	725	72.5	7.25
7250 mm	cm	dm	m

Exercises

Complete.

1. 14 000 m = 14 km 2. 3607 m = 36.07 hm 3. 100 m = 0.1 km
4. 1.5 m = 1500 mm 5. 0.75 m = 75 cm 6. 0.043 m = 0.43 dm

Give each length in metres.

7. 4200 mm 4.2 m 8. 0.56 km 560 m 9. 12 dm 1.2 m 10. 88 cm 0.88 m 11. 15 hm 1500 m
12. 160 dam 1600 m 13. 1.6 km 1600 m 14. 530 mm 0.53 m 15. 6 cm 0.06 m 16. 13 km 13 000 m

Which length is greatest,

17. 500 m, 1 km, or 0.6 dam? 18. 6.7 m, 6800 mm, or 69 cm?

Which unit of length would you use to measure each of these?

19. a marathon run kilometre or metre 20. Moncton to Halifax kilometre
21. a mosquito millimetre 22. your height centimetre, decimetre, or metre

Solve.

23. Greg estimates that he walks 2800 m each day delivering papers. About how many kilometres would he walk delivering papers for 20 d? 56

24. Susan made 90 ribbons for the school field day. Each ribbon is 14 cm long. How many metres of material did Susan use? 12.6

159

RELATED ACTIVITIES

- Have the students play the game "Inventory" described on page T 380 and list distances which they would measure in kilometres. Have them play the game again for each of the other units of length presented in this lesson.
- Ask each of ten students to cut a strip of paper that is 1 m long. Have them connect these metre strips to form a strip of paper one decametre long. Label the strip "one decametre" and "1 dam", and display it in the classroom.
- Students may use trundle wheels to find distances in and around the school that measure one hectometre.
- Students can prepare and display a list of places showing the distance in kilometres from their community to each of these places.
- For practice in multiplying by 0.1, 0.01, and 0.001, have the students include cards similar to the following for the game "Concentration" described on page T 379.

6 × 0.001	0.006

- The string cut to a length of 1 dam as suggested in *Before Using the Pages* may be useful for comparing lengths in the school. For example, students can determine whether the length of the classroom, the length of a corridor, the length and the width of the gymnasium, the length of the parking lot, and so on, are greater than, less than, or about the same as 1 dam.

The examples below the chart on page 158 review the following concepts.

1. When a larger unit is changed to a smaller unit, there will be more units.
2. When a smaller unit is changed to a larger unit, there will be fewer units.
3. Division by 10 (100, 1000) gives the same result as multiplication by 0.1 (0.01, 0.001).

These concepts can be recalled in a discussion of the example in which 5.4 km is expressed in turn as 54 hm, 540 dam, and 5400 m. Also, the arrows in that example help to show that multiplying by 10 (0.1) three times gives the same result as multiplying by 1000 (0.001).

Working Together: For Ex. 1-9, ask students to explain the use of multiplication or division. For example, for Ex. 2, there will be more units when a larger unit is changed to a smaller unit. The multiplier used is 1000 because 1 km = 1000 m. Similarly, for Ex. 7, a smaller unit (hm) is changed to a larger unit (km); multiplication by 0.1 or division by 10 will produce fewer units.

Exercises: After the students have completed Ex. 1-16, it would be beneficial to have them explain how they found the answers. For Ex. 17 and 18, it will be necessary to express the given lengths in the same unit and then compare the lengths. Ask students to explain their choice of units for the answers to Ex. 19-22.

Assessment

Complete.

1. 83 m = 0.083 km 2. 7.2 m = 720 cm

Give each length in metres.

3. 0.6 dm 0.06 m 4. 930 dam 93 m

Which length is greatest,

5. 8 hm, 745 m, or 7 km? 7 km
6. 5.4 dm, 570 mm, or 0.5 m? 570 mm

Solve.

7. A paper clip is 32 mm long. If 300 clips were placed end to end in a row, how long would the row be in metres? 9.6 m

Pages 160-161

LESSON OUTCOME

Estimate capacity in millilitres and in litres; express millilitres as litres and litres as millilitres; choose the millilitre or the litre as the better unit for measuring capacity

Materials

a variety of containers that hold 1 L, more than 1 L, and less than 1 L; measuring cups marked in litres and millilitres; water

Vocabulary

capacity, kilolitre, kL, hectolitre, hL, decalitre, daL, decilitre, dL, centilitre, cL

Prerequisite Skills

Multiply by 1000; divide by 1000; multiply by 0.001

Checking Prerequisite Skills

Multiply each number by 1000.

1. 6.792 6792 **2.** 38 38 000 **3.** 2 2000

Divide each number by 1000.

4. 7500 7.5 **5.** 410 0.41 **6.** 60 0.06

Multiply each number by 0.001.

7. 7500 7.5 **8.** 10 0.01 **9.** 550 0.55

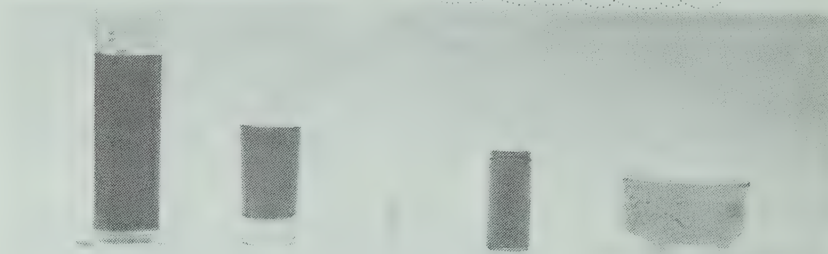
Capacity

The litre and the millilitre are the most common units for measuring **capacity**.

1 L = 1000 mL

1 mL = 0.001 L

When multiplying by 1000, each digit moves three places to the left. When dividing by 1000, or multiplying by 0.001, each digit moves three places to the right.



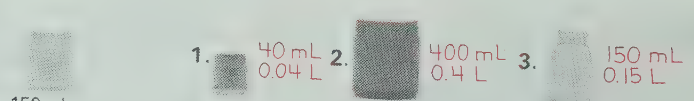
400 mL 10 mL 200 mL 1 L

1.5 L
= 1.5 × 1000 mL,
or 1500 mL

200 mL
= 200 × 0.001 L,
or 0.2 L

Working Together

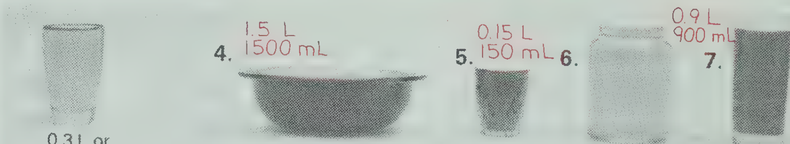
Estimate the capacity in millilitres.
Then give your estimate in litres. Answers will vary.



150 mL or 0.15 L

1. 40 mL 0.04 L **2.** 400 mL 0.4 L **3.** 150 mL 0.15 L

Estimate the capacity in litres.
Then give your estimate in millilitres. Answers will vary.



0.3 L or 300 mL

4. 1.5 L 1500 mL **5.** 0.15 L 150 mL **6.** 0.9 L 900 mL **7.** 0.4 L 400 mL

LESSON ACTIVITY

Before Using the Pages

- Display a measuring cup that is marked to show 1 L. Ask students to identify containers in the classroom that they think hold about 1 L of water. Use the one-litre measuring cup and water to measure the capacity of each container as it is named.

  Display a container that has a capacity of 4 L, for example, and ask the students to estimate its capacity in litres. Then have a student measure to check the estimates. Repeat the procedure for several other containers.
- Refer to the previous lesson and ask what unit names a length that is one-thousandth of a metre. Then ask what unit would name one-thousandth of a litre, to elicit the term *millilitre*. Ask how many millilitres would fill a one-litre container. Have students refer to containers that hold less than 1 L for estimating and measuring in millilitres in a way similar to that suggested in the preceding activity.

Using the Pages

- Ask a student to read the title and the statement below it. Associate the word *capacity* with the amount a container holds. Note the symbols L and mL, and discuss the two ways of expressing the relationship between the units *litre* and *millilitre*. Draw attention to the photograph for the example. The cylinder at the left is filled to the one-litre mark. Note the capacities shown for the other containers in the photograph. Compare these containers with the one-litre cylinder in the photograph and with the containers used in the measuring activities of *Before Using the Pages*.

  Discuss the example provided for expressing 1.5 L as millilitres. Because a larger unit is changed to a smaller unit, there will be more units; the multiplier is 1000 because 1 L = 1000 mL.

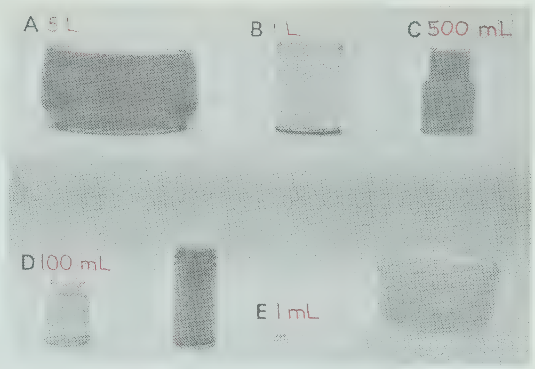
  For the example in which 200 mL is expressed as litres, a smaller unit is being changed to a larger unit, and therefore there will be fewer units; the multiplier is 0.001 because 1 mL = 0.001 L. Draw attention to the information in the "thought cloud" above the photograph. Then

Exercises

For the objects in the picture,

1. choose the best estimate for the capacity of each.

1 mL	100 mL
500 mL	1 L
5 L	



Copy and complete the chart.

2.

Millilitres	5000	500	4	25 ?	? 4800	? 700	250	30 ?
Litres	5 ?	0.5?	? 0.004	0.025	4.8	0.7	? 0.25	0.03

Which is a better unit for measuring the capacity of each of these, millilitre or litre?

3. a car gasoline tank litre
4. a soup can millilitre
5. a washing machine litre

Look around. Find 10 objects that could hold water.

If possible, measure to check your estimate.

6. Estimate the capacity of each.
Answers will vary

The names for different units of capacity use "litre".

Unit	Symbol	Relation to litre
kilolitre	kL	1000 L
hectolitre	hL	100 L
decalitre	daL	10 L
litre	L	1 L

Unit	Symbol	Relation to litre
litre	L	1 L
decilitre	dL	0.1 L
centilitre	cL	0.01 L
millilitre	mL	0.001 L

What is the total capacity of a container that holds

1. 1.2 kL, 2.6 L, 4.8 daL, and 3.5 hL? 1600.6 L, 160.06 daL, 16.006 hL, or 1.6006 kL
2. 17 dL, 21 cL, 40 mL, and 1 L? 2950 mL, 295 cL, 29.5 dL, or 2.95 L

try this

161

RELATED ACTIVITIES

• Provide containers marked in litres and millilitres, several other containers marked A, B, C, and so on, and water, sand, rice, or grain. For each container, have students decide whether to measure in litres or in millilitres, estimate and then measure the capacity in the selected unit. Have them write the results in a chart.

Container	Estimate	Measurement
A	250 mL	200 mL

• Have students collect such containers as boxes, cans, cartons, jars, and so on, for which the capacity is clearly marked in millilitres or in litres. These may be used for different activities as follows.

1. Prepare a display using the labels of the containers to show items that are sold by capacity in millilitres and in litres.
2. Have students express an amount shown in litres as millilitres and vice versa.
3. Have students arrange a set of containers in order of capacity from least to greatest.
4. Write word problems about some of the containers and have students solve the problems by using one or more of the operations addition, subtraction, multiplication, and division.

ask the students for another way to change millilitres to litres. Lead them to suggest that the number of millilitres can be divided by 1000 because dividing by 1000 and multiplying by 0.001 give the same result.

Working Together: In each of these two photographs, the capacity of the container at the left is given as a referent to help the students to estimate. Begin by asking the students to write an estimate in millilitres for Ex. 1 and to express the estimate in litres. When this has been done, state the actual capacity of the container. Providing the actual capacity for Ex. 1 at this time can help the students to estimate the capacity for Ex. 2.

Exercises: To help the students to estimate, two of the containers from the photograph at the top of page 160 are shown again in the photograph on page 161. Have the students compare each new container on page 161 with the previous ones and choose the best estimate for its capacity.

Try This: The units of capacity in these charts should be compared with the units of length in the chart at the top of page 158. The same prefixes are now seen with the word "litre". Have students read the name of each unit, describe how the unit is related to the litre, and note the symbol for the unit. For Ex. 1 and 2, the addends must be expressed in the same unit of capacity before addition can be performed. The students can select one unit in each exercise and express the other three amounts in that unit before adding.

Assessment

Complete the chart.

1.

Millilitres	800	950	470	1260
Litres	0.8	0.95	0.47	1.26

Which is the better estimate for measuring the capacity of each of these, millilitre or litre?

2. a thimble millilitre
3. a kitchen sink litre

Choose the best estimate for

1 mL
100 mL
1 L

4. the amount of soup that fills a soup ladle. 100 mL
5. the amount of ink in a pen. 1 mL

LESSON OUTCOME

Estimate mass in grams and in kilograms; express grams as kilograms and kilograms as grams; choose the gram or the kilogram as the better unit for measuring mass

Materials

several masses of 1 g and several of 1 kg, other masses from 1 g to 1 kg, balance scales and such objects as pencils, books, nails, corks, and stones

Vocabulary

mass, hectogram, hg, decagram, dag, decigram, dg, centigram, cg

Prerequisite Skills

Multiply by 1000; divide by 1000; multiply by 0.001

Checking Prerequisite Skills

Multiply each number by 1000.

1. 4.37 4370 **2.** 6 6000

Divide each number by 1000.

3. 28 0.028 **4.** 59 000 59

Multiply each number by 0.001.

5. 28 0.028 **6.** 340 0.34

Mass

The kilogram and the gram are the most common units for measuring **mass**.

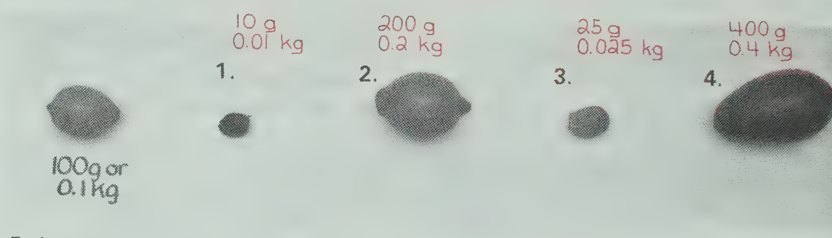
1 kg = 1000 g
1 g = 0.001 kg

2 kg
= 2 × 1000 g,
or 2000 g

50 g
= 50 × 0.001 kg,
or 0.05 kg

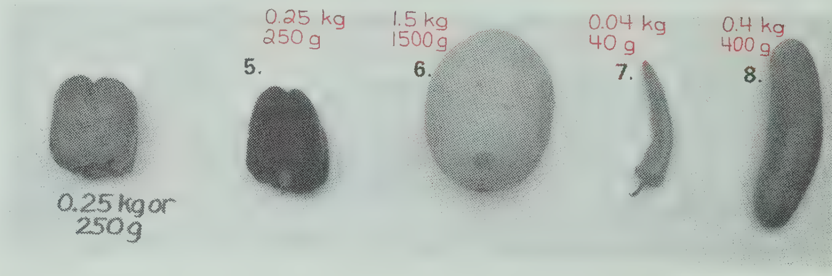
Working Together

Estimate the mass in grams.
Then give your estimate in kilograms. Answers will vary.



1. 10 g 0.01 kg **2.** 200 g 0.2 kg **3.** 25 g 0.025 kg **4.** 400 g 0.4 kg

Estimate the mass in kilograms.
Then give your estimate in grams. Answers will vary.



5. 0.25 kg 250 g **6.** 1.5 kg 1500 g **7.** 0.04 kg 40 g **8.** 0.4 kg 400 g

162

LESSON ACTIVITY

Before Using the Pages

- Display several one-kilogram masses and give each student an opportunity to hold one. Review that the word *mass* is associated with finding how heavy an object is. Ask a student to name an object in the classroom that has a mass of about 1 kg. Then use the balance scales to measure the mass of the object.

  Display an object that has a mass of 3 kg, for example, and ask the students to estimate its mass in kilograms. Then have a student measure to check the estimates. Repeat this for several other objects.

  Review that the term *kilometre* names a length of 1000 m. Ask what meaning can be given to the term *kilogram*, and lead the students to suggest 1000 g. Display several one-gram masses and give each student an opportunity to hold one. Provide objects having a mass less than 1 kg for estimating and measuring in grams, using the procedure described above.

Using the Pages

- Ask a student to read the title and the statement below it. Note the symbols kg and g, and discuss the two ways of expressing the relationship between the units *kilogram* and *gram*.

  The photograph at the top of page 162 shows a one-kilogram mass and different foods for which the masses are given. Compare the one-kilogram mass with the one used in *Before Using the Pages*. Have students identify the foods shown and compare their masses with those of objects measured earlier.

  Direct the students' attention to the example in which 2 kg is expressed as grams. Explain that there are more units because a larger unit is being changed to a smaller unit. The number of kilograms is multiplied by 1000 because 1 kg = 1000 g.

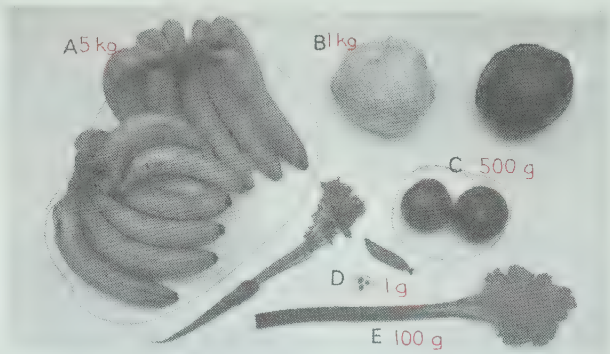
  For the example in which 50 g is expressed as kilograms, there will be fewer units because a smaller unit is being changed to a larger unit. The multiplier is 0.001 because 1 g = 0.001 kg. Ask the students for another way to

Exercises

For the objects in the picture,

1. choose the best estimate for the mass of each.

1 g	100 g
500 g	1 kg
5 kg	



Complete the chart.

2.

Grams	2000	? 5000	500	75 ?	8	? 8600	300?	7264	90?
Kilograms	2 ?	5	0.5?	0.075	? 0.008	8.6	0.3	? 7.264	0.09

Give the better unit, gram or kilogram, for measuring the mass of each of these.

3. yourself kilogram
4. a hockey puck gram
5. a loaf of bread gram

Look around. List 10 different objects.

6. Estimate the mass of each. If possible, measure to check your estimate.
Answers will vary

The names for different units of mass use "gram".

Unit	Symbol	Relation to gram
kilogram	kg	1000 g
hectogram	hg	100 g
decagram	dag	10 g
gram	g	1 g

Unit	Symbol	Relation to gram
gram	g	1 g
decigram	dg	0.1 g
centigram	cg	0.01 g
milligram	mg	0.001 g

A tonne (t) is a unit used for very large masses. 1 t = 1 000 000 g = 1 000 kg

try this

Try to think of three objects measured in
Answers will vary
1. milligrams.
2. tonnes.

163

RELATED ACTIVITIES

- Some students may benefit from using such non-standard units as coins, washers, and nails to measure the mass of an object such as a stone, a book, or a shoe.
- Provide balance scales, masses, and objects marked A, B, C, and so on. For each object, have students decide whether to determine its mass in kilograms or in grams. Have them estimate the mass, measure the mass, and write the results in a chart.

Object	Estimate	Measurement
A	500 g	400 g

- Have the students play the game "Inventory" described on page T 380 for masses they would measure in kilograms and then for masses they would measure in grams.
- Adapt the second activity on page T 175 for units of mass.
- Provide an opportunity for each student to make a ball of plasticine that has a mass of 1 g. Have them use balance scales to measure the mass of each ball, adjusting its size until it has a mass of 1 g. This can be repeated for 1 kg and for other units of mass named on page 163.

change grams to kilograms. Lead them to suggest that the number of grams can be divided by 1000 to express grams as kilograms, because dividing by 1000 gives the same result as multiplying by 0.001. Ask students which method is easier to use — multiplying by 0.001 or dividing by 1000.

Working Together: In each of these two photographs the mass of the item at the left is given as a referent to help the students to estimate. Begin by asking the students to record an estimate in grams for Ex. 1 and to express the estimate in kilograms. When this has been done, state the actual mass for Ex. 1. Giving the students the actual mass at this time can help them to estimate the mass for the next exercise.

Exercises: Items from the photograph at the top of page 162 are included in the photograph on page 163 as referents. Have the students compare each new item on page 163 with the previous ones. For Ex. 6, encourage them to think of objects in the classroom and outside the classroom. Provide balance scales and masses to enable the students to check their estimates by measuring.

Try This: The units of mass in these charts should be compared with the units of length in the chart at the top of page 158 and with the units of capacity in the charts at the bottom of page 161. Review the meaning of the prefixes and point out that they are used here with the word "gram". Review the term *tonne*, the symbol t, and the relationships among tonnes, grams, and kilograms.

Assessment

Complete the chart.

1.

Grams	6300	9	510	3250
Kilograms	6.3	0.009	0.51	3.25

Give the better unit, gram or kilogram, for measuring the mass of each of these.

2. a table kilogram
3. a pencil gram

Choose the best estimate for

4. the mass of a cat. 1 kg
5. the mass of a box of crayons. 100 g

1 g
100 g
1 kg

Pages 164-165

LESSON OUTCOME

Convert between measurements for volume and measurements for capacity; find the capacity in litres of containers for which the dimensions are given in centimetres

Materials

decimetre cube, ruler marked in decimetres and in centimetres, metre sticks (optional), materials indicated for the experiment described in the *Try This* feature

Prerequisite Skills

Find the volume of a rectangular prism; convert among centimetres, decimetres, and metres; convert among millilitres, litres, and kilolitres

Checking Prerequisite Skills

Find the volume for each rectangular prism.

1. base: 20 cm by 5 cm; height: 6 cm 600 cm^3
2. base: 7 cm by 7 cm; height: 7 cm 343 cm^3

Complete.

3.	9000 cm	900 dm	90 m
4.	32 000 cm	3200 dm	320 m
5.	500 cm	50 dm	5 m

6. 8500 mL = ▒ L 8.5
7. 1.4 L = ▒ mL 1400
8. 3 kL = ▒ L 3000
9. 700 L = ▒ kL 0.7

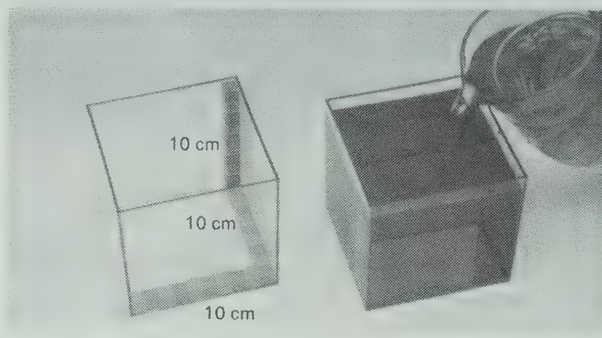
Volume and Capacity

Volume is the measure of the space taken by an object. It is given in cubic units.

This box takes 1000 cm^3, or 1 dm^3, of space.

Capacity is the measure of how much an object holds. The names for units of capacity use "litre".

1000 mL, or 1 L, will fill this box.



The space taken by 1000 cm^3, or 1 dm^3, can be filled with 1000 mL, or 1 L.

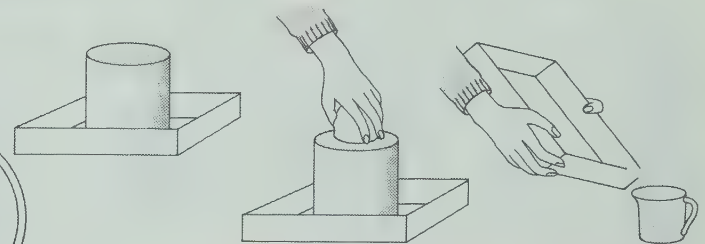
Think of a cube with 10 dm along each edge.

Volume	Capacity
1 cm^3 or 0.001 dm^3	1 mL or 0.001 L
1000 cm^3 or 1 dm^3	1000 mL or 1 L
1000 dm^3 or 1 m^3	1000 L or 1 kL

Here is a way to find the volume of an object, such as a rock.

1. Prepare for the experiment.
2. Place a rock in the water.
3. Measure the water that overflows.

try this



4. What will the amount of overflow show?
The overflow will show the volume of the object

164

LESSON ACTIVITY

Before Using the Pages

- Display several different containers for which the capacity is 1 L, such as a plastic ice-cream tub, a soft-drink bottle, and a measuring cup. One container should be an unmarked decimetre cube. Ask students to help measure the capacity of each container to note that it holds 1 L or 1000 mL. Then remove all the containers except the decimetre cube. Ask students to measure its length, width, and height to establish that the shape is a cube. The dimensions may be expressed in centimetres and in decimetres. Have the students determine the volume of the cube in cubic centimetres and in cubic decimetres. Summarize the results on the board, using the terms *volume* and *capacity*.

Volume	Capacity
1 dm^3 or 1000 cm^3	1 L or 1000 mL

Develop that 1000 mL of water would fill any container having a volume of 1000 cm^3, for example, a rectangular prism for which the base is 20 cm by 10 cm and the height is 5 cm. Ask how many millilitres of water would fill a centimetre cube. Have students explain their answers.

Review that each edge of a metre cube is the same length as 10 dm. Develop that the volume is 1 m^3 or 1000 dm^3. Ask how many litres of water would fill such a container.

Using the Pages

- Ask students to read the statements to note what is meant by volume and capacity and to note the corresponding units of measurement.

Direct the students' attention to the photograph and compare the decimetre cube in the photograph with the one suggested in *Before Using the Pages*. Ask students to read the statements above the photograph to review the volume and the capacity of the cube.

The chart below the photograph summarizes relationships between volume and capacity. Ask which line of the chart shows the relationship for the container in the

Working Together

How many millilitres, or litres, will fill

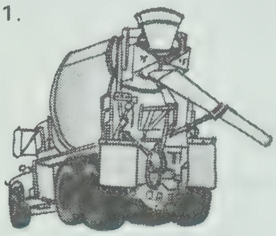
1. 3500 cm³? 3500 mL 3.5 L
2. 2 dm³? 2000 mL 2 L

How many cubic centimetres, or cubic decimetres, are needed

3. for 1800 mL? 1800 cm³ 1.8 dm³
4. for 7.6 L? 7600 cm³ 7.6 dm³

Exercises

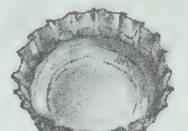
Give the volume of each of these in two ways.

1. 

6 kL 6000 dm³ or 6 m³

2. 

1.2 L 1200 cm³ or 1.2 dm³

3. 

3 mL 3 cm³ or 0.003 dm³

Give the capacity of each of these in two ways.

4. 

91 m³ 91 000 L or 91 kL

5. 

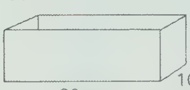
1.5 dm³ 1500 mL or 1.5 L

6. 

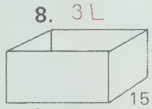
100 cm³ 100 mL or 0.1 L

Give the capacity in litres for each of these.

7. 3 L



8. 3 L



9. 2.5 L

165

RELATED ACTIVITIES

- Give the students work sheets with charts similar to the following. After they have completed the work sheets, ask the students to find patterns in the charts.

Volume	Capacity
1 cm³	____ mL
____ cm³	900 mL
45 cm³	____ mL
36 dm³	____ L
____ dm³	4.21 L
96 m³	____ kL
____ m³	784 kL
824 cm³	____ L
____ cm³	6 L
1273 dm³	____ kL
____ dm³	3.5 kL

- Have students estimate and measure the volume of each of several objects, such as rocks or paperweights, using the method shown in the *Try This* feature on page 164.
- Provide boxes for the students to measure the dimensions to the nearest centimetre, find the volume, and then find the capacity. If a waterproof container similar to the one shown in the photograph on page 164 is available, have the students calculate and then measure the capacity, using water and a container marked in millilitres.

photograph, noting that the volume and the capacity are given in two ways. Help students understand the other relationships given in the chart. For example, because each edge of a centimetre cube may also be expressed as 0.1 dm, the volume may be found by multiplying 0.1 by 0.1 by 0.1 to obtain 0.001 dm³. Similarly, each edge of a metre cube has a length of 10 dm and the volume may be given as 1000 dm³. Review the relationships 1 mL = 0.001 L, 1000 mL = 1 L, and 1000 L = 1 kL.

Working Together: For these exercises, each answer is to be expressed in two ways. Students may refer to the chart on page 164 for assistance.

Exercises: The illustrations for Ex. 1-6 can help students to relate these measurements to objects in their environment. Because there is more than one way to complete each exercise, you may wish to ask a few students to describe the procedure they used. For Ex. 5, for example, 1.5 dm³ may be expressed as 1.5 L, which may then be expressed as 1500 mL; an alternative approach would be to think of 1.5 dm³ as 1500 cm³ and express this first as 1500 mL and then as 1.5 L. For Ex. 7-9, students are to use multiplication to find the volume, which can then be used to determine the capacity.

Try This: When a container is filled with water and an object is then placed in the container, the amount of water that overflows indicates the volume of the object. That is, the object takes the place of an equal volume of water. The experiment is suggested here as a method for finding the volume of an irregular object.

Assessment

Give the volume of each of these in two ways.

1. 3 kL: 3000 dm³ or 3 m³ **2.** 8 L: 8000 cm³ or 8 dm³

Give the capacity of each of these in two ways.

3. 42 m³: 42 000 L or 42 kL **4.** 250 cm³: 250 mL or 0.25 L

Give the capacity in litres for each rectangular prism.

5. Base is 10 cm by 10 cm. Height is 15 cm. 1.5 L

6. Base is 40 cm by 50 cm. Height is 10 cm. 20 L

LESSON OUTCOME

Convert among measurements for volume, capacity, and mass of water; find the mass in grams and in kilograms of water held by containers for which the dimensions are given in centimetres; solve related word problems

Materials

balance scales, masses from 1 g to 1 kg, two identical boxes, an object for which the mass is to be measured, two identical one-litre containers, water

Prerequisite Skills

Convert between measurements for volume and capacity; find the volume of a rectangular prism

Checking Prerequisite Skills

Give the volume for each of these capacities in two ways.

1.	16 mL	16 cm^3	0.016 dm^3
2.	7 kL	7000 dm^3	7 m^3

Give the capacity for each of these volumes in two ways.

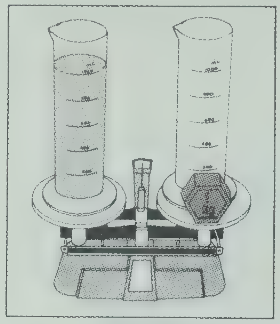
3.	2.8 m^3	2800 L	2.8 kL
4.	9 dm^3	9000 mL	9 L

Find the volume of the following rectangular prism.

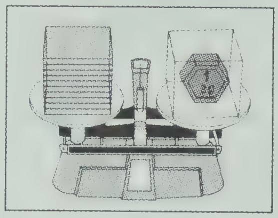
5. Base is 10 cm by 8 cm.
Height is 4 cm. 320 cm^3

Volume, Capacity, and Mass

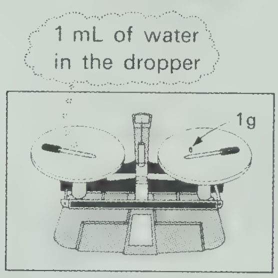
The mass of water is related to capacity and volume.



1 L of water has a mass of 1 kg.



1000 cm^3, or 1 dm^3, of water has a mass of 1 kg.



1 mL, or 1 cm^3, of water has a mass of 1 g.

Volume	Capacity	Mass (of water)
1 cm^3 or 0.001 dm^3	1 mL or 0.001 L	1 g or 0.001 kg
1000 cm^3 or 1 dm^3	1000 mL or 1 L	1000 g or 1 kg
1 000 000 cm^3 or 1000 dm^3 or 1 m^3	1 000 000 mL or 1000 L or 1 kL	1 000 000 g or 1000 kg or 1 t

Working Together

Complete.

	Volume	Capacity	Mass (of water)
1.	500 cm^3	500 mL	500 g
2.	3 cm^3	3 mL	3 g
3.	870 cm^3	870 mL	870 g
4.	3 dm^3	3 L	3 kg
5.	4.6 dm^3	4.6 L	4.6 kg
6.	0.8 dm^3 or 800 cm^3	0.8 L or 800 mL	0.8 kg or 800 g

166

LESSON ACTIVITY

Before Using the Pages

- Have students help to find the mass of an object by placing the object on one side of the balance scales and appropriate masses on the other side. Ask a student to determine the mass of the object. Then place the object in a box and place the box on one side of the scales, leaving the same masses on the other side. Ask the students why the scales are no longer balanced. Develop that one side was made heavier by including the box, whereas the other side was not changed. Place an identical box on the lighter side of the scales to restore balance.
- Display two identical one-litre containers, one of which is filled with water. Ask the students how to find the mass of one litre of water. The procedure in the previous activity will lead the students to suggest placing both containers on different sides of the scales and masses on the lighter side to give balance. Have students help to perform the experiment. Summarize that 1 L of water has a mass of 1 kg.

Using the Pages

- Ask a student to read the title and the statement below it. Remind the students that the relationship between capacity and volume is presented on pages 164 and 165. Emphasize that for water, the mass is also related to the capacity and the volume. Ask students to explain what is shown in the first illustration. Since a cylinder is placed on each side of the balance scales, the cylinders do not affect the balance, therefore the water and the 1 kg mass have equal masses.

The second illustration shows that 1 dm^3 of water has a mass of 1 kg. Review that a volume of 1000 cm^3, or 1 dm^3, is the same as the capacity of 1 L. Ask why two decimetre cubes are shown.

The third illustration shows that 1 mL, or 1 cm^3, of water has a mass of 1 g. Point out that there is 1 mL of water in the dropper, and remind the students that 1 mL of water has a volume of 1 cm^3. Discuss that the droppers balance each other on the scales and that the 1 mL of water is balanced by the one-gram mass.

Discuss the chart showing the relationships for volume,

Exercises

Complete.

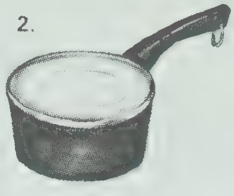
1\.

350 mL

350 g or 0.35 kg

350 cm^3 or 0.35 dm^3

2\.

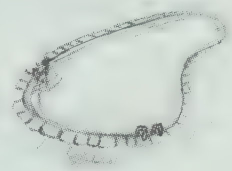


1.5 L

1500 g or 1.5 kg

1500 cm^3 or 1.5 dm^3

3\.



86 kL

86 000 kg or 86 t

86 000 dm^3 or 86 m^3

4\.

15 g

15 mL or 0.015 L

15 cm^3 or 0.015 dm^3

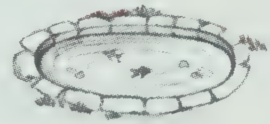
5\.

3 kg

3000 mL or 3 L

3000 cm^3 or 3 dm^3

6\.



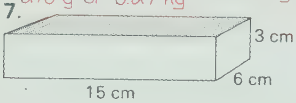
2 t

2000 L or 2 kL

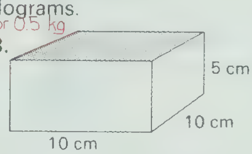
2000 dm^3 or 2 m^3

Give the mass of the water in each of these containers in grams and in kilograms.

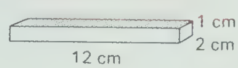
7\. 270 g or 0.27 kg



8\. 500 g or 0.5 kg



9\. 24 g or 0.024 kg



Solve.

10\. The mass of a volume of milk is about 1.03 times the mass of an equal volume of water. About how heavy is 1500 mL of milk? 1545 g

11\. A balloon and the water inside had a mass of 375 g. The mass of the balloon was 12 g. How many millilitres of water were in the balloon? 363

167

RELATED ACTIVITIES

- Adapt the first activity on page T 179 so that a column for the mass of water is included in the chart.
- If waterproof containers are available, extend the third activity on page T 179 and have the students calculate the mass of the water and then measure to check.
- Word problems related to balance scales may be assigned, for example, "Five identical books are placed on balance scales, three on one side and two on the other. When a mass of 100 g is placed with the two books, the scales balance. What is the mass of each book?" Students may be challenged to write and solve similar problems.

capacity, and mass of water. Refer to the illustrations to help explain the first two rows of the chart. You may wish to have the students compare this chart with the one on page 164.

Working Together: For Ex. 1, the students can find the capacity and the mass for 500 cm^3 of water, or they can find the capacity from the volume and use it to find the mass. Similarly, for each of the other exercises, the missing numbers can be found in more than one way.

Exercises: The illustrations for Ex. 1-6 can help students to relate the volume, the capacity, and the mass of water to familiar objects. The missing numbers can be found in more than one way. After the students have completed Ex. 1-6, read various measurements obtained and ask whether they name the volume, the capacity, or the mass of the water.

Assessment

Complete.

1\. 80 mL
80 g or 0.08 kg
80 cm^3 or 0.08 dm^3

2\. 3.5 kL
3500 kg or 3.5 t
3500 dm^3 or 3.5 m^3

3\. 26 g
26 mL or 0.026 L
26 cm^3 or 0.026 dm^3

4\. 4 kg
4000 mL or 4 L
4000 cm^3 or 4 dm^3

Give the mass of water held by each rectangular prism in grams and in kilograms.

5\. Base is 8 cm by 5 cm. Height is 10 cm. 400 g 0.4 kg

6\. Base is 10 cm by 6 cm. Height is 3 cm. 180 g 0.18 kg

Solve.

7\. The mass of a volume of sea water is about 1.05 times the mass of an equal volume of fresh water. About how heavy is 5000 mL of sea water? 5250 g

LESSON OUTCOME

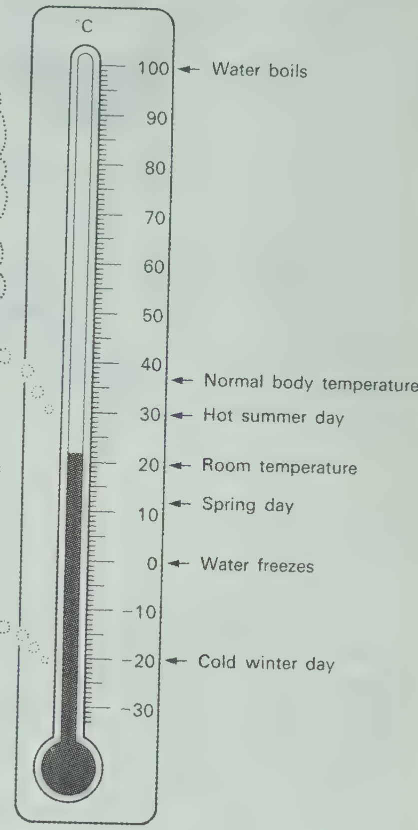
Relate temperatures in degrees Celsius to activities; read a thermometer

Materials

a demonstration thermometer (optional); a large sheet of paper (optional); a copy of page T 397 for each student

Temperature

Match sports and other activities with temperatures in degrees Celsius.



Working Together

Give two outdoor work activities for each temperature. Answers will vary.

1. 0°C 2. 10°C 3. 20°C 4. 30°C

Read the thermometer to give the temperature

5. for a hot summer day. 30°C

168

LESSON ACTIVITY

Before Using the Pages

- Begin by asking the students to describe today's weather. Ask if they would describe the day as being cold, cool, warm, or hot. Ask what device is used to measure temperature and have them give examples of temperatures. They may have heard these from radio or television, read them in a newspaper, or read a thermometer at home or at school. Ensure that they always say "degrees Celsius" in stating temperatures.

Using the Pages

- Begin with a discussion of the illustration of a thermometer. Ask what is significant about the temperatures 0°C and 100°C. Ask students to read the temperatures suggested for a cold winter day, a spring day, and so on. (The temperature − 20°C is read "twenty degrees below zero Celsius" or "minus twenty degrees Celsius".) Note that normal body temperature is 37°C. Review that the red liquid in a thermometer rises (falls) as the weather becomes warmer (cooler).

  Ask a student to read the statement below the title. Have students suggest activities for a few temperatures that you name. The illustrations can suggest ideas for the activities.

Working Together: You may wish to record each of the temperatures for Ex. 1-4 at the top of a large sheet of paper and have the students list several activities for each. These lists can be displayed for several days to help the students to relate the temperatures and the activities.

Exercises: After the students have completed their work, have them share their answers for Ex. 1-5 to make them aware of many activities for each temperature. Ask the students to refer to the thermometer on page 168 for Ex. 6-11. You may wish to locate the places named in Ex. 12-14 on the map of Canada on page 170.

For Ex. 15, some students may choose to draw a bar graph and others may decide to draw a broken-line graph to show Sandra's temperature. Provide each student with a

Exercises

Name two outdoor play activities for each temperature. Answers will vary

1. −10°C 2. 5°C 3. 12°C 4. 20°C 5. 30°C

Read the thermometer to give the temperature

6. at which water boils. 100°C
7. at which water freezes. 0°C
8. for a spring day. 12°C
9. for a normal body. 37°C
10. for room temperature. 20°C
11. for a cold winter day. −20°C

Match each place with a January temperature.

−28°C	−18°C	−4°C

12. Winnipeg, Manitoba −18°C
13. Yellowknife, N.W.T. −28°C
14. Windsor, Ontario −4°C

Draw a graph to show A graph is shown on page T 370

15. Sandra's temperature during an illness.

Day	1	2	3	4	5	6	7	8
Temperature	37.0°C	37.6°C	37.9°C	38.0°C	38.2°C	37.5°C	37.2°C	37.0°C

The first Canadian television program was broadcast on September 6, 1952.

This date could be written 1952 09 06.

The first four digits show the year, the next two show the month, and the last two show the day.

Give the month, day, and year for each of these.

1. The date that the first telephone message was sent in Canada was 1876 08 10. August 10, 1876
2. The date of the first Canadian airplane flight was 1909 02 23. February 23, 1909

Use 8 digits to show the date for each of these.

3. On April 23, 1851, the first postage stamp in Canada was issued. 1851 04 23
4. The first Canadian satellite was put into orbit on September 29, 1962. 1962 09 29
5. On March 23, 1752, the first newspaper in Canada was printed. 1752 03 23

try this

169

RELATED ACTIVITIES

• Label several containers A, B, C, and so on, and fill them with water at different temperatures. Placing different amounts of ice with the water in some of the containers will help to vary the temperatures. Have the students estimate the temperature of the water, measure the temperature, and write the results in a chart.

Container	Estimate	Measurement
A	20°C	25°C

• The temperatures for several places in Canada on a specific day can be found in many newspapers. Students may draw a bar graph to show the temperatures. They may also write the temperatures with the names of the places on a map of Canada. This activity may be adapted for temperatures in various parts of the world. Differences in temperatures of places on a map can lead to discussions concerning the causes. If the class is divided into small groups, each group can choose a different day. Displaying the graphs and the maps will help the students to learn about changes in temperature.

• Have the students record dates, such as their birthdays, dates that they have studied in history, or dates that are important in their community, in the two ways shown in the *Try This* feature on page 169.

copy of page T 397 on which to draw a graph. It may be necessary to review the procedure for drawing such graphs. Refer the students to pages 40, 41, 46, and 47 in Unit 3.

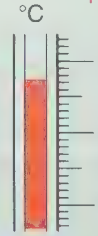
Try This: This feature presents *numeric dating*: for any date, the year is named first, followed by the month, and then the day. Because two digits must be used to show the month and two to show the day, eight digits are used in all. You may wish to have the students write today's date in two ways. To help the students to remember the second way, point out that the year, the longest unit of time in the sequence, is shown first, then the month, the next longest unit of time, and then the day is shown. You may wish to mention the fact that the numerals in numeric dating can be continued to show the hour, the minute, and the second. You may wish to discuss the events in the example and in the exercises and challenge the students to calculate how long ago the events occurred.

Assessment

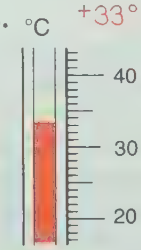
Answers will vary.

Give the temperature and name an outdoor activity for each.

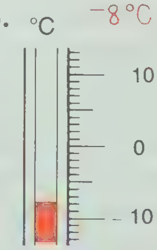
1. °C +7°C



2. °C +33°C



3. °C −8°C



Pages 170-171

LESSON OUTCOME

Write numerals for times to the second using a.m. or p.m. and as shown on 24-hour dial and digital clocks; read times from 24-hour dial and digital clocks; show a given time on dial and digital clock faces; express the time in one time zone in Canada as the time in another time zone in Canada

Materials

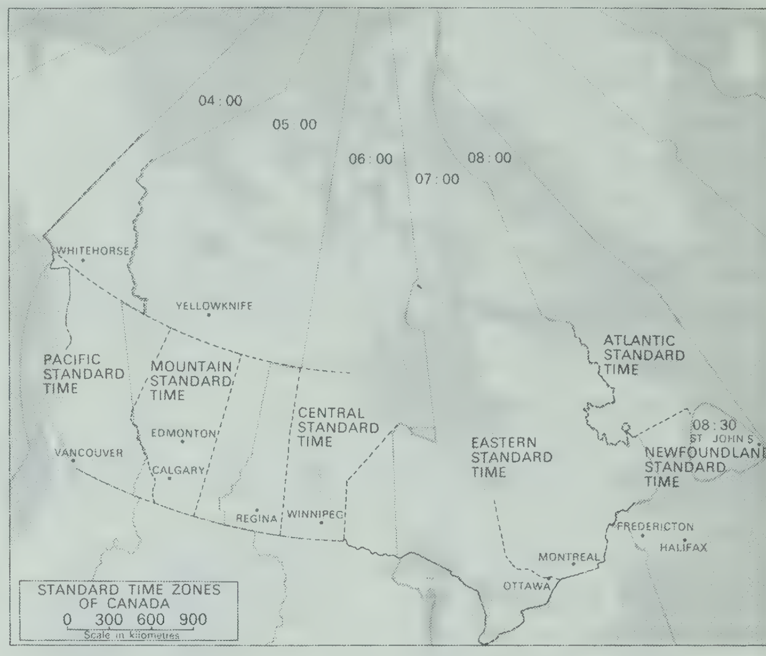
a dictionary for each student

Vocabulary

time zone, Pacific Standard Time, Mountain Standard Time, Central Standard Time, Eastern Standard Time, Atlantic Standard Time, Newfoundland Standard Time, standard time, daylight saving time, decade, century, millennium, fortnight, eon, twinkling, digital clock

Time

This map shows the time in each time zone across Canada when it is 06:00 in Winnipeg.



Standard time is the time used for most of the year.
Daylight saving time is used during the summer in many places.

Answers will vary for Ex. 1 and 2.

1. What is the purpose of daylight saving time?
2. Find out when daylight saving time begins and when it ends.
3. Find out what a.m. and p.m. mean.
   a.m. means ante meridiem (before noon)
   p.m. means post meridiem, (after noon)
4. Decade, century, millennium, fortnight, eon, and twinkling represent lengths of time. Find out what they mean.

PROBLEM SOLVING

4. decade, 10 years   century, 100 years
millennium, any period of 1000 years
fortnight, 2 weeks
eon, an extremely long, indefinite period of time; thousands and thousands of years
twinkling, the very brief time it takes to wink; an instant

170

LESSON ACTIVITY

Before Using the Pages

- Ask a student to read the time shown on the classroom clock to the nearest minute. Ask another student to write the numeral on the board. The numeral will likely be shown as for a 12-hour clock. Ask what the time will be 12 h from now, and write the numeral on the board. Remind the students that it is necessary to indicate which time shown is before noon and which is after noon. Lead them to suggest the use of a.m. and p.m. for the times shown on the board, for example, 9:45 a.m. and 9:45 p.m.

  Ask how many hours are in one day. Refer to the afternoon time shown on the board and ask how many hours and minutes have passed since midnight. For example, for 9:45 p.m., 21 h and 45 min have elapsed. Have a student show another numeral for the afternoon time using this information (21:45). Point out that p.m. is not used because for a 24-hour clock, the number of hours greater than or less than 12 shows whether the time of day is before noon or after noon. Write a few such numerals on the board and have students interpret the times using a.m. or p.m. Note that two digits are used for hours and two for minutes.

Using the Pages

- Begin with a discussion of the map on page 170. Establish which is the west coast of Canada and which is the east coast. Lead the students to realize that because the sun rises in the east and appears to move from east to west, the time is earlier in western Canada than in eastern Canada. Develop that there are different time zones in the world so that each time, for example, 8:00 a.m., refers to the same time in the day for each zone. Ask students to read the names of the time zones and note the lines dividing the zones. Have the students locate Winnipeg on the map and note the time shown for that time zone. Review that 06:00 means 6:00 a.m. Ask questions such as "If it is 6:00 a.m. in Winnipeg, what time is it in Vancouver? in Halifax?" "In which city will the time be one hour earlier than in

Working Together

For the map on page 170, give the time in each of these cities. Show the time on a dial clock. Then show the time on a 24-hour digital clock. Clocks are shown on page T370

1. Calgary 05 : 00 or 5:00 a.m.
2. Montreal 07 : 00 or 7:00 a.m.

The digital clock shows 20 s after 13:54, or 20 s after 1:54 p.m. 13:54:20

What would a dial clock show for each of these times?

3. 04:07:55
4. 15:43:17

Clocks are shown on page T370

Show a 24-hour digital clock

5. for 21 s after 3:45 a.m. 03:45:21
6. for 7 s after 9:30 p.m. 21:30:07

When it is 22:15:30 in Ottawa, what time is it

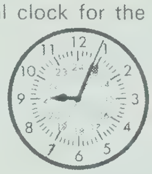
7. in Winnipeg? 21:15:30
8. in Halifax? 23:15:30
9. in Vancouver? 19:15:30

Exercises

Show the time right now to the nearest second in each of these ways. Answers will vary

1. a.m. or p.m.
2. on a dial clock
3. on a 24-hour clock

Draw a digital clock for the time shown on the dial clock

4. when it is morning.  09:04:35
5. when it is afternoon.  14:14:02

When a digital clock shows 02:33:45 in Edmonton, give the time in each of these cities using a.m. or p.m. Then show a dial clock and a 24-hour digital clock for each. Clocks are shown on page T370

6. Whitehorse 45 s after 1:33 a.m.
7. Regina 45 s after 3:33 a.m
8. Yellowknife 45 s after 2:33 a.m
9. Ottawa 45 s after 4:33 a.m.
10. St. John's 45 s after 6:03 a.m
11. Halifax 45 s after 5:33 a.m

"Hockey Night in Canada" begins at 20:00 in Montreal. At what time does it begin

12. in Calgary? 18:00
13. in Fredericton? 21:00
14. in Yellowknife? 18:00

171

RELATED ACTIVITIES

- Have students work in pairs so that as one student shows a time on a demonstration dial clock, the other student states the time shown in each of the ways presented on pages 170 and 171. Reverse this activity and have one student state a time for the other to show on the clock.
- Have each student prepare and display a 24-hour dial clock and a 24-hour digital clock so that each clock shows the same time.
- Have the students complete addition and subtraction exercises involving times. Begin with exercises that do not require regrouping and then extend the work to include exercises for which regrouping is required.

$$\begin{array}{r} 2\text{ d } \ 4\text{ h } 25\text{ min } \ 3\text{ s} \\ +\ 3\text{ d } \ 8\text{ h } 14\text{ min } 36\text{ s} \\ \hline \end{array}$$

$$\begin{array}{r} 5\text{ d } 18\text{ h } 55\text{ min } 18\text{ s} \\ +\ 9\text{ d } 26\text{ h } 17\text{ min } 46\text{ s} \\ \hline \end{array}$$

$$\begin{array}{r} 7\text{ d } 17\text{ h } 45\text{ min } 17\text{ s} \\ -\ 1\text{ d } \ 5\text{ h } 32\text{ min } \ 2\text{ s} \\ \hline \end{array}$$

$$\begin{array}{r} 8\text{ d } \ 2\text{ h } \ 4\text{ min } \ 8\text{ s} \\ -\ 3\text{ d } 20\text{ h } \ 4\text{ min } 27\text{ s} \\ \hline \end{array}$$

- If a globe is available, have students use it to note how the earth is divided into 24 time zones. The relationship between the sun and the earth as it rotates can be demonstrated to help students to understand the differences in time around the world.

Winnipeg? one hour later?" Ask how the Newfoundland time zone differs from the other time zones.

Working Together: Discuss the digits displayed on digital clocks. Discuss the example that shows how hours, minutes, and seconds are recorded on a digital clock. For Ex. 7, have the students refer to the map on page 170, decide whether the time in Winnipeg is earlier than or later than it is in Ottawa, and find the number of hours difference in time.

Exercises: For Ex. 10, the students must realize that the time in St. John's is three and one-half hours later than the time in Edmonton.

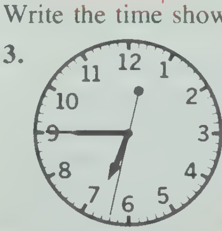
Problem Solving: Introduce the terms *standard time* and *daylight saving time*. Explain that if daylight saving time is used, the clocks are changed in the spring so that the time is one hour later; then they are changed in the autumn so that the time is one hour earlier. The expression "spring forward, fall back" helps to remember how the clocks are changed. Lead the students to realize that, although the clocks are changed, the amount of time remains the same. For Ex. 1, relate the purpose of daylight saving time to the term itself. That is, the amount of daylight time is not changed, but moving the clock one hour forward allows more daylight for the waking hours. Provide each student with a dictionary for Ex. 3 and 4.

Assessment

Write each time using a.m. or p.m.

1. 15:02:28 3:02:28 p.m.
2. 08:51:40 8:51:40 a.m.

Write the time shown on each clock using a.m. or p.m.

3.  the sun is rising 6:45:32 a.m.
4. 21:09:35 9:09:35 p.m.

Show each time on a dial clock and on a 24-hour digital clock.

5. 15 s after 1:20 a.m.
6. 38 s after 5:42 p.m.

Clocks are shown on page T370.

Page 172

OBJECTIVE

Find the information needed to solve a word problem

RELATED ACTIVITIES

- Have students write word problems based on the schedule on page 172 for others to solve. The word problems in the lesson may suggest some ideas. For example, the names of the cities can be changed in Ex. 1-4. The time and the name of the city can be changed for Ex. 6. Remind the students that in order to write a problem that can be answered, they must consider the schedule and the solution for the problem.
- Provide a schedule or several schedules for buses, railroads, and airlines. If possible, find ones that include your community. Ask the students to create word problems based on the schedules for themselves or for others to solve.

Finding the Information Needed

Bus, railroad, and airline schedules are useful to people who travel.

SAINT JOHN – MONTREAL – WINNIPEG
REGINA – CALGARY – BANFF – VANCOUVER

Traveling west: Read down. | Traveling east: Read up.

Kilometres	Example	Time		Daily		Time	Example	Time Zone
0	Thurs.	20:50	Dp	Saint John, N.B.	Ar	09:45	Fri.	Atlantic
763	Fri.	08:35	Ar	Montreal, Que.	Dp	20:05	Thurs.	Eastern
		11:27	Dp		Ar	19:50		"
778		11:33		Dorval, Que.		19:40		"
938		13:35		Ottawa, Ont.		17:55		"
1463		21:15	Ar	Sudbury, Ont.	Dp	10:00		"
	Fri.	22:25	Dp		Ar	09:20	Thurs.	"
2351	Sat.	12:55		Thunder Bay, Ont.		19:10	Wed.	Eastern
3025		20:15	Ar	Winnipeg, Man.	Dp	09:20		Central
	Sat.	20:45	Dp		Ar	08:45		"
3600	Sun.	03:55		Regina, Sask.		01:45	Wed.	Central
4365		12:20	Ar	Calgary, Alta.	Dp	15:35	Tues.	Mountain
		12:55	Dp		Ar	15:00		"
4497	Sun.	15:10		Banff, Alta.		13:00	Tues.	Mountain
5596	Mon.	08:25	Ar	Vancouver, B.C.	Dp	17:45	Mon.	Pacific

This schedule shows the time it takes to travel across Canada by train.

1. For traveling from Saint John, do you catch the train in the morning or in the evening? evening
2. How long does it take to go from Saint John to Montreal? 12 h 45 min
3. How long does it take to go from Regina to Calgary? 9h 25 min
4. How long does it take to go from Calgary to Regina? 9h 10 min
5. Is there much difference in your answers for Exercises 3 and 4? If so, why? Does your reason suggest a change in your answer for these exercises and for Exercise 2? Answers will vary.

Look in the last column of the schedule for a clue.

6. If you leave Thunder Bay on the train going west, where will you be 26 h later? Calgary

PROBLEM SOLVING

172

LESSON ACTIVITY

Before Using the Page

- Have the students help to show the schedule on the board for your class for a particular school day. Ask questions such as "How much time is allowed for mathematics?" Ask what kinds of schedules are useful for people who travel from province to province or from country to country.

Using the Page

- Ask a student to read the title and the statement below it. Allow a few moments for the students to study the schedule. Ask a student to read the statement below the schedule. Note the headings of the columns and the significance of reading down or reading up in the schedule. Ask what is meant by "Ar" and "Dp". Explain that the train stays in some places for such a short time that only one time is shown on the schedule, for example, in Dorval, Quebec. However, because the train stays for a significant time in other places, for example, in Sudbury, both the arrival time and the departure time are shown. Ask why the schedule shows a column for time zones. Have the students refer to the map on page 170 to review the names of the time zones and to note the location of places listed in the schedule. You may wish to point out where Thunder Bay, for example, would be shown on the map. To help the students read the schedule, ask questions such as
"At what time does the train travelling west leave St. John? When does it arrive in Montreal?"
"When does the train travelling east arrive in Calgary?"
"When does the train travelling east leave Calgary?"
"In what time zone is Thunder Bay?"
- Some students may realize how the time zones influence finding the time required to travel from one place to another and use this knowledge when completing Ex. 2-4. Explain to those who have difficulty with Ex. 5 how the time changes when travelling from one time zone to another. Refer to the map on page 170 and ask, "What time is it in Regina when it is 05:30 in Calgary?" Ex. 6 also requires that the students consider the time zones.

Checking Up

Choose the best unit, millimetre, centimetre, metre, or kilometre, for measuring each of these.

1. the edge of a page millimetre
2. the height of a house metre

Choose the better unit, millilitre or litre, for measuring the capacity of each of these.

3. a drinking straw millilitre
4. a cereal bowl millilitre

Choose the better unit, gram or kilogram, for measuring the mass of each of these.

5. a concrete block kilogram
6. a wristwatch gram

Complete.

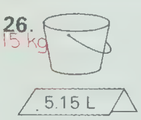
7. 4 m = 400 cm
8. 75 mm = 7.5 cm
9. 130 cm = 1.3 m
10. 8625 mm = 8.625 m
11. 12.3 cm = 123 mm
12. 11 111 m = 11.111 km
13. 500 mL = 0.5 L
14. 1375 cm³ = 1375 mL
15. 6 L = 6000 mL
16. 750 mL = 750 cm³
17. 15 000 cm³ = 15 L
18. 2.5 L = 2500 cm³
19. 2 kg = 2000 g
20. 2700 g = 2.7 kg
21. 0.25 kg = 250 g

Which is greater,

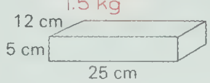
22. 26.1 cm or 2.62 m?
23. 7 km or 750 m?
24. 0.075 L or 705 mL?
25. 1060.3 g or 1.063 kg?

How heavy is the water?

26. 5.15 kg



27. 1500 g or 1.5 kg

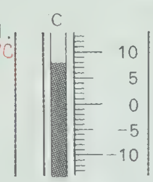


How much water is there?

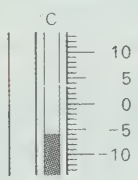
28.	675 g	675 mL	675 cm³
29.	3.5 kg	3500 mL	3500 cm³
30.	4250 g	4.25 L	4250 cm³

Give the temperature.

31. 8°C



32. −6°C



Write the time shown on the clock

33. when it is time for lunch. 12:05:14

173

Page 173

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Students having difficulty expressing one unit of length as another may benefit from completing exercises similar to the following.

1.		2.	
0.3 km		_____ km	
_____ hm		_____ hm	
_____ dam		_____ dam	
_____ m		_____ m	
_____ dm		400 dm	
_____ cm		_____ cm	
_____ mm		_____ mm	

- Have the students find their masses in kilograms and use the information to prepare a bar graph. The information can be organized first in a tally chart to show the number of students in each range for masses of 40 kg – 42 kg, 43 kg – 45 kg, and so on.
- For further work involving time and dates, provide exercises similar to each of the following. Find the amount of time between
  1. 14:28:37 and 16:14:02
  2. 3:48 a.m. and 2:15 p.m.
  3. 11:25 p.m. on Feb. 8 and 1:06 a.m. on Feb. 11
  4. 1908 07 24 and 1974 12 05

Challenge the students to write and solve other similar exercises.

Skills	Exercises	Related Pages
Choose the best unit for measuring length	1, 2	T 172-T 173
Choose the better unit for measuring capacity	3, 4	T 174-T 175
Choose the better unit for measuring mass	5, 6	T 176-T 177
Convert among units for measuring length	7-12	T 170-T 173
Express millilitres as litres and litres as millilitres	13, 15	T 174-T 175
Express grams as kilograms and kilograms as grams	19-21	T 176-T 177
Convert between measurements for volume and measurements for capacity	14, 16-18	T 178-T 179
Compare measurements of length	22, 23	T 170-T 173
Compare measurements of capacity	24	T 174-T 175
Compare measurements of mass	25	T 176-T 177
Convert among measurements for volume, capacity, and mass of water	26-30	T 180-T 181
Read a thermometer	31, 32	T 182-T 183
Read a clock	33	T 184-T 185

Comments

Understanding kinds of measurement and units of measurement is best developed through various activities, such as those suggested in *Related Activities* of this unit.

Students who have difficulty with Ex. 1-6 may find the answer obvious if they are asked to measure or to give a measurement for each object using the different units. Working with rulers, metre sticks, and measuring tapes can help students to remember the units for Ex. 7-12. Using containers and water to measure capacity may help students with Ex. 13-18. Balance scales and masses can be used to reinforce the understanding required for Ex. 19-21.

Note that for Ex. 22-25, the students must use one unit to express both measurements in order to compare the numbers.

Unit 9 Overview

Geometry

This unit begins with a study of lines, line segments, and rays and proceeds to a study of angles, polygons, and solids. Line symmetry is examined in work with polygons. The names of different solids are developed from their features, including the shapes of their end faces, or bases. The lengths of sides are considered in classifying three kinds of triangles. Parts of a circle are named and the relationship of circumference to diameter is explored. Congruent shapes are identified by tracings, and similar shapes are studied on grids. Grids are also used to copy pictures, to enlarge and reduce them and to distort them. Scale drawings are made by using grids, and a brief study is made of the scale of a map. Equations are presented as a means of structuring number relationships for the solution of word problems.

Prerequisite Skills

- draw rays
- read a scale
- multiply whole numbers
- multiply decimals and whole numbers

Unit Outcomes

- identify, name, and draw lines, line segments, and rays
- identify, name, and draw lines, line segments, and rays that are intersecting, perpendicular, or parallel
- identify, name, measure, and draw angles
- classify angles as acute, right, obtuse, or straight
- identify and show lines of symmetry
- identify objects having line symmetry
- identify the number of vertices and the number of sides for polygons
- name triangles; classify triangles as equilateral, isosceles, or scalene, according to the number of sides of equal length
- identify the circle and the terms *centre, radius, diameter, chord,* and *circumference*
- measure radii, diameters, and circumferences
- identify congruent shapes
- identify and draw similar shapes
- use a scale to find the distance represented on a map
- draw a diagram according to a given scale
- copy a picture from a square grid onto a grid with squares of a different size and onto grids with rectangles or parallelograms
- identify the number of vertices, edges, and faces of solid shapes; identify the shapes of the faces of solids
- draw a pattern for a given solid
- write an equation for information given in a word problem; solve a problem by writing and solving a related equation

Background

At the elementary level, students obtain most of their geometric concepts through manipulation and observation. In some instances they can make comparisons visually and draw conclusions, while in others they are obliged to use tracings and cutouts. In the course of their studies, they gradually acquire a number of abstract concepts and appropriate terms to identify them. Certain basic concepts underlie all aspects of geometry. Some of these concepts are undefined and basic assumptions are made concerning them.

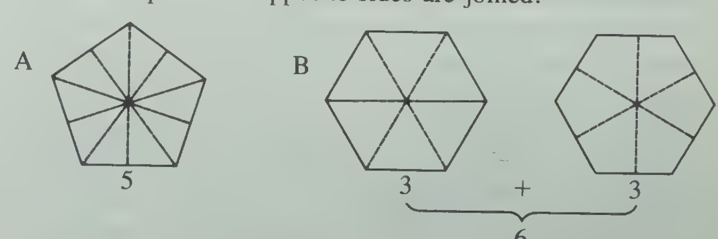
The term *point* is undefined. On the abstract level, a geometric point is a particular location in space. A point is immovable and has no length, width, or thickness. A close approximation to a point is the smallest dot on paper or the point of a pin. Other geometric terms are derived in terms of points. For example, *space* is considered to be the set of all points. A *plane* is the set of all points on a flat surface whose length and width extend endlessly. A *line* is thought of as a set of points forming a straight path that continues without end in opposite directions. A *line segment* is part of a line. It has two end points and includes all the points that lie between them.

The terms encountered in this unit are listed in *Vocabulary* on page T 189. Most of these terms are defined in the glossary on pages 338-341 of the textbook. The aim of this unit is for the students to understand the meaning of each term and to be able to use the term, not to memorize the definitions for the terms.

Polygons are closed figures composed of line segments. Although polygons are often represented by cutouts of paper, wood, or plastic, it is the shape — not the surface — made by the line segments that is the polygon. Models made of straws and pipe cleaners approximate the concepts of polygons more closely than cutouts. The least number of line segments, or *sides*, of a polygon is three. Regular polygons have sides of equal length and angles of equal measure. The most common regular polygons are the equilateral triangle, the square, the regular pentagon, the regular hexagon, and the regular octagon.

If two figures have the same size and shape, they are *congruent*; if they have the same shape but differ in size, they are *similar*. In congruent figures corresponding angles are of equal measure and corresponding sides are of equal length, whereas in similar figures only corresponding angles are of equal measure.

If a figure can be divided into two congruent parts, one the mirror image of the other, the figure has *line symmetry*. Line symmetry is seen in the outstretched wings of a butterfly or in a snowflake. The line that divides the shape into two identical parts is known as the *line of symmetry*. Polygons may be studied for their lines of symmetry. A triangle with three lines of symmetry is an *equilateral* triangle; a triangle with one line of symmetry is an *isosceles* triangle; and a triangle with no line of symmetry is a *scalene* triangle. A square has four lines of symmetry, a rectangle has two, and a rhombus also has two. For any regular polygon, the number of lines of symmetry is the same as the number of sides and thus, the number of vertices. If the number is an odd number (A), a line of symmetry extends from each vertex to the midpoint of the opposite side. If the number is an even number (B), there are two sets of lines of symmetry: for one set, opposite vertices are joined; for the other set, the midpoints of opposite sides are joined.



Some three-dimensional figures, or *solids*, consist of polygons sharing sides. Again, it should be emphasized that it is the shape, not the enclosed space, to which the term *solid* refers. In

a solid, two faces meet to form an edge, and three or more edges meet to form a vertex.

The least number of faces of a solid is four, and this shape is called a *triangular pyramid* (see page 340). A *pyramid* has one face as its base and the other faces meet at one point. These other faces are all triangles. The shapes of their bases give the names to pyramids, such as *square pyramid*, *rectangular pyramid*, *hexagonal pyramid*.

Another type of a solid is the *prism* which has two congruent parallel faces and three or more other faces which are rectangles or parallelograms (see page 340). It is the shapes of the bases that give the descriptive names to prisms, such as *triangular prism*, *rectangular prism*, *pentagonal prism*. A prism with each of its six faces a square is called a *cube*.

Solids which have circular faces and curved surfaces are the *cylinder*, the *cone*, and the *sphere*.

One approach to developing skills in problem solving involves the use of equations. The structure of the situation is translated into an equation by using the proper mathematical symbols. Numerals are used to represent what is known, and frames or letters are used to represent what is unknown. The inverse relationships between addition and subtraction, and between multiplication and division are used frequently in solving equations.

addend + addend = sum	factor × factor = product
sum − addend = addend	product ÷ factor = factor

Solving for an unknown in an equation may be achieved directly or indirectly. In the latter case, either the same operation is used with a rearrangement of the known quantities, or an inverse operation is used.

Directly	Indirectly	
$24 + 27 = n$	$78 - n = 33$	$6 \times n = 252$
$51 = n$	$n = 78 - 33$, or 45	$n = 252 \div 6$, or 42

Although there may be several equations for the same relationship, only one can actually be used to find the solution. For example, if the product of two numbers is 5.184 and one of the factors is 16, there are four equations possible, but only one that can be used directly.

$16 \times n = 5.184$	$5.184 \div n = 16$
$n \times 16 = 5.184$	$\boxed{5.184 \div 16 = n}$

Teaching Strategies

The vocabulary in this unit is quite extensive and includes many terms which are found only in the context of mathematics. Special devices can be used as aids to introduce the terms and to reinforce their meanings. Charts and models which show the salient features may be prepared in advance of the lessons, and separate name cards may be placed with the models during the lessons. These may then be displayed in the classroom for reference, review, and reinforcement. The students may also be involved in making posters of related topics, using diagrams similar to those in this unit. They may also make sets of cards to use for matching names and diagrams. The cards showing diagrams may also be used as flash cards.

It is important for the teacher to observe how the students respond to the exercises to ensure that they acquire accurate concepts. In some lessons it may be advisable to have smaller instructional groups. In such instances the other students may be assigned some of the exercises in the *Keeping Sharp* features on pages 181 and 190.

The work with scale drawings includes a study of maps. Care must be taken in selecting atlases and maps to ensure that they have scales in metric units. Older maps may not be suitable. If a recent highway map of the region is available, students may enjoy finding the difference of the distance from one place to another as stated on the map and the distance they calculate by measurement and use of the scale.

Regarding the use of equations to solve problems, it is important that students be able to identify the nature of the given numbers and of the unknown number, whether each represents an addend, a sum, a factor, or a product.

Results of the *Checking Up* page should be analyzed to note whether there are any misconceptions concerning the geometric ideas in the unit. Often a brief discussion with appropriate models or diagrams will correct such difficulties.

Materials

a straight edge and a protractor for each student
a pronged paper fastener and two narrow strips of cardboard
an overhead projector, a transparent plastic protractor, and transparent acetate marked with a square grid (optional)
a demonstration protractor
a sheet of plain paper and a piece of carbon paper, tracing paper, and copies of pages T 383-T 385 for each student (optional)
string; pins (optional); rulers, metre sticks, and measuring tapes marked in centimetres or trundle wheels
a copy of page T 397, a ruler, compasses, colored pencils, and a few large sheets of paper for each student
objects for demonstrating similar figures such as two pictures, photographs, or drawings for which one is an enlargement
a slide and slide projector (optional)
copies of pages T 395-T 397 for each student
2 cm graph paper (or copies of page T 382), copies of pages T 398 and T 399, and a sheet of blank paper for each student
models of the solids named on pages 192 and 193
paper model of a cube prepared from the pattern on page T 386
copies of page T 382 or page T 396, a straight edge, scissors, and tape for each student
a protractor, tracing paper, a ruler marked in centimetres, and a copy of page T 398 or page T 399 for each student

Vocabulary

line, line segment	scalene triangle
ray, end point	centre (of a circle)
intersecting lines	radius, diameter, chord
intersection	circumference, compasses
perpendicular lines	congruent, similar
parallel lines	scale, scale drawing
angle ($\angle$), vertex	point, plane, curve
degrees (of an angle)	solid, sphere, cone
protractor	cylinder
centre (of a protractor)	cube, cuboid
base line (of a protractor)	triangular prism
acute angle, right angle	rectangular prism
obtuse angle, straight angle	pentagonal prism
line symmetry, line of symmetry	hexagonal prism
vertex (of a polygon)	tetrahedron
vertices, side	triangular pyramid
rhombus, trapezoid	square pyramid
quadrilateral	octagonal pyramid
equilateral triangle	vertex, edge, face
isosceles triangle	equation

Pages 174-175

LESSON OUTCOME

Identify, name, and draw lines, line segments, and rays; identify, name, and draw lines, line segments, and rays that are intersecting, perpendicular, or parallel

Materials

a straight edge for each student

Vocabulary

line, line segment, ray, end point, intersecting lines, intersection, perpendicular lines, parallel lines

9 GEOMETRY

Lines, Line Segments, and Rays

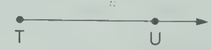
A **line** is straight. It continues without end in two directions.

Say line PQ or line QP. Write $\overleftrightarrow{PQ}$ or $\overleftrightarrow{QP}$.

A **line segment** is part of a line. It has two **end points**.

Say line segment RS or line segment SR. Write $\overline{RS}$ or $\overline{SR}$.

A **ray** is part of a line. It has one end point and continues without end in one direction.

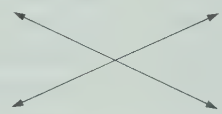


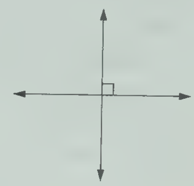
Say ray TU. Write $\overrightarrow{TU}$.

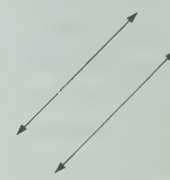
Two lines that meet are **intersecting lines**. The point where they meet is their **intersection**.

Two lines or parts of a line that meet and form a square corner are **perpendicular lines**.

These two lines never meet. They are **parallel lines**.







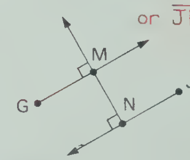
Working Together

Complete.

1.	A B	line AB or line BA ?	?	$\overleftrightarrow{AB}$ or $\overleftrightarrow{BA}$
2.	C ? D	line segment CD or line segment DC	?	$\overline{CD}$ or $\overline{DC}$
3.	E ? F	ray EF ?	$\overrightarrow{EF}$	

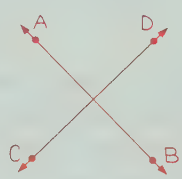
For the rays shown,

4. name two that are parallel. $\overrightarrow{GM}$, $\overrightarrow{JN}$
5. name two that are perpendicular. $\overrightarrow{GM}$ and $\overrightarrow{NM}$, or $\overrightarrow{JN}$ and $\overrightarrow{NM}$



Draw and label

6. a pair of intersecting lines.
Answers will vary.



174

LESSON ACTIVITY

Using the Pages

- Ask a student to read the title of the chapter on page 174. You may wish to have the students consult a dictionary for the meaning of the word *geometry*, which comes from Greek words that mean ‘‘earth measurement’’. In this unit, various shapes, their properties, and relationships will be studied.

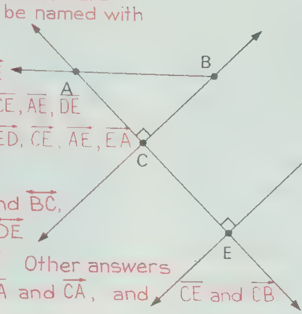
  Ask a student to read the title of the lesson. Many students will have encountered the terms *line*, *line segment*, and *ray* in earlier work and may be able to suggest how they are alike and how they are different. Ask students to read the descriptions for these terms at the top of page 174. Relate each to the corresponding diagram, discussing the significance of the arrowheads on the diagrams for a ray and a line. (They suggest that the straight path continues without end in the indicated directions.) Point out that there is just one way of naming a ray for two given points on the ray, and that the first point named is the *end point*. Ask a student to describe or to draw a diagram to illustrate how ray TU and ray UT differ. Summarize that rays, lines, and line segments are all straight paths.

  To relate these concepts to the environment, name a long, straight street near the school and ask how it is similar to a line. (It seems to go on without end in opposite directions.) Name a section of that street, for example, the length of one block, and ask how it suggests a line segment. Ask how a long, straight road can suggest a ray. For example, students may think of standing on the road and looking down the road in one direction.

  Have the students study the descriptions and diagrams for *intersecting lines*, *perpendicular lines*, and *parallel lines*. Discuss that perpendicular lines are ‘‘special’’ intersecting lines because their *intersection* forms a square corner. Ask students to name two long, straight roads that suggest perpendicular lines and two that suggest parallel lines, such as divided highway lanes which run side by side, remaining the same distance apart.

Exercises For lines and line segments, the letters may be given in either order. Lines may be named with different letters.

For this picture,



1. name two lines. $\overleftrightarrow{BC}$, $\overleftrightarrow{DE}$, $\overleftrightarrow{AC}$, $\overleftrightarrow{CE}$, or $\overleftrightarrow{AE}$
2. name five line segments. $\overline{AB}$, $\overline{AC}$, $\overline{BC}$, $\overline{CE}$, $\overline{AE}$, $\overline{DE}$
3. name three rays. $\overrightarrow{CA}$, $\overrightarrow{CB}$, $\overrightarrow{BA}$, $\overrightarrow{BC}$, $\overrightarrow{DE}$, $\overrightarrow{ED}$, $\overrightarrow{CE}$, $\overrightarrow{AE}$, $\overrightarrow{EA}$
4. name two parallel lines. $\overleftrightarrow{BC}$, $\overleftrightarrow{DE}$
5. name two perpendicular lines. $\overleftrightarrow{AE}$ and $\overleftrightarrow{BC}$, or $\overleftrightarrow{AE}$ and $\overleftrightarrow{DE}$
6. name a line segment and a ray that intersect. $\overline{DE}$ and $\overrightarrow{CE}$ Other answers are possible, such as $\overline{AC}$ and $\overrightarrow{BC}$, $\overline{BA}$ and $\overrightarrow{CA}$, and $\overline{CE}$ and $\overrightarrow{CB}$

Draw and label each of these. Answers will vary

7. $\overleftrightarrow{XY}$ X Y
8. $\overline{EF}$ E F
9. $\overrightarrow{JK}$ J K
10. two intersecting line segments A B C D
11. ray CD parallel to line HR
12. $\overrightarrow{CD}$ perpendicular to $\overrightarrow{CR}$

In this picture, the shore line suggests a line. The rungs on the railing suggest line segments. The corners of the door frame suggest perpendicular line segments. The yellow beams from the lighthouse suggest rays.

11 C D H R

12 C D R

Look around. Name two things that suggest each of these. Answers will vary

13. a ray
14. a line
15. a line segment
16. intersecting line segments
17. perpendicular line segments
18. parallel line segments
19. a ray intersecting with a line segment

175

RELATED ACTIVITIES

• Have students help to prepare a large chart showing geometric figures and terms introduced in this unit. As new concepts are introduced they may be included in this chart.

Name	Diagram
line	
ray	
end points	
line segment	

• Have the students use two rulers, pencils, or other similar objects to demonstrate each of the following.

1. two perpendicular line segments
2. two intersecting line segments that are not perpendicular
3. two parallel line segments
4. two line segments that share one end point
5. two line segments that are not perpendicular, not parallel, and do not intersect

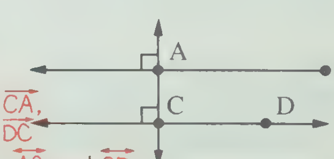
• Extend the preceding activity to provide a challenge. Ask students to think of their pencils not as line segments but as lines, reviewing that a line continues without end in opposite directions. Ask if it is possible for two lines never to intersect yet not be parallel. If the lines are in the same plane, that is, if the pencils remain on the desk, it is not possible. However, a student may suggest holding one of the pencils above the desk. Such lines are described as *skew* lines.

Working Together: Provide the students with straight edges for drawing the diagrams. Pay particular attention to the order of the letters used to name each ray for Ex. 4 and 5. Note that two answers are possible for Ex. 5. For Ex. 6, remind the students to use capital letters to label the lines.

Exercises: Discuss the different answers that are possible for Ex. 1-3, 5, and 6. Note that the same two points can be used to name a line segment, a ray, and a line, for example, $\overline{CA}$, $\overrightarrow{CA}$, and $\overleftrightarrow{CA}$. This can help to emphasize that a line segment and a ray are parts of a line. Explain that each of $\overleftrightarrow{AC}$, $\overleftrightarrow{CE}$, and $\overleftrightarrow{AE}$ names the same line, but $\overline{AC}$, $\overline{CE}$, and $\overline{AE}$ are different line segments. A line includes all the points in the line and continues without end, but a line segment includes only the points from one end point to the other. Remind the students to use a straight edge for Ex. 7-12. For Ex. 12, $\overrightarrow{CD}$ and $\overrightarrow{CR}$ meet at point C. You may wish to discuss the illustration of the lighthouse and the statements related to it. For Ex. 13-19, ask the students to name things in the classroom and outside the classroom.

Assessment

For this picture,



1. name a line segment. $\overline{AB}$, $\overline{AC}$, $\overline{CD}$
2. name two parallel rays. $\overrightarrow{BA}$, $\overrightarrow{AC}$, $\overrightarrow{CA}$, $\overrightarrow{CD}$, $\overrightarrow{DC}$
3. name two intersecting lines. $\overleftrightarrow{AC}$ and $\overleftrightarrow{CD}$

Draw and label each of these. Diagrams will vary.

4. $\overrightarrow{XY}$ Y X
5. line segment PQ P Q
6. line EF perpendicular to ray EG E F G
7. ray HJ parallel to line segment KL H J K L

Pages 176-177

LESSON OUTCOME

Identify, name, measure, and draw angles; classify angles as acute, right, obtuse, or straight

Materials

a pronged paper fastener and two narrow strips of cardboard; an overhead projector and a transparent plastic protractor (optional); a demonstration protractor; a protractor and a straight edge for each student

Vocabulary

angle (∠), vertex, degrees (of an angle), protractor, centre (of a protractor), base line (of a protractor), acute angle, right angle, obtuse angle, straight angle

Prerequisite Skills

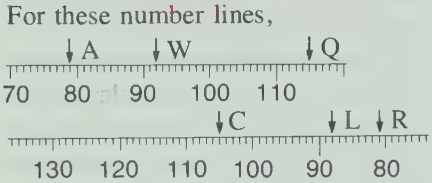
Draw rays; read a scale

Checking Prerequisite Skills

Draw and label each of these.

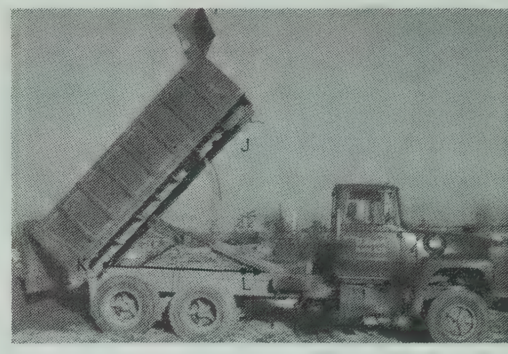
1. ray EF E F 2. $\overrightarrow{HG}$ H G

For these number lines,



3. which letter names the point for 79? A
4. which letter names the point for 92? W
5. which letter names the point for 115? Q

Angles



Two rays that have the same end point form an **angle**.

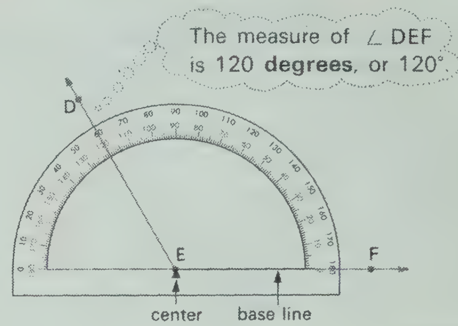
For the angle suggested in the picture, you could say angle JKL or angle LKJ. You could write ∠ JKL or ∠ LKJ.

The common end point is the **vertex** of the angle.

K is the vertex of the angle in the picture.

Working Together

To measure an angle, place the center of a **protractor** on the vertex of the angle. Line up the base line of the protractor with one ray of the angle. Start at 0 on the base line. Read the number of degrees shown at the other ray.



An angle greater than 0° but less than 90° is an **acute angle**.

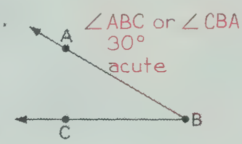
An angle of 90° is a **right angle**.

An angle greater than 90° but less than 180° is an **obtuse angle**.

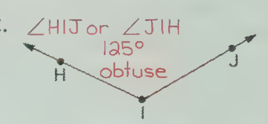
An angle of 180° is a **straight angle**.

Name each angle. Measure the angle. Tell whether it is acute, right, obtuse, or straight.

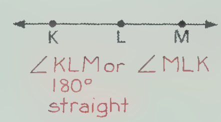
1.



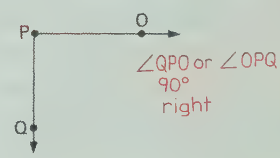
2.



3.



4.

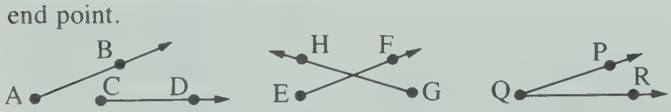


176

LESSON ACTIVITY

Before Using the Pages

- Draw the following diagrams on the board. For each diagram, ask students to name two rays. For the third diagram, ask what is special about the two rays. (They share an end point.) Introduce the term *angle* for two rays that share an end point.



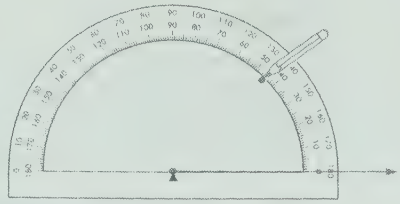
Display a pronged paper fastener and two narrow strips of cardboard, each cut to a point at one end and having a hole in the other end. Tell the students that each strip represents a ray and the hole represents the end point of the ray. As the students watch, use the fastener to join the strips to represent two rays with the same end point, that is, an angle. Position the strips so that one overlaps the other. Slowly rotate the top strip to form several different angles. Ask students to name examples of angles in the classroom. Open the classroom door slightly to illustrate an angle that is not a square corner. For some of the examples, ask which of two angles is larger (smaller) and use the cardboard strips to match the angles. In this way, it can be demonstrated that the length of each strip (ray) does not affect the size of an angle.

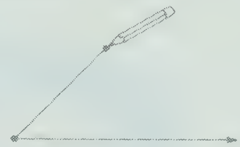
Using the Pages

- Draw attention to the spelling of the word *angle*. Ask how the object in the photograph can demonstrate angles of different sizes. Have the students use their fingers to trace ∠ JKL in the photograph, starting first at point J and then again at point L. This will help them to understand why the name of the *vertex* is always the middle letter in the two ways of naming an angle. Use the strips to form an angle measuring less than 90° and relate it to the symbol ∠ for ''angle''.

Working Together: The steps described on page 176 for measuring an angle and on page 177 for drawing an angle may be demonstrated on an overhead projector using a

To draw an angle of a certain measurement, such as 45°, first draw a ray. Place the center of a protractor on the end point of the ray. Line up the base line of the protractor with the ray. Mark a dot at 45°. Draw a ray from the end point through the dot.





Use a protractor and draw an angle for each measurement.

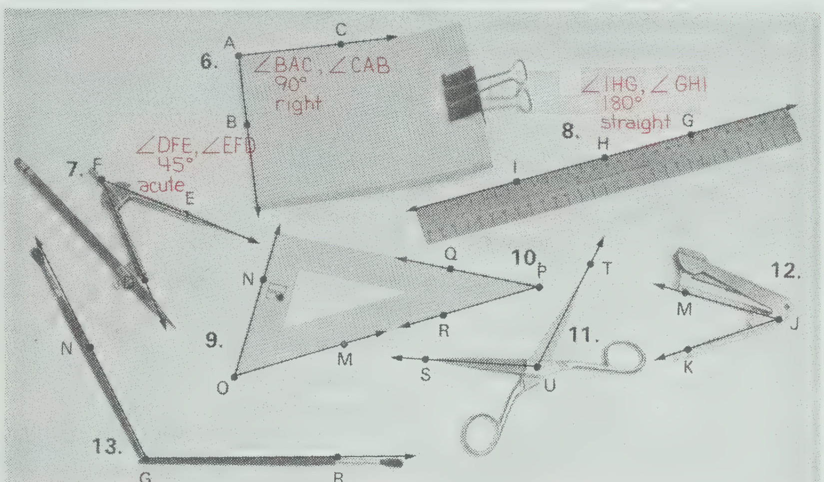
5. 30° 6. 107°

Exercises

Use a protractor and draw an angle for each measurement.

1. 10° 2. 95° 3. 180° 4. 86° 5. 162°

Name each angle in two ways. Write its measurement. Tell whether it is acute, right, obtuse, or straight.



9. ∠NOM, ∠MON 57° acute
10. ∠QPR, ∠RPQ 28° acute
11. ∠SUT, ∠TUS 113° obtuse
12. ∠KJM, ∠MJK 34° acute
13. ∠NGR, ∠RGN 116° obtuse

RELATED ACTIVITIES

• Many students will enjoy showing that line segments can form a curve through the process of "curve stitching". A pattern and directions for its use are given on page T 380.

• New terms introduced in this lesson may be included in the chart suggested in *Related Activities* on page T 191.

• Have the students draw two or more line segments intersecting at the same point, measure all the angles formed, and find the sum of the measurements. Repeating this a few times can lead to the discovery that the sum is always 360°. The students can also discover that vertically opposite angles are equal.

• Have students use protractors to measure angles for various objects in the room. Ask them to classify the angles as acute, right, obtuse, or straight. The results may be written in a chart.

Angle	Size of angle	Kind of angle
corner of a table	90°	right

Then, for each measurement in the list, have them draw the angle and compare it with the angle for the object.

• Have students draw angles on copies of page T 395, measure the angles, and classify each as acute, right, obtuse, or straight.

transparent plastic protractor, or on the chalkboard using a large demonstration protractor. Introduce the term *degrees* and the symbol °. Point out that there is an inner scale and an outer scale on the protractor and have the students follow each scale from 0° to 180°. Note that the marks for 90° are at the same place on each scale and that angles measuring 90° form square corners. Develop that angles which are greater than (less than) a square corner measure more than (less than) 90°.

Provide the students with protractors and ask them to compare them with the protractor shown on page 176, noting particularly the centre and the base line. Review that each ray of an angle is thought of as continuing without end in the indicated direction, but that this has no effect on the measure of the angle.

Introduce the terms *acute angle*, *right angle*, *obtuse angle*, and *straight angle*, and point out that it is often possible to identify the kind of angle without measuring it. You may wish to have the students classify the angles first for Ex. 1-4 and then check their answers by measuring.

Before the students complete Ex. 5 and 6, have them follow the steps to draw an angle of 45° and, if you wish, an angle of 135°.

Exercises: Emphasize the steps for measuring an angle that are given in *Working Together* before the students begin the exercises.

Assessment

Use a protractor and draw an angle for each measurement.

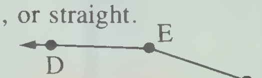
1. 55° 2. 142°

Name each angle in two ways. Write its measurement. Tell whether it is acute, right, obtuse, or straight.

3.

∠ABC
∠CBA
90°, right

4.

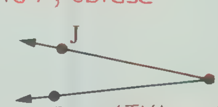


∠DEF, ∠FED
164°, obtuse

5.

∠GHI, ∠IHG
180°, straight

6.



∠JKL, ∠LKJ
18°, acute

LESSON OUTCOME

Identify and show lines of symmetry; identify objects having line symmetry; identify the number of vertices and the number of sides for polygons

Materials

a sheet of plain paper and a piece of carbon paper for each student, tracing paper and a straight edge for each student, copies of pages T383-T385 for each student (optional)

Vocabulary

line symmetry, line of symmetry, vertex (of a polygon), vertices, side, rhombus, trapezoid

Line Symmetry

A picture has **line symmetry** if it can be folded in half so that one part matches the other part.

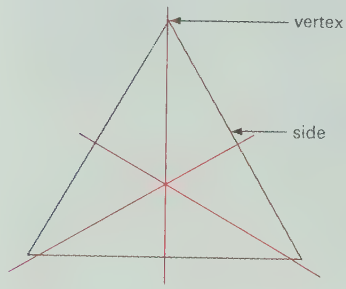
A line that separates the picture into two matching parts is a **line of symmetry**.

This picture has line symmetry.

Find points on one side of the line of symmetry for this picture that match points on the other side of the line of symmetry.

Working Together

Check for line symmetry. First trace the shape. Then fold the tracing in half. If the two halves can be made to match, the shape has line symmetry. The fold shows a line of symmetry.



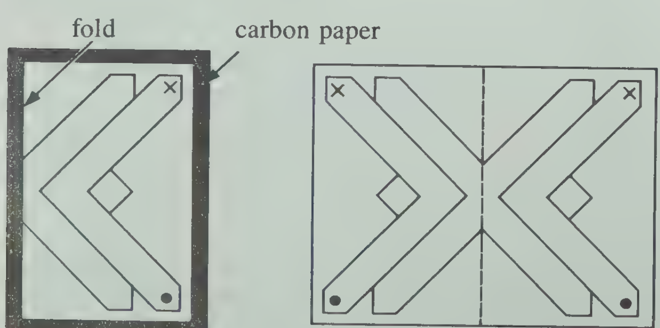
1. Check this triangle for line symmetry. yes
2. How many lines of symmetry can you find? 3
3. Show the lines of symmetry in your tracing.
4. Give the number of vertices. 3
5. Give the number of sides. 3

178

LESSON ACTIVITY

Before Using the Pages

- Give each student a sheet of plain paper and a piece of carbon paper about half the size of the plain paper. Ask the students to fold the plain paper in half, place it on top of the carbon paper with the carbon side up, and draw a design around the folded edge. Discuss the design formed when the paper is unfolded.



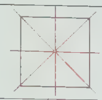
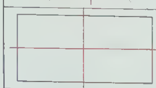
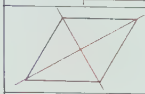
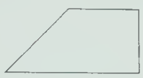
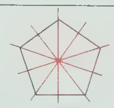
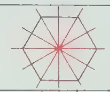
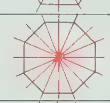
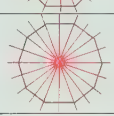
Using the Pages

- Draw attention to the illustration and ask how it is similar to the designs formed in the preliminary activity. Lead the students to suggest that it shows two matching parts. Ask which part of the illustration suggests a fold line. Then introduce the term *line symmetry*. Ask students to read the statements above and below the illustration, noting the term *line of symmetry*. Have the students match several points on one side of the line of symmetry with the corresponding points on the other side of the line of symmetry.

  Draw a diagram similar to the following on the board to show two identical parts that do not match by folding. Ask if the diagram shows a line of symmetry and have students give reasons for their answers.

Exercises

Complete this chart.

	Kind of polygon	Number of vertices	Number of sides	Number of lines of symmetry
1.	 Square	4 ?	4 ?	4 ?
2.	 Rectangle	4 ?	4 ?	2 ?
3.	 Rhombus	4 ?	4 ?	2 ?
4.	 Parallelogram	4 ?	4 ?	0 ?
5.	 Trapezoid	4 ?	4 ?	0 ?
6.	 Regular pentagon	5 ?	5 ?	5 ?
7.	 Regular hexagon	6 ?	6 ?	6 ?
8.	 Regular octagon	8 ?	8 ?	8 ?
9.	 Regular decagon	10 ?	10 ?	10 ?

Trace the polygons for Exercises 1 to 9.

10. Show the lines of symmetry.

Look around. Find objects with line symmetry.

11. Give ten examples.
Answers will vary

179

RELATED ACTIVITIES

- Have each student draw one half of a design or picture on copies of page T 397 or page T 398 so that a grid line can form a line of symmetry. Then have the students exchange papers and complete the drawings.
- A semitransparent plexiglass mirror is useful in a study of line symmetry. As a mirror, it shows the reflection of an object. Being semitransparent, it enables students to match the reflection in testing for line symmetry or to trace the reflection in drawing a symmetrical shape.

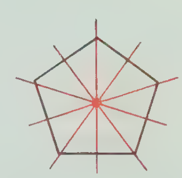
  Have students draw part of a shape and trace its reflection in the mirror to form a shape having a line of symmetry. Provide the students with magazines and catalogs to search for pictures that suggest line symmetry. Have them use a mirror to test for line symmetry in the pictures.
- Terms introduced in this lesson may be included in the chart suggested in *Related Activities* on page T 191. Include examples of line symmetry, line of symmetry, and polygons, both regular and irregular.
- Have each student fold a copy of page T 396 or page T 397 in half and color some or all of the squares in one half various colors to create a design. Then have each student use the fold as a line of symmetry and color the squares on the other half of the paper to make a design that has line symmetry.

Working Together: Ask students to refer to the diagram and to explain the terms *vertex* and *side* of a triangle in their own words. Draw attention in Ex. 4 to the word *vertices*, which is the plural form of *vertex*. Have the students read the procedure described for checking for line symmetry. Then ask, "What is the first step?" "What is the second step?" "How will this show if the shape has line symmetry?" "Can there be more than one way to fold the tracing?" "Can there be more than one line of symmetry for a shape?" Provide each student with tracing paper and a straight edge for Ex. 1 and 3.

Exercises: With the exception of the rhombus (Ex. 3) and the trapezoid (Ex. 5), the names of these polygons were encountered earlier in Unit 6 on pages 110 and 111. You may wish to refer students to those pages to recall what is meant, for example, by a regular pentagon. Note that diagrams for the polygons named in Ex. 1-9 are provided on pages T 383-T 385. You may wish to provide the students with copies of those pages for completing Ex. 10, rather than having them trace the shapes. Also, the students may find it easier to complete the column at the right of the chart for Ex. 1-9 after folding the shapes to show lines of symmetry for Ex. 10. With regard to Ex. 1, 7, 8, and 9, note that some lines of symmetry join opposite vertices and other lines of symmetry join the midpoints of opposite sides.

Assessment

For this shape, give



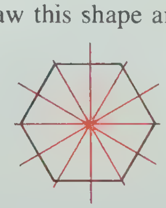
1. the number of vertices. 5

2. the number of sides. 5

3. the number of lines of symmetry. 5

Draw this shape and show the lines of symmetry.

4.



Pages 180-181

LESSON OUTCOME

Name triangles; classify triangles as equilateral, isosceles, or scalene, according to the number of sides of equal length

Materials

a ruler, a protractor, and tracing paper for each student

Vocabulary

equilateral triangle, isosceles triangle, scalene triangle

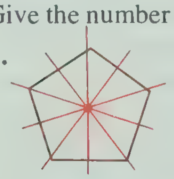
Prerequisite Skills

Identify lines of symmetry; measure angles

Checking Prerequisite Skills

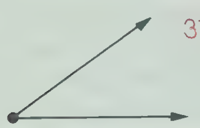
Give the number of lines of symmetry.

1.



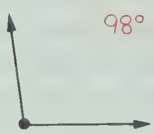
Measure each angle.

2.



37°

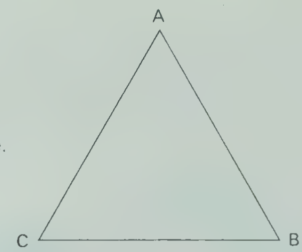
3.



98°

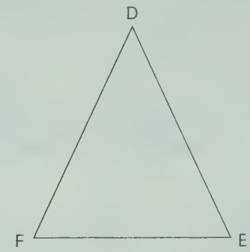
Triangles

A triangle with three sides of equal length is an **equilateral triangle**.



Triangles are polygons.

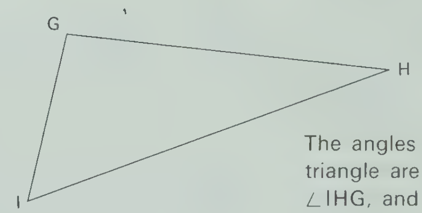
A triangle with two sides of equal length is an **isosceles triangle**.



A triangle can be named using its three vertices. Two names for this triangle are △DEF and △EFD.

What are its other names?

A triangle with all sides of different length is a **scalene triangle**.



The angles of this triangle are ∠GIH, ∠IHG, and ∠HGI.

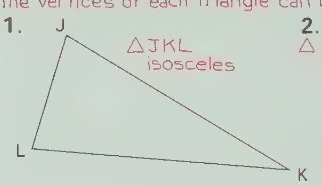
Exercises

Name each triangle. Tell whether it is equilateral, isosceles, or scalene.

You can measure the sides to find out.

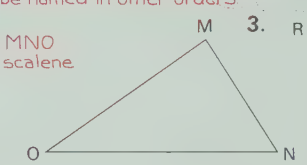
The vertices of each triangle can be named in other orders.

1.



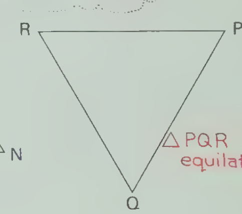
△JKL isosceles

2.



△MNO scalene

3.



△PQR equilateral

180

LESSON ACTIVITY

Before Using the Pages

- Begin by asking how many wheels there are on a tricycle and ask which part of the word indicates the number three. Ask for similar words that start with the prefix *tri* and have students explain each word, for example, triplets, trio, and tricolor. If no one suggests the word *triangle*, ask which polygon is named using the prefix *tri* and discuss the relationship of *tri* and *angle* to the shape of the polygon. Ask for ways in which triangles are alike and ways in which they may be different.

Using the Pages

- Draw attention to triangle ABC. Ask what appears to be true about the sides of the triangle. Ask a student to read the statement to the left of triangle ABC. Help, if necessary, with the pronunciation of *equilateral*.

  For triangle DEF, ask whether all three sides appear to be of equal length. If students suggest that just two of the three sides appear equal in length, ask which two sides these might be. Introduce the term *isosceles triangle*. Ask a student to read the statements to the right of the triangle. Ask for two other ways to name triangle DEF. Then ask how many ways in all there are to name a triangle.

  Discuss triangle GHI in a similar manner to introduce the concept of a *scalene triangle*. For the statement to the right of triangle GHI, discuss that the order of the letters may vary when naming each angle, but that the second letter in the name must name the vertex of the angle.

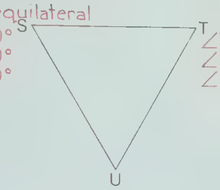
- Have the students use their rulers to measure the sides of triangles ABC, DEF, and GHI to check the number of sides of equal length in each triangle. Then ask them to show the six ways of naming triangle ABC.

△ ABC	△ CAB	△ ACB
△ CBA	△ BAC	△ BCA

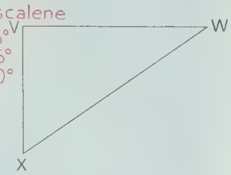
Exercises: Provide each student with a ruler to measure the sides of the triangles and a protractor to measure the angles. For each triangle, ask the students to guess before measuring

Tell whether each triangle is equilateral, isosceles, or scalene. Measure the angles.

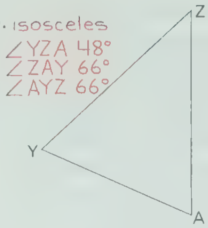
4. equilateral
∠STU 60°
∠TUS 60°
∠UST 60°



5. scalene
∠VWX 35°
∠WXV 55°
∠XVW 90°



6. isosceles
∠YZA 48°
∠ZAY 66°
∠AYZ 66°



Complete this chart for the nine triangles on pages 180 and 181. Answers are given on page T 371

7.

Name of triangle	Kind of triangle	Number of sides of equal length	Number of lines of symmetry	Angle measurements
△ ABC	equilateral	3	3	60°, 60°, 60°

For this chart, what do you notice about

8. the number of sides of equal length for each kind of triangle?

9. the number of lines of symmetry for each kind of triangle?

10. the angle measurements of equilateral triangles? all 60°

11. the sum of the angle measurements for each triangle? 180°

Multiply.

1. 916	2. 570	3. 814	4. 963	5. 185	6. 967
10	90	73	100	932	610
9160	51 300	59 422	96 300	172 420	589 870

Divide. Give the quotient and remainder.

7. 8)51 984 6498 8. 10)12 170 1217 9. 70)45 458 649 R28 10. 31)19 496 628 R28

11. 75)22 935 305 R60 12. 100)48 008 480 R8 13. 589)64 364 109 R163 14. 421)16 739 39 R320

Multiply or divide. Write only the results.

15. 9732 × 100 973 200 16. 644 000 ÷ 10 64 400 17. 1000 × 6514 6 514 000

18. 814 000 ÷ 100 8140 19. 473 000 ÷ 1000 473 20. 10 × 8407 84 070

21. 60 300 ÷ 10 6030 22. 100 × 2010 201 000

KEEPING SHARP

8 equilateral triangles-3
isosceles triangles-2
scalene triangles-0

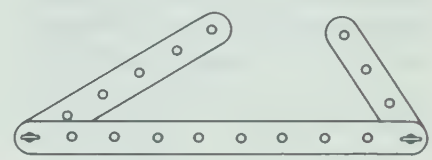
9 equilateral triangles-3
isosceles triangles-1
scalene triangles-0

181

RELATED ACTIVITIES

• Have the students include examples of an equilateral triangle, an isosceles triangle, and a scalene triangle in the chart suggested in *Related Activities* on page T 191.

• Provide the students with geostrips, D-Stix, or straws and pipe cleaners to form triangular shapes. Lead them to discover that a triangle can be formed only when two sides together are longer than the third side. For example, for strips that are 5 cm, 9 cm, and 3 cm long, a triangle cannot be formed: 5 + 9 is greater than 3, 3 + 9 is greater than 5, but 5 + 3 is less than 9.



• Students can demonstrate that the sum of the measures of the angles of a triangle is 180° without measuring the angles, as follows. A triangular shape is cut from a piece of paper. The three corners are torn off and are arranged side by side as shown. They can be seen to form a straight line (angle).

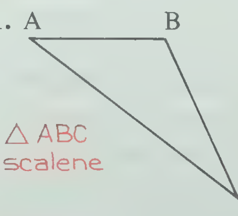
whether the triangle is equilateral, isosceles, or scalene. Ensure that the students understand what is required for Ex. 7. For assistance with the column labeled "Number of lines of symmetry", provide the students with tracing paper. They may trace the shapes and fold the tracings as needed. The students should measure the angles of triangle ABC to check the answers given in the chart for that triangle. The results of Ex. 8-11 are important and should be discussed.

Keeping Sharp: These exercises review multiplication and division of whole numbers. Point out that the students are to write only the results for Ex. 15-22. Note that although Ex. 12 involves a divisor of 100, the instructions indicate that any remainders are to be shown with the quotient; thus, decimal quotients are not involved at this time.

Assessment

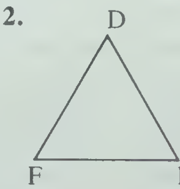
Name each triangle. Tell whether it is equilateral, isosceles, or scalene.

1.



△ ABC
scalene

2.



△ DEF
equilateral

3.

△GHI
isosceles

The order of the letters in the names will vary.

Pages 182-183

LESSON OUTCOME

Identify the circle and the terms *centre, radius, diameter, chord,* and *circumference*; measure radii, diameters, and circumferences

Materials

string; pins (optional); rulers, metre sticks, and measuring tapes marked in centimetres; a copy of page T397, a ruler, compasses, colored pencils, and a few large sheets of paper for each student

Vocabulary

centre (of a circle), radius, diameter, chord, circumference, compasses

Circles

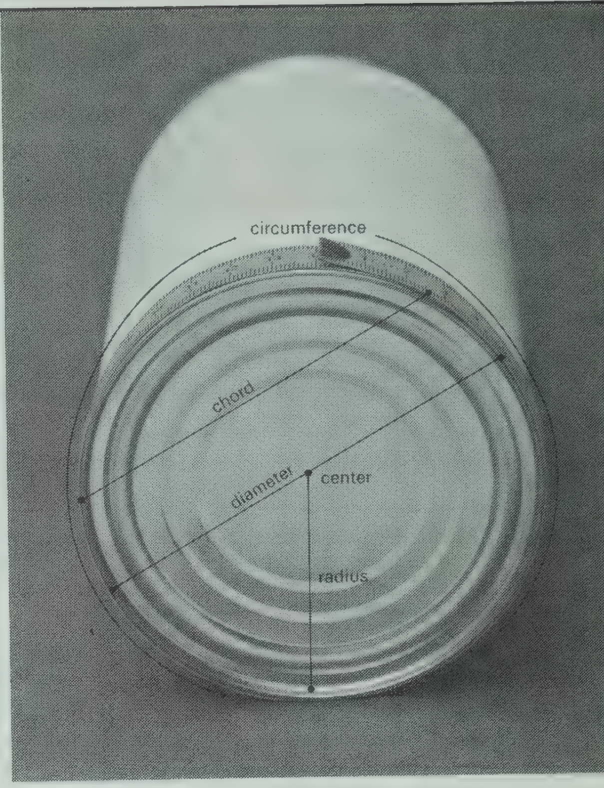
The points in a circle are all the same distance from one point called the **center** of the circle.

A **radius** of a circle is a line segment with one end point at the center of the circle and the other end point on the circle.

A **diameter** of a circle is a line segment having both end points on the circle and containing the center.

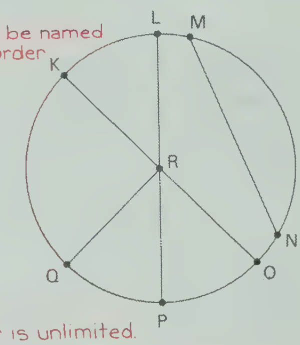
A **chord** of a circle is a line segment with both end points on the circle.

The distance around a circle is the **circumference**. The circumference of this can is about 34 cm.



Exercises

For this circle, Diameters, radii, and chords can be named by using the end points in either order.



1. name the center. R
2. name two diameters. $\overline{LP}$, $\overline{KO}$
3. measure each diameter. 51 mm
4. name each radius. $\overline{RL}$, $\overline{RO}$, $\overline{RP}$, $\overline{RQ}$, $\overline{RK}$
5. measure each radius. 25.5 mm
6. name three chords. $\overline{MN}$, $\overline{LP}$, $\overline{KO}$
7. name two lines of symmetry. $\overline{LP}$, $\overline{KO}$
*8. give the number of lines of symmetry that are possible for a circle. The number is unlimited.

182

LESSON ACTIVITY

Before Using the Pages

- Have the students work in pairs using a piece of string about 50 cm long. Have them tie the ends of the string together to form a loop. Ask them to place the string on a flat surface, such as cork board, and to use their fingers or pins to stretch the loop to form each of the following shapes in turn: scalene triangle, isosceles triangle, equilateral triangle, square, rectangle, parallelogram, pentagon, hexagon. Review that the preceding shapes are examples of polygons, and that polygons have sides which are line segments.

  Ask if a *circle* is a polygon. Have the students name examples of circles, such as rings, hoops, and the shape of the classroom clock. Discuss that the loop of string cannot be arranged very easily to represent a circle because a circle is a smooth curve. Use one of the loops to demonstrate the following procedure for drawing a circle. Fasten a large sheet of paper to a surface such as a cork board. Use two pencils to hold the loop of string firmly against the paper. Keep one pencil stationary and move the other pencil on the paper, always keeping the string taut. The resulting shape is a circle. Ask how the length of the string affects the size of the circle. If you wish to have the students try the procedure themselves, they will require a large sheet of paper on which to draw the circle.

Using the Pages

- The photograph shows the circular end of a can. Ask students to read the statements that explain the terms *centre, radius, diameter, chord,* and *circumference*. Relate each to the corresponding part of the photograph. Then ask students to identify these parts for the circle drawn earlier with a loop of string. It will be necessary to draw a diameter, a chord, and a radius. The students should understand and be able to use each of these terms, but it is not necessary for them to be able to state a definition for each.

Exercises: The students will require rulers for Ex. 3 and 5. For Ex. 6, discuss why a diameter is also a chord. Ex. 8 is starred because the number of lines of symmetry for a circle

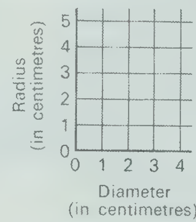
Look around. Make a chart like this for six different objects. Answers will vary

9.

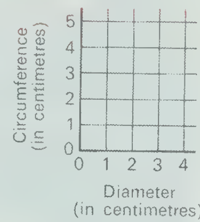
Examples of circles	Radius measurement	Diameter measurement	Circumference measurement
the lid of a can	4 cm	8 cm	25.1 cm

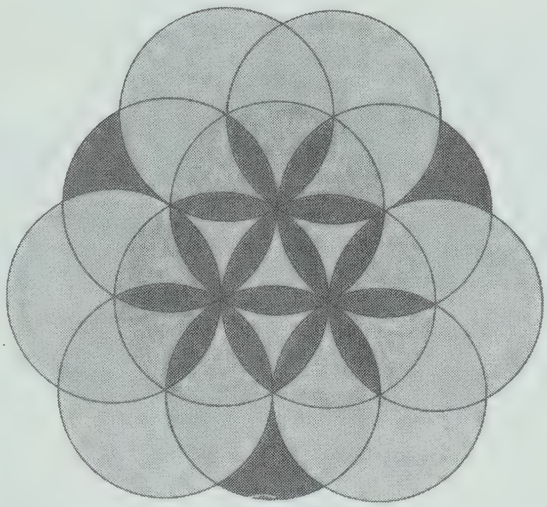
Use the information in the chart and complete these line graphs to show Line graphs are shown on page T371

10. the radius measurements and the diameter measurements.



11. the diameter measurements and the circumference measurements.





Make a design

1. using compasses.
2. using compasses and a ruler.

try this

RELATED ACTIVITIES

• Have students prepare a display showing pictures of objects that suggest circles, for example, the lid of a can. Have them draw and label the center, radius, diameter, chord, and circumference for several pictures.

• You may wish to have the students draw a line graph using the radius measurements and the circumference measurements for Ex. 9 on page 183.

• For enrichment, have students use compasses to draw circles having a given radius or a given diameter.

• Give each student a circular filter paper or a copy of the circle on page T 385 to perform the following activities in sequence.

1. Fold the circular shape in half and unfold it to note that the fold line marks a diameter.
2. Fold the shape in half another way and unfold it to note another diameter, the centre, and four radii.
3. Fold the shape once but not in half, and unfold it to illustrate a chord.
4. Fold the shape so that at least six lines of symmetry are shown.

For the above activities, emphasize that the boundary of the shape represents the circle. The centre of a circle is not a point of the circle; it is in the interior region of the circle.

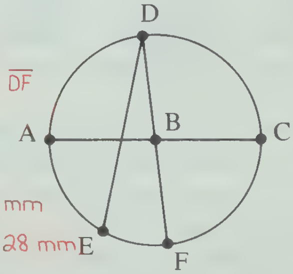
is unlimited. Before the students begin Ex. 9, decide whether you wish to have them measure to the nearest tenth of a centimetre or to the nearest centimetre. Provide metre sticks and measuring tapes for the students to use. If there are insufficient measuring tapes, string may be wrapped around a circular object to find its circumference, then the length of the string can be measured with a metre stick. Provide copies of page T 397 for Ex. 10 and 11. Display several completed graphs and discuss the information they show. The results of the chart and the graphs show that the length of a diameter of a circle is twice the length of a radius, and that the circumference is a little more than three times the length of a diameter.

Try This: These exercises offer an opportunity for the students to experiment and practice with compasses. Demonstrate how to use compasses to draw circles. Then encourage the students to make creative designs for Ex. 1 and 2. These exercises may be assigned during an art lesson and the resulting designs may be displayed and discussed. For example, for the design illustrated on page 183, ask how many circles there are (12). A design for Ex. 2 might be discussed in terms of polygons and parts of a circle. Provide the students with large pieces of paper, compasses, rulers, and colored pencils.

Assessment

For this circle,

1. name the center. B
2. name a diameter. $\overline{AC}$, $\overline{DF}$
3. name a radius.
4. name two chords.
5. measure a radius. 14 mm
6. measure a diameter. 28 mm



3. $\overline{BA}$, $\overline{BC}$, $\overline{BD}$, $\overline{BF}$
4. $\overline{DE}$, $\overline{DF}$, $\overline{AC}$

Pages 184-185

LESSON OUTCOME

Identify congruent shapes

Materials

several sheets of tracing paper and a straight edge for each student

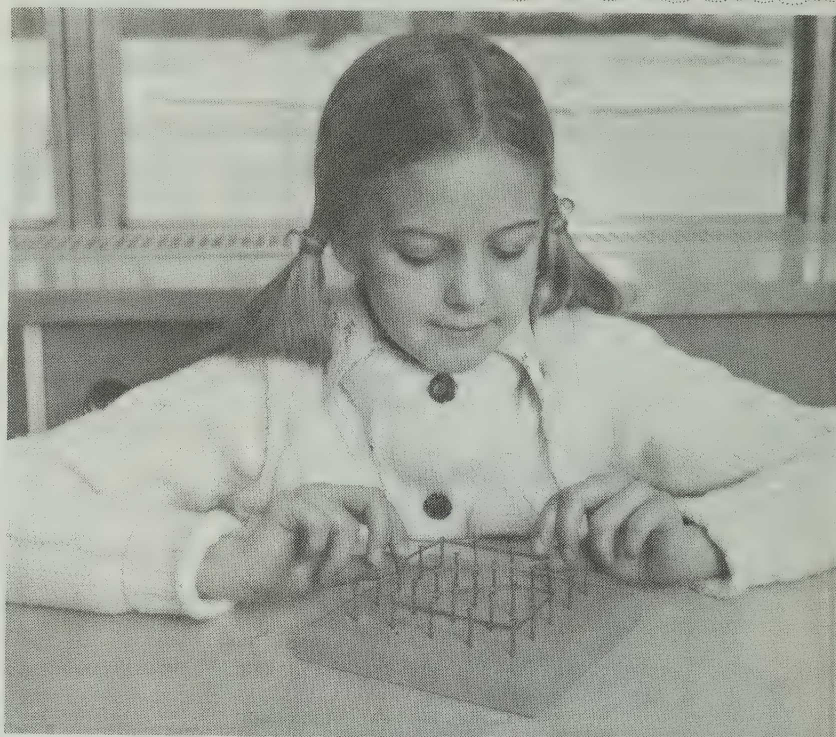
Vocabulary

congruent, quadrilateral

Congruent Shapes

Margaret Ann made two **congruent** shapes on a geoboard.

Objects that are congruent have the same shape and the same size.

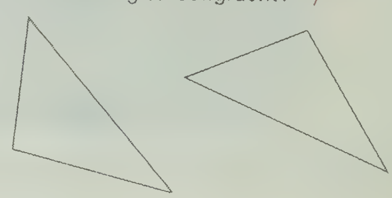


Working Together

Check for congruent shapes by first tracing one of the shapes. Then place the tracing over the other shape. If the tracing can be made to fit exactly, the shapes are congruent.

Are these triangles congruent? yes

1.



184

LESSON ACTIVITY

Before Using the Pages

- Have the students work in pairs for the following activity. One student uses a straight edge to draw a polygon on a sheet of paper. The other student uses a straight edge and tracing paper to trace the shape. Discuss that each original polygon and its tracing are identical because they have the same shape and the same size.

  Tape several of the students' drawings to the chalkboard. Place the corresponding tracings in a separate pile. Ask a student to choose one of the tracings and identify the matching polygon on the board. Repeat the procedure until all the polygons on the board have been matched with their tracings. Tell the students that there is a word to describe figures that are the same size and the same shape. Some students may recall the word. If not, have them turn to page 184.

Using the Pages

- Ask students to read the title of the lesson and the statements at the top of page 184. Ask, for example, why the pages of a book are congruent. Have students name other examples of congruent shapes, such as postage stamps, envelopes, and cookies cut from the same cutter. Discuss ways of showing whether two shapes, on a geoboard similar to the one shown, are congruent. Students may suggest copying the shapes on geopaper and finding whether a tracing of one shape will match the other shape.

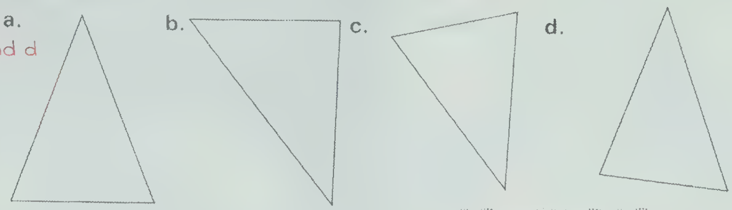
  If appropriate, demonstrate with tracing paper or overhead transparencies the idea of *corresponding parts* of the congruent triangles. Show how corresponding parts of congruent figures may be indicated by identical markings (tic marks for sides, arcs for angles), or by lettering (ABC and XYZ, or ABC and A'B'C', for example).

Working Together: Provide the students with a straight edge and tracing paper to check whether the triangles are congruent. Emphasize that a tracing of one shape must match the other shape exactly in order for the two shapes to be considered congruent. Note that for this example, the tracing paper must be flipped.

Exercises

Use tracing paper and find two congruent triangles.

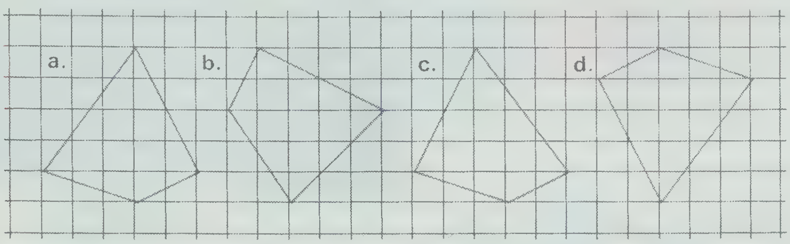
1. c and d



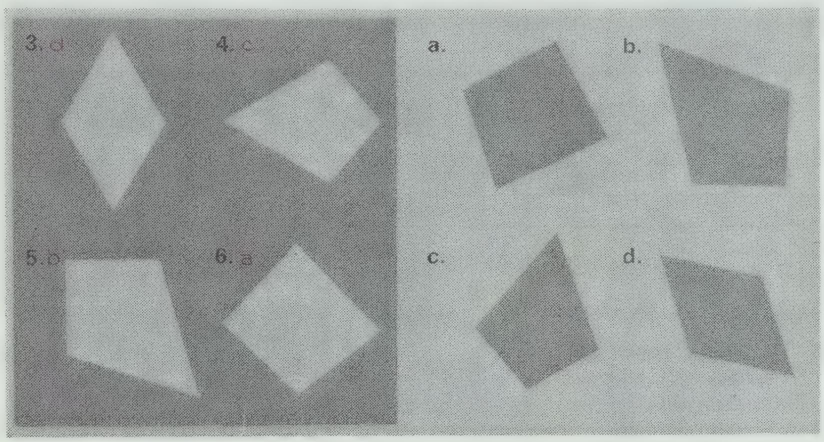
Find two congruent quadrilaterals.

A quadrilateral is a polygon with four sides.

2. a and d



Use tracing paper to find out which puzzle piece fits in each space in the puzzle.



185

RELATED ACTIVITIES

- Students may work individually or in pairs using geoboards and rubber bands for the following activities. The activities may be adapted for use with copies of page T 396 or page T 397.
  1. Show a polygon on the geoboard.
  2. Show a polygon congruent to the first one (a) in the same position on another geoboard (b) in a different position on the same geoboard.
  3. How many shapes congruent to the first one can be shown on the same geoboard?
- The students may create puzzles similar to the one shown on page 185 to use in spare moments.
- The following activity relates the concepts of congruence, area, and division. Provide each student with a decimetre square from copies of page T 392. Have them express the area in square centimetres (100 cm^2). Tell the students to fold the square in half, then in half again, and again two more times. Have them unfold the paper and note that the fold lines mark the decimetre square into 16 small congruent squares, or triangles if the first fold was along a diagonal. Discuss that the 16 shapes are equal in area because they are congruent. Complete the division $16\overline{)100}$ to find the area of each small shape.
- Provide students with pictures of pairs of congruent figures (including at least one pair of congruent regular polygons). Have them mark the figures to indicate the vertices, sides, and angles that correspond for each congruence.

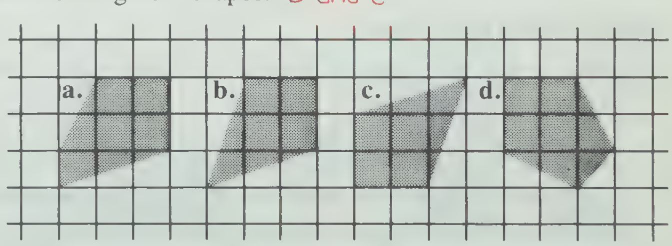
Exercises: Draw attention to the statement in the "thought cloud" for Ex. 2 to review the meaning of the word *quadrilateral*. Discuss that a square, a rectangle, a rhombus, a parallelogram, and a trapezoid are quadrilaterals which are special; for example, a rhombus is a quadrilateral for which the four sides are equal in length. Ask which part of the word *quadrilateral* suggests the number four. The use of tracing paper is suggested for Ex. 1 and Ex. 3-6 to find which shapes are congruent. For Ex. 2, ask the students to decide which two of the four quadrilaterals are congruent by noting their positions with respect to lines of the grid. You may wish to have them use tracing paper to check their answers.

If appropriate, have students identify the corresponding parts of the congruent figures in a manner suggested in the second paragraph of "Using the Pages."

Assessment

Find two congruent shapes. b and c

1.



Pages 186-187

LESSON OUTCOME

Identify and draw similar shapes

Materials

objects for demonstrating similar figures such as two pictures, photographs, or drawings for which one is an enlargement of the other; a slide and slide projector, or an overhead projector and a triangular shape; copies of pages T 395 - T 397 and a straight edge for each student

Vocabulary

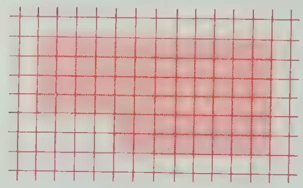
similar

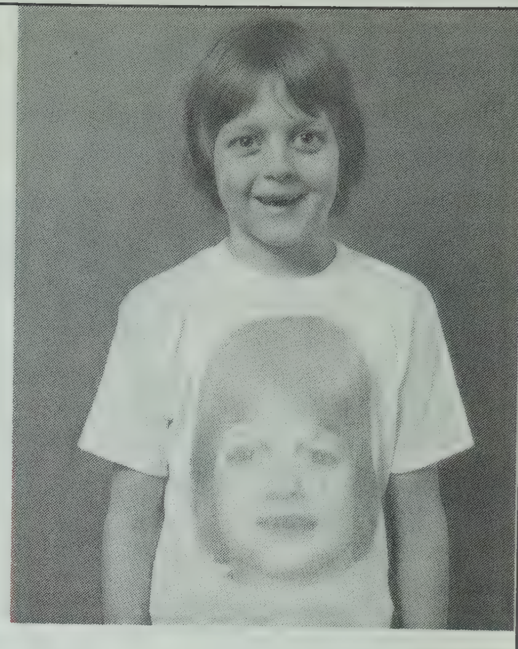
Similar Shapes

Todd's face in this picture and the face on the T-shirt are **similar**.

Similar objects have the same shape, but they are not always the same size.

a.

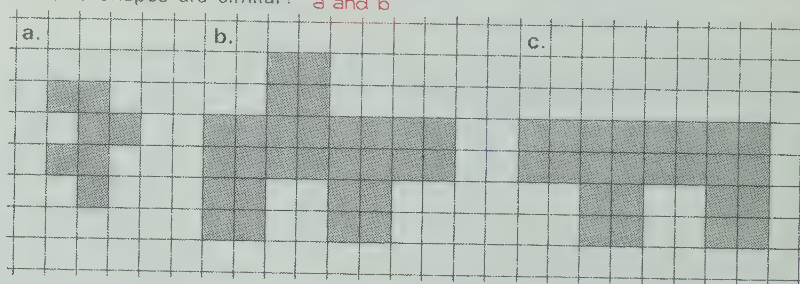




Working Together

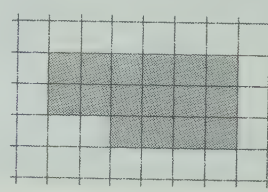
Which two shapes are similar? a and b

1.



Copy this shape on grid paper.

2. On the grid paper, draw a similar shape with each side two times as long.



LESSON ACTIVITY

Before Using the Pages

- Adapt the following activity for one or more of the items suggested under *Materials*. Present two pictures for which one is an enlargement of the other and ask whether the pictures are congruent. Develop that they are not congruent because they are not the same size. Point out that they are, however, the same shape; that is, each point in one picture matches a corresponding point in the other picture. Tell the students that in mathematics, we use the word *similar* to describe figures that have the same shape but not necessarily the same size.

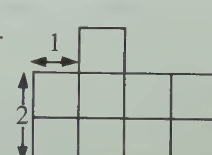
  Ask the students for examples of similar shapes such as similar labels on different sizes of ketchup bottles.

Using the Pages

- Ask students to describe how the photograph suggests similar figures. Emphasize the difference in what is meant by *similar figures* and what is meant by *congruent figures*.

Working Together: Of the three shapes shown for Ex. 1, it will likely be obvious that shape c. is not similar to shapes a. and b. (Shape c. suggests the letter F; the other two shapes do not.) Students may need to perform the following activity to reassure themselves that shapes a. and b. are similar. Have the students work in pairs so that one student's book is turned 90° counterclockwise and placed on the upper half of the other student's book in the normal position for reading page 186. This will help in comparing shape a. in the book on top with shape b. in the book below, because the two shapes will be oriented the same way. In this way, corresponding sides can be compared more easily to note that each side of shape b. is twice as long as the corresponding side of shape a.

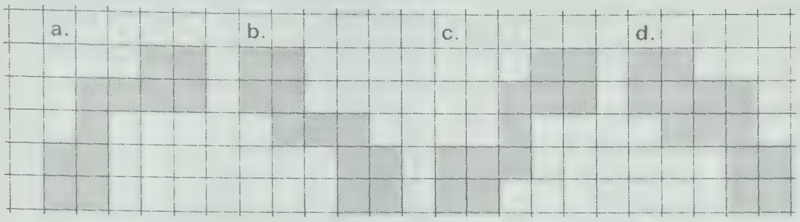
a.



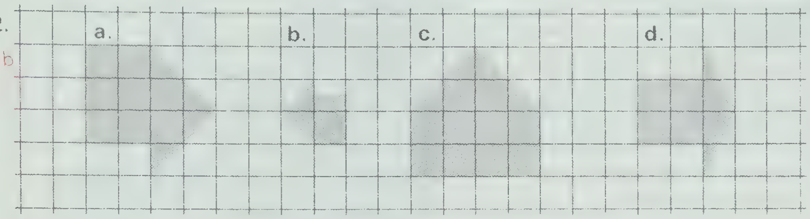
Exercises

For each of these, which two shapes are similar?

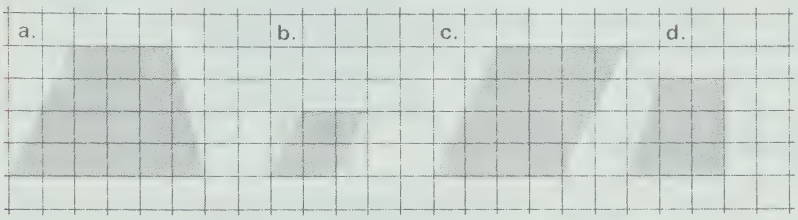
1. b and c



2. a and b

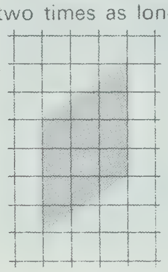


3. b and c

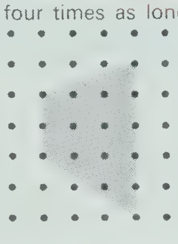


Copy each of these shapes. Then draw a similar shape with each side Answers are shown on page T371

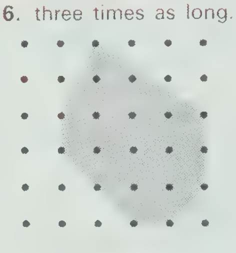
4. two times as long.



5. four times as long.



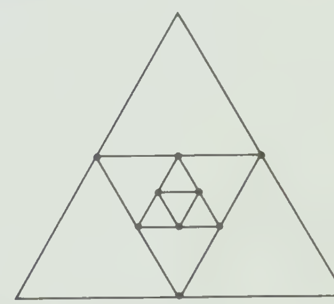
6. three times as long.



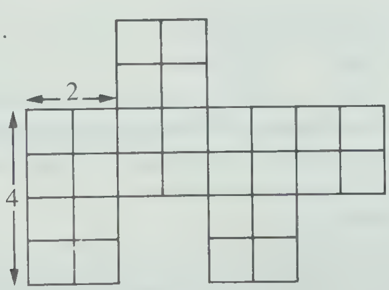
187

RELATED ACTIVITIES

- Include examples of congruent shapes and of similar shapes in the chart suggested in *Related Activities* on page T 191.
- Have students search magazines and catalogs for pictures of similar shapes. The shapes may be cut out and pasted in their notebooks or placed on the display board.
- Have students work in pairs using two geoboards and rubber bands. One student shows a shape on one geoboard. The other shows a similar shape on the other geoboard. You may wish to tell the students that the second shape is to have sides that are two or three times as long as those of the first shape. This activity can be adapted for use with geopaper or graph paper.
- Give each student a copy of the equilateral triangle on page T 383. Have the students find the midpoint of each side of the triangle and join the points to note that a similar triangle is obtained. Repeat the procedure for this new triangle. This activity may be adapted for an isosceles triangle and a scalene triangle.



b.



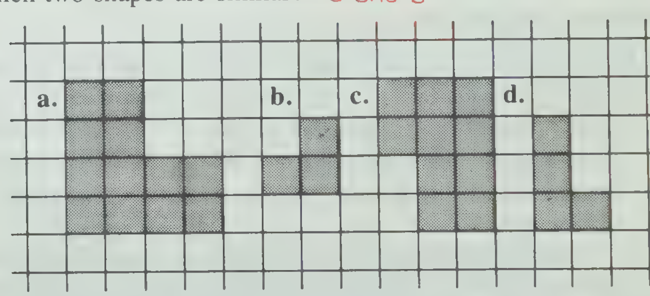
Provide each student with a straight edge and a copy of page T 396 or page T 397 for Ex. 2.

Exercises: Point out that for Ex. 1, the shapes that are similar are also congruent. The students should consider the shapes carefully in each exercise, noting clues which indicate that two shapes are not similar. For example, the trapezoid in Ex. 3d has a right angle, whereas the trapezoid in Ex. 3a does not. Have the students use a straight edge and a copy of page T 396 or page T 397 for Ex. 4, and a copy of page T 395 for Ex. 5 and 6. The results for these exercises can be displayed as examples of similar shapes.

Assessment

Which two shapes are similar? a and b

1.



Copy this shape. Then draw a similar shape with each side three times as long.

2.

Answer is shown on page T371.

LESSON OUTCOME

Use a scale to find the distance represented on a map; draw a diagram according to a given scale

Materials

a ruler marked in centimetres and tenths of centimetres for each student; metre sticks, measuring tapes, or trundle wheels

Vocabulary

scale, scale drawing, point, plane, curve

Prerequisite Skills

Multiply whole numbers; multiply decimals and whole numbers

Checking Prerequisite Skills

Multiply.

1. $\begin{array}{r} 13 \\ \underline{40} \end{array}$ 520
2. $\begin{array}{r} 24.8 \\ \underline{100} \end{array}$ 2480
3. 50×7.9 395.0
4. 20×16 320

Scale Drawings

What is the real distance between Vancouver and Rock Bay?

On the map, Vancouver and Rock Bay are 3.3 cm apart.

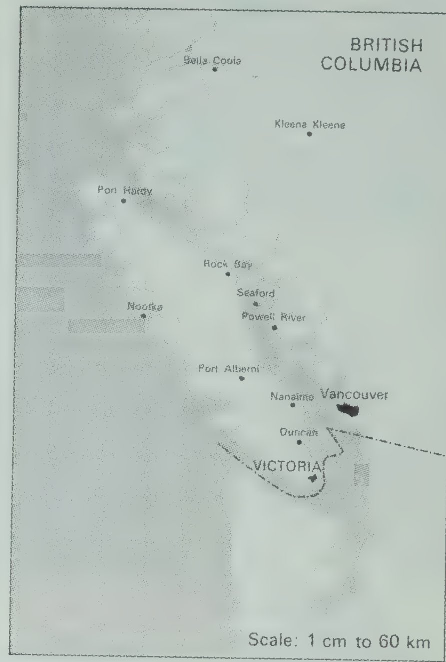
The scale for the map shows that each centimetre on the map represents 60 km.

Multiply to find the real distance.

3.3 × 60, or 198 km

the number of centimetres on the map times the number of kilometres represented by each centimetre

The real distance between Vancouver and Rock Bay is 198 km.



Working Together

Use the map.

1. On the map, what is the distance in centimetres from Seaford to Nanaimo? 2.1 cm
2. What numbers should be multiplied to find the real distance from Seaford to Nanaimo? 2.1 and 60
3. What is the real distance from Seaford to Nanaimo? 126 km

Follow these steps to make a scale drawing of your classroom.

4. Measure its length and width. Answers will vary
5. Choose a scale, such as 1 cm to 2 m.
6. Make the scale drawing.

For the scale 1 cm to 2 m, each centimetre length you draw represents 2 m of real length.

7. In your drawing, show the chalkboard.

188

LESSON ACTIVITY

Before Using the Pages

- Ask the students to turn to page 115 in their books. Have them use a ruler marked in centimetres to measure each of the sides marked on the diagram for Ex. 1. Discuss that each side is the actual length that is indicated for it. Have them measure the unmarked sides and give the lengths in centimetres.

  Draw attention to Ex. 5, pointing out that the indicated unit is kilometres. Ask the students to measure each of the marked sides of the diagram in centimetres. They will note that the side marked 3 km has an actual length of 3 cm, the side marked 1 km has a length of 1 cm, and so on. Establish that for this diagram, each centimetre represents 1 km. Have them measure the two unmarked sides in centimetres and give the lengths in kilometres. You may wish to repeat the procedure for Ex. 6 for which 1 mm represents 1 m, and for Ex. 9 for which 4 mm represent 1 km.

  Write the following scales on the board.

  1 cm to 1 km   4 mm to 1 km   1 mm to 1 m

  Discuss that the symbol = cannot be written between the units. Write the word "to" between the units in each pair and say, for example, "The scale is 1 cm *to* 1 km".

Using the Pages

- Begin with a discussion of the map shown on page 188. Ask several students to read the names of places shown on the map. Draw attention to the scale shown at the bottom of the map and ask a student to interpret it. Ask questions such as "If two places on the map are 2 cm apart, what is the real distance between them?" "What operation is used to find this distance?" "If the distance between two places is 300 km, how far apart are the places on the map?" "What operation did you use?"
- Lead the students through the worked example. Have them measure the distance on the map between Vancouver and Rock Bay to check that it is 3.3 cm. Ask a student to show the multiplication 3.3 × 60 on the board. For the

Exercises

Use the map to find the real distance for each of these. Answers may vary

1. Nootka to Powell River 150 km
2. Port Hardy to Port Alberni 246 km
3. Bella Coola to Duncan 438 km
4. Victoria to Seaford 210 km
5. Kleena Kleene to Port Hardy 228 km
6. Powell River to Kleena Kleene 228 km

Complete.

	Scale on a map	Distance on the map	Real distance	Places on the map
7.	1 cm to 20 km	12.75 cm	255 km	Jasper to Banff
8.	1 cm to 50 km	16.3 cm	815 km	Montreal to Charlottetown
9.	1 cm to 10 km	28.5 cm	285 km	Edmonton to Calgary
10.	1 cm to 100 km	3.8 cm	380 km	Niagara Falls to Ottawa
11.	1 cm to 50 km	14.3 cm	715 km	Saskatoon to Winnipeg

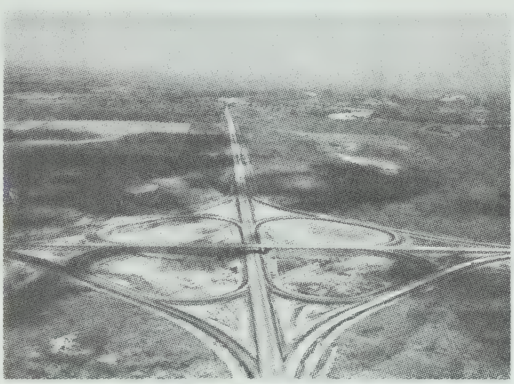
Use a scale different from the one on page 188.
Answers will vary

12. Make another scale drawing of your classroom. Include scale drawings of the classroom furniture.

Show your scale.

In this picture, each car suggests a **point**.
The fields suggest a **plane**.
A turn in a road suggests a **curve**.

A plane is a flat surface.



Look around.
Find five objects that suggest each of these.
Answers will vary

1. a point
2. a plane
3. a curve
4. a line

try this

189

RELATED ACTIVITIES

• Prepare a work sheet similar to the following and have the students use an atlas to find the actual distances.

Cities	Distance
Halifax to Charlottetown	
Regina to Winnipeg	

• Have the students measure and then make a scale drawing of one or more rooms in their homes.

• Several students may work together to make a large scale model of their neighborhood.

• Provide students with a chart showing actual distances and the corresponding distances on a map. Have them use the information to find the scale for each map.

Real distance	Distance on a map	Scale
145 km	5 cm	1 cm to ____ km

concluding statement, emphasize that the unit is kilometres.

Working Together: For Ex. 4, students may use a metre stick, a measuring tape, or a trundle wheel to measure the length and the width of the classroom. Discuss possible scales for Ex. 5 and help the students select an appropriate scale.

Exercises: Ex. 1-6 refer to the map on page 188. Measurements should be made to the nearest tenth of a centimetre. Point out that Ex. 7-11 involve different scales. Note that Ex. 10 and 11 require the use of division. This concept was suggested earlier in questioning the students about the map. The real distance divided by the number of kilometres represented by each centimetre gives the distance on the map in centimetres. This will help the students to complete the scale drawing for Ex. 12. Provide measuring tapes and metre sticks for Ex. 12.

Try This: The concepts of *point*, *plane*, and *curve* are suggested in the photograph on page 189. For example, seen from a height, a car on the road appears very small and suggests a point. It is important to discuss that just as a line is considered as continuing without end in opposite directions, a plane continues without end. The concept of different planes can be suggested by different levels in the same house and different playing levels for such games as three-dimensional chess or Tic Tac Toe. For Ex. 1-4, encourage the students to consider objects both inside and outside the classroom.

Assessment

Complete.

	Scale on the map	Distance on the map	Real distance
1.	1 cm to 30 km	25 cm	750 km
2.	1 cm to 100 km	3.7 cm	370 km
3.	1 cm to 50 km	8.9 cm	445 km

Choose a scale and make a scale drawing

4. of the top of your desk. Answers will vary.

LESSON OUTCOME

Copy a picture from a square grid onto a grid with squares of a different size and onto grids with rectangles or parallelograms

Materials

overhead projector and transparent acetate marked with a square grid (optional); 2 cm graph paper (or copies of page T 382), copies of pages T 398 and T 399, a sheet of blank paper, and a straight edge for each student

Multiply.

1. 87.6 30 2628	2. 1.71 2.9 4.959	3. 36.8 4.62 170.016
4. 516 4.1 2115.6	5. 0.61 0.1 0.061	6. 5.8 5.67 32.886
7. 0.82 0.2 0.164	8. 800 8.51 6808	9. 4.52 10 45.2
10. 0.93 0.01 0.0093	11. 0.24 0.34 0.0816	12. 0.87 0.1 0.087

Divide. Use extra zeros if needed.

13. 9)83.844 9.316	14. 54)3258.9 60.35
15. 685)265.78 0.388	16. 80)724 9.05
17. 32)25.824 0.807	18. 230)19.228 0.0836
19. 100)158 1.58	20. 90)67.59 0.751
21. 6)2883 480.5	22. 941)7151.6 7.6
23. 12)106.08 8.84	24. 475)35.815 0.0754
25. 10)4800.9 480.09	26. 600)294 0.49

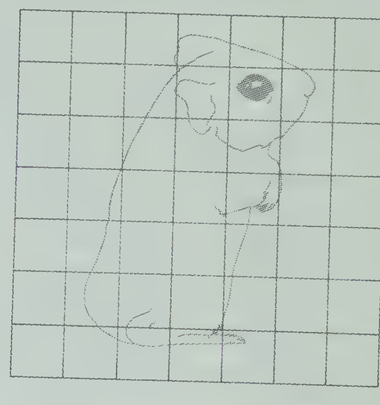
Do these in your head.

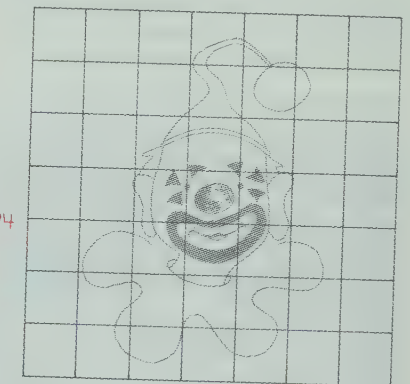
27. 0.1 × 9641 964.1	28. 29.4 ÷ 1000 0.0294
29. 8348 ÷ 10 834.8	30. 100 × 1.26 126
31. 4.37 × 10 43.7	32. 203 × 0.01 2.03
	33. 20 ÷ 1000 0.02
	34. 0.001 × 77 0.077
	35. 595 ÷ 100 5.95
	36. 0.01 × 400 4
	37. 146 ÷ 10 14.6

KEEPING SHARP

Copying Pictures Using Grids

Each of these pictures is drawn on a grid that uses squares.





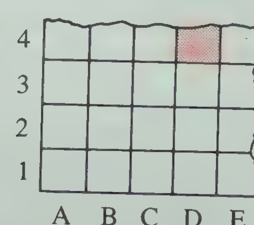
190

LESSON ACTIVITY

Using the Pages

- This lesson may be approached as one of exploration and enjoyment. The required skill is the ability to match points and join them as in the original figure.

  If you wish, prepare a copy of the picture of the clown on an overhead transparency marked with a square grid. Have the students trace the shape on the page at the same time as you trace the shape on the overhead projector. Discuss that a grid with smaller squares produces a smaller image, and that a grid with squares larger than the original grid produces a larger image. This concept can be demonstrated by moving the projector closer to or farther from the screen and adjusting the focus. Point out that all images have the same shape but are different sizes; that is, the images are similar shapes. Emphasize that each part of the clown is shown in the corresponding square on each grid. For example, the clown's nose appears in the fourth square of the fourth row. Because the students have studied ordered pairs previously, they may label the rows and columns of squares with letters and numbers, and use ordered pairs to identify regions. For the following way of labeling the grid, the clown's nose appears in the region identified by (D,4).

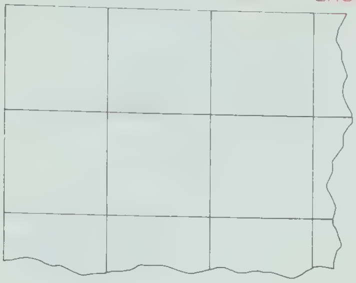


Exercises: Before the students begin, discuss the six grids shown on page 191. Ask how they are alike and how they are different. For the grids in Ex. 2-6, ask students to describe the image that will likely be obtained. For example, the image for Ex. 2 will not be the same size as the original shape, and it will be *distorted*; that is, it will appear narrower and taller.

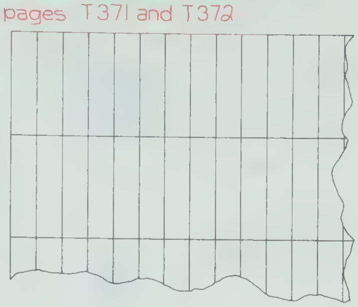
Exercises

Copy a picture from page 190 on grids that look like these. Pictures for the hampster are shown on pages T371 and T372

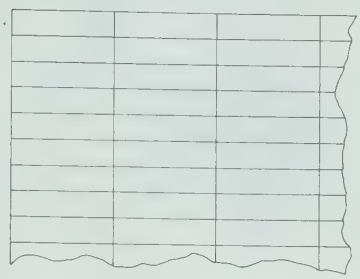
1.



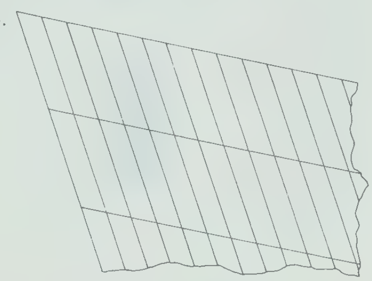
2.



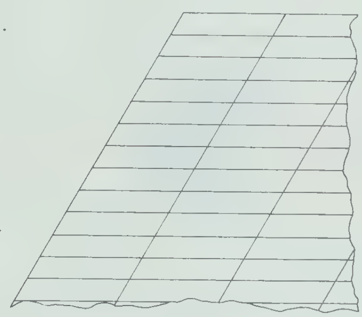
3.



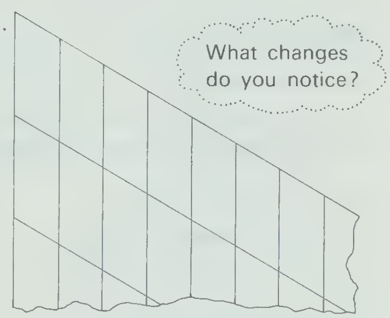
4.



5.



6.



7. Use a straight edge and draw a grid. Copy a picture from page 190 on your grid. Answers will vary for Ex 7 and 8

8. Draw a picture on a grid that uses squares. Copy your picture on grids that look like the ones on this page.

191

RELATED ACTIVITIES

- Have the students draw a shape on a copy of page T 398 or page T 399 and then draw a similar shape with each side two, three, or four times as long on a copy of the same page.
- Have the students draw distorted images of their favorite cartoon characters on copies of page T 398 or page T 399. Lines for a square grid are drawn on the original picture or transparent acetate marked with a square grid is placed over the picture. The picture may then be copied on the grid selected by the student for drawing a distorted image.

Provide the students with 2 cm graph paper for Ex. 1. If 2 cm graph paper is not available, copies of page T 382 may be used, although the sides of the squares of that grid are shorter than 2 cm. The students will require two copies of page T 398 for Ex. 2, 3, 4, and 5, and one copy of page T 399 for Ex. 6. For Ex. 7, the students are to use a straight edge to create their own grid. For Ex. 8, they may choose one grid from copies of pages T 398 and T 399.

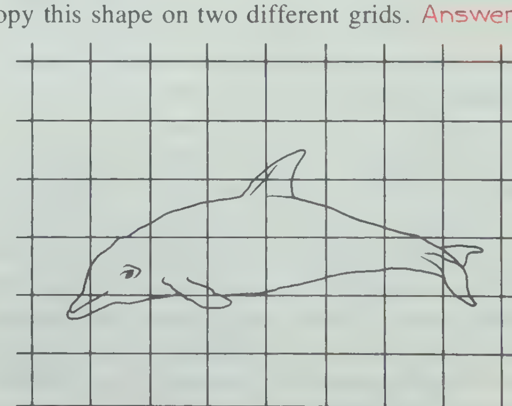
The results of these exercises can motivate a discussion as to where students have seen distorted images such as these. Some examples are the distorted images in curved mirrors at a fair, in the shiny curved surface of a kettle or a spoon, and words or pictures on inflated balloons. Discuss the changes noticed by the students in comparing the pictures for these exercises with those on page 190.

Keeping Sharp: Ex. 1 - 12 provide practice with multiplication of decimals (Unit 7). Ex. 13 - 26 involve division with decimals which also appears in Unit 7. Ex. 27 - 37 review multiplying by 0.1, 0.01, 0.001, 10, and 100 and dividing by 10, 100, and 1000. Have the students write their answers without written computation for Ex. 5, 9, 10, 12, 19, 25, and Ex. 27 - 37. It would be advisable to have the students write these answers first and then complete the other exercises.

Assessment

Copy this shape on two different grids. Answers will vary.

1.



Pages 192-193

LESSON OUTCOME

Identify the number of vertices, edges, and faces of solid shapes; identify the shapes of the faces of solids; draw a pattern for a given solid

Materials

models of the solids named on pages 192 and 193; paper model of a cube prepared from the pattern on page T 386; copies of page T 382 or page T 396, a straight edge, scissors, and tape for each student; copies of pages T 383-T 385 and compasses for each student (optional)

Vocabulary

solid, sphere, cone, cylinder, triangular prism, cube, rectangular prism, cuboid, pentagonal prism, hexagonal prism, triangular pyramid, tetrahedron, square pyramid, octagonal pyramid, vertex, edge, face

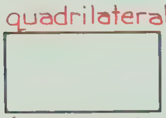
Prerequisite Skills

Identify polygons

Checking Prerequisite Skills

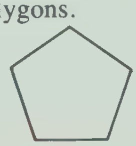
Name each of these polygons.

1. quadrilateral



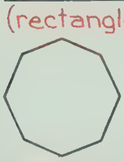
(rectangle)

2.



pentagon

3.

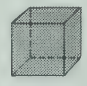


octagon

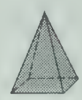
Solid Shapes

A solid shape can be made by folding a pattern for the shape.

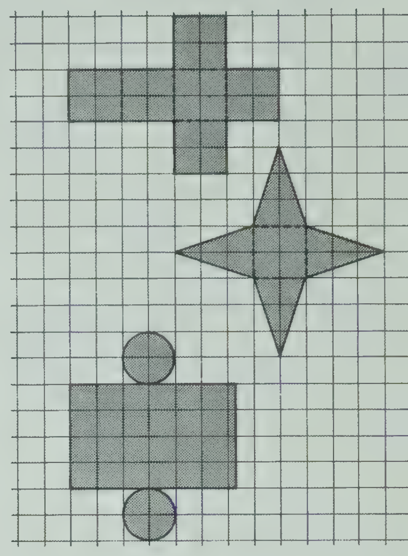
Cube



Square pyramid



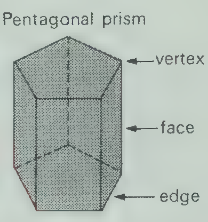
Cylinder



Working Together

To draw a pattern for the pentagonal prism,

1. how many faces would you draw? 7
2. what shape would each face have? 2 pentagons 5 rectangles

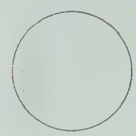


For a pentagonal prism,

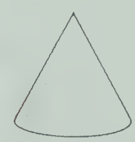
3. give the number of edges. 15
4. give the number of vertices. 10

Match these objects with the solids they suggest.

5. an ice cream cone b
6. a balloon a



a. a sphere



b. a cone

192

LESSON ACTIVITY

Before Using the Pages

- Display a set of solids for several days in advance of the lesson. Provide opportunities for the students to examine them and to observe their likenesses and differences. For example, some of them have curved surfaces which enable them to roll. Others can be stacked one on top of another. Still others have a sharp point at the "top" and cannot be stacked.

  The terms *solid*, *base*, *face*, *edge*, *vertex*, and *vertices* may be introduced informally. For one or more of the solids, ask students to count the vertices, edges, and faces. Note the shapes of the faces, having students trace around all the faces of a solid for assistance in identifying them. Ask questions such as "Which solid suggests the shape of a sugar cube?" "the shape of a bubble?" Have students point to the solids to answer the questions. Also, point to each solid in turn and ask them to name an object that has the same shape as the solid.

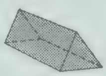
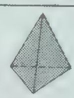
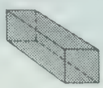
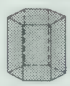
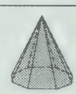
- Have students help to arrange the models into two groups — those having a curved surface and those having no curved surface. Then separate the latter group into two groups — those which rise to a sharp point (pyramids) and those which do not (prisms). Ensure that each prism is placed to stand on one of its two identical end faces.

Using the Pages

- Read each of the following terms: *cube*, *square pyramid*, *cylinder*, *pentagonal prism*, *sphere*, *cone*, *triangular prism*, *triangular pyramid (tetrahedron)*, *rectangular prism (cuboid)*, *hexagonal prism*, and *octagonal pyramid*. As each term is mentioned, have the students find the illustration for that solid on page 192 or page 193. Ask one student to match the illustration with the corresponding model displayed in the classroom. Note that the lateral faces of each pyramid are triangular and that the name of a pyramid is derived from the shape of its base. Discuss that the lateral faces of each prism are rectangular and that the name of a prism is derived from the shape of its two

Exercises

Complete.

	Kind of solid		Number of vertices	Number of edges	Number of faces	Number and shapes of the faces
1.		Triangular prism	6 ?	9 ?	5 ?	2 triangles 3 rectangles
2.		Triangular pyramid (tetrahedron)	4 ?	6 ?	4 ?	? 4 triangles
3.		Rectangular prism (cuboid)	8 ?	12 ?	6 ?	? 6 rectangles
4.		Hexagonal prism	12 ?	18 ?	8 ?	2 hexagons ? 6 rectangles
5.		Octagonal pyramid	9 ?	16 ?	9 ?	1 octagon ? 8 triangles

For each pattern on page 192,

6. enlarge the pattern and make a solid shape.

Draw a pattern for each of these.
Then make the solid shape.

7. a rectangular prism See the pattern on page T387
8. a triangular pyramid See the pattern on page T388
9. a cone See the pattern on page T386

Which solid is suggested by each of these?

10. a spruce tree cone
11. a baseball sphere
12. a juice can cylinder

Look around. Make a chart like this. Answers will vary

13.	Examples of solids	Kind of solid suggested
	a can of paint	cylinder

193

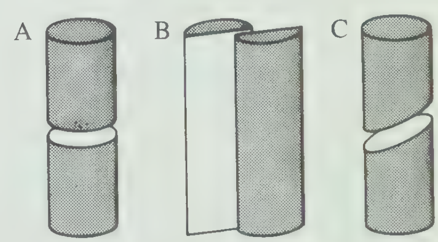
RELATED ACTIVITIES

• Print the names of the solids on pieces of paper and tape each name to the appropriate model. Display them for several days to enable the students to refer to them. Have students collect objects that resemble the shapes of the different solids.

• Students may work in pairs so that one student points to a model and the other student names the solid. If the name is taped to the solid, it can provide a check.

• Students may make solid shapes using the patterns from copies of pages T386-T389. The completed shapes may be displayed as a mobile, decorated to resemble objects that have the same shape, or glued together to create a robot-like character.

• Have students use plasticine or modeling clay to construct models of prisms, pyramids, cylinders, cones, and spheres. With supervision, some of the models may be sliced into two parts to observe the resulting shapes. For example, a horizontal cut for a cylinder produces two cylinders (A), whereas a vertical cut does not (B). An oblique cut (C) shows a curved surface that is not circular.



identical end faces. The shape of the base or the shape of the end faces determines the number of lateral faces for a pyramid or a prism.

• Draw attention to the patterns illustrated on page 192 for a cube, a square pyramid, and a cylinder. Ask what the dotted lines indicate on the patterns for the first two solids.

Display a paper model of a cube prepared from the pattern on page T386. Unfold the model to reveal the pattern for the cube and have the students compare it with the one shown at the top of page 192. Then fold the pattern to show the cube again.

Working Together: Some students may need to refer to the model of a pentagonal prism to help answer Ex. 1-4. Although the students are not required to draw the pattern, it would be beneficial to have them attempt one at this time. They can discover that more than one pattern is possible for this solid.

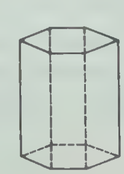
Exercises: Display the models to help the students complete the exercises. Provide them with copies of page T382 or page T396, straight edges, scissors, and tape for Ex. 6-9. To draw the patterns for Ex. 7-9, they may trace the faces of the models, use the procedure shown in the examples on page 192, or use polygons from copies of pages T383-T385 for some of the faces.

Assessment

Complete.

	Solid	Number of vertices	Number of edges	Number of faces	Number and shape of the faces
1.		8	12	6	6 square faces

Draw a pattern for this solid shape. See the pattern on page T388 for a hexagonal prism.

2. 

OBJECTIVE

Write an equation for information given in a word problem; solve the problem by writing and solving a related equation

Vocabulary

equation

RELATED ACTIVITIES

- Students may play a game using statements similar to the ones below. The first student to suggest the correct equation and the solution gives similar information for the next equation.

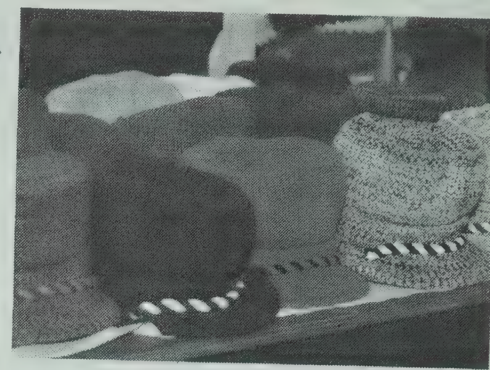
  I am thinking of a number. When I multiply it by 8, the result is 72. What is the number?

10. $n \times \$3 = \$66$
$\$66 \div \$3 = n$
$n = 22$

11. $\$3 - n = \$1.98$
$\$3 - \$1.98 = n$
$n = \$1.02$

12. $n \div 12 = 2$
$2 \times 12 = n$
$n = 24$

Writing and Solving Equations



Ann and her family sold hats at the fair. They had 62 hats. After selling for 3 h, they had 48 hats left. How many hats did they sell?

This equation tells the story.

$62 - n = 48$

This related equation helps complete the solution.

$62 - 48 = n$

$62 - 48 = 14$

They sold 14 hats.

Here are examples of related number sentences.

Addition and Subtraction		Multiplication and Division	
$14 + 48 = 62$	$48 + 14 = 62$	$8 \times 9 = 72$	$9 \times 8 = 72$
$62 - 48 = 14$	$62 - 14 = 48$	$72 \div 8 = 9$	$72 \div 9 = 8$

Copy each equation. Write a related equation. Then find the solution.

1. $n - 10 = 78$ $78 + 10 = n$ $n = 88$
2. $6 \times n = 42$ $42 \div 6 = n$ $n = 7$
3. $n \div 11 = 9$ $9 \times 11 = n$ $n = 99$
4. $31 \times n = 620$ $620 \div 31 = n$ $n = 20$
5. $n + 15 = 73$ $73 - 15 = n$ $n = 58$
6. $n - 25 = 100$ $100 + 25 = n$ $n = 125$
7. $n \div 10 = 76$ $76 \times 10 = n$ $n = 760$
8. $n \times 90 = 900$ $900 \div 90 = n$ $n = 10$
9. $21 + n = 53$ $53 - 21 = n$ $n = 32$

Write an equation for each of these. Then write a related equation. Find the solution.

10. Each hat sold for \$3. On the first day, Ann's family collected \$66. How many hats did they sell on the first day?

Answers are given at the left.

11. On the last day, the reduced price was \$1.98. How much was the reduction in price?

12. Ann worked many hours making hats for 12 d before the fair. The average time she spent each day was 2 h. How many hours did she spend making hats?

PROBLEM SOLVING

194

LESSON ACTIVITY

Before Using the Page

- Write the sentence $12 - 5 = 7$ on the board. Review that a number sentence that has the symbol = is called an *equation*. Ask a student to write another equation using subtraction and the same three numbers, 12, 5, and 7. Ask students to write two more equations using the same three numbers and addition. Review that the four equations form a family of related number sentences.

  Write the numbers 5, 8, and 40 on the board and ask whether equations can be written for them. Develop the four related sentences for multiplication and division.
- Write the sentence $n \times 8 = 48$ on the board and ask how this equation differs from the previous ones. Develop that one of the three numbers is not named, but it is represented by the letter n. Ask students to write the related sentences for $n \times 8 = 48$ using n, 8, and 48.

$n \times 8 = 48$	$48 \div n = 8$
$8 \times n = 48$	$48 \div 8 = n$

Point out that the related sentence $48 \div 8 = n$ is helpful in finding that n represents the number 6.

Using the Page

- Ask a student to read the word problem at the top of the page. Ask how the equation $62 - n = 48$ tells the story of the word problem. Ask what information in the word problem is not needed to solve the problem. Refer to the examples of related number sentences and discuss how the related equation used in the worked example helps to complete the solution. Emphasize that related sentences involve either addition and subtraction, or multiplication and division.
- For Ex. 1-9, the students are to use related equations to find the solutions. For Ex. 10-12, the students must write and solve equations and write concluding statements.

Pages 195-196

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

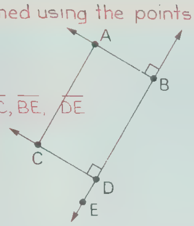
Materials

a protractor, tracing paper, a ruler marked in centimetres, and a copy of page T 398 or page T 399 for each student

Checking Up

Line segments and lines can be named using the points in either order

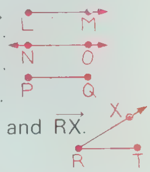
For this picture, name



1. a line. $\overleftrightarrow{BD}$, $\overleftrightarrow{DE}$
2. a line segment. $\overline{BD}$, $\overline{DC}$, $\overline{AB}$, $\overline{AC}$, $\overline{BE}$, $\overline{DE}$
3. a ray. $\overrightarrow{BA}$, $\overrightarrow{DC}$, $\overrightarrow{DB}$, $\overrightarrow{BD}$, $\overrightarrow{DE}$, $\overrightarrow{BE}$
4. two parallel rays. $\overrightarrow{BA}$ and $\overrightarrow{DC}$
5. two perpendicular rays. $\overrightarrow{DC}$ and $\overrightarrow{DB}$, or $\overrightarrow{BA}$ and $\overrightarrow{BD}$

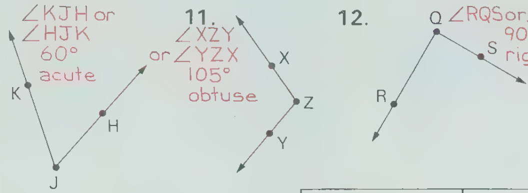
Draw and label

6. $\overleftrightarrow{LM}$.
7. $\overleftrightarrow{NO}$.
8. $\overline{PQ}$.
9. $\overrightarrow{RT}$ and $\overrightarrow{RX}$.



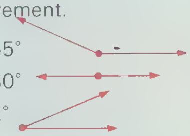
Name each angle. Measure the angle. Tell whether it is acute, right, obtuse, or straight.

10. ∠KJH or ∠HJK 60° acute
11. ∠XZY or ∠YZX 105° obtuse
12. ∠RQS or ∠SQR 90° right

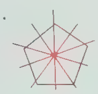
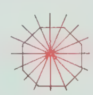
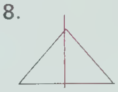


Draw an angle for each measurement.

13. 155°
14. 180°
15. 22°



Complete.

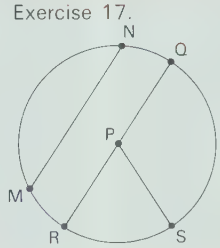
	16. 	17. 	18. 
Number of vertices	5 ?	8 ?	3 ?
Number of sides	5 ?	8 ?	3 ?
Number of lines of symmetry	5 ?	8 ?	1 ?

Trace and then show a line of symmetry for the polygon in All the lines of symmetry are shown

19. Exercise 16.
20. Exercise 17.
21. Exercise 18.

For this circle, name

22. the center. P
23. a diameter. $\overline{RQ}$
24. a radius. $\overline{PS}$, $\overline{PQ}$, $\overline{PR}$
25. two chords. $\overline{MN}$, $\overline{RQ}$
26. a line of symmetry. $\overline{RQ}$



Turn the page for more exercises.

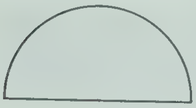
195

Skills	Exercises	Related Pages
Identify and name lines, line segments, and rays	1-3	T 190 - T 191
Identify and name rays that are parallel or perpendicular	4, 5	T 190 - T 191
Draw lines, line segments, and rays	6-9	T 190 - T 191
Name angles, measure angles, and classify them as acute, right, obtuse, or straight	10-12	T 192 - T 193
Draw angles	13-15	T 192 - T 193
Identify vertices, sides, and lines of symmetry for polygons	16-18	T 194 - T 195
Show lines of symmetry	19-21	T 194 - T 195
Identify parts of a circle	22-26	T 198 - T 199
Identify congruent shapes	27	T 200 - T 201
Identify similar shapes	28	T 202 - T 203
Use the scale of a map to find the real distance	29	T 204 - T 205
Draw a diagram to a given scale	30	T 204 - T 205
Copy a picture on a grid	31	T 206 - T 207
Draw a pattern for a solid shape	32	T 208 - T 209
Identify vertices, edges, and faces of a solid shape	33-35	T 208 - T 209
Identify the shapes of the faces of a solid shape	36	T 208 - T 209

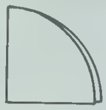
RELATED ACTIVITIES

• Give each student a copy of the circle on page T 385 or a circular filter paper. Have them fold the shape using these steps to divide the circumference of a circle into 12 equal parts.

1. Fold

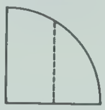


2. Fold



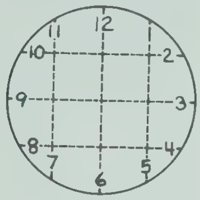
3. Fold

4. Unfold



5. Fold

6. Unfold



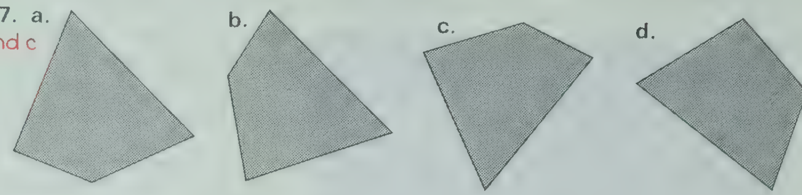
Have the students number the points on the edge as shown. Ask them to show all the ways in which points may be joined to form

a. an equilateral triangle.
b. a square.
c. a regular hexagon.
d. a regular dodecagon (12-sided polygon).
e. an isosceles triangle for which the common vertex of the equal sides is at 12.

The above exercises may be adapted for a geoboard with 12 nails that are arranged to form a regular dodecagon.

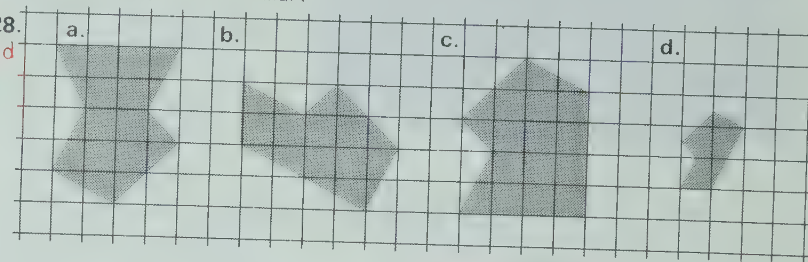
Use tracing paper and find two congruent shapes.

27. b and c



Which two shapes are similar?

28. b and d



Solve.

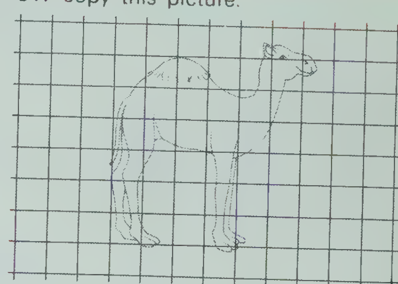
29. The distance on a map between two cities is 8 cm. The scale on the map is 1 cm to 50 km. What is the real distance between the cities? 400 km

Use a scale 1 cm to 5 m.

30. Make a scale drawing of a rectangle that is 30 m long and 25 m wide. Answer is shown below.

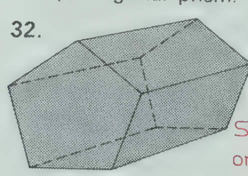
Using a different grid,
One picture is shown on page T372.

31. copy this picture.



Draw a pattern for this pentagonal prism.

32.



See the pattern on page T387.

For the solid shape in Exercise 32,

33. how many vertices are there? 10
34. how many edges are there? 15
35. how many faces are there? 7
36. what is the shape of each face? 2 pentagons 5 rectangles

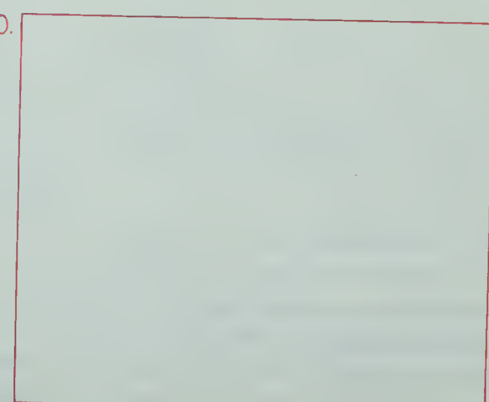
196

Comments

Because several exercises involve measuring, copying, and tracing, allow the students ample time so that they are not rushed in completing the exercises. Provide them with tracing paper for Ex. 19-21 and 27, and a copy of page T 398 or page T 399 for Ex. 31.

Students having difficulty with Ex. 16-21 may benefit from more practice in folding shapes cut from copies of pages T 383-T 385. The *Related Activities* on pages T 195 and T 197 suggest some ideas for reinforcement. Working with maps in an atlas can help students with the concept in Ex. 29. Those who find Ex. 30 difficult can measure familiar objects and make scale drawings to help them understand the relationship between a distance on a scale drawing and the real distance. More experience in tracing around the faces of models for solid shapes can lead to the understanding required for Ex. 32-36.

30.



Checking Skills

Multiply.

1.	974	7	6818
2.	638	9	5742
3.	4573	3	13 719
4.	46	40	1840
5.	502	10	5020
6.	489	60	29 340
7.	59	87	5133
8.	610	13	7930
9.	615	72	44 280
10.	267	900	240 300
11.	508	978	496 824
12.	496	100	49 600
13.	893	160	142 880
14.	137	809	110 833
15.	629	585	367 965
16.	4.2	80	336.0
17.	4.71	35	164.85
18.	27.3	700	19 110.0
19.	39	5.7	222.3
20.	2.7	0.1	0.27
21.	98.7	6.3	621.81
22.	2.094	6.8	14.2392
23.	54.7	3.15	172.305
24.	3.84	0.01	0.0384
25.	0.4	0.1	0.04
26.	0.81	0.9	0.729
27.	0.473	0.2	0.0946
28.	0.782	0.7	0.5474
29.	0.56	0.01	0.0056
30.	0.91	0.04	0.0364

31. 47 × 21.48
32. 0.001 × 3.4
33. 9.6 × 6.6
34. 3.1 × 2.365
35. 0.01 × 0.1
36. 5.3 × 8.6
37. 0.42 × 0.27
38. 9.02 × 0.01
39. 0.9 × 0.001
40. 6.73 × 5.48
41. 0.3 × 0.276
42. 0.1 × 6.66

31 1009.56
32 0.0034
33 63.36
34 7.3315
35 0.001
36 45.58
37 0.1134
38 0.0902
39 0.0009
40 37.2092
41 0.0828
42 0.666

Divide. Give the quotient and remainder.

1.	7)64	9 R1
2.	6)697	116 R1
3.	8)1675	209 R3
4.	9)58 500	6 500
5.	70)4350	62 R10
6.	80)6347	79 R27
7.	32)489	15 R9
8.	66)735	11 R9
9.	54)32 700	605 R30
10.	10)20 936	2 093 R6
11.	600)27 000	45
12.	700)31 280	44 R480
13.	482)29 895	62 R11
14.	327)46 385	141 R278
15.	150)92 104	614 R4
16.	903)60 093	66 R495

Divide. Use extra zeros if needed.

17.	5)2.65	0.53
18.	4)618	154.5
19.	7)49.28	7.04
20.	8)7.336	0.917
21.	58)60.436	1.042
22.	40)78.4	1.96
23.	71)7.313	0.103
24.	10)301	30.1
25.	30)11.46	0.382
26.	62)942.09	15.195
27.	100)692.5	6.925
28.	568)1448.4	2.55
29.	336)789.6	2.35
30.	825)1105.5	1.34
31.	654)624.57	0.955
32.	500)173	0.346

33. 465.2 ÷ 100
34. 5882 ÷ 85
35. 850.7 ÷ 94
36. 743 ÷ 100
37. 100 ÷ 800
38. 6.48 ÷ 45
39. 293.33 ÷ 10
40. 1323 ÷ 84
41. 125.8 ÷ 37
42. 53.9 ÷ 100

33 4.652
34 69.2
35 9.05
36 7.43
37 0.125
38 0.144
39 29.333
40 15.75
41 3.4
42 0.539

197

Page 197

OBJECTIVE

Demonstrate competence in multiplication and division skills

RELATED ACTIVITIES

- Have each student write a word problem for which the solution would be found by completing an exercise on page 197. You may wish to provide pictures to help the students think of topics for the problems. For example, the following problem would be solved by the multiplication in Ex. 17. "One roll of film costs \$4.71. What is the price for 35 rolls of the same film?"
- The activity "Multipatterns" described on page T 380 relates multiplication, addition, and geometry. Many students will find the activity an enjoyable experience.

Using the Page

- The exercises are arranged in two sets as follows.

	Multiplication	Division
Whole numbers	Ex. 1-15	Ex. 1-16
Decimals	Ex. 16-42	Ex. 17-42

Remind the students to place the decimal point in the dividend and to use extra zeros if necessary in divisions with decimals. Note that for certain exercises, written computation is not necessary, for example, when multiplying by 1000 or 0.01, and when dividing by 100. Earlier in this unit, the *Keeping Sharp* feature on page 181 provided review for multiplying and dividing with whole numbers, and the *Keeping Sharp* feature on page 190 provided review for multiplying and dividing with decimals.

The two sets of exercises may be assigned in several parts so that the students' abilities and needs may be identified more specifically. For example, you may wish to assign exercises in multiplication and division with whole numbers before assigning similar exercises involving decimals. Adapt the suggestions for the lessons and for the *Related Activities* on the appropriate pages of Unit 4 and Unit 7.

Unit 10 Overview

Fractions

In this unit the various concepts and terms associated with fractions and numbers in mixed form are presented. The terms *numerator* and *denominator* are introduced in the first lesson with reference to equal parts of a whole and equal parts of a set. Both multiplication and division are used to find equivalent fractions. Cross products are used to determine whether fractions are equivalent and also to find a missing term in a pair of equivalent fractions. Improper fractions are converted to whole numbers or to numbers in mixed form, and vice versa. Like denominators for fractions are found by multiplying their unlike denominators and by finding their least common multiple. Fractions with unlike denominators are compared by considering the numerators of equivalent forms of the fractions having like denominators. The *Problem Solving* lesson directs attention to the use of logical thinking to find solutions. Special features in the unit include two *Problem Solving* features and one *Keeping Sharp* feature to maintain skills in the basic operations with whole numbers. There are also two *Try This* features; one introduces prime numbers and composite numbers and the other the use of division by common factors to obtain fractions in lowest terms.

Prerequisite Skills

- relate multiplication and division
- identify factors of a number
- simplify an expression involving multiplication and addition
- divide whole numbers

Unit Outcomes

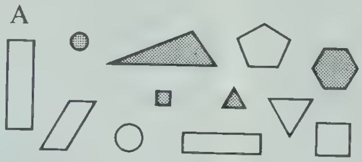
- identify the numerator and the denominator of a fraction
- write numerals for fractions less than one for part of a whole and part of a set
- write equivalent fractions for diagrams showing part of a whole
- use multiplication to find equivalent fractions
- use division to find equivalent fractions; find the greatest common factor of two numbers; write fractions in lowest terms
- use cross products to determine whether two fractions are equivalent
- use cross products to find the missing term in two equivalent fractions
- express a whole number as an improper fraction; express a number in mixed form as an improper fraction
- express an improper fraction as a whole number or as a number in mixed form
- use the product of unlike denominators to find equivalent fractions with like denominators for two or three fractions
- use the least common multiple of unlike denominators to find equivalent forms with like denominators for fractions and numbers in mixed form
- compare two fractions with unlike denominators; compare two numbers in mixed form for which the whole numbers are the same
- solve a problem through a process of logical thinking

Background

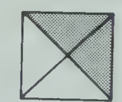
Early records show the development of numbers from simple tally systems, such as notches on a stick, to numeration systems using a variety of symbols and styles of notation. Numeration systems for whole numbers were developed, but they did not meet the need for representing part of a whole, so fractional numbers were created. Ancient civilizations used different methods for dealing with fractions. For instance, the Egyptians used only unit fractions, fractions with 1 as numerators, and to express other fractions such as $\frac{3}{4}$, they indicated the addition of $\frac{1}{2}$ and $\frac{1}{4}$. The Babylonians used a better system based on sixty and denominators were factors and multiples of sixty. The Romans used denominators based on twelve and multiples of twelve. To express a fraction the Hindus wrote one numeral above another, $\begin{smallmatrix}3\\4\end{smallmatrix}$, and the Arabs introduced the use of the horizontal bar, $\frac{3}{4}$.

The word *fraction* comes from the Latin word *frangere* meaning to break; hence a fraction represents part of a whole or a set. When a fraction represents part of a whole, the *denominator* shows the number of equal parts of the whole and the *numerator* shows the number of parts being considered. When a fraction represents part of a set, the denominator shows the number of items in the set and the numerator shows the number of items that are being considered. The items in a set are not necessarily the same; for instance, the fraction $\frac{5}{12}$ may be used to represent 5 gray shapes in a set of 12 shapes, although the shapes are not alike (A). It is apparent that while zero may be a numerator, it cannot be a denominator because no object or set could have zero equal parts.

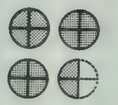
A



B



C



A fraction may be represented in various equivalent forms. From concrete models or from prepared diagrams, it is seen, for instance, that $\frac{1}{2}$ and $\frac{2}{4}$ are equivalent (B) and that $\frac{14}{4}$, $\frac{7}{2}$, and $3\frac{1}{2}$ are equivalent (C). Comparison of whole numbers is relatively easy since cardinal numbers are an ordered set in which each number is one greater than the preceding number. Thus, 7 is greater than 6, and 8 is greater than 7. Comparison of fractions with like denominators is also easy because only the numerators which represent the cardinal numbers of parts need to be considered. Because 7 is greater than 6, it is obvious that $\frac{7}{8}$ is greater than $\frac{6}{8}$. Comparison of fractions with unlike denominators is more complex, however, because the sizes indicated by the denominators must be considered as well as the number of parts represented by each numerator.

Comparison of fractions with unlike denominators is usually achieved by rewriting them as equivalent fractions having the same denominator. The most commonly used procedure for finding an equivalent fraction involves multiplying both the numerator and the denominator of a fraction by the same number. This method changes the form of the fractional number represented, but it does not change its value. With whole numbers, the identify element for multiplication is 1; that is, any number multiplied by 1 is unchanged ($6 \times 1 = 6$, and $1 \times 31 = 31$). The number 1 may be used in a variety of fractional forms, such as $\frac{2}{2}$, $\frac{3}{3}$, $\frac{5}{5}$, and so on, and this is the basis for the method of finding equivalent fractions. For example, if $\frac{2}{3}$ is multiplied by 1 in these forms, equivalent fractions are

obtained (D). Equivalent fractions in lower terms may also be found by dividing by the identity element, 1, in any appropriate form (E). Thus an equivalent fraction may be found by multiplying (dividing) by the number 1 in any fractional form, or, as it is more commonly stated, by multiplying (dividing) both the numerator and the denominator by the same number.

D $\boxed{\frac{2}{3} \times 1}$ $\frac{2}{3} \times \frac{2}{2} = \frac{4}{6}$ E $\boxed{\frac{16}{20} \div 1}$ $\frac{16}{20} \div \frac{2}{2} = \frac{8}{10}$

The same application of the identity element underlies the use of cross products to test whether two fractions are equivalent. For example, to check whether $\frac{3}{4}$ and $\frac{18}{24}$ are equivalent fractions (F), they may be rewritten so that the product of 4 and 24 is the common denominator (96). The cross-product method does not involve writing the common denominator, but the factors used are identical to those used to obtain the numerators in the preceding method, namely, 3×24 and 4×18 (G).

F $\frac{3}{4} \times \frac{24}{24}$ $\frac{18}{24} \times \frac{4}{4}$

$\frac{72}{96}$ $\frac{72}{96}$

$\frac{72}{96} = \frac{72}{96}$, so $\frac{3}{4} = \frac{18}{24}$

G $\frac{3}{4}$ $\frac{18}{24}$

3×24 4×18

$72 = 72$

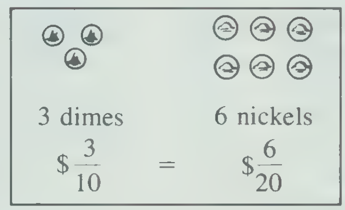
$\frac{3}{4} = \frac{18}{24}$

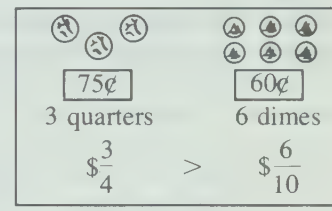
The set of whole numbers is an ordered set starting from zero with each number one greater than its predecessor. There is no whole number between any two successive whole numbers; their difference is always 1. However, between any two whole numbers and between any two fractions there is an unlimited number of fractions, that is, between any two fractions another can always be inserted. This property of fractions is referred to as *density*. For example, between $\frac{1}{4}$ and $\frac{3}{8}$ there is the fraction $\frac{5}{16}$, and between $\frac{5}{16}$ and $\frac{3}{8}$ there is the fraction $\frac{1}{3}$. If the fractions $\frac{5}{16}$ and $\frac{1}{3}$ are expressed as equivalent fractions with the denominator 96 ($\frac{5}{16} = \frac{30}{96}$ and $\frac{1}{3} = \frac{32}{96}$), then the fraction $\frac{31}{96}$ is between them.

Any whole number may be expressed as the product of two or more whole-number factors. The number 0 may be represented in an unlimited number of ways, but one of the factors must be 0, as in 2×0, 765×0, and $3 \times 2 \times 0$. The number 1 has only itself as a factor and it may be used any number of times, as in 1×1, $1 \times 1 \times 1 \times 1$, and so on. Numbers greater than 1 may be of two types: some have only two factors, while others have more than two factors. For instance, 24 may be represented by 4×6, 3×8, 2×12, and 1×24, and thus 24 has eight factors, 1, 2, 3, 4, 6, 8, 12, and 24. In contrast, a number such as 23 has only two factors, 1 and itself, because it can be shown only as 1×23. A number which has only two factors is a *prime* number; and a number which has more than two factors is a *composite* number. The number 1 is neither prime nor composite. The first prime number is 2; 3 is also prime, but 4 is composite. All even numbers greater than 2 are composite. All prime numbers greater than 2 are odd, but not all odd numbers are prime. For instance, 5 (1×5) and 7 (1×7) are prime numbers, but 9 (1×9, 3×3) is a composite number. In the third century B.C. a famous Greek geographer devised a method for finding prime numbers. It is called the *sieve of Eratosthenes*. He arranged the numbers in order on a parchment sheet and, beginning with the number 2, he left it and punched out all the other multiples of 2. Then he did the same for 3, for 5, for 7, and so on. When he had finished, the parchment looked somewhat like a sieve and only prime numbers remained. In the set of numbers to 100 there are 25 prime numbers (2, 3, 5, 7, 11, 13, 17, 19, 23, 29, 31, 37, 41, 43, 47, 53, 59, 61, 67, 71, 73, 79, 83, 89, 97).

Teaching Strategies

Since the concepts associated with fractions are more complex and are met less frequently in daily life than those for whole numbers, it is important that a variety of models and teaching aids be used in the lessons. It is suggested that the customary circular shapes and square shapes be supplemented by such devices as number lines, egg cartons, and coins. Number lines with two or more scales can be used to visually identify equivalent fractions and to compare fractions with unlike denominators. For the same purpose, cardboard strips with equal unit lengths may be marked to show fractional parts. An egg carton may be used to illustrate equivalent fractions, such as $\frac{1}{2} = \frac{6}{12}$, $\frac{5}{6} = \frac{10}{12}$, and $\frac{2}{3} = \frac{8}{12}$. Coins, expressed as fractional parts of a dollar may be used as shown below.





Students should have access to the materials listed for the lessons and to those indicated in the *Related Activities* for use during the lessons and in their free time. For slower students who may not have much free time, it may be necessary to schedule such activities so that they may also have direct experiences with concrete materials.

Although this unit and Unit 11 deal with fractions and operations with them, regular practice with whole numbers and with decimals should be arranged to maintain students' skills. Suitable exercises are given on pages 328-331.

It is important that any errors or misconceptions which are discovered in the *Checking Up* exercises on page 215 be corrected, since many of the concepts and skills developed in this unit are prerequisites for the work in Unit 11.

Materials

shapes with parts colored blue to represent $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{1}{5}$
three blue items, such as pencils, crayons, or markers, and seven items of other colors
a blue crayon, a copy of the decimetre square on page T 392, and a piece of paper for each student
six red markers and four blue markers
models for wholes, halves, thirds, fourths, and fifths prepared from copies of the squares on page T 392 or the circle on page T 385
a copy of page T 394 for each student (optional)
a straight edge for each student

Vocabulary

fraction	improper fraction
numerator	proper fraction
denominator	mixed form
equivalent fractions	prime number
common factor	composite number
lowest terms	unlike denominators
greatest common factor	like denominators
cross products	common denominators
is not equal to (≠)	multiple
names of fractions less than one	common multiple
	least common multiple

LESSON OUTCOME

Identify the numerator and the denominator of a fraction; write numerals for fractions less than one for part of a whole and part of a set

Materials

shapes with parts colored blue to represent $\frac{3}{4}$, $\frac{2}{3}$, and $\frac{1}{5}$; three blue items, such as pencils, crayons, or markers, and seven items of other colors

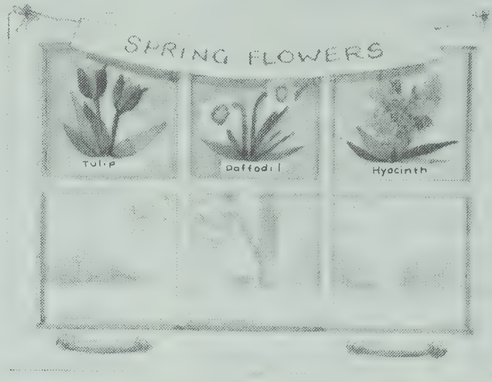
Vocabulary

fraction, numerator, denominator, names of fractions less than one

10 FRACTIONS

Writing Fractions

The students decorated the windows for spring. Brad has 3 of 6 parts of his window decorated.



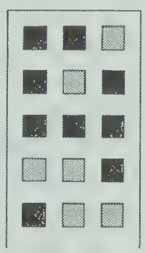
The fraction $\frac{3}{6}$ shows what part of the window Brad has decorated.

The fraction $\frac{1}{2}$ ← numerator, ← denominator

also shows how much is decorated.

Working Together

For the windows in this building,



1. give a fraction that shows how many of the windows have a light shining. $\frac{6}{15}$

For the fraction you gave, which number

2. is the numerator? 6
3. is the denominator? 15

Complete each sentence with a fraction.

4. $\frac{\blacksquare\,2}{\blacksquare\,8}$ of the letters in the word "Manitoba" are the letter "a".
5. $\frac{\blacksquare\,3}{\blacksquare\,11}$ <u>of the underlined words in this sentence begin with the letter "t".</u>

198

LESSON ACTIVITY

Before Using the Pages

- Present several examples to enable the students to recall previous work with fractions for part of a whole and part of a set. For example, fold a sheet of paper into four equal parts and color three of the four parts blue. Ask how many parts there are, whether the parts of the whole are the same size, and how many parts are blue. Elicit the fraction "three-fourths" to describe the part of the whole that is blue and, if possible, have a student write the numeral $\frac{3}{4}$ on the board. Review that the numeral below the bar shows the number of equal parts, and that the numeral above the bar shows the number of special parts. Use other similar examples. Display a whole that shows four unequal parts, three of which are blue. Ask whether three-fourths of the whole is blue and ask students to explain their answers.

  For part of a set, display two blue markers and three red markers. Ask how many markers there are in the set and how many of these are blue. Elicit the fraction "two-fifths" to describe the part of the set that is blue, and ask a student to write the numeral $\frac{2}{5}$ on the board. For other examples, have students help to show the sets and write the numerals for parts of the sets on the board. For instance, display 5 red markers. Ask what fraction describes the part of the set that is red ($\frac{5}{5}$) and the part that is blue ($\frac{0}{5}$). Ask what name describes such numbers as $\frac{3}{4}$ and $\frac{2}{5}$.

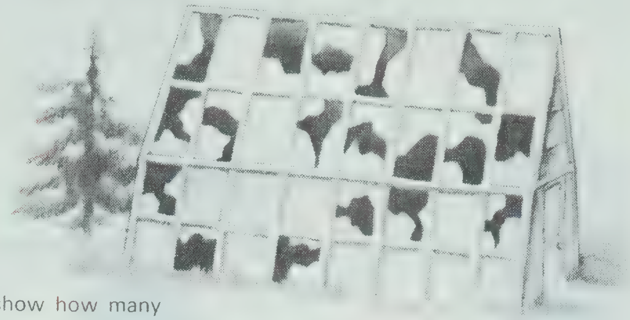
Using the Pages

- Have the students note the title of the lesson on page 198. Draw attention to the illustration, noting that the window shows six equal parts, three of which have been decorated. Ask students to read the statement that relates $\frac{3}{6}$ to the illustration. Ask which part of the frame separates the window into two equal parts. Students may suggest that the upper half of the window has been decorated, and therefore the fraction $\frac{1}{2}$ shows how much is decorated. Summarize that $\frac{1}{2}$ and $\frac{3}{6}$ are different names for the number that shows what part of the window is decorated. Review the terms *numerator* and *denominator*.

Exercises

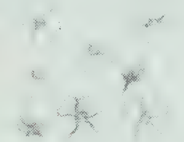
The hailstorm broke many panes of glass on one side of the greenhouse. Write a fraction to show how many panes have been broken in each row

1. first row $\frac{5}{8}$
2. second row $\frac{7}{8}$
3. third row $\frac{4}{8}$
4. fourth row $\frac{2}{8}$

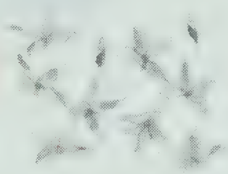


Write fractions to show how many of the buds have blossomed.

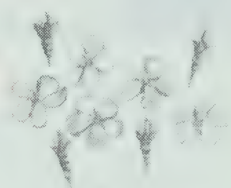
5. $\frac{6}{8}$



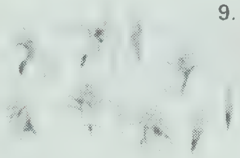
6. $\frac{8}{10}$



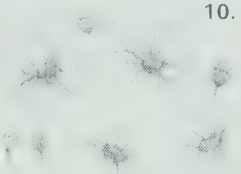
7. $\frac{5}{9}$



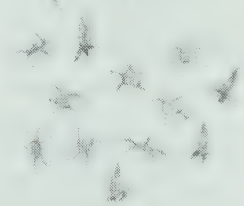
8. $\frac{6}{9}$



9. $\frac{4}{8}$



10. $\frac{8}{12}$



Write the fraction

11. with numerator 3 and denominator 7. $\frac{3}{7}$
12. with denominator 12 and numerator 7. $\frac{7}{12}$

Write a sentence using a fraction for each of these.

13. The room has 5 windows. 3 windows are open. $\frac{3}{5}$ of the windows are open
14. There are 16 windows. 12 have been washed. $\frac{12}{16}$ of the windows have been washed

*15. 6 window frames were painted. 8 frames are still unpainted.
$\frac{6}{14}$ of the window frames were painted
$\frac{8}{14}$ of the window frames are still unpainted

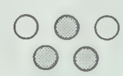
*16. 7 windows are broken. 3 windows are not broken.
$\frac{7}{10}$ of the windows are broken
$\frac{3}{10}$ of the windows are not broken.

199

RELATED ACTIVITIES

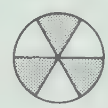
- For Ex. 5-10 on page 199, students may write fractions to show how many of the buds have not blossomed.
- Have the students draw pictures or color parts of shapes to represent several fractions less than one. Encourage them to include drawings that show part of a whole and drawings that show part of a set. For part of a whole, students may fold regular polygons cut from copies of pages T 383-T 385. Include such fractions as $\frac{0}{3}$ and $\frac{3}{3}$. Display several drawings for each fraction.
- Play a game by asking, for example, "What fraction represents the number of students in this class who are boys?" The first student to name the correct fraction asks the next question. In preparation for the game, ask the students to write questions with the answers on cards.
- Have each student write a sentence using a fraction, for example, "Ann ran $\frac{3}{4}$ of the way home."
- To help students who tend to reverse the numerator and the denominator in a fraction, use exercises similar to the following.

1.



2 of the 5 marbles are white. $\frac{\blacksquare}{\blacksquare}$ of the set of marbles is white.

2.

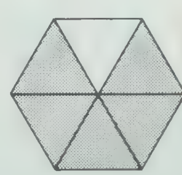


■ of the 6 parts of the whole are grey. $\frac{\blacksquare}{6}$ of the whole is grey.

Working Together: Ask students to describe what the numerator and the denominator represent in Ex. 2-5. Students may write other fractions for these exercises to show, for example, the number of windows that are dark, the number of underlined words that begin with the letter "w", the letter "x", and so on.

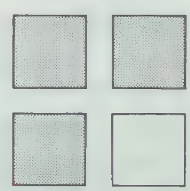
Exercises: Ex. 15 and 16 are starred because the students must use addition to find the number of windows in all for the denominator. Note that two answers are possible for each of these exercises.

5.



$\frac{5}{6}$

6.



$\frac{3}{4}$

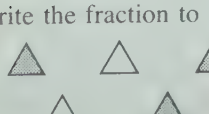
Assessment

For the fraction $\frac{2}{3}$, which number

1. is the numerator? 2
2. is the denominator? 3

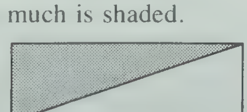
Write the fraction to show how much is shaded.

3.



$\frac{3}{5}$

4.



$\frac{1}{2}$

LESSON OUTCOME

Write equivalent fractions for diagrams showing part of a whole; use multiplication to find equivalent fractions

Materials

a blue crayon and a copy of the decimetre square on page T 392, a piece of paper for each student, tracing paper (optional)

Vocabulary

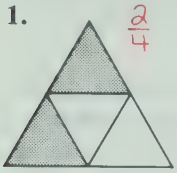
equivalent fractions

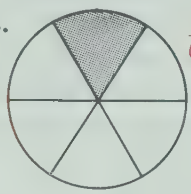
Prerequisite Skills

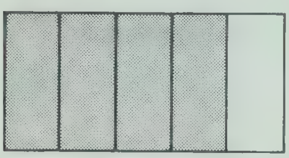
Write numerals for fractions less than one

Checking Prerequisite Skills

Write the fraction to show how much is shaded.

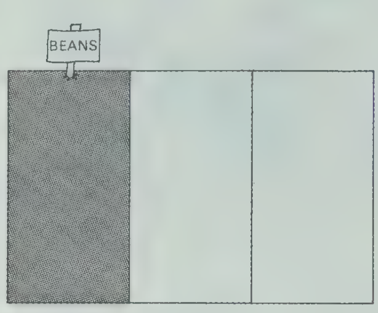
1.  $\frac{2}{4}$

2.  $\frac{1}{6}$

3.  $\frac{4}{5}$

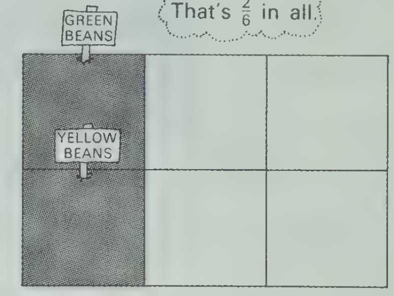
Multiplying to Find Equivalent Fractions

Fred wanted to plant two kinds of beans in $\frac{1}{3}$ of his garden.



To do this, he first doubled the number of sections in the garden. Then he planted green beans in $\frac{1}{6}$ of the garden and yellow beans in $\frac{1}{6}$ of the garden.

That's $\frac{2}{6}$ in all.



$$\frac{1}{3} \underset{3 \times 2}{\overset{1 \times 2}{=}} \frac{2}{6}$$

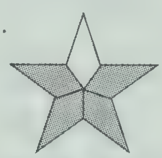
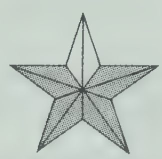
$\frac{1}{3}$ and $\frac{2}{6}$ both name the same amount.

$\frac{1}{3}$ and $\frac{2}{6}$ are **equivalent fractions.**

Multiplying both the numerator and the denominator by the same number gives an equivalent fraction.

Working Together

Complete.

1.  

$$\frac{4}{5} \underset{5 \times 2}{\overset{4 \times 2}{=}} \frac{\blacksquare}{\blacksquare} \quad \frac{8}{10}$$

Multiply both the numerator and the denominator by 4 to find equivalent fractions.

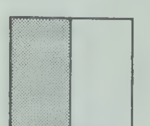
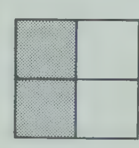
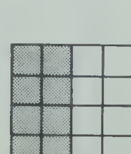
2. $\frac{1}{2}$ $\frac{4}{8}$ 3. $\frac{3}{5}$ $\frac{12}{20}$ 4. $\frac{2}{3}$ $\frac{8}{12}$ 5. $\frac{5}{8}$ $\frac{20}{32}$

200

LESSON ACTIVITY

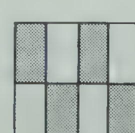
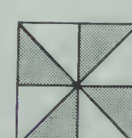
Before Using the Pages

- Give each student a decimetre square cut from copies of page T 392. Ask one group of students to fold their squares in half and color one half blue. Ask a second group to fold their squares into fourths and color two fourths blue. Direct a third group to fold their squares into eighths and color four eighths. Ask the remaining students to fold their squares to show sixteenths and color eight sixteenths blue. For these folds, direct the students not to fold along a diagonal at this time. Select four examples, one from each group, and tape them to the board.

Develop that the same amount on each of the congruent shapes is colored, and thus the fractions $\frac{1}{2}$, $\frac{2}{4}$, $\frac{4}{8}$, and $\frac{8}{16}$ name the same amount. Ask the students, "If there were six equal parts, how many parts would be blue?" "What fraction is another name for one-half?" Ask a student to draw two lines on the diagram for one-half to show six equal parts, three of which are blue. Ask students to suggest other fractions that name the same amount as $\frac{1}{2}$, and have them write the numerals on the board ($\frac{2}{4}$, $\frac{3}{6}$, $\frac{4}{8}$, $\frac{8}{16}$, $\frac{10}{20}$, $\frac{25}{50}$, . . .).

You may wish to have the students repeat one of the folding and coloring activities to show diagrams for one-half similar to the following.

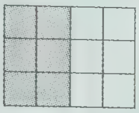
Ask if $\frac{9}{17}$ is another name for one-half and ask students to explain their answers. Then ask the students if they recall the mathematical term that describes fractions that name the same amount.

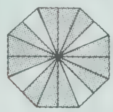
Exercises

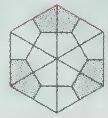
Write four equivalent fractions for each of these by multiplying both the numerator and the denominator by the same number. First use 2, then 3, then 4, and then 5.

1. $\frac{1}{4}$, $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$ **2.** $\frac{1}{6}$, $\frac{2}{12}, \frac{3}{18}, \frac{4}{24}, \frac{5}{30}$ **3.** $\frac{2}{5}$, $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$ **4.** $\frac{4}{7}$, $\frac{8}{14}, \frac{12}{21}, \frac{16}{28}, \frac{20}{35}$ **5.** $\frac{5}{8}$, $\frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}$

Write two equivalent fractions for each picture.

6.  $\frac{6}{12}$ and $\frac{1}{2}$

7.  $\frac{12}{16}$ and $\frac{3}{4}$ or $\frac{4}{16}$ and $\frac{1}{4}$

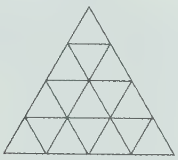
8.  $\frac{6}{18}$ and $\frac{1}{3}$ or $\frac{12}{18}$ and $\frac{2}{3}$

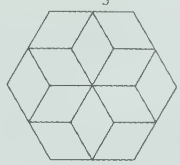
Write five fractions that are equivalent to each of these. Other answers are possible

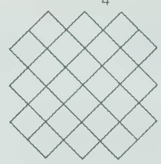
9. $\frac{1}{5}$ **10.** $\frac{5}{6}$ **11.** $\frac{3}{8}$ **12.** $\frac{4}{9}$ **13.** $\frac{7}{10}$

Copy and color each picture three different ways Answers will vary

14. to show $\frac{1}{2}$. **15.** to show $\frac{2}{3}$. **16.** to show $\frac{3}{4}$.







How large do numbers get when you double them again and again?

Make a choice:

$10 allowance each week for half a year

1¢ allowance the first week,
2¢ allowance the second week,
4¢ allowance the third week,
8¢ allowance the fourth week,
and so on for half a year.

1. How much would you have in half a year for each choice?
$260, $671 088.63

Fold a piece of paper in half. Then fold it in half again.

Answers will vary for Ex. 2 and 3

2. How many times can you keep folding the paper?

3. How thick does the folded paper get?

4. If a piece of paper is 0.1 mm thick unfolded, how thick would it get if you could fold it in half 20 times? 104 857.6 mm

PROBLEM SOLVING

9. $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}, \frac{6}{30}$ 10. $\frac{10}{12}, \frac{15}{18}, \frac{20}{24}, \frac{25}{30}, \frac{30}{36}$ 11. $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \frac{15}{40}, \frac{18}{48}$

12. $\frac{8}{18}, \frac{12}{27}, \frac{16}{36}, \frac{20}{45}, \frac{24}{54}$ 13. $\frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}, \frac{42}{60}$

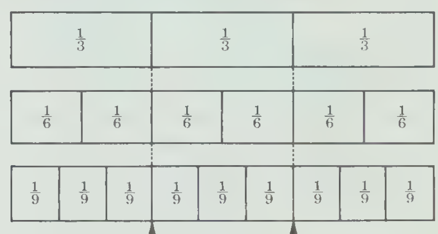
201

RELATED ACTIVITIES

- Students can play the game "Name the Fraction" described on page T 381.
- Give each student a copy of page T 382 on which to complete a multiplication table for the numbers 1 to 9 as factors. Demonstrate how the table can be used to help name equivalent fractions. For example, the row for 2 and the row for 5 can reveal fractions that are equivalent to $\frac{2}{5}$ ($\frac{4}{10}$, $\frac{6}{15}$, $\frac{8}{20}$, . . .). It may also be seen, for example, that the numerator and the denominator of $\frac{2}{5}$ are each multiplied by 2 to obtain $\frac{4}{10}$, by 3 to obtain $\frac{6}{15}$, and so on.

×	1	2	3	4
1				
2	2	4	6	8
3				
4				
5	5	10	15	20

- Cardboard strips of equal length may be marked to show halves, thirds, fourths, fifths, sixths, eighths, ninths, tenths, and twelfths. By placing strips together, students can find equivalent fractions for $\frac{1}{2}, \frac{1}{3}, \frac{2}{3}, \frac{1}{4}, \frac{3}{4}, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}$.



$\frac{1}{3} = \frac{2}{6} = \frac{3}{9}$ $\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$

Using the Pages

- Have the students note the title at the top of page 200. Ask a student to read the first statement below it. Note that the first diagram shows thirds and that one-third is shaded. Draw attention to the second diagram and ask students to interpret how Fred likely solved his problem. Then ask a student to read the statement above the second diagram. Summarize that the diagrams show that $\frac{1}{3}$ and $\frac{2}{6}$ name the same amount. Then ask how this can be shown using multiplication. Emphasize that both the numerator and the denominator of the fraction $\frac{1}{3}$ are multiplied by 2 to obtain the fraction $\frac{2}{6}$. Introduce the term *equivalent fractions* and discuss the information about equivalent fractions in the "thought cloud".

Working Together: For Ex. 1, the students may use the diagrams to derive the equivalent fractions. However, discuss that both the numerator and the denominator of $\frac{4}{5}$ are multiplied by 2 to obtain the second fraction.

Exercises: Ensure that the students understand what is required for Ex. 1-5. The multipliers are applied to the original fractions each time. After the students have completed the exercises, discuss different answers suggested for Ex. 6-16. Students may find it easier to copy the pictures in Ex. 14-16 on tracing paper.

Problem Solving: The answers for these exercises may surprise the students. For Ex. 1, the pattern 1, 2, 4, 8,. . . can be thought of as 1, 2, 2 × 2, 2 × 2 × 2, and so on. In other words, the number of cents in the tenth week's allowance, for example, would be found by using 2 as a factor nine times.

Assessment

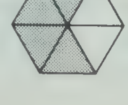
Answers may vary.

Write four equivalent fractions for each of these by multiplying both the numerator and the denominator by the same number.

1. $\frac{1}{5}$, $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}$ **2.** $\frac{3}{4}$, $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}$ **3.** $\frac{7}{10}$, $\frac{14}{20}, \frac{21}{30}, \frac{28}{40}, \frac{35}{50}$

Write two equivalent fractions for each picture.

4.  $\frac{2}{4}, \frac{1}{2}$

5.  $\frac{4}{6}, \frac{2}{3}$

Pages 202-203

LESSON OUTCOME

Use division to find equivalent fractions; find the greatest common factor of two numbers; write fractions in lowest terms

Materials
six red markers and four blue markers

Vocabulary
common factor, lowest terms, greatest common factor

Prerequisite Skills
Use multiplication to find equivalent fractions; relate multiplication and division; identify factors of a number

Checking Prerequisite Skills
Write four equivalent fractions for each of these by multiplying both the numerator and the denominator by the same number. First use 2, then 3, then 4, and then 5.

1. $\frac{3}{5}$ **2.** $\frac{1}{4}$ **3.** $\frac{2}{3}$

Complete.

4. $3 \times$ 4 $= 12$, $12 \div$ 4 $= 3$

5. $2 \times$ 5 $= 10$, $10 \div$ 5 $= 2$

Ring the factors of 8.

6. 2, 9, 3, 5, 16, 1, 7, 4

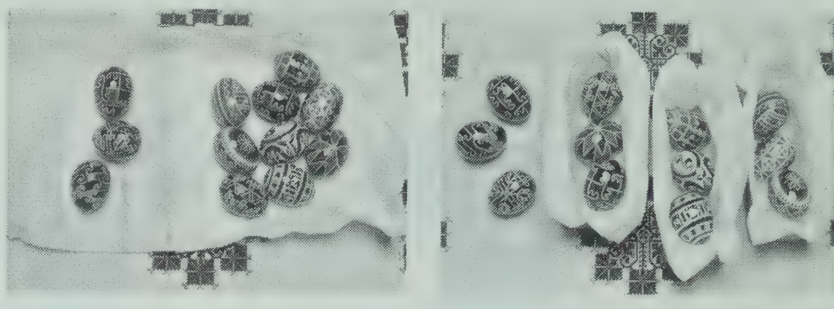
1. $\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{15}{25}$

2. $\frac{1}{4}, \frac{2}{8}, \frac{3}{12}, \frac{4}{16}, \frac{5}{20}$

3. $\frac{2}{3}, \frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{12}{15}$

Dividing to Find Equivalent Fractions

Gail decided to give 9 of the 12 eggs to her friends.
She divided the 9 eggs into 3 gift boxes.



$\frac{9}{12}$ and $\frac{3}{4}$ are equivalent fractions.

$$\frac{9}{12} \overset{9 \div 3}{\underset{12 \div 3}{=}} \frac{3}{4}$$

Gail gave $\frac{3}{4}$ of the eggs to her friends.

9 and 12 are each divided by 3 in the above example. 3 is a **common factor** of 9 and 12.

When the numerator and the denominator have a common factor, division will give an equivalent fraction.

Use the common factor as the divisor.

Take another look:

4 is a common factor of 24 and 36, so

$$\frac{24}{36} \text{ is equivalent to } \frac{6}{9}. \quad (24 \div 4,\ 36 \div 4)$$

3 is a common factor of 6 and 9, so

$$\frac{6}{9} \overset{6 \div 3}{\underset{9 \div 3}{=}} \frac{2}{3}$$

2 and 3 have no common factor greater than 1. The fraction $\frac{2}{3}$ is in **lowest terms**.

202

LESSON ACTIVITY

Before Using the Pages

- Display six red markers and four blue markers (A). Ask what fraction names the part of the set that is blue ($\frac{4}{10}$). As the students watch, arrange the ten markers into sets of two so that there are three sets of two red markers and two sets of two blue markers (B). Establish that of the five sets, two sets are blue. Write $\frac{2}{5}$ beside $\frac{4}{10}$ on the board.

A

B

Ask what multiplier can be used to show that $\frac{2}{5}$ and $\frac{4}{10}$ are equivalent fractions. Ask how division can be used to explain that $\frac{4}{10}$ and $\frac{2}{5}$ are equivalent fractions. Ask a student to use multiplication to obtain another fraction equivalent to $\frac{2}{5}$, for example, $\frac{12}{30}$, without naming the multiplier. Ask another student to use division to explain that $\frac{12}{30}$ is equivalent to $\frac{2}{5}$.

Using the Pages

- Ask a student to read the title at the top of page 202 and the first statement below it and relate it to the photograph at the left. Have a student read the second statement and relate it to the photograph at the right. Develop that the eggs that are in gift boxes can be represented by $\frac{3}{4}$, or by $\frac{9}{12}$, and therefore, $\frac{9}{12}$ and $\frac{3}{4}$ are equivalent fractions.

  Ask for and list on the board the factors of 9 (1, 3, 9) and the factors of 12 (1, 2, 3, 4, 6, 12). Point out that 3 is a factor of 9 and of 12 and introduce the term *common factor*. Explain that dividing both the numerator and the denominator of a fraction by one of their common factors results in an equivalent fraction.

  Have students help to explain the steps for showing $\frac{24}{36} = \frac{6}{9}$, and $\frac{6}{9} = \frac{2}{3}$. Develop that $\frac{24}{36}$ is equivalent to $\frac{2}{3}$. Introduce the concept *lowest terms*. Have the students divide the numerator and the denominator of $\frac{2}{3}$ by 1. Establish that the fraction is still $\frac{2}{3}$. Therefore, unless there is a common factor greater than 1, the fraction is in lowest terms. Write the fraction $\frac{24}{36}$ on the board and have the

Working Together

Give one factor of each.

1. 6 2 or 3 or 6 2. 14 2 or 7 or 14

Give a common factor of the numerator and the denominator.

3. $\frac{9}{24}$ 3 4. $\frac{10}{40}$ 10 or 5 or 2

Divide the numerator and the denominator by a common factor to get an equivalent fraction.

5. $\frac{3}{9}$ $\frac{1}{3}$ 6. $\frac{12}{16}$ $\frac{6}{8}$ or $\frac{3}{4}$ 7. $\frac{10}{25}$ $\frac{2}{5}$

Are the fractions you get in lowest terms? $\frac{1}{3}$, $\frac{3}{4}$, $\frac{2}{5}$ are in lowest terms

Exercises

Divide the numerator and the denominator by a common factor to get an equivalent fraction.

Ring the fractions you get that are in lowest terms.

1. $\frac{4}{6}$ 2. $\frac{8}{64}$ 3. $\frac{10}{45}$ 4. $\frac{12}{30}$ 5. $\frac{20}{32}$ 6. $\frac{9}{21}$ 7. $\frac{5}{30}$

8. $\frac{18}{30}$ 9. $\frac{70}{80}$ 10. $\frac{10}{24}$ 11. $\frac{7}{28}$ 12. $\frac{28}{49}$ 13. $\frac{36}{40}$ 14. $\frac{15}{20}$

Finding the **greatest common factor** of the numerator and the denominator allows you to write a fraction in lowest terms in one step.

The factors of 24 are 1, 2, 3, 4, 6, 8, 12, 24.

The factors of 40 are 1, 2, 4, 5, 8, 10, 20, 40.

The common factors of 24 and 40 are 1, 2, 4, 8.

The greatest common factor is 8. $\frac{24}{40} = \frac{3}{5}$ ($24 \div 8$, $40 \div 8$) lowest terms

15. Find the greatest common factor of

28 and 36	30 and 36	30 and 40	35 and 40	35 and 84	48 and 84
4	6	10	5	7	12

16. Write the fraction in lowest terms that is equivalent to

$\frac{28}{36}$	$\frac{30}{36}$	$\frac{30}{40}$	$\frac{35}{40}$	$\frac{35}{84}$	$\frac{48}{84}$
$\frac{7}{9}$	$\frac{5}{6}$	$\frac{3}{4}$	$\frac{7}{8}$	$\frac{5}{12}$	$\frac{4}{7}$

1. $\frac{2}{3}$ 2. $\frac{4}{32}$ or $\frac{2}{16}$ or $\frac{1}{8}$ 3. $\frac{2}{9}$ 4. $\frac{6}{15}$ or $\frac{4}{10}$ or $\frac{2}{5}$ 5. $\frac{10}{16}$ or $\frac{5}{8}$ 6. $\frac{3}{7}$ 7. $\frac{1}{6}$

203

8. $\frac{9}{15}$ or $\frac{6}{10}$ or $\frac{3}{5}$ 9. $\frac{35}{40}$ or $\frac{14}{16}$ or $\frac{7}{8}$ 10. $\frac{5}{12}$ 11. $\frac{1}{4}$ 12. $\frac{4}{7}$ 13. $\frac{18}{20}$ or $\frac{9}{10}$ 14. $\frac{3}{4}$

RELATED ACTIVITIES

• Provide students with copies of shapes from pages T 383-T 385. Have them draw lines to divide the shapes into equal parts and color the parts in different ways to represent such fractions as $\frac{1}{4}$ and $\frac{3}{5}$. Label and display several ways to illustrate each fraction.

• Have students complete a work sheet similar to the following. For each row, have them check the equivalent fraction that is in lowest terms.

Ring the equivalent fractions in each row.

$\frac{1}{2}$, $\frac{2}{5}$, $\frac{3}{6}$, $\frac{50}{100}$, $\frac{7}{12}$, $\frac{2}{4}$, $\frac{4}{8}$, $\frac{6}{10}$

$\frac{8}{12}$, $\frac{4}{6}$, $\frac{10}{15}$, $\frac{60}{100}$, $\frac{2}{3}$, $\frac{9}{12}$, $\frac{6}{9}$, $\frac{4}{5}$

• Have students use multiplication, division, or number patterns to complete sets of equivalent fractions similar to the following.

$\frac{3}{5}$	$\frac{6}{\blacksquare}$	$\frac{\blacksquare}{15}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{\blacksquare}$	$\frac{\blacksquare}{50}$

$\frac{48}{72}$	$\frac{\blacksquare}{24}$	$\frac{8}{\blacksquare}$	$\frac{\blacksquare}{3}$	$\frac{4}{\blacksquare}$	$\frac{\blacksquare}{30}$	$\frac{10}{\blacksquare}$

• The students can cut shapes and color some of them blue to show fractions representing parts of a set in different ways. For example, to show $\frac{1}{2}$, they can color one of two shapes, two of four shapes, three of six shapes, and so on. Have them display each way of representing the fraction on a separate piece of paper.

students show different ways of obtaining the equivalent fraction in lowest terms.

Working Together: After the students have completed the exercises, have them name all the factors for Ex. 1 and 2. There is only one common factor for Ex. 3, but there are three for Ex. 4. For Ex. 5 and 7, the equivalent fractions will be in lowest terms, but two equivalent fractions are possible for Ex. 6.

Exercises: For Ex. 2, 4, 5, 8, 9, and 13, answers will vary because there are two or more common factors in each case.

The example at the bottom of page 202 shows a method using more than one step to obtain a fraction in lowest terms. The example preceding Ex. 15 and 16 on page 203 shows how to write a fraction in lowest terms in only one step.

Discuss the list of factors for 24 and 40 and the list of their common factors. Have students express the meaning of the term *greatest common factor* in their own words and relate it to the term *common factor*. In Ex. 15, students perform the first step required for completing Ex. 16. For Ex. 15, it is helpful to write the row of factors of a number to fill both ends of the row simultaneously. For example, for 84, write the least and the greatest factors first (1 and 84), the second least and the second greatest factors next, (2 and 42), and so on, to obtain 1, 2, 3, 4, 6, 7, 12, 14, 21, 28, 42, 84. It can be seen that no factors can be written after the pair 7 and 12 because the numbers between 7 and 12 (8, 9, 10, 11) are not factors of 84.

Assessment

Divide the numerator and the denominator by a common factor to get an equivalent fraction.

1. $\frac{9}{12}$ $\frac{3}{4}$ 2. $\frac{8}{20}$ $\frac{2}{5}$ 3. $\frac{12}{18}$ $\frac{2}{3}$

Find the greatest common factor for each pair.

4. 10 and 50 10 5. 15 and 45 15 6. 42 and 70 14

Write each fraction in lowest terms.

7. $\frac{10}{50}$ $\frac{1}{5}$ 8. $\frac{15}{45}$ $\frac{1}{3}$ 9. $\frac{42}{70}$ $\frac{3}{5}$

Pages 204-205

LESSON OUTCOME

Use cross products to determine whether two fractions are equivalent; use cross products to find the missing term in two equivalent fractions

Vocabulary

cross products, is not equal to ($\neq$)

Cross Products

Cross products can be used to check whether two fractions are equivalent.

$\frac{4}{7} \quad \frac{12}{21}$ → 7×12, → 4×21

For the fractions $\frac{4}{7}$ and $\frac{12}{21}$, both cross products are 84.

When the cross products for two fractions are equal, the fractions are equivalent.

$$\frac{4}{7} = \frac{12}{21}$$

When the cross products for two fractions are not equal, the fractions are not equivalent.

$\frac{5}{6} \quad \frac{3}{4}$ → 6×3, → 5×4

For $\frac{5}{6}$ and $\frac{3}{4}$ the cross products are 20 and 18.

18 — 6×3 20 — 5×4

$$\frac{5}{6} \neq \frac{3}{4}$$

The symbol $\neq$ means "is not equal to".

Cross products can be used to find a number that will give two equivalent fractions.

$\frac{7}{14} = \frac{4}{■}$ → 14×4, → $7 \times ■$

For equivalent fractions, $7 \times ■$ must equal 14×4.

For $7 \times ■ = 56$, 8 is the value for ■.

$7 \times 8 = 56$

Write $\frac{7}{14} = \frac{4}{8}$ and use cross products to check.

$\frac{7}{14} = \frac{4}{8}$ → 56, → 56

The cross products are equal. $\frac{7}{14}$ and $\frac{4}{8}$ are equivalent.

204

LESSON ACTIVITY

Before Using the Pages

- Write pairs of equivalent fractions on the board and one or two pairs of non-equivalent fractions.

  $\frac{2}{3}$ and $\frac{8}{12}$ $\qquad$ $\frac{12}{27}$ and $\frac{4}{9}$ $\qquad$ $\frac{2}{5}$ and $\frac{8}{10}$

  Ask the students to determine which pairs do not show equivalent fractions and to explain the method used. For example, for $\frac{2}{3}$ and $\frac{8}{12}$, they will likely suggest $2 \times 4 = 8$ and $3 \times 4 = 12$. For $\frac{12}{27}$ and $\frac{4}{9}$, they will likely suggest $12 \div 3 = 4$ and $27 \div 3 = 9$. For $\frac{2}{5}$ and $\frac{8}{10}$, they will likely suggest $2 \times 4 = 8$, and then realize that 5×4 is not equal to 10.

  For the first example, point to the 2 and to the 12 and ask for the product. Then point to the 3 and to the 8 and ask for the product. Have the students find similar products for the remaining pairs of fractions. Lead them to discover that equal products are obtained for equivalent fractions and that unequal products are obtained for non-equivalent fractions. They may wish to test the procedure by using two fractions of their own choice for equivalent or non-equivalent fractions.

Using the Pages

- Have students explain the meaning of *cross products* in their own words. Ask students to explain how cross products are used to check whether $\frac{4}{7}$ and $\frac{12}{21}$ are equivalent, and whether $\frac{5}{6}$ and $\frac{3}{4}$ are equivalent. (Note that the symbol = is not shown because it is not known whether the fractions are equivalent until the cross products are checked.) Note the symbol $\neq$ for "is not equal to".

  Draw attention to the example showing $\frac{7}{14} = \frac{4}{■}$. Discuss the use of cross products to find the missing term (the number that will make the two fractions equivalent). Because one cross product, 14×4, equals 56, the other cross product, $7 \times ■$, must also equal 56 for the two fractions to be equivalent. Some students may suggest the division $7\overline{)56}$ to obtain 8 as the missing term. Others may recall the basic multiplication fact $7 \times 8 = 56$.

Working Together

Find the cross products. Are the fractions equivalent?

1. $\frac{2}{3}, \frac{6}{9}$ → 18, 18 yes
2. $\frac{7}{8}, \frac{5}{6}$ → 45, 42 no
3. $\frac{6}{8}, \frac{9}{12}$ → 72, 72 yes

Write a sentence showing equal cross products.

Example: For $\frac{4}{5} = \frac{12}{■}$, write $4 \times ■ = 5 \times 12$.

Find the missing term.

Example: For $5 \times ■ = 7 \times 15$, or 105, you can divide 105 by 5 to find the value for ■.

4. $\frac{6}{21} = \frac{2}{■}$ $6 \times ■ = 21 \times 2$
5. $\frac{6}{9} = \frac{■}{21}$ $6 \times 21 = 9 \times ■$
6. $8 \times ■ = 2 \times 20$ 5
7. $\frac{4}{12} = \frac{5}{■}$ 15

Exercises

Find the cross products. Are the fractions equivalent?

1. $\frac{15}{20}, \frac{3}{4}$ → 60, 60 yes
2. $\frac{2}{5}, \frac{4}{9}$ → 20, 18 no
3. $\frac{1}{6}, \frac{6}{36}$ → 36, 36 yes
4. $\frac{7}{12}, \frac{5}{9}$ → 60, 63 no
5. $\frac{8}{16}, \frac{5}{10}$ → 80, 80 yes
6. $\frac{7}{28}, \frac{4}{16}$ → 112, 112 yes
7. $\frac{9}{15}, \frac{12}{21}$ → 180, 189 no
8. $\frac{9}{24}, \frac{6}{16}$ → 144, 144 yes

Use cross products to find the missing term.

9. $\frac{1}{5} = \frac{7}{■}$ 35
10. $\frac{3}{5} = \frac{■}{25}$ 15
11. $\frac{3}{7} = \frac{6}{■}$ 14
12. $\frac{5}{9} = \frac{■}{27}$ 15
13. $\frac{3}{12} = \frac{■}{20}$ 5
14. $\frac{8}{12} = \frac{10}{■}$ 15
15. $\frac{3}{6} = \frac{■}{4}$ 2
16. $\frac{6}{15} = \frac{10}{■}$ 25
17. $\frac{14}{16} = \frac{21}{■}$ 24
18. $\frac{15}{18} = \frac{■}{12}$ 10
19. $\frac{14}{24} = \frac{21}{■}$ 36
20. $\frac{12}{16} = \frac{■}{24}$ 18

Kevin is $\frac{1}{7}$ as old as Ida.

1. How will Kevin's age compare with Ida's in another year? $\frac{1}{5}$
2. How will Kevin's age compare with Ida's in two years? $\frac{1}{4}$
3. How many years will it be until Kevin is $\frac{1}{3}$ as old as Ida? 4
4. How many years will it be until Kevin is $\frac{1}{2}$ as old as Ida? 10

PROBLEM SOLVING

205

Working Together: Ex. 1-3 provide practice in using cross products to check whether fractions are equivalent. Ex. 4-7 develop the steps for using cross products to find a missing term. The example for Ex. 6 and 7 shows how to find the missing term for equal cross products.

Exercises: You may wish to direct the students to use either = or ≠ to show their answers to Ex. 1-8 as indicated below for Ex. 2.

$$2 \times 9 = 18 \qquad \frac{2}{5} \neq \frac{4}{9}$$
$$5 \times 4 = 20$$

Problem Solving: The students can count the candles on the cakes in the photograph to find the ages of Kevin (2) and Ida (14). After the students have completed the exercises, have them explain how they found the answers. A chart similar to the following can be helpful.

	Age				
	now	in 1 year	in 2 years	in 3 years	in
Kevin	2	3	4	5	
Ida	14	15	16	17	

RELATED ACTIVITIES

- Because facility with basic multiplication facts is necessary for work with fractions, students who do not have rapid recall of these facts should be given an opportunity each day to practice them. To determine which facts are not well known, use a copy of page T 382. Write the factors 1 to 9 in the appropriate squares. Name a multiplication, for example, 4 × 9. If the students respond rapidly and correctly, they may color inside the square for that product. Students may work in pairs for this activity.
- Students having difficulty may benefit from illustrating pairs of equivalent fractions for some of the exercises on page 205. They may color part of a shape from pages T 383-T 385 in different ways or they may draw pictures. Display some of these for several days.
- For the third activity on page T 221, ask the students to use cross products to check that pairs of fractions in a set are equivalent.
- Have students find the missing factors for exercises similar to the following.

1. $2 \times ⬚ = 3 \times 6$
2. $⬚ \times 8 = 1 \times 16$
3. $9 \times 4 = ⬚ \times 2$
4. $5 \times 9 = 3 \times ⬚$

Assessment

Find the cross products. Are the fractions equivalent?

1. $\frac{1}{4}, \frac{6}{24}$ 24, 24 yes
2. $\frac{32}{40}, \frac{9}{15}$ 360, 480 no
3. $\frac{4}{6}, \frac{10}{15}$ 60, 60 yes

Use cross products to find the missing term.

4. $\frac{5}{10} = \frac{■}{100}$ 50
5. $\frac{20}{25} = \frac{12}{■}$ 15
6. $\frac{8}{12} = \frac{■}{15}$ 10

Pages 206-207

LESSON OUTCOME

Express a whole number as an improper fraction; express a number in mixed form as an improper fraction

Materials

models for wholes, halves, thirds, fourths, and fifths prepared from copies of the squares on page T 392 or the circle on page T 385

Vocabulary

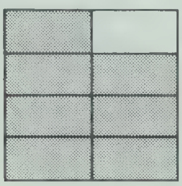
improper fraction, proper fraction, mixed form

Prerequisite Skills

Write fractions for numbers less than one; simplify an expression involving multiplication and addition

Checking Prerequisite Skills

Write the fraction.

1.  $\frac{7}{8}$ is shaded.

2. The numerator is 6 and the denominator is 7. $\frac{6}{7}$

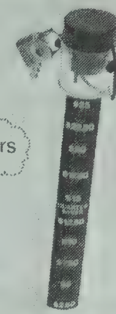
Complete.

3. $(7 \times 8) + 5$ 61
4. $(6 \times 4) + 3$ 27
5. $(3 \times 8) + 1$ 25
6. $(6 \times 12) + 7$ 79

Changing to Improper Fractions

Emilie earned \$5.75 on Saturday. She was paid with 5 one-dollar bills and 3 quarters. To put her money into her quarter bank, she must change it all to quarters. How many quarters would she have for her bank?

$5\frac{3}{4}$ dollars



$5\frac{3}{4}$ = $\frac{20}{4}$ and $\frac{3}{4}$

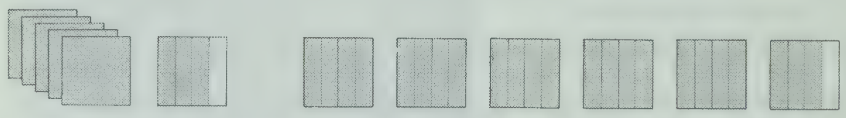
$$5\frac{3}{4} = \frac{23}{4}$$

Emilie would have 23 quarters for her quarter bank.

$\frac{23}{4}$ is an **improper fraction.**
$5\frac{3}{4}$ is a number in **mixed form.**

In an improper fraction, the numerator is greater than or equal to the denominator. In a proper fraction, the numerator is less than the denominator. A number in mixed form is a whole number together with a fraction.

To change a number in mixed form, such as $5\frac{3}{4}$, to an improper fraction,



$5\frac{3}{4}$ = 5×4 fourths + 3 fourths

23 fourths

multiply the whole number and the denominator. Then add the numerator.

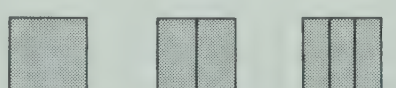
$$5\frac{3}{4} = \frac{23}{4}$$

206

LESSON ACTIVITY

Before Using the Pages

- Develop that fractions may be used to name whole numbers. For example, use models to show that each of $\frac{2}{2}$, $\frac{3}{3}$, and $\frac{4}{4}$ represents 1 whole, and thus are names for the number 1. Ask students to name 1 whole as fifths, as eighths, and as tenths.



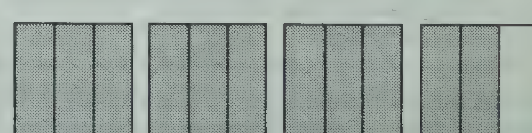
1 $\frac{2}{2}$ $\frac{3}{3}$

Similarly, develop that 2 wholes may be thought of as 4 halves, 6 thirds, 8 fourths, and so on, and 5 wholes may be thought of as 10 halves, 15 thirds, and 20 fourths. Write numerals on the board to show different names for the same whole number.

$$1 = \frac{2}{2} = \frac{3}{3} = \frac{4}{4} \qquad 5 = \frac{10}{2} = \frac{15}{3} = \frac{20}{4}$$

Display models to represent 3. Ask for the number of wholes shown. Ask for the number of fourths that would result if the models were cut into four equal parts. If necessary, cut the three models into fourths to demonstrate that $\frac{12}{4}$ is another name for 3. Ask the students how the fraction $\frac{12}{4}$ can be obtained without using the models.

- Use models to lead students to recall such numbers as $1\frac{1}{4}$ and $3\frac{2}{3}$. For example, display three models of wholes and a model of two-thirds. Ask how many wholes there are and what part of a whole is shown. Write the numeral $3\frac{2}{3}$ on the board and review that it is read "three and two-thirds". Mark the wholes into thirds and ask a student to count the thirds. Emphasize that $3\frac{2}{3}$ and $\frac{11}{3}$ are different names for the same number.



$3\frac{2}{3}$ or $\frac{11}{3}$

Working Together

Multiply the whole number and the denominator to find the numerator of the improper fraction.

1. $3 = \frac{■}{4}$ 12 2. $7 = \frac{■}{3}$ 21 3. $4 = \frac{■}{5}$ 20

Multiply the whole number and the denominator. Then add the numerator to change each of these to an improper fraction.

Example: For $3\frac{1}{8}$, use $(3 \times 8) + 1$ and write $3\frac{1}{8} = \frac{25}{8}$.

4. $7\frac{1}{2}$ $\frac{15}{2}$ 5. $4\frac{2}{3}$ $\frac{14}{3}$ 6. $5\frac{4}{7}$ $\frac{39}{7}$ 7. $8\frac{7}{12}$ $\frac{103}{12}$

Exercises

Write 4 as an improper fraction

1. showing halves. $\frac{8}{2}$ 2. showing thirds. $\frac{12}{3}$ 3. showing tenths. $\frac{40}{10}$

Write 5 as an improper fraction

4. showing fourths. $\frac{20}{4}$ 5. showing eighths. $\frac{40}{8}$ 6. showing twelfths. $\frac{60}{12}$

Write each as an improper fraction.

7. $3\frac{1}{3}$ $\frac{10}{3}$	8. $1\frac{3}{8}$ $\frac{11}{8}$	9. $8\frac{1}{5}$ $\frac{41}{5}$	10. $7\frac{5}{6}$ $\frac{47}{6}$	11. $4\frac{4}{5}$ $\frac{24}{5}$
12. $6\frac{5}{12}$ $\frac{77}{12}$	13. $5\frac{2}{7}$ $\frac{37}{7}$	14. $7\frac{5}{8}$ $\frac{61}{8}$	15. $2\frac{1}{4}$ $\frac{9}{4}$	16. $9\frac{4}{9}$ $\frac{85}{9}$
17. $7\frac{2}{5}$ $\frac{37}{5}$	18. $5\frac{3}{4}$ $\frac{23}{4}$	19. $10\frac{5}{9}$ $\frac{95}{9}$	20. $15\frac{3}{10}$ $\frac{153}{10}$	21. $12\frac{3}{5}$ $\frac{63}{5}$
22. $10\frac{7}{9}$ $\frac{97}{9}$	23. $4\frac{7}{8}$ $\frac{39}{8}$	24. $21\frac{7}{10}$ $\frac{217}{10}$	25. $4\frac{5}{7}$ $\frac{33}{7}$	26. $3\frac{11}{12}$ $\frac{47}{12}$

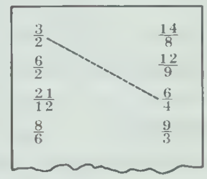
Add, subtract, multiply, or divide. Show your work.

1. 5232 + 5232 10 464 2. 10 464 − 9 293 1171 3. 1171 × 72 84 312
4. 84 312 ÷ 24 3513 5. 3513 − 1814 1699 6. 1699 × 234 397 566
7. 397 566 ÷ 78 5097 8. 5 097 + 24 903 30 000 9. 30 000 − 29 624 376
10. 376 × 148 55 648 11. 55 648 + 11 564 67 212 12. 67 212 ÷ 12 5601
13. 5601 − 634 4967 14. 4967 + 6778 11 745

KEEPING SHARP

RELATED ACTIVITIES

- Have students draw diagrams to illustrate the whole number and the improper fraction for one of Ex. 1-6, and also the number in mixed form and the improper fraction for one of Ex. 7-26 on page 207.
- Prepare a work sheet with pairs of improper fractions. Discuss that cross products can be used to check whether improper fractions are equivalent. Then have the students use cross products to match equivalent fractions.



- For enrichment, ask students to write any pair of equivalent fractions, for example, $\frac{2}{3}$ and $\frac{14}{21}$. Ask them to write two new equivalent fractions using the same four numbers 2, 3, 14, and 21. Cross products may be used to check whether the fractions in a pair are equivalent. For 2, 3, 14, and 21, other pairs of equivalent fractions are as follows.

1. $\frac{2}{14}$ and $\frac{3}{21}$
2. $\frac{3}{2}$ and $\frac{21}{14}$
3. $\frac{14}{2}$ and $\frac{21}{3}$

Using the Pages

- Ask a student to read the word problem at the top of page 206. Direct the students' attention to the 5 one-dollar bills and the 3 quarters in the photograph. Ask how many quarters there would be in all if each dollar bill is changed to quarters for the bank. Note that the photograph shows 5 stacks of 4 quarters and 3 more quarters to give 23 quarters altogether.

  Discuss the meanings for *improper fraction*, *proper fraction*, and *number in mixed form* and ask for other examples for each of these.

  The models illustrated at the bottom of page 206 can be compared with the photograph at the top of the page. Each shows that $5\frac{3}{4}$ and $\frac{23}{4}$ name the same amount. Ask how $\frac{23}{4}$ can be derived from $5\frac{3}{4}$ without using models. Discuss that the whole number 5 and the denominator 4 are multiplied, and then the numerator 3 is added to obtain the number of fourths for an improper fraction. Emphasize that $5\frac{3}{4}$ and $\frac{23}{4}$ name the same number.

Working Together: For Ex. 1-3, whole numbers are changed to improper fractions; for Ex. 4-7, numbers in mixed form are changed to improper fractions. Discuss the example for Ex. 4-7.

Exercises: Ensure that the students take care in writing numerals for fractions. The numerator and the denominator of a fraction are separated by a horizontal bar and, in mixed form, the whole number is written so that it is separate from and larger than the numerals for the numerator and the denominator.

Keeping Sharp: For these exercises, the answer for one exercise is the same as the first number in the next exercise; the students will likely observe this pattern and use it as a check for their work.

Assessment

Write 6 as an improper fraction

1. showing thirds. $\frac{18}{3}$ 2. showing fifths. $\frac{30}{5}$

Write each as an improper fraction.

3. $2\frac{3}{4}$ $\frac{11}{4}$ 4. $4\frac{1}{2}$ $\frac{9}{2}$ 5. $1\frac{5}{8}$ $\frac{13}{8}$

Pages 208-209

LESSON OUTCOME

Express an improper fraction as a whole number or as a number in mixed form

Materials
models for wholes, thirds, fourths, and other fractions prepared from copies of the squares on page T 392 or the circle on page T 385; a copy of page T 394 for each student (optional)

Vocabulary
prime number, composite number

Prerequisite Skills
Divide whole numbers; write fractions in lowest terms

Checking Prerequisite Skills
Find the quotient and the remainder.

1. $7\overline{)96}$ 13 R5
2. $12\overline{)140}$ 11 R8
3. $15\overline{)49}$ 3 R4
4. $10\overline{)62}$ 6 R2

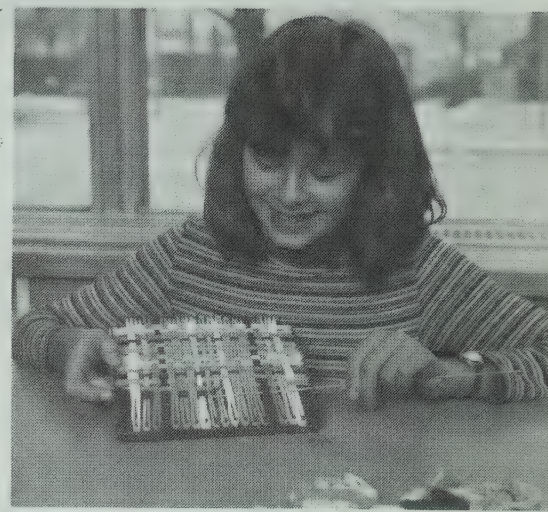
Write each in lowest terms.

5. $\frac{16}{20}$ $\frac{4}{5}$
6. $\frac{6}{8}$ $\frac{3}{4}$
7. $\frac{10}{12}$ $\frac{5}{6}$

Changing Improper Fractions

Claudette needs one quarter-hour to weave each pot holder. How much time does she need to weave 13 pot holders?

How long is 13 quarter-hours?



To change an improper fraction, such as $\frac{13}{4}$, to a number in mixed form, divide the numerator by the denominator.

$$\begin{array}{r} 3 \\ 4\overline{)13} \\ 12 \\ \hline 1 \end{array}$$

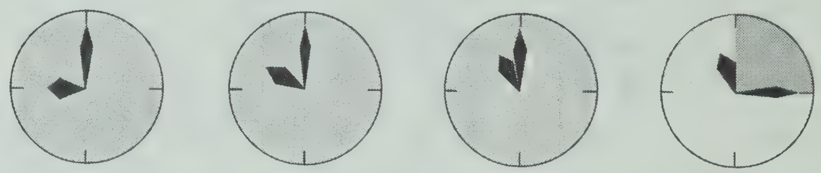
3 wholes

1 fourth of another whole

$3\frac{1}{4}$

The improper fraction $\frac{13}{4}$ is equal to $3\frac{1}{4}$ when it is changed to mixed form.

Claudette needs 3 h and 1 quarter of another hour to weave 13 pot holders.



Working Together

Divide the numerator by the denominator. Show the result as a number in mixed form or as a whole number.

Example: For $\frac{17}{3}$, use $3\overline{)17}$ 5 R2 and write $5\frac{2}{3}$.

1. $\frac{12}{5}$ $2\frac{2}{5}$
2. $\frac{54}{8}$ $6\frac{6}{8}$ or $6\frac{3}{4}$
3. $\frac{36}{9}$ 4
4. $\frac{76}{6}$ $12\frac{4}{6}$ or $12\frac{2}{3}$

208

LESSON ACTIVITY

Before Using the Pages

- Display models of one whole and two-thirds of another whole and ask what number is shown. Have a student write the numeral $1\frac{2}{3}$ on the board. Ask why it is described as a number in mixed form. Establish that it names a number of wholes and part of another whole. Review the use of multiplication and addition to express $1\frac{2}{3}$ as the improper fraction $\frac{5}{3}$.
- Display nine fourths cut from models of fourths. Ask students to arrange the fourths to show the number of wholes that can be formed and the number of fourths for part of a whole. Write the numerals on the board and emphasize that $\frac{9}{4}$ and $2\frac{1}{4}$ are names for the same number. Use other examples as required. Then ask how the answers can be obtained without the use of models or diagrams. Have students write examples on the board to demonstrate their ideas.

Using the Pages

- Use the photograph to introduce the word problem. Develop that 4 fourths equal 1 whole. Thus, when the numerator of $\frac{13}{4}$ is divided by the denominator, the quotient shows the number of wholes and the remainder shows the part of a whole left over, in this case, the number of fourths. Ask students to explain how the clocks illustrated on page 208 show $\frac{13}{4}$, or $3\frac{1}{4}$. Summarize that the use of division is an efficient method for expressing a number in mixed form.

Working Together: Before the students begin the exercises, discuss the example provided, which deals with interpreting the divisor, quotient, and remainder in a division as a number in mixed form. When they have finished their work, note that the remainder for the division for Ex. 3 is zero. Discuss that there are no parts of a whole left over when the remainder is zero: the result is a whole number. Draw attention to the answer $6\frac{6}{8}$ for Ex. 2. Discuss that $\frac{6}{8}$ can be expressed in lowest terms as $\frac{3}{4}$, and therefore $6\frac{6}{8}$ may be expressed as $6\frac{3}{4}$. Discuss the answer for Ex. 4 in a similar manner.

Exercises

Write each improper fraction as a number in mixed form or as a whole number.

1. $\frac{19}{5}$ $3\frac{4}{5}$	2. $\frac{15}{2}$ $7\frac{1}{2}$	3. $\frac{40}{5}$ 8	4. $\frac{108}{10}$ $10\frac{8}{10}$ or $10\frac{4}{5}$	5. $\frac{24}{9}$ $2\frac{6}{9}$ or $2\frac{2}{3}$	6. $\frac{117}{12}$ $9\frac{9}{12}$ or $9\frac{3}{4}$
7. $\frac{66}{8}$ $8\frac{2}{8}$ or $8\frac{1}{4}$	8. $\frac{27}{3}$ 9	9. $\frac{86}{7}$ $12\frac{2}{7}$	10. $\frac{26}{4}$ $6\frac{2}{4}$ or $6\frac{1}{2}$	11. $\frac{42}{6}$ 7	12. $\frac{69}{15}$ $4\frac{9}{15}$ or $4\frac{3}{5}$
13. $\frac{84}{9}$ $9\frac{3}{9}$ or $9\frac{1}{3}$	14. $\frac{34}{4}$ $8\frac{2}{4}$ or $8\frac{1}{2}$	15. $\frac{30}{2}$ 15	16. $\frac{144}{12}$ 12	17. $\frac{34}{6}$ $5\frac{4}{6}$ or $5\frac{2}{3}$	18. $\frac{25}{10}$ $2\frac{5}{10}$ or $2\frac{1}{2}$

For Exercises 1 to 18,

19. list the results in lowest terms.

Write a sentence using a number in mixed form.

20. The boy ate 9 quarter waffles.
The boy ate $2\frac{1}{4}$ waffles.

21. She played for 51 quarters of last season's basketball games.
She played $12\frac{3}{4}$ of last season's basketball games

A whole number greater than 1 that has itself and 1 as its only whole-number factors is a **prime number**.

$37 = 1 \times 37$

This is the only way to show 37 as a product of whole numbers. 37 is a prime number.

A number that is not a prime number is a **composite number**.

$35 = 1 \times 35 \quad 35 = 7 \times 5$

35 has other whole-number factors besides 1 and 35. 35 is a composite number.

List the numbers from 2 to 200.

1. After 2, every second number is not prime. Why? After 2, cross every second number off the list.
2. After 3, every third number is not prime. Why? After 3, cross every third number off the list.
3. Complete: 5 is the next prime number after 3. After 5, every fifth number is not prime. Why? After 5, every fifth number has 5 as a factor After 5, cross every 5 th number off the list.
4. Continue the pattern begun with steps 1 to 3. Then tell what is true about the numbers that are not crossed off. They are all prime numbers
5. Tell what is true about all the numbers that are crossed off. They are all composite numbers.

try this

1. After 2, every second number has 2 as a factor.
2. After 3, every third number has 3 as a factor.

209

RELATED ACTIVITIES

- When students play games that involve game boards with numbered squares and/or dice, suggest that the following rules apply.
  1. If the number obtained by tossing the dice is a prime number, the player takes another turn.
  2. If the number in the square on which a player's marker lands is a prime number, the player moves her/his marker back two squares.
- Provide students with unmarked number lines from copies of page T 390. Ask them to mark the lines to show whole numbers above the lines and fractions below. By referring to the number line, students can rename improper fractions as numbers in mixed form and vice versa.
- Have students use copies of pages T 383-T 385 to prepare pictures that represent a number in mixed form and the corresponding improper fraction, similar to the following.

$\frac{9}{4} \quad = \quad 2\frac{1}{4}$

- For Ex. 44-56 on page 329, have the students interpret the remainders for these divisions as fractions.
- You may wish to have students write different numbers in mixed form for a given improper fraction as shown below.

$\frac{18}{5}$	$1\frac{13}{5}$, $2\frac{8}{5}$, $3\frac{3}{5}$

Exercises: Before the students begin, point out that some of the answers for Ex. 1-18 will be in lowest terms and others will not. Then draw attention to the instructions for Ex. 19.

Try This: Have students express the explanation for *prime number* and then for *composite number* in their own words and give other examples of each. Discuss the answers for Ex. 1 and develop that they are composite numbers, not prime numbers, because every second number after 2 has 2 as a factor. When the students understand what is required, have them continue with Ex. 2-5. You may wish to provide each student with a copy of page T 394 on which to show the numbers from 1 to 200 in the squares. The procedure of crossing out numbers will be facilitated by the pattern of the numbers in the squares. The students should cross 1 off the list.

~~1~~	2	3	~~4~~	5	~~6~~	7	~~8~~	~~9~~	~~10~~
11	~~12~~	13	~~14~~	~~15~~					

Assessment

Write each improper fraction as a number in mixed form or as a whole number.

1. $\frac{18}{4}$ $4\frac{2}{4}$ $\left(4\frac{1}{2}\right)$ **2.** $\frac{28}{7}$ 4 **3.** $\frac{24}{5}$ $4\frac{4}{5}$ **4.** $\frac{21}{6}$ $3\frac{3}{6}$ $\left(3\frac{1}{2}\right)$

For Ex. 1-4,

5. list the results in lowest terms.

Page 210

LESSON OUTCOME

Use the product of unlike denominators to find equivalent fractions with like denominators for two or three fractions

Vocabulary

unlike denominators, like denominators, common denominators

Prerequisite Skills

Use multiplication to find equivalent fractions

1. $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}, \frac{5}{25}$

Checking Prerequisite Skills

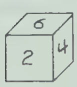
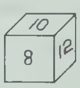
Write four equivalent fractions for each of these. Answers will vary.

1. $\frac{1}{5}$ **2.** $\frac{3}{8}$ **3.** $\frac{2}{7}$ $\frac{4}{14}, \frac{6}{21}, \frac{8}{28}, \frac{10}{35}$

2. $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}, \frac{15}{40}$

RELATED ACTIVITIES

- Mark one die to show 2, 3, 4, 5, 6, 8, and another die to show 5, 6, 8, 9, 10, 12. Have students work in pairs as follows. One student tosses one die and writes a proper fraction for which the denominator is the number shown on the die (A). The other student performs the same procedure using the second die (B). For the two fractions obtained, both students write equivalent fractions with like denominators (C).

A  $\frac{5}{6}$ B  $\frac{7}{10}$ C $\frac{50}{60}, \frac{42}{60}$

Finding Like Denominators

$\frac{1}{2}$ and $\frac{2}{3}$ have **unlike denominators**.
The unlike denominators are 2 and 3.

$\frac{1}{2} \xrightarrow[2 \times 3]{1 \times 3} \frac{3}{6}$ $\frac{1}{2} = \frac{3}{6}$

$2 \times 3 = 3 \times 2$

$\frac{2}{3} \xrightarrow[3 \times 2]{2 \times 2} \frac{4}{6}$ $\frac{2}{3} = \frac{4}{6}$

$\frac{1}{2}$ is equivalent to $\frac{3}{6}$.

$\frac{2}{3}$ is equivalent to $\frac{4}{6}$.

$\frac{3}{6}$ and $\frac{4}{6}$ have **like denominators**.

Like denominators are often called **common denominators**.

To find like denominators for two fractions that have unlike denominators, multiply the unlike denominators.

Working Together

For each pair, use the product of the unlike denominators to help you find equivalent fractions with like denominators.

1. $\frac{2}{3}, \frac{1}{5}$ $\frac{10}{15}, \frac{3}{15}$ **2.** $\frac{3}{4}, \frac{1}{10}$ $\frac{30}{40}, \frac{4}{40}$ **3.** $2\frac{4}{5}, 1\frac{5}{6}$ $2\frac{24}{30}, 1\frac{25}{30}$

Exercises

For each pair, find equivalent fractions with like denominators.

1. $\frac{1}{3}, \frac{1}{4}$ $\frac{4}{12}, \frac{3}{12}$ **2.** $\frac{1}{2}, \frac{1}{3}$ $\frac{3}{6}, \frac{2}{6}$ **3.** $\frac{1}{8}, \frac{1}{5}$ $\frac{5}{40}, \frac{8}{40}$ **4.** $\frac{2}{3}, \frac{3}{4}$ $\frac{8}{12}, \frac{9}{12}$ **5.** $\frac{1}{2}, \frac{4}{5}$ $\frac{5}{10}, \frac{8}{10}$

6. $\frac{5}{8}, \frac{2}{3}$ $\frac{15}{24}, \frac{16}{24}$ **7.** $\frac{7}{9}, \frac{3}{4}$ $\frac{28}{36}, \frac{27}{36}$ **8.** $\frac{5}{9}, \frac{1}{2}$ $\frac{10}{18}, \frac{9}{18}$ **9.** $\frac{2}{3}, \frac{3}{5}$ $\frac{10}{15}, \frac{9}{15}$ **10.** $\frac{7}{8}, \frac{8}{9}$ $\frac{63}{72}, \frac{64}{72}$

11. $\frac{1}{4}, \frac{5}{6}$ $\frac{6}{24}, \frac{20}{24}$ **12.** $\frac{2}{9}, \frac{1}{6}$ $\frac{12}{54}, \frac{9}{54}$ **13.** $\frac{5}{8}, \frac{5}{12}$ $\frac{60}{96}, \frac{40}{96}$ **14.** $\frac{5}{6}, \frac{9}{10}$ $\frac{50}{60}, \frac{54}{60}$ **15.** $\frac{1}{9}, \frac{1}{12}$ $\frac{12}{108}, \frac{9}{108}$

16. $1\frac{1}{2}, 2\frac{4}{5}$ $1\frac{5}{10}, 2\frac{8}{10}$ **17.** $2\frac{3}{4}, \frac{2}{5}$ $2\frac{15}{20}, \frac{8}{20}$ **18.** $3\frac{1}{5}, 1\frac{5}{6}$ $3\frac{6}{30}, 1\frac{25}{30}$ **19.** $1\frac{5}{6}, 3\frac{3}{8}$ $1\frac{40}{48}, 3\frac{18}{48}$ **20.** $1\frac{3}{10}, 2\frac{1}{6}$ $1\frac{18}{60}, 2\frac{10}{60}$

Find three fractions with like denominators that are equivalent to

21. $\frac{1}{2}, \frac{1}{3}$, and $\frac{1}{4}$. $\frac{12}{24}, \frac{8}{24}, \frac{6}{24}$ **22.** $\frac{2}{3}, \frac{3}{5}$, and $\frac{1}{6}$. $\frac{60}{90}, \frac{54}{90}, \frac{15}{90}$ **23.** $\frac{5}{6}, \frac{1}{2}$, and $\frac{3}{8}$. $\frac{80}{96}, \frac{48}{96}, \frac{36}{96}$

210

LESSON ACTIVITY

Before Using the Page

- Review that a number has many different names. For example, $\frac{1}{3}$, $\frac{2}{6}$, $\frac{3}{9}$, and $\frac{4}{12}$ are different names for the number "one-third". Similarly, $\frac{3}{4}$, $\frac{6}{8}$, $\frac{9}{12}$, and $\frac{12}{16}$ are names for "three-fourths". Write the preceding fractions on the board, reviewing that $\frac{1}{3}$ and $\frac{3}{4}$ name the fractions in lowest terms. For each example, emphasize that although the names differ, the values are the same. Ask a student to ring the names for $\frac{1}{3}$ and $\frac{3}{4}$ on the board for which the denominators are the same number. Ask how the number 12 in the denominator is related to the denominators 3 and 4.

Using the Page

- Ask the students to explain why the denominators 2 and 3 are called *unlike denominators*. Discuss each step for the worked example. Develop that the unlike denominators 2 and 3 are multiplied to find the number which may be used as a denominator for $\frac{1}{2}$ and for $\frac{2}{3}$. Have the students refer to $\frac{3}{6}$ and $\frac{4}{6}$ and explain the meaning of *like denominators* and *common denominators*. Read the final statement to summarize the procedure.

Working Together: For each pair of fractions, remind the students to multiply both the numerator and the denominator of each fraction by the denominator of the other fraction. Pay particular attention to Ex. 3, noting that the whole-number part of each number in mixed form does not change.

Exercises: Each of Ex. 21-23 involves three fractions. The students are required to multiply the numerator and the denominator of one fraction by the product of the denominators of the other two fractions.

Assessment

For each pair, find equivalent fractions with like denominators.

1. $\frac{1}{2}, \frac{1}{6}$ $\frac{6}{12}, \frac{2}{12}$ **2.** $\frac{2}{5}, \frac{2}{3}$ $\frac{6}{15}, \frac{10}{15}$ **3.** $\frac{5}{6}, \frac{3}{10}$ $\frac{50}{60}, \frac{18}{60}$

Find three fractions with like denominators that are equivalent to

4. $\frac{1}{2}, \frac{1}{4}$, and $\frac{1}{5}$. $\frac{20}{40}, \frac{10}{40}, \frac{8}{40}$ **5.** $\frac{2}{5}, \frac{1}{3}$, and $\frac{3}{4}$. $\frac{24}{60}, \frac{20}{60}, \frac{45}{60}$

Common Multiples as Like Denominators

Multiples of 4 are	4,	8,	12,	16,	20,	24,	28,	32,	36, and so on.
Multiples of 6 are		6,	12,	18,		24,	30,		36, and so on.
Common multiples of 4 and 6 are			12,			24,			36, and so on.

Any common multiple can be used for like denominators.

For $\frac{3}{4}$ and $\frac{5}{6}$, $\frac{3}{4} = \frac{9}{12}$ $\frac{5}{6} = \frac{10}{12}$

$\frac{3}{4} = \frac{18}{24}$ $\frac{5}{6} = \frac{20}{24}$

Sometimes there are common multiples that are less than the product of the unlike denominators.

Working Together

Give a common multiple of the denominators that is *less than* their product.

1. $\frac{1}{3}, \frac{1}{6}$ 6 **2.** $\frac{1}{4}, \frac{3}{10}$ 20 **3.** $\frac{5}{8}, \frac{7}{12}$ 24

Find equivalent fractions with like denominators. Use the common multiples from Exercises 1 to 3.

4. $\frac{1}{3}, \frac{1}{6}$ $\frac{2}{6}, \frac{1}{6}$ **5.** $\frac{1}{4}, \frac{3}{10}$ $\frac{5}{20}, \frac{6}{20}$ **6.** $1\frac{5}{8}, 2\frac{7}{12}$ $1\frac{15}{24}, 2\frac{14}{24}$

Exercises

Find equivalent fractions with like denominators. Use denominators that are *less than* the product of the unlike denominators.

1. $\frac{1}{2}, \frac{1}{4}$ $\frac{2}{4}, \frac{1}{4}$ **2.** $\frac{1}{8}, \frac{3}{10}$ $\frac{5}{40}, \frac{12}{40}$ **3.** $\frac{3}{4}, \frac{7}{8}$ $\frac{6}{8}, \frac{7}{8}$ **4.** $\frac{1}{4}, \frac{1}{6}$ $\frac{3}{12}, \frac{2}{12}$ **5.** $\frac{5}{12}, \frac{1}{2}$ $\frac{5}{12}, \frac{6}{12}$

6. $\frac{3}{8}, \frac{5}{12}$ $\frac{9}{24}, \frac{10}{24}$ **7.** $\frac{5}{6}, \frac{7}{9}$ $\frac{15}{18}, \frac{14}{18}$ **8.** $1\frac{1}{5}, \frac{1}{10}$ $1\frac{2}{10}, \frac{1}{10}$ **9.** $3\frac{2}{3}, 2\frac{7}{9}$ $3\frac{6}{9}, 2\frac{7}{9}$ **10.** $1\frac{7}{12}, 2\frac{5}{9}$ $1\frac{21}{36}, 2\frac{20}{36}$

In the charts at the top of this page, 12 is the **least common multiple** of 4 and 6.

List multiples for each. Ring the least common multiple.

11. 6, 10 **12.** 8, 4 **13.** 4, 10 **14.** 6, 8 **15.** 9, 12

For each pair, write equivalent fractions using the least common multiple as the common denominator.

16. $\frac{1}{6}, \frac{3}{10}$ $\frac{5}{30}, \frac{9}{30}$ **17.** $\frac{5}{8}, \frac{3}{4}$ $\frac{5}{8}, \frac{6}{8}$ **18.** $\frac{3}{4}, \frac{7}{10}$ $\frac{15}{20}, \frac{14}{20}$ **19.** $\frac{5}{6}, \frac{5}{8}$ $\frac{20}{24}, \frac{15}{24}$ **20.** $\frac{4}{9}, \frac{5}{12}$ $\frac{16}{36}, \frac{15}{36}$

11. 6, 12, 18, 24, 30, 36, 42
10, 20, 30, 40, 50
12. 8, 16, 24, 32
4, 8, 12, 16
13. 4, 8, 12, 16, 20, 24, 28
10, 20, 30, 40
14. 6, 12, 18, 24, 30, 36
8, 16, 24, 32, 40
15. 9, 18, 27, 36, 45, 54
12, 24, 36, 48, 60

211

Page 211

LESSON OUTCOME

Use the least common multiple of unlike denominators to find equivalent forms with like denominators for fractions and numbers in mixed form

Vocabulary

multiple, common multiple, least common multiple

Prerequisite Skills

Use multiplication to find equivalent fractions; use cross products to find the missing term in two equivalent fractions

Checking Prerequisite Skills

Write four equivalent fractions for each of these. Answers may vary.

1. $\frac{1}{3}$ **2.** $\frac{2}{5}$ **3.** $\frac{5}{8}$

Use cross products to find the missing term.

4. $\frac{4}{6} = \frac{■}{9}$ 6 **5.** $\frac{7}{14} = \frac{5}{■}$ 10

1. $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}, \frac{5}{15}$

RELATED ACTIVITIES

- Have students adapt the activity described on page T 228 so that the like denominators are the least common multiples of the numbers obtained by tossing the dice.

A	B	C
$\frac{5}{6}$	$\frac{7}{10}$	$\frac{25}{30}, \frac{21}{30}$

2. $\frac{4}{10}, \frac{6}{15}, \frac{8}{20}, \frac{10}{25}$

3. $\frac{10}{16}, \frac{15}{24}, \frac{20}{32}, \frac{25}{40}$

LESSON ACTIVITY

Before Using the Page

- Write the fractions $\frac{5}{8}$ and $\frac{7}{12}$ on the board. Ask the students to rename the fractions to show like denominators. They will likely use the procedure of the previous lesson and show $\frac{60}{96}$ and $\frac{56}{96}$. Draw attention to the fact that 96 is a large number for the denominator of a fraction. Ask if there are other names for $\frac{5}{8}$ and $\frac{7}{12}$ which have like denominators that are less than 96.

Using the Page

- Relate the term *multiples* to the word "multiply". For example, to obtain multiples of 4, 4 is multiplied by 1, 2, 3, and so on. Ask what is meant by the term *common multiples*. Have students read the multiples of 4, of 6, and their common multiples shown at the top of the page.

  Develop that any common multiple can be used for like denominators. Ask how each pair of equivalent fractions with like denominators is found. For example, for $\frac{3}{4}$ and $\frac{5}{6}$, 12 is a common multiple of the denominators 4 and 6; the missing terms (numerators) for $\frac{3}{4} = \frac{■}{12}$ and $\frac{5}{6} = \frac{■}{12}$ can be found by using cross products. The denominator 24 is a common multiple of 4 and 6 as well as the product of 4 and 6. Thus, the numerators for $\frac{3}{4} = \frac{■}{24}$ and $\frac{5}{6} = \frac{■}{24}$ can be found by using either cross products or the procedure shown at the top of page 210.

Working Together: Ex. 1-3 prepare for the work in Ex. 4-6. Note, for example, that the same fractions are used in Ex. 1 and 4, and in Ex. 2 and 5. In Ex. 6, however, numbers in mixed form are presented.

Exercises: Discuss the instructions for Ex. 1-10 before the students begin the exercises. Ex. 11-20, highlighted in blue, are of particular importance because they introduce the concept of *least common multiple*.

Assessment

For each pair, write equivalent fractions using the least common multiple as the common denominator.

1. $\frac{2}{3}, \frac{5}{6}$ $\frac{4}{6}, \frac{5}{6}$ **2.** $\frac{9}{10}, \frac{7}{8}$ $\frac{36}{40}, \frac{35}{40}$ **3.** $\frac{5}{12}, \frac{4}{9}$ $\frac{15}{36}, \frac{16}{36}$ **4.** $3\frac{5}{8}, 2\frac{1}{4}$ $3\frac{5}{8}, 2\frac{2}{8}$

LESSON OUTCOME

Compare two fractions with unlike denominators; compare two numbers in mixed form for which the whole numbers are the same

Prerequisite Skills

Find equivalent fractions with like denominators for two fractions and for two numbers in mixed form

Checking Prerequisite Skills

For each pair, find equivalent fractions with like denominators.

1. $\frac{4}{9}$, $\frac{2}{3}$ $\frac{4}{9}$, $\frac{6}{9}$ **2.** $1\frac{5}{8}$, $1\frac{7}{12}$ $1\frac{15}{24}$, $1\frac{14}{24}$

RELATED ACTIVITIES

- Prepare several cards showing fractions that are not equivalent. A student may draw two cards and determine the greater fraction. To challenge more capable students, have them draw three (four) cards and order the fractions from least to greatest.

  The fraction strips described in *Related Activities* on page T 219 may be used for a visual comparison of two given fractions.
- The above activity can be used as a game in which two or more players select the same number of cards and order the fractions from least to greatest. The player who finishes first scores three points and the player who finishes second scores one point.

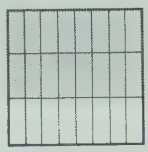
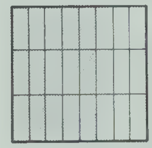
Comparing Fractions

Which is greater, $\frac{2}{3}$ or $\frac{5}{8}$?

3×8, or 24, is a common multiple of 3 and 8.

$\frac{2}{3}$   $\frac{5}{8}$

is equivalent to   is equivalent to

$\frac{16}{24}$  $16 > 15$ $\frac{16}{24} > \frac{15}{24}$  $\frac{15}{24}$

$$\frac{2}{3} > \frac{5}{8}$$

Working Together

Give equivalent fractions with like denominators for each pair.

1. $\frac{5}{8}$, $\frac{3}{4}$ $\frac{5}{8}$, $\frac{6}{8}$ **2.** $\frac{1}{6}$, $\frac{3}{10}$ $\frac{5}{30}$, $\frac{9}{30}$

Which is greater? Use equivalent fractions with like denominators.

3. $\frac{3}{4}$, $\frac{2}{3}$ $\frac{9}{12} > \frac{8}{12}$ $\frac{3}{4} > \frac{2}{3}$ **4.** $\frac{4}{6}$, $\frac{5}{8}$ $\frac{16}{24} > \frac{15}{24}$ $\frac{4}{6} > \frac{5}{8}$

Exercises

Use >, < or = to make true statements.

1. $\frac{1}{3} < \frac{4}{9}$	**2.** $\frac{3}{5} < \frac{2}{3}$	**3.** $\frac{5}{8} > \frac{3}{5}$	**4.** $\frac{4}{5} = \frac{8}{10}$
5. $\frac{2}{9} < \frac{1}{4}$	**6.** $\frac{7}{8} > \frac{5}{6}$	**7.** $\frac{5}{6} > \frac{8}{10}$	**8.** $\frac{3}{4} < \frac{4}{5}$
9. $3\frac{2}{4}$ ◯ $3\frac{3}{6}$	**10.** $1\frac{2}{5} < 1\frac{3}{7}$	**11.** $2\frac{3}{8} < 2\frac{5}{12}$	**12.** $7\frac{2}{3} > 7\frac{5}{9}$

On a job, would you rather

13. do $\frac{4}{7}$ of the work or do $\frac{3}{5}$ of the work? $\frac{4}{7}$

14. earn $\frac{4}{7}$ of the money or earn $\frac{3}{5}$ of the money? $\frac{3}{5}$

212

LESSON ACTIVITY

Before Using the Page

- Draw diagrams on the board or use models for fractions such as $\frac{3}{8}$, $\frac{5}{8}$, and $\frac{7}{8}$. Ask students to identify which of the two fractions is greater for the pairs $\frac{3}{8}$ and $\frac{5}{8}$, $\frac{3}{8}$ and $\frac{7}{8}$, and $\frac{5}{8}$ and $\frac{7}{8}$. Ask the students how they can tell which of two fractions is greater without referring to models or diagrams. They will likely suggest that they can compare the numerators. Have them use that procedure to compare $\frac{1}{8}$ and $\frac{3}{8}$, and then $2\frac{1}{8}$ and $2\frac{3}{8}$. Then present the fractions $\frac{1}{2}$ and $\frac{3}{8}$. In this case, although 3 is greater than 1, $\frac{3}{8}$ is less than $\frac{1}{2}$. Ask students to suggest why the procedure of checking the numerators cannot apply in this last example. Lead them to realize that the numerators reveal which fraction is greater only if the denominators are identical. Have students express $\frac{1}{2}$ as $\frac{4}{8}$ and then compare $\frac{4}{8}$ and $\frac{3}{8}$.

Using the Page

- The worked example demonstrates that the fractions $\frac{2}{3}$ and $\frac{5}{8}$ may be compared by comparing equivalent fractions with like denominators. Diagrams are provided to help students understand the process. Draw attention to the use of the symbol >.

Working Together: The students can find like denominators by multiplying the denominators of the two fractions or by using any common multiple of the two denominators.

Exercises: Point out the three symbols in the instructions for Ex. 1-12. Remind the students that changes in the fractions for Ex. 9-12 will not affect the whole numbers. Ex. 13 and 14 require the students to compare fractions and consider the situation involved.

Assessment

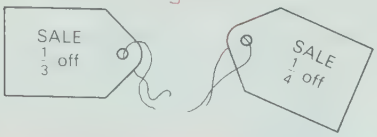
Use >, <, or = to make true statements.

1. $\frac{1}{6}$ ◯ $\frac{1}{8}$ > **2.** $\frac{3}{4}$ ◯ $\frac{9}{12}$ = **3.** $2\frac{3}{5}$ ◯ $2\frac{2}{3}$ <

Practice

Which is the better sale

1. $\frac{1}{3}$ off or $\frac{1}{4}$ off? $\frac{1}{3}$ off



2. prices marked down to $\frac{2}{3}$, or to $\frac{3}{5}$ of the original price? $\frac{3}{5}$

*3. $\frac{2}{5}$ off, or prices marked down to $\frac{3}{4}$ of the original price? $\frac{2}{5}$ off

Other answers are possible

Write three fractions equivalent to

4. $\frac{1}{2}$. $\frac{2}{4}, \frac{3}{6}, \frac{4}{8}$ 5. $\frac{3}{4}$. $\frac{6}{8}, \frac{9}{12}, \frac{12}{16}$ 6. $\frac{5}{7}$. $\frac{10}{14}, \frac{15}{21}, \frac{20}{28}$

Write a fraction in lowest terms for

7. $\frac{18}{24}$. $\frac{3}{4}$ 8. $\frac{14}{42}$. $\frac{1}{3}$ 9. $\frac{30}{75}$. $\frac{2}{5}$

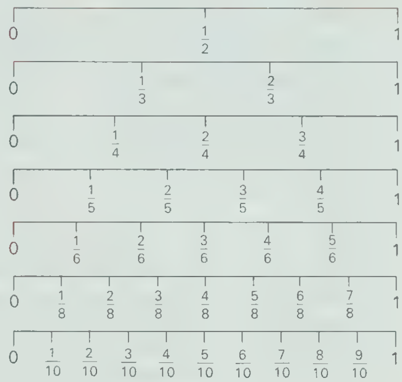
Write an improper fraction for

10. $7\frac{2}{3}$. $\frac{23}{3}$ 11. $4\frac{7}{8}$. $\frac{39}{8}$ 12. $3\frac{5}{12}$. $\frac{41}{12}$

Write a number in mixed form for

13. $\frac{25}{4}$. $6\frac{1}{4}$ 14. $\frac{39}{7}$. $5\frac{4}{7}$ 15. $\frac{23}{10}$. $2\frac{3}{10}$

Use the number lines. Find an equivalent fraction for each of these.



$\frac{2}{4}$ or $\frac{3}{6}$ 16. $\frac{1}{2}$ or $\frac{4}{8}$ or $\frac{5}{10}$ 17. $\frac{1}{3}$ $\frac{2}{6}$ 18. $\frac{1}{4}$ $\frac{2}{8}$

19. $\frac{6}{8}$ $\frac{3}{4}$ 20. $\frac{4}{10}$ $\frac{2}{5}$ 21. $\frac{4}{6}$ $\frac{2}{3}$

Use > or < to make true statements.

Use the number lines to help you.

22. $\frac{2}{3} < \frac{4}{5}$ 23. $\frac{3}{4} > \frac{7}{10}$

24. $\frac{1}{3} > \frac{3}{10}$ 25. $\frac{1}{3} < \frac{3}{8}$

Here is a way to write a fraction in lowest terms.

$$\frac{42}{56} = \frac{\overset{1}{\cancel{2}} \times 3 \times \overset{1}{\cancel{7}}}{\underset{1}{\cancel{2}} \times 2 \times 2 \times \underset{1}{\cancel{7}}} \qquad \frac{42}{56} = \frac{3}{4}$$

For each of these, write the numerator and denominator as products of factors that are prime numbers. Then divide by the common factors to find a fraction in lowest terms.

1. $\frac{6}{15}$ $\frac{2}{5}$ 2. $\frac{15}{24}$ $\frac{5}{8}$ 3. $\frac{24}{36}$ $\frac{2}{3}$ 4. $\frac{36}{42}$ $\frac{6}{7}$ 5. $\frac{42}{70}$ $\frac{3}{5}$ 6. $\frac{70}{84}$ $\frac{5}{6}$

try this

213

Page 213

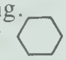
OBJECTIVE

Demonstrate competence in writing equivalent fractions, in expressing a number in mixed form as an improper fraction, in expressing an improper fraction as a number in mixed form, and in comparing fractions

Materials

straight edge for each student

RELATED ACTIVITIES

- For enrichment, have students complete exercises similar to the following.
If  is $\frac{1}{6}$, draw a shape to show $\frac{5}{6}$. One solution is given below. Copies of shapes from pages T383-T385 may be used. To present a greater challenge, have students show $\frac{2}{3}$, 1, $1\frac{1}{6}$, or $1\frac{1}{3}$ for the given exercise.

- Provide students with number lines from copies of page T390. Have them write three names for each of several different points from 0 to 3. For example, 0 may be shown as $\frac{0}{1}$, $\frac{0}{2}$, and $\frac{0}{3}$; 1 as $\frac{3}{3}$, $\frac{5}{5}$, and $\frac{6}{6}$; and $\frac{1}{2}$ as $\frac{1}{2}$, $\frac{2}{4}$, and $\frac{4}{8}$.
- Students may apply the method shown in *Try This* for writing fractions on page 203 in lowest terms.

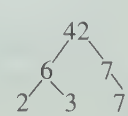
LESSON ACTIVITY

Using the Page

- Before the students begin the exercises, draw attention to the number lines illustrated. Ask how many lines there are and ask how they are similar and how they are different. Each line represents the segment from 0 to 1, but the segments are marked into a different number of equal parts. Develop that the first line shows halves, the second shows thirds, and so on. Ask the students to place a straight edge on the page to align the points for $\frac{1}{2}$ and $\frac{2}{4}$. Because the number lines are aligned so that the points for 0 match and the points for 1 match, points for equivalent fractions will also be vertically aligned. The students may use a straight edge for assistance in answering Ex. 16-25.

For Ex. 1 and 2, the students must compare the fractions and then consider the situations in which the fractions are used. For example, for Ex. 1, the greater fraction indicates the better sale because subtracting the greater amount creates the lower sale price. For Ex. 2, the lesser fraction indicates the lower sale price. Ex. 3 is starred because the answer is not found by comparing the given fractions $\frac{2}{5}$ and $\frac{3}{4}$, but by comparing $\frac{2}{5}$ and $\frac{1}{4}$ (from $1 - \frac{3}{4}$), or $\frac{3}{5}$ (from $1 - \frac{2}{5}$) and $\frac{3}{4}$.

Try This: The example shows a procedure for writing a fraction in lowest terms. First, the numerator and the denominator are written as products of factors that are prime numbers. The process reveals common factors of the numerator and the denominator. Then, the numerator and the denominator are divided by each common factor in turn. In this example, both the numerator and the denominator are divided first by 2, and then by 7, resulting in quotients of 1 which are recorded as shown. Because the numerator and the denominator are divided by the same number, the number named by the fraction does not change. The students may find it helpful to show "factor tree" diagrams to obtain prime factors.



OBJECTIVE

Solve a problem through a process of logical thinking

RELATED ACTIVITIES

- Students may enjoy the challenge of writing and solving word problems similar to those on page 214. Problems written by some students can be displayed on cards for others to solve. The problems may differ only slightly from those in the lesson or they may be very different.
- For exercises in logical reasoning, activities involving the use of attribute blocks is recommended (see page xxiv). Several activities are described on page T 381.

Logical Thinking

Solving these problems requires careful thinking.

1. Alice, Byron, Cathy, Dinah, and Edmund want to catch the bus at 18:00.

   Alice's watch is 5 min fast. She thinks it is 10 min slow.

   Byron's watch is 5 min slow. He thinks it is 10 min fast.

   Cathy's watch is 10 min slow. She thinks it is 5 min fast.

   Dinah's watch is 5 min fast. She thinks it is 10 min fast.

   Edmund's watch is 5 min slow. He thinks it is 10 min slow.

   Each student leaves home allowing exactly enough time to catch the bus according to the time each thinks it is.

   Who will miss the bus? Byron, Cathy, Dinah

2. All the trains from Portville go to Barton. From Barton, some trains go to Clark Valley. Some trains go from Barton to Durham and then to Elk River. Other trains go from Barton to Fairview and then to Murray. The fare is \$3 to Clark Valley, Elk River, or Murray, \$1 to Barton, and \$2 to the other places.

   Ivy is in a hurry. She bought a ticket for \$2 at the Portville train station. The first train is going to Elk River. Ivy did not get on the first train. Where is Ivy going? Fairview

3. Karen, Lea, Matt, and Nate each live on a different floor of a four-storey apartment building. Their ages are 7, 10, 11, and 12, but not necessarily in that order. Karen lives directly above the 11-year-old and directly below the 10-year-old. When Lea leaves her apartment, she passes the floor where the 7-year-old lives. Lea is more than one floor away from Nate. Nate is more than a year younger than Lea.

   What is the age of each student? On which floor does each student live?

Lea	10	4th floor
Karen	12	3rd floor
Matt	11	2nd floor
Nate	7	1st floor

PROBLEM SOLVING

214

LESSON ACTIVITY

Before Using the Page

- Write the following word problem on the board. "Anita lives farther from the school than Brian, but not as far as Carl. Who lives farthest from the school?" Develop that a diagram may help to solve the problem. For "Anita lives farther from the school than Brian" show the following sequence.

  school B A

  Then illustrate "but not as far as Carl".

  school B A C

  Discuss how the diagram helps in solving the word problem. Students may suggest a different diagram for the problem.

Using the Page

- Encourage the students to draw diagrams or to record partial solutions in the process of solving the three word problems. For Ex. 1, for example, if it is assumed that each person plans to be at the bus stop at 18:00 according to her/his watch, statements similar to the following can be written for each person.

  Alice: To be at the bus stop at 18:00, she will plan to be there when her watch shows 17:50. However, when her watch shows 17:50, the correct time is 17:45. She will not miss the bus.

For Ex. 2, a map may be drawn to show train routes and stops. Because students may find this exercise easier to solve than Ex. 1, you may wish to direct the students to begin with Ex. 2. For Ex. 3, diagrams of a four-storey apartment building can be drawn for the different possibilities. Some of these will be rejected as facts are tested, until only one possibility remains.

Floor	Age	Name	Age	Name	Age	Name
4	10		10		12	
3	7	Karen	12	Karen	10	
2	11		11		7	Karen
1	12		7		11	

Checking Up

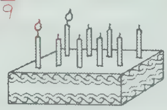
For the fraction $\frac{4}{5}$,

1. what is the numerator? 4

2. what is the denominator? 5

Write a fraction that shows how many candles are burning.

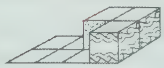
3. $\frac{2}{9}$



4. $\frac{7}{12}$

Write a fraction that shows how much cake is left.

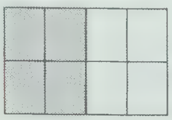
5. $\frac{3}{8}$



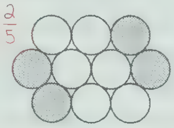
6. $\frac{1}{10}$

Write two equivalent fractions that show how much of each picture is blue.

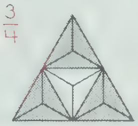
7. $\frac{4}{8}, \frac{1}{2}$



8. $\frac{4}{10}, \frac{2}{5}$



9. $\frac{9}{12}, \frac{3}{4}$



Write four fractions that are equivalent to each of these.

10. $\frac{1}{7}$ 11. $\frac{2}{3}$ 12. $\frac{5}{10}$

Write the fraction in lowest terms that is equivalent to

13. $\frac{10}{12}$ $\frac{5}{6}$ 14. $\frac{6}{18}$ $\frac{1}{3}$ 15. $\frac{28}{36}$ $\frac{7}{9}$

Are the two fractions equivalent? Use cross products.

16. $\frac{1}{4}, \frac{3}{12}$ yes 17. $\frac{15}{24}, \frac{10}{16}$ yes 18. $\frac{2}{3}, \frac{5}{7}$ no 19. $\frac{6}{9}, \frac{8}{12}$ yes

Find the missing term.

20. $\frac{3}{6} = \frac{■}{8}$ 4 21. $\frac{6}{10} = \frac{9}{■}$ 15 22. $\frac{5}{9} = \frac{■}{36}$ 20 23. $\frac{6}{8} = \frac{12}{■}$ 16

Write 3 as an improper fraction

24. showing halves. $\frac{6}{2}$ 25. showing thirds. $\frac{9}{3}$ 26. showing fourths. $\frac{12}{4}$

Write each of these as an improper fraction.

27. $3\frac{1}{2}$ $\frac{7}{2}$ 28. $1\frac{7}{10}$ $\frac{17}{10}$ 29. $7\frac{3}{4}$ $\frac{31}{4}$

Write as a whole number or as a number in mixed form.

30. $\frac{10}{3}$ $3\frac{1}{3}$ 31. $\frac{36}{12}$ 3 32. $\frac{14}{5}$ $2\frac{4}{5}$

For each pair, find equivalent fractions with like denominators.

33. $\frac{1}{2}, \frac{3}{5}$ $\frac{5}{10}, \frac{6}{10}$ 34. $\frac{5}{6}, \frac{7}{8}$ $\frac{20}{24}, \frac{21}{24}$ or $\frac{40}{48}, \frac{42}{48}$ 35. $1\frac{3}{4}, 3\frac{1}{3}$ $1\frac{9}{12}, 3\frac{4}{12}$

Use >, <, or = to make true statements.

36. $\frac{3}{4} > \frac{5}{7}$ 37. $\frac{5}{12} < \frac{4}{9}$

10. $\frac{2}{14}, \frac{3}{21}, \frac{4}{28}, \frac{5}{35}$ 11. $\frac{4}{6}, \frac{6}{9}, \frac{8}{12}, \frac{10}{15}$ 12. $\frac{10}{20}, \frac{15}{30}, \frac{20}{40}, \frac{25}{50}$

Other answers are possible.

215

Page 215

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Have students compare fractions having equal numerators but different denominators, for example, $\frac{3}{4}$ and $\frac{3}{8}$. They can discover that the fraction with the greater denominator always names the lesser number.
- Ask each student to print her/his name and show what fraction represents the letters that are vowels and what fraction represents the letters that are consonants. Have several students show their solutions on the board. Cross products can be used to determine equivalent fractions. Pairs of fractions may be compared. Some fractions may be rewritten in lowest terms.
- Prepare 20 sets of three cards for each topic for the game "Match Up" described on page T 381. An example of one set for each topic is shown below.

Equivalent fractions:	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$
Whole numbers and improper fractions:	2	$\frac{8}{4}$	$\frac{6}{3}$
Numbers in mixed form and improper fractions:	$1\frac{2}{3}$	$1\frac{1}{6}$	$\frac{5}{3}$

Skills	Exercises	Related Pages
Identify the numerator and the denominator of a fraction	1, 2	T 216-T 217
Write numerals for fractions	3-6	T 216-T 217
Write equivalent fractions for diagrams	7-9	T 218-T 219
Find equivalent fractions	10-12	T 218-T 221
Write fractions in lowest terms	13-15	T 220-T 221
Use cross products to determine equivalent fractions	16-19	T 222-T 223
Find the missing term in two equivalent fractions	20-23	T 222-T 223
Express a whole number as an improper fraction	24-26	T 224-T 225
Express a number in mixed form as an improper fraction	27-29	T 224-T 225
Express an improper fraction as a whole number or as a number in mixed form	30-32	T 226-T 227
Find equivalent fractions with like denominators	33-35	T 228-T 229
Compare fractions	36, 37	T 230

Comments

Students having difficulty with fractions may benefit by working with models or by drawing pictures to illustrate fractions. The models or pictures can be used to show exercises that are causing difficulty and to show similar exercises. After the students have completed the exercises, discuss different answers for Ex. 7-12.

Many of the concepts and skills presented in this unit are applied in Unit 11. For example, fractions with unlike denominators must be expressed as equivalent fractions with like denominators prior to addition or subtraction. Also, fractions obtained after performing any of the four operations must be expressed as equivalent fractions in lowest terms. Thus, errors and misconceptions which are detected in the exercises on page 215 should be corrected before the work of Unit 11 is begun.

Unit 11 Overview

Operations with Fractions

This unit on operations with fractions builds upon the foundation of the previous unit in which the underlying concepts and principles concerning fractions are developed. The four basic operations are performed with combinations of fractions, numbers in mixed form, and whole numbers. Equivalent fractions are required in finding common denominators for addition and subtraction, in renaming sums and minuends, and in writing results in lowest terms. Division involving fractions is presented by using the reciprocal of the divisor as a multiplier. Decimal equivalents of fractions are obtained by dividing the numerators by the denominators and students find that sometimes the equivalence is only approximate. Decimal equivalents for fractions are then used in the four operations and students compare the results and use them to check their computations. The lesson on the use of the calculator emphasizes the speed with which decimal equivalents can be obtained and the ease of performing calculations with numbers in this form. The lessons in the unit include opportunities for students to solve related word problems. The *Problem Solving* lesson emphasizes the value of restating a problem situation in other ways to help in choosing the required data and in selecting the correct operation(s) to use.

Prerequisite Skills

- write fractions
- find equivalent fractions with like denominators, no regrouping
- write fractions in lowest terms
- express an improper fraction as a number in mixed form or as a whole number
- express a number in mixed form as an improper fraction
- express a whole number as an improper fraction
- divide whole numbers using extra zeros in the dividend
- round quotients to two or three decimal places
- order decimals
- add, subtract, multiply, and divide decimals

Unit Outcomes

- add fractions and numbers in mixed form with like denominators, no regrouping; subtract fractions and numbers in mixed form with like denominators, no regrouping
- add two or three fractions with unlike denominators, no regrouping
- express sums of fractions in lowest terms
- add two fractions or two numbers in mixed form with like or unlike denominators, regrouping
- subtract fractions with unlike denominators, no regrouping
- express differences of fractions in lowest terms
- add to check subtraction of fractions
- subtract fractions or numbers in mixed form from numbers in mixed form with unlike denominators, regrouping
- subtract fractions or numbers in mixed form from whole numbers
- multiply fractions; multiply numbers in mixed form
- express the products of fractions and numbers in mixed form in lowest terms
- multiply a fraction and a whole number
- identify reciprocals; find reciprocals of fractions, of numbers in mixed form, and of whole numbers
- divide a proper fraction by a proper fraction; divide a proper fraction by a whole number; divide a whole number by a proper fraction
- express the quotients from dividing with fractions in lowest terms; multiply to check division with fractions
- divide the numerator of a fraction by the denominator to express the fraction as a decimal, round the quotient to two or three decimal places
- compare and order fractions using their decimal equivalents
- add, subtract, multiply, and divide fractions and their decimal equivalents or their approximate decimal equivalents, and then compare the results
- solve word problems involving fractions
- use a calculator to express fractions as decimals; use a calculator to multiply whole numbers and fractions
- restate word problems

Background

In the Overview for Unit 10, it is pointed out that the *numerator* of a fraction indicates the *number* of parts that are being considered and that the *denominator* designates the *size* of each part or the *name* of each part. These concepts are essential for understanding why only the numerators are added or subtracted in performing these operations. Consider how only the numbers are added (subtracted) if numbers are applied to concrete objects, such as 7 green apples and 3 red apples. The total number of apples is found by adding the numbers 7 and 3, not the names. To find how many more green apples there are than red apples, again, only the numbers 7 and 3 are used. Similarly, if two fractions have the same name (denominator), only their numbers (numerators) are added or subtracted.

$$\begin{array}{r} 7 \text{ eighths} \\ +\ 3 \text{ eighths} \\ \hline 10 \text{ eighths} \end{array} \quad \frac{7}{8} + \frac{3}{8} = \frac{10}{8} \qquad\qquad \begin{array}{r} 7 \text{ eighths} \\ -\ 3 \text{ eighths} \\ \hline 4 \text{ eighths} \end{array} \quad \frac{7}{8} - \frac{3}{8} = \frac{4}{8}$$

$$\begin{array}{r} \frac{7}{8} \\ +\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ +\ \frac{6}{8} \\ \hline \frac{13}{8} \end{array} \qquad \begin{array}{r} \frac{7}{8} \\ -\ \frac{3}{4} \\ \hline \end{array} \quad \begin{array}{r} \frac{7}{8} \\ -\ \frac{6}{8} \\ \hline \frac{1}{8} \end{array}$$

Considering fruit again, numbers of apples and pears cannot be added (subtracted) unless they are given a common name, such as fruit. Similarly, fractions with different denominators (names) cannot be added or subtracted, unless they are given a common denominator (name). Multiplication and division do not require that fractions have common denominators. In these operations, both the numerators and the denominators are used in the operations.

In connection with whole numbers, there are several basic properties, including the commutative and associative properties for addition and multiplication, and the inverse relationships between addition and subtraction and between multiplication and division. These properties also apply in operations with fractions. The commutative properties make it possible for two addends or two factors to be interchanged without affecting the sums or products. The associative properties make it possible for three or more addends or factors to be associated in different ways without affecting the sums or products. The inverse operations may be used to check the accuracy of work — addition to check subtraction, and multiplication to check division.

$$\begin{array}{r} 4\frac{5}{6} \\ -\ 2\frac{1}{3} \\ \hline \end{array} \qquad \begin{array}{r} 4\frac{5}{6} \\ -\ 2\frac{2}{6} \\ \hline 2\frac{3}{6}, \text{ or } 2\frac{1}{2} \end{array} \qquad \text{Check:} \quad \begin{array}{r} 2\frac{1}{2} \\ +\ 2\frac{1}{3} \\ \hline \end{array} \qquad \begin{array}{r} 2\frac{3}{6} \\ +\ 2\frac{2}{6} \\ \hline 4\frac{5}{6} \end{array}$$

$$\begin{aligned} \tfrac{8}{9} \div \tfrac{2}{3} &= \tfrac{8}{9} \times \tfrac{3}{2} \\ &= \tfrac{24}{18} \\ &= 1\tfrac{6}{18}, \text{ or } 1\tfrac{1}{3} \end{aligned}$$

$$\text{Check: } 1\tfrac{1}{3} \times \tfrac{2}{3} = \tfrac{4}{3} \times \tfrac{2}{3} = \tfrac{8}{9}$$

In this unit, division of fractions is achieved by using the reciprocal of the divisor as a multiplier. This is closely related to the understanding which students have had in their previous work, that a unit fraction of a number can be obtained by division. For example, $\frac{1}{3}$ of 48 ($\frac{1}{3} \times 48$ or $48 \times \frac{1}{3}$) is found by the division $48 \div 3$. In other words, $48 \div 3$ and $48 \times \frac{1}{3}$ represent the same number, 16. Similarly, $72 \div 4$ and $72 \times \frac{1}{4}$ represent the same number, 18. Thus, to divide by a number one can multiply by the reciprocal of the divisor. The following explanation shows why this procedure is valid. The easiest number to divide by is the number 1, since the quotient is always identical to the dividend ($14 \div 1 = 14$). It would be easy to perform a division, such as $24 \div \frac{3}{4}$, if the divisor were equivalent to 1. By multiplying the divisor, $\frac{3}{4}$, by its reciprocal, $\frac{4}{3}$, the divisor becomes equivalent to 1; however, to maintain the original relationship between the dividend and the divisor, it is necessary to also multiply the dividend, 24, by $\frac{4}{3}$. The example then becomes one in which $24 \times \frac{4}{3}$ is divided by 1, and the result is obtained by merely multiplying the original dividend, 24, by the reciprocal of the divisor. This is essentially the same as the method suggested on page 234, namely, "dividing by a number is the same as multiplying by its reciprocal".

$$\begin{aligned} & 24 \div \tfrac{3}{4} \\ &= (24 \times \tfrac{4}{3}) \div (\tfrac{3}{4} \times \tfrac{4}{3}) \\ &= (24 \times \tfrac{4}{3}) \div 1 \\ &= 24 \times \tfrac{4}{3} \end{aligned}$$

The decimal equivalents of fractions are sometimes exact, but many are only approximate. These different results are obtained because of our decimal numeration system. Ten and powers of ten are not exactly divisible by such numbers as 3, 6, 9, and 12, which often occur in the denominators of fractions. In many of these cases, the decimal equivalents have repeating patterns of digits. The repetition is indicated by drawing a horizontal bar over the digits in the pattern.

$$\begin{aligned} \tfrac{1}{3} &= 0.333\ldots & \tfrac{2}{3} &= 0.666\ldots \\ &= 0.\overline{3} & &= 0.\overline{6} \\ \tfrac{1}{6} &= 0.1666\ldots & \tfrac{5}{6} &= 0.8333\ldots \\ &= 0.1\overline{6} & &= 0.8\overline{3} \\ \tfrac{5}{9} &= 0.555\ldots & \tfrac{8}{9} &= 0.888\ldots \\ &= 0.\overline{5} & &= 0.\overline{8} \\ \tfrac{5}{12} &= 0.41666\ldots & \tfrac{7}{12} &= 0.58333\ldots \\ &= 0.41\overline{6} & &= 0.58\overline{3} \end{aligned}$$

Because some decimal equivalents are only approximate, results of operations with them differ slightly from those which are obtained by working with the fractions themselves. For example, $\frac{1}{3} \times 24$ is equal to 8, but 0.33×24 is equal to 7.92.

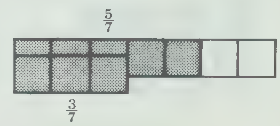
Teaching Strategies

If there was a considerable range in the students' achievements in Unit 10, it may be advisable to group the students for instruction in this unit. A thorough background of the concepts and skills which were developed in that unit is necessary so that the operations with fractions may be performed meaningfully.

For finding common denominators, students may find charts of multiples, like the one shown below, useful. This is an extension of the one on page 211 to include the numbers which are used most frequently as denominators. A quick glance across the rows for 4 and 6, for instance, reveals that 12 is the least common multiple of both of these numbers.

2	2		4		6	8		10	12	14		16	18	20	
3		3			6		9		12		15		18		21
4			4			8			12			16		20	
5				5				10			15			20	
6					6				12				18		
8						8						16			
9							9						18		

It should be pointed out that there are mainly two types of subtractive situations and these may be classified as "removal" subtraction and comparative subtraction. The lesson suggestions on pages T 236 and T 237 show a removal type in *Before Using the Pages*. In this type there is a remainder after one eighth of the three eighths is removed. In the lesson, the subtractive situations illustrated on page 216 and in Ex. 2 of *Working Together* on page 217 are of the comparative type. There are no removals, but there are comparisons of numbers to find how much greater (less) one number is than the other. This type of subtraction can be shown best if one model is placed on top of another, as shown for $\frac{5}{7} - \frac{3}{7} = \frac{2}{7}$.



Ex. 26-31 on page 219 involve the addition of three fractions. These provide an opportunity for students to test the associative and commutative properties of addition. Some of the students may be instructed to change the order of addends from that given in the book and to compare their sums with those obtained by other students. Some students may be challenged by extending the multiplication of fractions to three factors, such as $\frac{3}{4} \times \frac{1}{2} \times \frac{5}{6}$ and $2\frac{2}{3} \times 1\frac{1}{4} \times 1\frac{4}{5}$. By grouping the factors in different ways they may discover that the associative property of multiplication applies to fractions and numbers in mixed form as well as to whole numbers.

The lesson on the use of the calculator shows how to find decimal equivalents for fractions and how to perform the multiplication of a fraction and a whole number on a calculator. If no calculators are available for use by the students, the lesson on these pages may be of limited value because the paper-and-pencil calculations for the exercises would be very time-consuming. If only a few calculators are available, it may be necessary to schedule these exercises over several days.

Materials

sheets of paper, a red pencil, a blue pencil, and crayons for each student

two sheets of paper, one marked into sixths and one marked into eighths for each student

models for $3\frac{2}{3}$ prepared from copies of page T 392

a model for $\frac{7}{10}$ prepared from a copy of page T 393

models prepared for the lesson on pages T 240 and T 241 (optional)

Vocabulary

prime factors　　　　reciprocals

Pages 216-217

LESSON OUTCOME

Add fractions and numbers in mixed form with like denominators, no regrouping; subtract fractions and numbers in mixed form with like denominators, no regrouping; solve related word problems

Materials

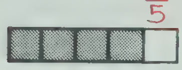
a sheet of paper, a red pencil, and a blue pencil for each student

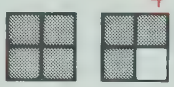
Prerequisite Skills

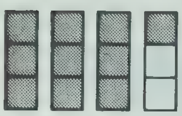
Write fractions

Checking Prerequisite Skills

Write the fraction to show how much is shaded.

1.  $\frac{4}{5}$

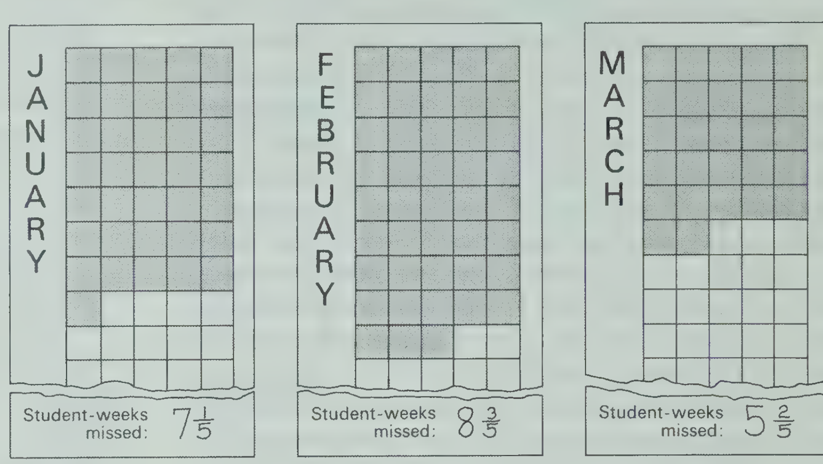
2.  $1\frac{3}{4}$

3.  $3\frac{1}{3}$

11 OPERATIONS WITH FRACTIONS

Adding and Subtracting with Like Denominators

Industry counts personnel days lost if they want to know how many days of work were missed by any or all of their workers. The students kept a similar record and recorded "student-weeks" lost by classmates.



How many student-weeks of school were missed in January and February together?

Add $7\frac{1}{5}$ and $8\frac{3}{5}$.

Add fifths. Then add ones.

$$\begin{array}{r} 7\frac{1}{5} \\ 8\frac{3}{5} \\ \hline 15\frac{4}{5} \end{array}$$

The students missed $15\frac{4}{5}$ student-weeks of school in January and February.

How many fewer student-weeks were missed in March than in February?

Subtract $5\frac{2}{5}$ from $8\frac{3}{5}$.

Subtract fifths. Then subtract ones.

$$\begin{array}{r} 8\frac{3}{5} \\ 5\frac{2}{5} \\ \hline 3\frac{1}{5} \end{array}$$

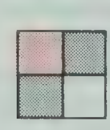
The students missed $3\frac{1}{5}$ fewer student-weeks in March than in February.

216

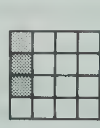
LESSON ACTIVITY

Before Using the Pages

- Give each student a sheet of paper. Direct one group of students to fold the sheets into fourths, another group to fold the sheets into eighths, and the remaining students to fold the sheets into sixteenths. Ask each student to color one of the equal parts red and two of the equal parts blue. Display one example from each group on the board.


$\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$


$\frac{1}{8} + \frac{2}{8} = \frac{3}{8}$


$\frac{1}{16} + \frac{2}{16} = \frac{3}{16}$

For the first example, discuss that $\frac{1}{4}$ was colored first, then $\frac{2}{4}$, and thus, $\frac{3}{4}$ was colored in all. Lead the students to suggest the addition sentence $\frac{1}{4} + \frac{2}{4} = \frac{3}{4}$ for this example. Develop the addition sentences for the other examples. Write each sentence below the corresponding diagram. Point out that the numerators in each sentence suggest the whole-number addition sentence $1 + 2 = 3$. Relate this to the number of red parts and the number of blue parts. Relate the denominator to the size of the parts for each example.

- Adapt the preceding activity for subtraction. For example, ask the students to use the opposite side of their sheets of paper, color three parts red, and cross out one of the three parts to represent the removal of one part.


$\frac{3}{8} - \frac{1}{8} = \frac{2}{8}$

Using the Pages

- Read the information at the top of page 216 and ask students to express the meaning of *student-weeks* in their own words. For example, if one student is absent for five days, this is equivalent to one student-week. If five students are absent for one day, this is also equivalent to one student-week.

Working Together

Complete the addition sentence.

1. 

$3\frac{1}{5}$ + ■ $1\frac{2}{5}$ = ■ $4\frac{3}{5}$

Complete the subtraction sentence.

2. 

■ $\frac{6}{7}$ − ■ $\frac{4}{7}$ = ■ $\frac{2}{7}$

Add.

3. $\begin{array}{r} \frac{1}{4} \\ \frac{2}{4} \\ \hline \frac{3}{4} \end{array}$

4. $\frac{4}{9} + \frac{2}{9}$ $\frac{6}{9}$

5. $4\frac{3}{12} + 1\frac{7}{12}$ $5\frac{10}{12}$

Subtract.

6. $\begin{array}{r} \frac{5}{8} \\ \frac{3}{8} \\ \hline \frac{2}{8} \end{array}$

7. $\frac{3}{4} - \frac{1}{4}$ $\frac{2}{4}$

8. $4\frac{2}{5} - 2\frac{1}{5}$ $2\frac{1}{5}$

Exercises

Add.

1. $\begin{array}{r} \frac{1}{6} \\ \frac{3}{6} \\ \hline \frac{4}{6} \end{array}$

2. $\begin{array}{r} 1\frac{1}{3} \\ 2\frac{1}{3} \\ \hline 3\frac{2}{3} \end{array}$

3. $\begin{array}{r} \frac{2}{7} \\ \frac{3}{7} \\ \hline \frac{5}{7} \end{array}$

4. $\frac{2}{5} + \frac{2}{5}$ $\frac{4}{5}$

5. $3\frac{1}{8} + 5\frac{3}{8}$ $8\frac{4}{8}$

Subtract.

6. $\begin{array}{r} \frac{4}{7} \\ \frac{1}{7} \\ \hline \frac{3}{7} \end{array}$

7. $\begin{array}{r} 9\frac{11}{12} \\ 5\frac{6}{12} \\ \hline 4\frac{5}{12} \end{array}$

8. $\begin{array}{r} \frac{9}{10} \\ \frac{3}{10} \\ \hline \frac{6}{10} \end{array}$

9. $\frac{5}{6} - \frac{2}{6}$ $\frac{3}{6}$

10. $4\frac{2}{3} - 3\frac{1}{3}$ $1\frac{1}{3}$

Solve.

11. Percy painted $\frac{3}{4}$ of a wall in the morning and $\frac{1}{4}$ of the wall in the afternoon. How much more of the wall did he paint in the morning than in the afternoon? $\frac{2}{4}$

12. Percy spent $\frac{1}{3}$ of the morning practicing the piano and $\frac{1}{3}$ of the morning playing hockey. How much of the morning did he spend on both activities? $\frac{2}{3}$

try this

Write the next three numbers in each pattern.
Then tell whether the numbers in each pattern are increasing, decreasing, or remaining the same.

1. $\frac{1}{12}, \frac{2}{12}, \frac{3}{12}, \frac{4}{12}, \ldots$
2. $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \ldots$
3. $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}, \frac{4}{8}, \ldots$
4. $\frac{2}{4}, \frac{3}{9}, \frac{4}{16}, \frac{5}{25}, \frac{6}{36}, \frac{7}{49}, \ldots$
5. $\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}, \ldots$
6. $\frac{2}{3}, \frac{3}{5}, \frac{4}{7}, \frac{5}{9}, \ldots$
7. $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

1. $\frac{5}{12}, \frac{6}{12}, \frac{7}{12}$; increasing 2. $\frac{1}{6}, \frac{1}{7}, \frac{1}{8}$; decreasing 3. $\frac{5}{10}, \frac{6}{12}, \frac{7}{14}$; same
4. $\frac{8}{64}, \frac{9}{81}, \frac{10}{100}$; decreasing 5. $\frac{5}{11}, \frac{6}{13}, \frac{7}{15}$; increasing 6. $\frac{6}{11}, \frac{7}{13}, \frac{8}{15}$; decreasing 7. $\frac{5}{6}, \frac{6}{7}, \frac{7}{8}$; increasing

217

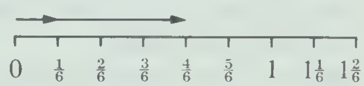
RELATED ACTIVITIES

• For further practice, you may wish to have students complete Ex. 1 - 12 on page 335.

• Students having difficulty adding and subtracting fractions, especially if they add or subtract the denominators, would benefit from illustrating exercises, such as those on page 217, with models. They can draw diagrams or arrange models made with construction paper to illustrate an exercise. Copies of the shapes on pages T 383 - T 385 may be used for models.

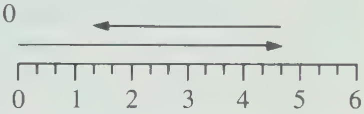
• Demonstrate addition and subtraction of fractions on a number line. Have the students use copies of page T 390 to show some of the exercises on page 217 on a number line.

Ex. 1



$\frac{1}{6} + \frac{3}{6} = \frac{4}{6}$

Ex. 10



$4\frac{2}{3} - 3\frac{1}{3} = 1\frac{1}{3}$

• Prepare sets of cards showing fractions for which the sum is one, for example, $\frac{1}{5}, \frac{2}{5}, \frac{2}{5}$. Include a few of each set for the game "Total Action" described on page T 379. The required sum for a set of cards would be one.

• You may wish to have students express the results in lowest terms for Ex. 1, 5, 8, 9, and 11 on page 217.

Ask how each model shows the indicated number of student-weeks missed.

Ask a student to read the first question. Elicit from the students that the answer is found by adding $7\frac{1}{5}$ and $8\frac{3}{5}$. Point out that the addends are numbers in mixed form, and that the addition is performed by adding the fifths first and then adding the ones. Note that the sum is a number in mixed form involving fifths. Ask a student to read the concluding statement.

In a similar manner, discuss the example involving the subtraction $8\frac{3}{5} - 5\frac{2}{5}$. Emphasize that for each operation, fractions are dealt with before the whole numbers.

Working Together: Ex. 1 and 2 provide models to help the students to add or subtract. For each exercise, emphasize that the two fractions have like denominators. Although students are not directed at this time to express the results in lowest terms, you may wish to discuss this concept for Ex. 4-7 with some or all of the students in your class.

Exercises: Remind the students to show their work for Ex. 11 and 12 and answer with a concluding statement.

Try This: For each exercise, ask the students to use the pattern for the numerators and the pattern for the denominators to write the next three numbers. It may be necessary to find equivalent fractions with like denominators for consecutive pairs of fractions and compare the fractions to determine whether the numbers are increasing, decreasing, or remaining the same.

Assessment

Add.

1. $\begin{array}{r} \frac{1}{7} \\ \frac{1}{7} \\ \hline \frac{2}{7} \end{array}$

2. $\frac{2}{5} + \frac{1}{5}$ $\frac{3}{5}$

3. $1\frac{5}{8} + 2\frac{1}{8}$ $3\frac{6}{8}$ $(3\frac{3}{4})$

Subtract.

4. $\begin{array}{r} \frac{3}{4} \\ \frac{2}{4} \\ \hline \frac{1}{4} \end{array}$

5. $\frac{9}{10} - \frac{1}{10}$ $\frac{8}{10}$ $(\frac{4}{5})$

6. $3\frac{4}{6} - 1\frac{1}{6}$ $2\frac{3}{6}$ $(2\frac{1}{2})$

Solve.

7. Alice missed $\frac{1}{5}$ of one student-week and $\frac{2}{5}$ of another student-week. How many student-weeks did she miss? $\frac{3}{5}$

8. Max missed $\frac{3}{5}$ of a student-week and Lee missed $\frac{1}{5}$ of a student-week. How many more student-weeks did Max miss than Lee? $\frac{2}{5}$

Pages 218-219

LESSON OUTCOME

Add two or three fractions with unlike denominators, no regrouping; express sums of fractions in lowest terms; solve related word problems

Materials

two sheets of paper, one marked into sixths and one marked into eighths for each student

Prerequisite Skills

Find equivalent fractions with like denominators, no regrouping; write fractions in lowest terms

Checking Prerequisite Skills

For each pair, find equivalent fractions with like denominators.

1. $\frac{1}{9}, \frac{1}{3}$ **2.** $\frac{2}{5}, \frac{3}{4}$ **3.** $\frac{1}{6}, \frac{3}{8}$

Write each fraction in lowest terms.

4. $\frac{6}{8}$ $\frac{3}{4}$ **5.** $\frac{3}{9}$ $\frac{1}{3}$ **6.** $\frac{12}{20}$ $\frac{3}{5}$

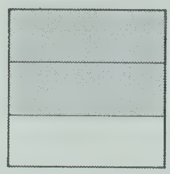
1. $\frac{1}{9}, \frac{3}{9}$ 2. $\frac{8}{20}, \frac{15}{20}$ 3. $\frac{4}{24}, \frac{9}{24}$

Adding Fractions with Unlike Denominators

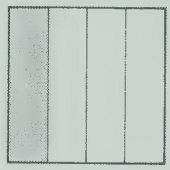
Find the sum of $\frac{2}{3}$ and $\frac{1}{4}$.

To add fractions with unlike denominators, find equivalent fractions with like denominators. Then add the numerators.

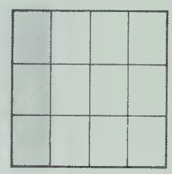
Add.



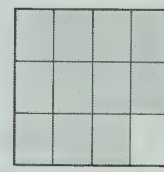
$\frac{2}{3} = \frac{8}{12}$



$\frac{1}{4} = \frac{3}{12}$



$\frac{11}{12}$



The sum of $\frac{2}{3}$ and $\frac{1}{4}$ is $\frac{11}{12}$.

Working Together

For the pair of fractions in each addition, give equivalent fractions that have like denominators.

1. $\frac{1}{2} + \frac{1}{4} = \frac{■}{■} + \frac{■}{■}$ $\frac{2}{4}$ $\frac{1}{4}$

2. $\frac{1}{8} + \frac{2}{3} = \frac{■}{■} + \frac{■}{■}$ $\frac{3}{24}$ $\frac{16}{24}$

Find equivalent fractions with like denominators. Then add.

3. $\frac{2}{5}$ $\frac{8}{20}$ / $+\frac{1}{4}$ $\frac{5}{20}$ / $\frac{13}{20}$

4. $\frac{3}{4}$ $\frac{9}{12}$ / $+\frac{1}{12}$ $\frac{1}{12}$ / $\frac{10}{12}$

5. $\frac{3}{7} + \frac{1}{3}$ $\frac{9}{21} + \frac{7}{21} = \frac{16}{21}$

6. $\frac{1}{2} + \frac{1}{10}$ $\frac{5}{10} + \frac{1}{10} = \frac{6}{10}$

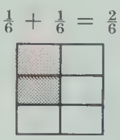
218

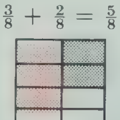
LESSON ACTIVITY

Before Using the Pages

- Give each student a sheet of paper marked into sixths and another marked into eighths. Ask the students to color the appropriate sheet to illustrate the following additions. Have them write the addition sentences on the sheets.

$\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ $\frac{3}{8} + \frac{2}{8} = \frac{5}{8}$





Write the exercises $\frac{1}{6} + \frac{1}{2}$ and $\frac{1}{8} + \frac{3}{4}$ on the board and draw attention to the unlike denominators. Ask the students to color the opposite side of the appropriate sheet to help in finding the sums. Discuss the procedure. For example, for $\frac{1}{2}$, it is necessary to color three of the sixths; thus, the sum of $\frac{1}{6}$ and $\frac{1}{2}$ is obtained by adding $\frac{1}{6}$ and $\frac{3}{6}$. Review that $\frac{1}{2}$ and $\frac{3}{6}$ are known as equivalent fractions and that they are different names for the same number. For this reason, $\frac{3}{6}$ may be used in place of $\frac{1}{2}$ in the addition. Similarly, $\frac{6}{8}$ is used in place of $\frac{3}{4}$ to complete the addition $\frac{1}{8} + \frac{3}{4}$.

Using the Pages

- The worked example demonstrates that it is necessary to find equivalent fractions with like denominators when adding fractions with unlike denominators. Diagrams are provided to help show the equivalent fractions and their sum. Ask students to explain how the equivalent fractions can be derived by multiplication. Point out that 12 is the product of 3 and 4 and also the least common multiple of 3 and 4. You may wish to refer to the examples in the preliminary activity for which the least common multiple is not the product of the denominators.

Working Together: Note that students may express the answers for these exercises in different ways as indicated below for Ex. 4. You may wish to write sets of equivalent fractions on the board and have students match fractions with like denominators. Then discuss the advantages of using the first pair, that is, the pair showing the least common multiple of the denominators, to find the sum.

Exercises

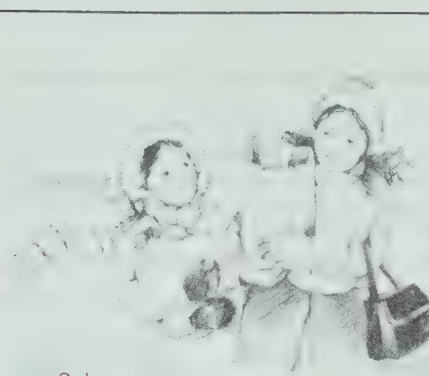
Add.

1. $\frac{1}{3}$ $\frac{1}{2}$ $\frac{5}{6}$
2. $\frac{1}{2}$ $\frac{3}{10}$ $\frac{8}{10}$ $\left(\frac{4}{5}\right)$
3. $\frac{1}{5}$ $\frac{1}{2}$ $\frac{7}{10}$
4. $\frac{3}{4}$ $\frac{1}{20}$ $\frac{16}{20}$ $\left(\frac{4}{5}\right)$
5. $\frac{3}{8}$ $\frac{1}{6}$ $\frac{13}{24}$
6. $\frac{5}{12}$ $\frac{1}{4}$ $\frac{8}{12}$ $\left(\frac{2}{3}\right)$
7. $\frac{2}{5}$ $\frac{1}{10}$ $\frac{5}{10}$ $\left(\frac{1}{2}\right)$
8. $\frac{3}{20}$ $\frac{3}{4}$ $\frac{18}{20}$ $\left(\frac{9}{10}\right)$
9. $\frac{1}{8}$ $\frac{3}{4}$ $\frac{7}{8}$

10. $\frac{7}{20}+\frac{2}{5}$ $\frac{15}{20}$ $\left(\frac{3}{4}\right)$
11. $\frac{2}{3}+\frac{1}{12}$ $\frac{9}{12}$ $\left(\frac{3}{4}\right)$
12. $\frac{1}{7}+\frac{2}{3}$ $\frac{17}{21}$
13. $\frac{1}{3}+\frac{1}{6}$ $\frac{3}{6}$ $\left(\frac{1}{2}\right)$
14. $\frac{1}{6}+\frac{3}{4}$ $\frac{11}{12}$
15. $\frac{3}{10}+\frac{1}{5}$ $\frac{5}{10}$ $\left(\frac{1}{2}\right)$
16. $\frac{1}{3}+\frac{5}{12}$ $\frac{9}{12}$ $\left(\frac{3}{4}\right)$
17. $\frac{3}{5}+\frac{3}{20}$ $\frac{15}{20}$ $\left(\frac{3}{4}\right)$
18. $\frac{1}{2}+\frac{1}{6}$ $\frac{4}{6}$ $\left(\frac{2}{3}\right)$
19. $\frac{1}{4}+\frac{3}{8}$ $\frac{5}{8}$
20. $\frac{7}{20}+\frac{1}{4}$ $\frac{12}{20}$ $\left(\frac{3}{5}\right)$
21. $\frac{1}{6}+\frac{7}{12}$ $\frac{9}{12}$ $\left(\frac{3}{4}\right)$

A fraction is in lowest terms if both the numerator and the denominator have been divided by all their common factors.

22. List the sums for Exercises 1 to 21 in lowest terms.



Solve.

23. Kelly's mother spent $\frac{1}{4}$ of her time in the grocery store and $\frac{1}{8}$ of her time in the drugstore. How much of her time did she spend in both stores? $\frac{3}{8}$
24. Kelly's father spent $\frac{1}{3}$ of his time in the grocery store and $\frac{1}{8}$ of his time in the drugstore. How much of his time did he spend in both stores? $\frac{11}{24}$
25. Kelly spent $\frac{1}{5}$ of her time in the grocery store and $\frac{2}{3}$ of her time in the department store. How much of her time did she spend in both stores? $\frac{13}{15}$

Add the three fractions.

Example: $\frac{3}{8}+\frac{1}{4}+\frac{5}{16}$
$=\frac{6}{16}+\frac{4}{16}+\frac{5}{16}$, or $\frac{15}{16}$

26. $\frac{1}{3}+\frac{1}{6}+\frac{4}{9}$ $\frac{17}{18}$
27. $\frac{1}{3}+\frac{1}{4}+\frac{1}{12}$ $\frac{8}{12}$ $\left(\frac{2}{3}\right)$
28. $\frac{1}{6}+\frac{1}{4}+\frac{3}{8}$ $\frac{19}{24}$
29. $\frac{1}{3}+\frac{2}{9}+\frac{5}{18}$ $\frac{15}{18}$ $\left(\frac{5}{6}\right)$
30. $\frac{1}{2}+\frac{1}{3}+\frac{1}{12}$ $\frac{11}{12}$
31. $\frac{1}{4}+\frac{3}{8}+\frac{1}{3}$ $\frac{23}{24}$

Other answers are possible for Ex 1-8

PROBLEM SOLVING

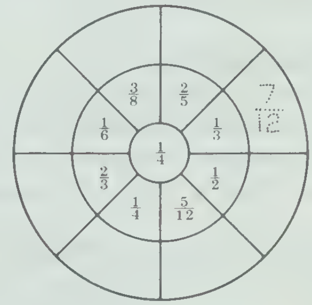
Can you find a number between

1. $\frac{2}{5}$ and $\frac{4}{5}$? $\frac{3}{5}$
2. $\frac{1}{4}$ and $\frac{3}{4}$? $\frac{2}{4}$
3. $\frac{1}{3}$ and $\frac{2}{3}$? $\frac{3}{6}$
4. $\frac{5}{7}$ and $\frac{6}{7}$? $\frac{11}{14}$
5. $\frac{1}{4}$ and $\frac{1}{3}$? $\frac{7}{24}$
6. $\frac{1}{2}$ and $\frac{2}{3}$? $\frac{7}{12}$
7. $\frac{5}{12}$ and $\frac{2}{5}$? $\frac{49}{120}$
8. $\frac{9}{10}$ and 1? $\frac{19}{20}$

219

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 13-36 on page 335.
- For reinforcement, have students draw diagrams to illustrate one or more of Ex. 1-21 on page 219.
- Have students complete number wheels for addition on copies of page T 391.



- To help students find the least common multiple for like denominators, have them refer to multiplication tables prepared from copies of page T 382. In the table illustrated below, 24 is seen as the least common multiple of 6 and 8.

×	1	2	3	4	5	6
1						
6	6	12	18	24	30	36
7						
8	8	16	24	32	40	48
9						

$\frac{3}{4}, \frac{6}{8}, \frac{9}{12}, \frac{12}{16}, \frac{15}{20}, \frac{18}{24}, \frac{21}{28}, \frac{24}{32}, \frac{27}{36}, \frac{30}{40}, \frac{33}{44}, \frac{36}{48}, \ldots$

$\frac{1}{12}, \frac{2}{24}, \frac{3}{36}, \frac{4}{48}, \ldots$

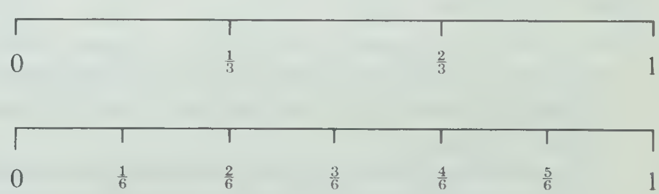
$$\frac{3}{4}+\frac{1}{12}=\frac{9}{12}+\frac{1}{12} = \frac{10}{12} \text{ (or } \tfrac{5}{6})$$

$$\frac{3}{4}+\frac{1}{12}=\frac{36}{48}+\frac{4}{48} = \frac{40}{48} \text{ (or } \tfrac{5}{6})$$

Exercises: For Ex. 22, establish that more, but not all, of the sums for Ex. 1-21 will be in lowest terms if the least common multiple of the unlike denominators is used as the like denominator.

Ex. 26-31 involve the sum of three fractions with unlike denominators and an example is provided. Develop that the procedure of finding equivalent fractions with like denominators and then adding is used. Discuss that it is preferable to find the least common multiple of the denominators, because the product of three unlike denominators can give a large number.

Problem Solving: For Ex. 3-8, the students will need to express the two fractions as equivalent fractions with like denominators. Number lines can help students realize, for example, that although there is no fraction with a denominator of 3 between $\frac{1}{3}$ and $\frac{2}{3}$, $\frac{3}{6}$ is between $\frac{2}{6}$ $(\frac{1}{3})$ and $\frac{4}{6}$ $(\frac{2}{3})$.



Assessment

Add. Show the sums in lowest terms.

1. $\frac{1}{12}$ $\frac{3}{4}$ $\frac{5}{6}$
2. $\frac{2}{3}$ $\frac{1}{5}$ $\frac{13}{15}$
3. $\frac{3}{10}+\frac{1}{2}$ $\frac{4}{5}$
4. $\frac{1}{6}+\frac{1}{3}+\frac{1}{4}$ $\frac{3}{4}$

Solve.

5. Cam spent $\frac{1}{2}$ of his time at the park and $\frac{1}{4}$ of his time in the schoolyard. How much of his time did he spend in both places? $\frac{3}{4}$

Pages 220-221

LESSON OUTCOME

Add two fractions with like or unlike denominators, regrouping; add two numbers in mixed form with like or unlike denominators, regrouping

Materials

models for $3\frac{2}{3}$ prepared from copies of the squares on page T 392

Prerequisite Skills

Add two fractions, no regrouping, sums expressed in lowest terms; express an improper fraction as a number in mixed form or as a whole number

Checking Prerequisite Skills

Add. Show the sums in lowest terms.

1. $\frac{3}{8} + \frac{1}{8}$ $\frac{1}{2}$
2. $\frac{1}{4} + \frac{1}{2}$ $\frac{3}{4}$
3. $\frac{3}{5} + \frac{3}{20}$ $\frac{3}{4}$
4. $\frac{1}{6} + \frac{1}{4}$ $\frac{5}{12}$

Write each as a number in mixed form or as a whole number.

5. $\frac{8}{5}$ $1\frac{3}{5}$ 6. $\frac{5}{4}$ $1\frac{1}{4}$ 7. $\frac{3}{3}$ 1   8. $\frac{27}{20}$ $1\frac{7}{20}$

Adding Fractions, Regrouping

Stuart found two boxes filled with paint bottles. One box had $1\frac{1}{4}$ bottles of green paint and $3\frac{1}{2}$ bottles of blue paint. The other box had $2\frac{1}{2}$ bottles of green paint and $1\frac{2}{3}$ bottles of blue paint.
How many bottles of green paint were in both boxes?
How many bottles of blue paint were in both boxes?

For the total amount of green paint, add $1\frac{1}{4}$ and $2\frac{1}{2}$.



Write $\begin{array}{r} 1\frac{1}{4} \\ 2\frac{1}{2} \\ \hline \end{array}$ with like denominators. $\begin{array}{r} 1\frac{1}{4} \\ 2\frac{2}{4} \\ \hline \end{array}$ Add fourths. Then add ones. $\begin{array}{r} 1\frac{1}{4} \\ 2\frac{2}{4} \\ \hline 3\frac{3}{4} \end{array}$

There were $3\frac{3}{4}$ bottles of green paint.

For the total amount of blue paint, add $3\frac{1}{2}$ and $1\frac{2}{3}$.



Write $\begin{array}{r} 3\frac{1}{2} \\ 1\frac{2}{3} \\ \hline \end{array}$ with like denominators. $\begin{array}{r} 3\frac{3}{6} \\ 1\frac{4}{6} \\ \hline \end{array}$ Add sixths. Then add ones. $\begin{array}{r} 3\frac{3}{6} \\ 1\frac{4}{6} \\ \hline 4\frac{7}{6} \end{array}$ or $5\frac{1}{6}$

Remember that $\frac{7}{6} = 1\frac{1}{6}$, so $4\frac{7}{6} = 5\frac{1}{6}$.

There were $5\frac{1}{6}$ bottles of blue paint.

220

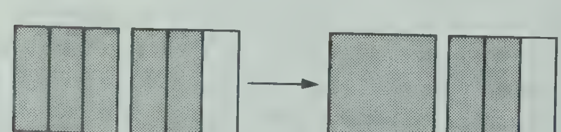
LESSON ACTIVITY

Before Using the Pages

- Use copies of the squares on page T 392 to prepare models to represent $3\frac{2}{3}$. Mark the opposite side of one of the whole models into thirds so that it may be used to show $\frac{3}{3}$ or, when flipped, 1 whole.

  Display models for $\frac{5}{3}$ and write the fraction on the board. Ask a student to express the improper fraction as a number in mixed form. Illustrate this by reversing the model for $\frac{3}{3}$ to show 1 whole, explaining that 3 thirds are regrouped as 1 whole.

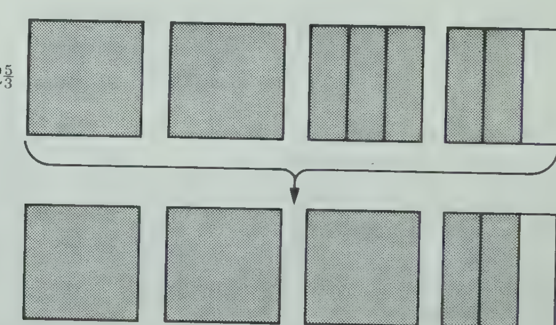
$\frac{5}{3}$



$1\frac{2}{3}$

Display models for $1\frac{5}{3}$ and repeat the activity. Then repeat the activity for $2\frac{5}{3}$.

$2\frac{5}{3}$



$3\frac{2}{3}$

Ask how $5\frac{4}{3}$ and $7\frac{3}{3}$ may be written without showing an improper fraction. Explain that such mixed numbers often occur as sums when adding fractions.

Using the Pages

- Ask a student to read the word problem at the top of page 220. Elicit from the students that addition must be used to find the total amount of each color of paint. Review that for numbers in mixed form, the fractions are added first and then the whole numbers. For the first example, ask what

Working Together

Regroup the sum so that it has no improper fraction.

1. $2\frac{3}{4}$ + $1\frac{2}{4}$ = $3\frac{5}{4}$ $4\frac{1}{4}$
2. $1\frac{1}{2} + 3\frac{1}{2} = 4\frac{2}{2}$ 5
3. $3\frac{6}{7} + 2\frac{5}{7} = 5\frac{11}{7}$ $6\frac{4}{7}$

Add. Regroup the sum if it shows an improper fraction.

4. $2\frac{2}{3}$ + $4\frac{2}{3}$ $7\frac{1}{3}$
5. $2\frac{3}{5} + 4\frac{2}{5}$ 7
6. $1\frac{5}{6} + \frac{4}{6}$ $2\frac{3}{6}$ $\left(2\frac{1}{2}\right)$

Find equivalent fractions with like denominators. Add. Regroup the sum if it shows an improper fraction.

7. $4\frac{1}{2}$ + $3\frac{2}{3}$ $8\frac{1}{6}$
8. $2\frac{4}{5}$ + $\frac{3}{4}$ $3\frac{11}{20}$
9. $2\frac{1}{3}$ + $4\frac{1}{6}$ $6\frac{3}{6}$ $\left(6\frac{1}{2}\right)$
10. $1\frac{3}{4} + \frac{3}{12}$ 2
11. $3\frac{4}{9} + 5\frac{2}{3}$ $9\frac{1}{9}$

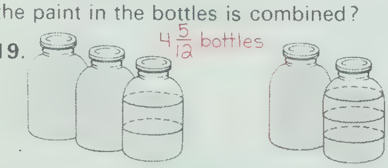
Exercises

Add. Regroup the sum if it shows an improper fraction.

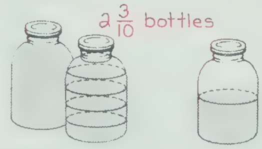
1. $1\frac{2}{3}$ + $2\frac{7}{9}$ $4\frac{4}{9}$
2. $5\frac{3}{4}$ + $6\frac{3}{5}$ $12\frac{7}{20}$
3. $5\frac{1}{6}$ + $3\frac{11}{12}$ $9\frac{1}{12}$
4. $1\frac{5}{6}$ + $\frac{2}{3}$ $2\frac{3}{6}$ $\left(2\frac{1}{2}\right)$
5. $1\frac{4}{5}$ + $3\frac{7}{10}$ $5\frac{5}{10}$ $\left(5\frac{1}{2}\right)$
6. $4\frac{1}{4}$ + $3\frac{2}{3}$ $7\frac{11}{12}$
7. $2\frac{7}{8} + \frac{3}{4}$ $3\frac{5}{8}$
8. $\frac{1}{2} + \frac{3}{5}$ $1\frac{1}{10}$
9. $4\frac{5}{8} + 5\frac{5}{6}$ $10\frac{11}{24}$
10. $3\frac{3}{4} + 3\frac{1}{2}$ $7\frac{1}{4}$
11. $1\frac{1}{2} + 4\frac{1}{2}$ 6
12. $1\frac{1}{6} + 2\frac{8}{9}$ $4\frac{1}{18}$
13. $8\frac{2}{3} + \frac{1}{3}$ 9
14. $\frac{3}{8} + 2\frac{7}{12}$ $2\frac{23}{24}$
15. $\frac{5}{6} + \frac{7}{12}$ $1\frac{5}{12}$
16. $3\frac{1}{6} + 2\frac{3}{4}$ $5\frac{11}{12}$
17. $7\frac{7}{10} + 5\frac{3}{10}$ 13
18. $4\frac{5}{12} + \frac{3}{4}$ $5\frac{2}{12}$ $\left(5\frac{1}{6}\right)$

How much paint will there be when the paint in the bottles is combined?

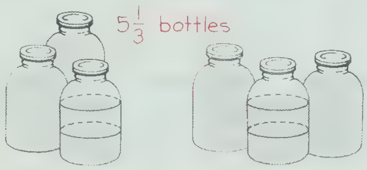
19. $4\frac{5}{12}$ bottles



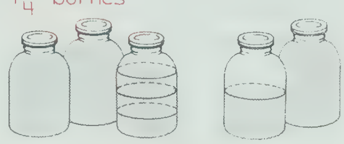
20. $2\frac{3}{10}$ bottles



21. $5\frac{1}{3}$ bottles



22. $4\frac{1}{4}$ bottles



221

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 37-58 on page 335.
- Students can draw models to illustrate some of the exercises on page 221.
- Prepare addition squares similar to the following on copies of page T 382. The sum in the lower right square provides a check.

+ →

$1\frac{5}{6}$	$3\frac{3}{4}$	
$2\frac{1}{2}$	$1\frac{2}{3}$	

- Have students prepare and play the game "King of Fractions" for addition. The game is described on page T 381.
- For practice in regrouping sums, adapt the second and third of the *Related Activities* on page T 227.

steps are necessary for adding $\frac{1}{4}$ and $\frac{1}{2}$. Point out that $2\frac{1}{2}$ is rewritten as $2\frac{2}{4}$, noting that the whole number is included with the equivalent fraction $\frac{2}{4}$. Ask students to explain the addition process and relate it to the illustration.

Discuss the second example in a similar way. Note that the sum shows an improper fraction. Ask students to explain how to regroup the sum.

Working Together: Ex. 1-3 provide practice in regrouping sums. For each of these exercises, ask students to explain the addition that is shown and then have the students regroup the sum. The fractions in Ex. 4-6 have like denominators. The students are required to add and then regroup the sums. For Ex. 7-11, the students must find equivalent fractions with like denominators, add, and regroup the sums. Depending on the ability of the students in your class, you may wish to have them express the results in lowest terms.

Exercises: For each of Ex. 19-22, have the students write the numbers represented by the models, show the addition, and write a concluding statement.

Assessment

Add. Regroup the sum if it shows an improper fraction.

1. $1\frac{2}{5}$ + $1\frac{3}{4}$ $3\frac{3}{20}$
2. $\frac{7}{8}$ + $\frac{1}{2}$ $1\frac{3}{8}$
3. $2\frac{8}{9} + 3\frac{1}{3}$ $6\frac{2}{9}$
4. $4\frac{3}{4} + 1\frac{5}{6}$ $6\frac{7}{12}$

LESSON OUTCOME

Subtract fractions with unlike denominators, no regrouping; express differences of fractions in lowest terms; add to check subtraction of fractions; solve related word problems

Materials

a model for $\frac{7}{10}$ prepared from a copy of page T 393

Prerequisite Skills

Subtract fractions with like denominators, no regrouping; add fractions with unlike denominators, no regrouping; write fractions in lowest terms

Checking Prerequisite Skills

Subtract.

1. $\frac{4}{5} - \frac{2}{5} = \frac{2}{5}$
2. $\frac{2}{3} - \frac{1}{3} = \frac{1}{3}$

Add.

3. $\frac{1}{4} + \frac{3}{20} = \frac{8}{20}$ $\left(\frac{2}{5}\right)$
4. $\frac{1}{5} + \frac{1}{2} = \frac{7}{10}$

Write each fraction in lowest terms.

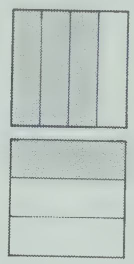
5. $\frac{8}{12}$ $\frac{2}{3}$
6. $\frac{5}{10}$ $\frac{1}{2}$

Subtracting Fractions with Unlike Denominators

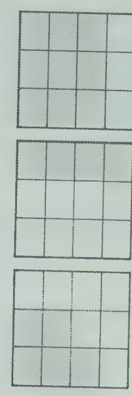
Find the difference of $\frac{3}{4}$ and $\frac{1}{3}$.

To subtract fractions with unlike denominators, find equivalent fractions with like denominators. Then subtract the numerators.

Subtract.



$$\frac{3}{4} = \frac{9}{12}$$
$$-\frac{1}{3} = \frac{4}{12}$$
$$\frac{5}{12}$$



The difference of $\frac{3}{4}$ and $\frac{1}{3}$ is $\frac{5}{12}$.

To check that $\frac{3}{4} - \frac{1}{3} = \frac{5}{12}$, add $\frac{5}{12}$ and $\frac{1}{3}$.

$$\frac{5}{12} = \frac{5}{12}$$
$$+\frac{1}{3} = \frac{4}{12}$$
$$\frac{9}{12}, \text{ or } \frac{3}{4}$$

If this number does not match the first number in the subtraction, there is a mistake in the work.

Working Together

For the pair of fractions in each subtraction, give equivalent fractions that have like denominators.

1. $\frac{5}{6} - \frac{1}{2} = \blacksquare - \blacksquare$ $\frac{5}{6}$ $\frac{3}{6}$
2. $\frac{7}{8} - \frac{1}{3} = \blacksquare - \blacksquare$ $\frac{21}{24}$ $\frac{8}{24}$

Find equivalent fractions with like denominators. Then subtract. Add to check your work.

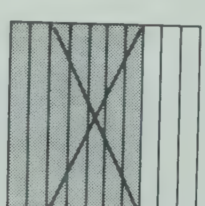
3. $\frac{3}{4} - \frac{2}{3}$ $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$
4. $\frac{7}{12} - \frac{1}{4}$ $\frac{7}{12} - \frac{3}{12} = \frac{4}{12}$
5. $\frac{1}{2} - \frac{2}{5}$ $\frac{5}{10} - \frac{4}{10} = \frac{1}{10}$
6. $\frac{3}{8} - \frac{1}{6}$ $\frac{9}{24} - \frac{4}{24} = \frac{5}{24}$

222

LESSON ACTIVITY

Before Using the Pages

- Write the subtraction $\frac{7}{10} - \frac{1}{2}$ on the board and display a model for $\frac{7}{10}$. Ask how to subtract $\frac{1}{2}$ from $\frac{7}{10}$. Lead the students to suggest that because $\frac{1}{2}$ is equivalent to $\frac{5}{10}$, the subtraction can be demonstrated by crossing out five of the 7 blue tenths, leaving 2 tenths. Have students help to show the corresponding subtraction on the board. Ask whether the difference is in lowest terms. Ask a student to explain how to write the difference in lowest terms.



$$\frac{7}{10} - \frac{1}{2}$$
$$= \frac{7}{10} - \frac{5}{10}$$
$$= \frac{2}{10}, \text{ or } \frac{1}{5}$$

Using the Pages

- Ask students to explain why equivalent fractions are necessary to perform the subtraction. Direct the students' attention to the models and discuss how they show the difference. Note that 12 is the least common multiple of the denominators 4 and 3. Ask how multiplication can be used to derive the equivalent fractions $\frac{9}{12}$ and $\frac{4}{12}$. Relate the difference, $\frac{5}{12}$, to the corresponding model.

  Review that addition can be used to check subtraction because of the inverse relationship between addition and subtraction. Ask students to explain each step of the addition.

Working Together: For Ex. 1 and 2, the students must give equivalent fractions that have like denominators. For Ex. 3-6, they are required to find the equivalent fractions, subtract, and then add to check. Remind the students that it would be preferable to use the least common denominator for Ex. 4 and 6, rather than the product of the two denominators.

Exercises

The students colored the designs they made from a hexagon. Subtract fractions to answer these questions.

1. John has colored $\frac{1}{2}$ of his hexagon. Mani has colored $\frac{1}{3}$ of hers. How much more of the hexagon has John colored? $\frac{1}{6}$

2. Brian has colored $\frac{5}{6}$ of his hexagon. Sandi has colored $\frac{5}{9}$ of hers. How much more of the hexagon has Brian colored? $\frac{5}{18}$

3. How much more of the hexagon has Sandi colored than John? $\frac{1}{18}$

4. How much more of the hexagon has Brian colored than Mani? $\frac{3}{6}$ $(\frac{1}{2})$

Subtract. Add to check.

5. $\frac{1}{2} - \frac{1}{6}$ $\frac{2}{6}$ $(\frac{1}{3})$
6. $\frac{4}{5} - \frac{3}{10}$ $\frac{5}{10}$ $(\frac{1}{2})$
7. $\frac{1}{3} - \frac{1}{4}$ $\frac{1}{12}$
8. $\frac{9}{10} - \frac{2}{5}$ $\frac{5}{10}$ $(\frac{1}{2})$
9. $\frac{3}{4} - \frac{1}{6}$ $\frac{7}{12}$
10. $\frac{7}{12} - \frac{1}{3}$ $\frac{3}{12}$ $(\frac{1}{4})$
11. $\frac{3}{8} - \frac{1}{4}$ $\frac{1}{8}$
12. $\frac{5}{6} - \frac{1}{3}$ $\frac{3}{6}$ $(\frac{1}{2})$
13. $\frac{7}{8} - \frac{2}{3}$ $\frac{5}{24}$
14. $\frac{2}{3} - \frac{5}{12}$ $\frac{3}{12}$ $(\frac{1}{4})$
15. $\frac{5}{6} - \frac{1}{4}$ $\frac{7}{12}$
16. $\frac{2}{3} - \frac{1}{2}$ $\frac{1}{6}$
17. $\frac{7}{9} - \frac{2}{3}$ $\frac{1}{9}$
18. $\frac{9}{10} - \frac{1}{2}$ $\frac{4}{10}$ $(\frac{2}{5})$
19. $\frac{3}{4} - \frac{1}{12}$ $\frac{8}{12}$ $(\frac{2}{3})$
20. $\frac{2}{3} - \frac{1}{6}$ $\frac{3}{6}$ $(\frac{1}{2})$
21. $\frac{1}{2} - \frac{1}{10}$ $\frac{4}{10}$ $(\frac{2}{5})$
22. $\frac{9}{20} - \frac{1}{4}$ $\frac{4}{20}$ $(\frac{1}{5})$
23. $\frac{2}{3} - \frac{5}{9}$ $\frac{1}{9}$
24. $\frac{4}{5} - \frac{3}{10}$ $\frac{5}{10}$ $(\frac{1}{2})$

A fraction is in lowest terms if both the numerator and the denominator have been divided by all their common factors.

25. List the differences for Exercises 5 to 24 in lowest terms.

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 1-12 on page 336.
- Have students color hexagons in ways similar to those in the photograph on page 223. Then have them refer to their diagrams to write word problems similar to Ex. 1-4. For the diagrams, copies of the hexagon on page T 385 or the triangular grid on page T 400 may be used.
- You may wish to use copies of page T 391 to provide subtractions similar to the following. For the exercises suggested below, have the students begin by writing fractions equivalent to $\frac{1}{3}$. This can motivate some students to complete the subtractions mentally and write only the differences.

—	$\frac{1}{3}$
$\frac{4}{9}$	$\frac{1}{9}$
$\frac{7}{12}$	
$\frac{5}{6}$	
$\frac{2}{3}$	

Exercises: Before the students begin Ex. 1-4, you may wish to discuss the designs in the photograph. Relate each design to the pair of equivalent fractions written below it.

Assessment

Subtract. Show the differences in lowest terms. Add to check.

1. $\frac{9}{10} - \frac{2}{5}$ $\frac{1}{2}$
2. $\frac{1}{3} - \frac{1}{6}$ $\frac{1}{6}$
3. $\frac{3}{4} - \frac{5}{12}$ $\frac{1}{3}$
4. $\frac{11}{12} - \frac{2}{3}$ $\frac{1}{4}$

Solve.

5. Sandy has $\frac{1}{4}$ of a jar of red paint and $\frac{7}{8}$ of a jar of blue paint. How much more blue paint has she than red paint?

$\frac{5}{8}$ of a jar

Pages 224-225

LESSON OUTCOME

Subtract fractions or numbers in mixed form from numbers in mixed form with unlike denominators, regrouping; subtract fractions or numbers in mixed form from whole numbers

Materials

models prepared for the lesson on pages T 240 and T 241 (optional)

Prerequisite Skills

Subtract fractions with unlike denominators, no regrouping; add to check subtraction of fractions; add numbers in mixed form with unlike denominators, regrouping

Checking Prerequisite Skills

Subtract. Add to check.

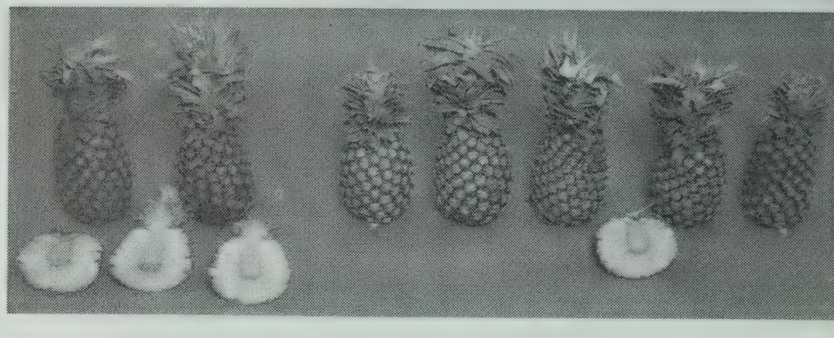
1. $\frac{3}{5} - \frac{1}{10} = \frac{5}{10}\ (\frac{1}{2})$ **2.** $\frac{8}{9} - \frac{2}{3} = \frac{2}{9}$

Add.

3. $2\frac{3}{4} + 1\frac{1}{2} = 4\frac{1}{4}$ **4.** $3\frac{2}{3} + 2\frac{7}{12} = 6\frac{3}{12}\ (6\frac{1}{4})$

Subtracting Fractions, Regrouping

The chef in a restaurant had 8 pineapples. After using $2\frac{3}{4}$ pineapples to make pineapple-upside-down cakes, how many pineapples were left?



Subtract $2\frac{3}{4}$ from 8.

For the subtraction $\begin{array}{r} 8 \\ \underline{2\frac{3}{4}} \end{array}$ regroup 8 as $7\frac{4}{4}$. $\begin{array}{r} 7\frac{4}{4} \\ \underline{2\frac{3}{4}} \end{array}$ Subtract fourths. Then subtract ones. $\begin{array}{r} 7\frac{4}{4} \\ \underline{2\frac{3}{4}} \\ 5\frac{1}{4} \end{array}$

There were $5\frac{1}{4}$ pineapples left.

Then, after using $3\frac{1}{2}$ pineapples for a fruit salad, how many pineapples were left?

Subtract $3\frac{1}{2}$ from $5\frac{1}{4}$.

Write $\begin{array}{r} 5\frac{1}{4} \\ \underline{3\frac{1}{2}} \end{array}$ with like denominators. $\begin{array}{r} 5\frac{1}{4} \\ \underline{3\frac{2}{4}} \end{array}$ Then regroup $5\frac{1}{4}$ as $4\frac{5}{4}$. Subtract fourths. Then subtract ones. $\begin{array}{r} 4\frac{5}{4} \\ \underline{3\frac{2}{4}} \\ 1\frac{3}{4} \end{array}$

After making a fruit salad, there were $1\frac{3}{4}$ pineapples left.

224

LESSON ACTIVITY

Before Using the Pages

- Review that $\frac{2}{2}$, $\frac{3}{3}$, $\frac{4}{4}$, $\frac{5}{5}$, and so on, are names for the number 1. Use models or draw diagrams to show that $1\frac{2}{2}$, $1\frac{3}{3}$, and so on, are different names for the number 2. Then have students determine without models or diagrams how to express 2 as a number in mixed form showing fifths, 7 as a number in mixed form showing fourths, and so on.
- Use models prepared for the lesson on pages T 240 and T 241 or draw diagrams. Develop that $1\frac{2}{3}$ is equivalent to $\frac{5}{3}$, $2\frac{2}{3}$ is equivalent to $1\frac{5}{3}$, and $3\frac{2}{3}$ is equivalent to $2\frac{5}{3}$. For each example, emphasize that 1 whole is regrouped as 3 more thirds.

$2\frac{2}{3}$ $1\frac{5}{3}$

Write $5\frac{1}{3}$ on the board and ask the students to regroup 1 whole as 3 more thirds. Write $3\frac{5}{8}$ on the board and ask the students to regroup 1 whole as 8 more eighths. Explain that renaming numbers in this way is often required in subtracting fractions.

Using the Pages

- Draw attention to the photograph. Note that there are seven whole pineapples and one pineapple that has been cut into fourths. Ask a student to read the word problem above the photograph. Ask what operation must be used to solve the problem. Recall that the order of subtraction is to subtract the fractions before the whole numbers. Note that there are no fourths from which to subtract $\frac{3}{4}$. Ask how 8 can be renamed as a number in mixed form showing fourths. Relate $7\frac{4}{4}$ to the pineapples in the photograph. Explain that the denominator 4 is used because 4 is the denominator of the fraction in $2\frac{3}{4}$. Emphasize that $7\frac{4}{4}$ names the same number of pineapples as 8. Ask students to explain the subtraction process.

Working Together

Regroup $5\frac{1}{5}$ as $4\frac{6}{5}$.
Then subtract.

1. $5\frac{1}{5} - 3\frac{4}{5}$ $\quad 4\frac{6}{5} - 3\frac{4}{5} = 1\frac{2}{5}$

2. $5\frac{1}{5} - 2\frac{2}{5}$ $\quad 4\frac{6}{5} - 2\frac{2}{5} = 2\frac{4}{5}$

3. $5\frac{1}{5} - \frac{3}{5}$ $\quad 4\frac{6}{5} - \frac{3}{5} = 4\frac{3}{5}$

Regroup 7 as 6 and a fraction.
Then subtract.

4. $7 - 5\frac{3}{8}$ $\quad 6\frac{8}{8} - 5\frac{3}{8} = 1\frac{5}{8}$

5. $7 - 2\frac{5}{9}$ $\quad 6\frac{9}{9} - 2\frac{5}{9} = 4\frac{4}{9}$

6. $7 - \frac{7}{12}$ $\quad 6\frac{12}{12} - \frac{7}{12} = 6\frac{5}{12}$

Find equivalent fractions with like denominators.
Regroup the first number if needed. Then subtract.

7. $4\frac{1}{2} - 2\frac{2}{3}$ $\quad 1\frac{5}{6}$
8. $1\frac{3}{4} - \frac{7}{8}$ $\quad \frac{7}{8}$
9. $6 - 1\frac{4}{5}$ $\quad 4\frac{1}{5}$
10. $7\frac{1}{6} - 2\frac{11}{12}$ $\quad 4\frac{3}{12}\left(4\frac{1}{4}\right)$

Exercises

Subtract. Add to check.

1. $7\frac{1}{3} - 5\frac{7}{9}$ $\quad 1\frac{5}{9}$
2. $2\frac{1}{2} - 1\frac{5}{8}$ $\quad \frac{7}{8}$
3. $8 - 5\frac{1}{4}$ $\quad 2\frac{3}{4}$
4. $1\frac{1}{3} - \frac{5}{6}$ $\quad \frac{3}{6}\left(\frac{1}{2}\right)$
5. $10\frac{4}{7} - 3\frac{3}{4}$ $\quad 6\frac{23}{28}$
6. $3\frac{4}{9} - 1\frac{7}{9}$ $\quad 1\frac{6}{9}\left(1\frac{2}{3}\right)$

7. $7\frac{2}{5} - 3\frac{7}{10}$ $\quad 3\frac{7}{10}$
8. $9\frac{1}{4} - 1\frac{1}{2}$ $\quad 7\frac{3}{4}$
9. $3\frac{2}{9} - \frac{2}{3}$ $\quad 2\frac{5}{9}$
10. $3 - 1\frac{7}{10}$ $\quad 1\frac{3}{10}$
11. $5\frac{3}{10} - 1\frac{4}{5}$ $\quad 3\frac{5}{10}\left(3\frac{1}{2}\right)$
12. $8\frac{2}{3} - 4\frac{11}{12}$ $\quad 3\frac{9}{12}\left(3\frac{3}{4}\right)$
13. $6\frac{3}{8} - 3\frac{2}{3}$ $\quad 2\frac{17}{24}$
14. $8\frac{1}{2} - 6\frac{3}{4}$ $\quad 1\frac{3}{4}$
15. $5 - \frac{5}{12}$ $\quad 4\frac{7}{12}$
16. $6\frac{4}{12} - 5\frac{3}{4}$ $\quad \frac{7}{12}$
17. $2\frac{5}{8} - \frac{11}{12}$ $\quad 1\frac{17}{24}$
18. $4\frac{1}{6} - 2\frac{2}{3}$ $\quad 1\frac{3}{6}\left(1\frac{1}{2}\right)$

The chef had 23 apples. He wanted to use $\frac{1}{2}$ of the apples for apple sauce, $\frac{1}{3}$ of the apples for baked apples, and $\frac{1}{8}$ of the apples for tarts.

Instead of cutting the apples, he placed an apple from his own lunch with the 23 apples. Then he made apple sauce with $\frac{1}{2}$ of the 24 apples, baked $\frac{1}{3}$ of the 24 apples, and made tarts with $\frac{1}{8}$ of the 24 apples.

When he had finished cooking, he ate his own lunch, having his apple for dessert.

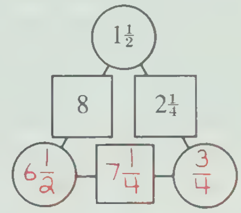
1. What do you think of that?
Answers will vary

PROBLEM SOLVING

225

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 13-32 on page 336.
- As enrichment, students may be able to change the fractions in the *Problem Solving* feature so that a similar situation results. For example, the problem may be altered by replacing 23 by 17 and $\frac{1}{8}$ by $\frac{1}{9}$.
- Use copies of page T 391 to prepare diagrams similar to the following for students to complete. Addends are shown in the circles and sums are shown in the squares.



- To provide practice in regrouping fractions, prepare cards similar to the following for the game "Dominoes" described on page T 379.

3	$1\frac{4}{3}$
$2\frac{1}{3}$	$6\frac{5}{5}$
7	$4\frac{3}{8}$

For the second example, discuss the need for equivalent fractions with like denominators before subtracting. Note that $\frac{2}{4}$ cannot be subtracted from $\frac{1}{4}$ and that it is necessary to regroup $5\frac{1}{4}$ to show 4 more fourths, in other words, $5\frac{1}{4}$ is expressed as $4\frac{5}{4}$. Relate this to cutting one of the five pineapples into fourths so that there would be 4 whole pineapples and 5 fourths. Ask students to explain the subtraction process. For each example on this page you may wish to have the students use addition to check the answer.

Working Together: Discuss the need for regrouping $5\frac{1}{5}$ as $4\frac{6}{5}$ in Ex. 1-3. Similarly, lead the students to realize the need for regrouping 7 as 6 and a fraction for Ex. 4-6. Ask the students to explain how they can determine the denominators for the fractions. For each of Ex. 7-10, have the students explain the need for regrouping the first number and then tell how the number is regrouped.

Exercises: Remind the students to use addition to check the subtractions. Explain that it may be necessary to express a sum in lowest terms to determine whether it is the same as the first number in the subtraction.

Problem Solving: Have the students calculate how many apples were used for the apple sauce, how many apples were baked, and how many apples were used for tarts. Some students may find it helpful to draw a diagram. Provide time, perhaps a few days, for the students to think about the problem. Then develop that $\frac{1}{2} + \frac{1}{3} + \frac{1}{8} = \frac{12}{24} + \frac{8}{24} + \frac{3}{24}$, or $\frac{23}{24}$. Therefore, $\frac{1}{2}$ of 24 apples, $\frac{1}{3}$ of 24 apples, and $\frac{1}{8}$ of 24 apples would be 23 apples.

Assessment

Subtract. Add to check.

1. $4\frac{1}{3} - 2\frac{3}{4}$ $\quad 1\frac{7}{12}$
2. $3\frac{1}{9} - \frac{2}{3}$ $\quad 2\frac{4}{9}$
3. $1 - \frac{3}{5}$ $\quad \frac{2}{5}$
4. $6\frac{3}{8} - 5\frac{1}{2}$ $\quad \frac{7}{8}$

Pages 226-227

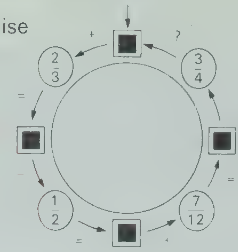
OBJECTIVE

Demonstrate competence in adding and subtracting fractions; solve related word problems

Practice

Copy this picture or use tracing paper.
Write a fraction of your choice here.

1. Go counterclockwise around the circle. Complete each ■. Answers will vary.



2. What should go in {?}, <, >, or = ? =
3. Choose another fraction and go around the circle again. Answers will vary.

Find the sums.

4. $1\frac{2}{7}$, $3\frac{4}{7}$ $4\frac{6}{7}$ **5.** $\frac{1}{4}$, $\frac{3}{4}$ 1 **6.** $3\frac{5}{8}$, $6\frac{7}{8}$ $10\frac{4}{8}$ $\left(10\frac{1}{2}\right)$ **7.** $7\frac{3}{5}$, $1\frac{4}{5}$ $9\frac{2}{5}$

8. $3\frac{2}{3}$, $\frac{5}{6}$ $4\frac{3}{6}$ $\left(4\frac{1}{2}\right)$ **9.** $\frac{3}{4}$, $2\frac{7}{12}$ $3\frac{4}{12}$ $\left(3\frac{1}{3}\right)$ **10.** $2\frac{5}{6}$, $1\frac{3}{4}$ $4\frac{7}{12}$ **11.** $5\frac{4}{5}$, $4\frac{7}{10}$ $10\frac{5}{10}$ $\left(10\frac{1}{2}\right)$

For Exercises 4 to 11,
12. list the sums in lowest terms.

Find the differences.

13. $2\frac{2}{9}$, $5\frac{5}{9}$ $3\frac{3}{9}$ $\left(3\frac{1}{3}\right)$ **14.** 1, $\frac{1}{2}$ $\frac{1}{2}$ **15.** $3\frac{2}{3}$, 5 $1\frac{1}{3}$ **16.** $7\frac{11}{12}$, $7\frac{5}{12}$ $\frac{6}{12}$ $\left(\frac{1}{2}\right)$

17. $3\frac{1}{6}$, $1\frac{1}{2}$ $1\frac{4}{6}$ $\left(1\frac{2}{3}\right)$ **18.** $6\frac{5}{12}$, $5\frac{2}{3}$ $\frac{9}{12}$ $\left(\frac{3}{4}\right)$ **19.** $1\frac{5}{6}$, $7\frac{3}{4}$ $5\frac{11}{12}$ **20.** $2\frac{7}{9}$, $3\frac{1}{3}$ $\frac{5}{9}$

For Exercises 13 to 20,
21. list the differences in lowest terms.

Add.

22. $4\frac{1}{10} + \frac{3}{10} + 1\frac{1}{10}$ $5\frac{5}{10}$ $\left(5\frac{1}{2}\right)$ **23.** $3\frac{7}{12} + 1\frac{1}{12} + \frac{7}{12}$ $5\frac{3}{12}$ $\left(5\frac{1}{4}\right)$ **24.** $1\frac{8}{9} + 1\frac{8}{9} + 1\frac{8}{9}$ $5\frac{6}{9}$ $\left(5\frac{2}{3}\right)$

25. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ $1\frac{1}{12}$ **26.** $3\frac{1}{2} + 1\frac{2}{5} + 4\frac{1}{10}$ 9 **27.** $2\frac{3}{4} + 3\frac{1}{2} + 1\frac{5}{8}$ $7\frac{7}{8}$

For Exercises 22 to 27,
28. list the sums in lowest terms.

Find the fraction that gives a sum of 1.

29. $\frac{1}{3} + \frac{\blacksquare}{\blacksquare} = 1$ $\frac{2}{3}$ **30.** $\frac{\blacksquare}{\blacksquare} + \frac{2}{5} = 1$ $\frac{3}{5}$ **31.** $\frac{7}{12} + \frac{\blacksquare}{\blacksquare} = 1$ $\frac{5}{12}$

32. $\frac{1}{2} + \frac{1}{3} + \frac{\blacksquare}{\blacksquare} = 1$ $\frac{1}{6}$ **33.** $\frac{2}{5} + \frac{\blacksquare}{\blacksquare} + \frac{1}{5} = 1$ $\frac{2}{5}$ **34.** $\frac{\blacksquare}{\blacksquare} + \frac{2}{3} + \frac{1}{4} = 1$ $\frac{1}{12}$

35. $\frac{3}{8} + \frac{\blacksquare}{\blacksquare} + \frac{1}{6} = 1$ $\frac{11}{24}$ **36.** $\frac{\blacksquare}{\blacksquare} + \frac{1}{6} + \frac{2}{9} = 1$ $\frac{11}{18}$ **37.** $\frac{1}{4} + \frac{1}{3} + \frac{\blacksquare}{\blacksquare} = 1$ $\frac{5}{12}$

226

LESSON ACTIVITY

Before Using the Pages

- Review any aspect of adding and subtracting fractions that students have found difficult. For addition of fractions, you may wish to review that the following steps are considered:
  1. write the fractions with like denominators;
  2. add the fractions;
  3. add the whole numbers;
  4. regroup the sum if it has an improper fraction;
  5. write the sum in lowest terms.

  For subtraction of fractions, the following steps may be necessary:
  1. write the fractions with like denominators;
  2. if the minuend is a whole number, regroup the minuend so that it contains a fraction having a denominator the same as that of the subtrahend;
  3. if the fraction in the minuend is less than the fraction in the subtrahend, regroup 1 whole of the minuend as an improper fraction;
  4. subtract the fractions;
  5. subtract the whole numbers;
  6. write the difference in lowest terms.

Using the Pages

- Before the students begin, review the instructions for each group of exercises, particularly for Ex. 1-3. For Ex. 1, the students can discover that the fraction used to start the path around the circle is equivalent to the fraction that ends the path.

  For Ex. 29-37, ask the students what they notice about the given sums. Review that 1 represents 1 whole which, in turn, is equivalent to 2 halves ($\frac{2}{2}$), 3 thirds ($\frac{3}{3}$), and so on. To help students who have difficulty, draw diagrams and number lines. Some examples are shown below. For Ex. 32-37, discuss that the given fractions must first be expressed with like denominators before the missing fractions can be found.

A 747 jet airliner takes about 4 h to fly between Toronto and Vancouver.

38. Of the fuel that a 747 *uses* in such a flight, $\frac{1}{5}$ is burned during takeoff. What fraction of the fuel it uses is burned during the rest of the flight? $\frac{4}{5}$

39. Of the fuel that a 747 uses, about $\frac{3}{16}$ is burned each hour that it cruises. What fraction of the fuel it uses would be burned after it takes off and cruises for an hour? $\frac{31}{80}$

40. Of the fuel that it uses, about $\frac{1}{5}$ is burned on takeoff and $\frac{3}{4}$ is burned while cruising between Toronto and Vancouver. What fraction of the fuel that it uses is burned during landing? $\frac{1}{20}$

41. Of the *total amount* of fuel that a 747 can carry, $\frac{1}{14}$ is burned on takeoff. What fraction of its fuel would be left for the rest of the flight? $\frac{13}{14}$

42. Of the total amount of fuel that a 747 can carry, $\frac{1}{14}$ is burned on takeoff and $\frac{1}{4}$ is burned while cruising between Toronto and Vancouver. What fraction of the fuel would be left when it is ready to land? $\frac{19}{28}$

43. Between Toronto and Vancouver, a 747 uses $\frac{1}{14}$ of the amount of fuel it can carry on takeoff, $\frac{1}{4}$ of the fuel while cruising, and $\frac{1}{80}$ of the fuel for landing. What fraction of the fuel will it use for the complete flight? $\frac{187}{560}$

44. On a recent flight, about $\frac{4}{7}$ of the passengers were men, about $\frac{2}{5}$ were women, and the rest were children. About what fraction of the passengers were children? $\frac{1}{35}$

Can you find a simple fraction with a one-digit denominator that is close to the fraction you found in Exercise 43? $\frac{1}{3}$

227

RELATED ACTIVITIES

• For practice with addition and subtraction, prepare number squares similar to the following on copies of page T 382. The number in the lower right square provides a check.

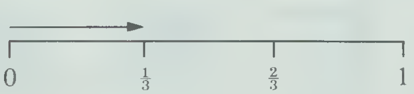
$+ \rightarrow$ / $- \downarrow$		
$3\frac{1}{6}$	4	
$1\frac{3}{4}$	$3\frac{1}{3}$	

• Ask students to determine why a fraction that starts the path in Ex. 1 also ends the path. If each of the four given fractions is expressed with 12 as the denominator, they can see that a complete path involves adding $\frac{15}{12}$ to the original fraction and also subtracting $\frac{15}{12}$. Some students may enjoy writing other similar paths by replacing the given fractions with other fractions.

• Ask each of several students to write a proper fraction on the board or on a large sheet of paper. Point to each fraction and ask students to name the fraction which, when added to the indicated fraction, gives a sum of 1.

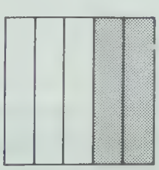
• Students may work in groups of two or three for the following activity. Each student writes a fraction or a number in mixed form for which the denominator is less than or equal to 10. Then each student finds the sum of the numbers and the results are compared.

Ex. 29



$\frac{1}{3} + \frac{\blacksquare}{\blacksquare} = 1$

Ex. 30



$\frac{\blacksquare}{\blacksquare} + \frac{2}{5} = 1$

Ex. 43 will challenge students in finding a like denominator for the three given fractions. Because 80 is a multiple of 4, it is necessary to find the least common multiple of only 14 and 80. A process similar to the one shown at the top of page 211 may be used. To answer the question in the "thought cloud", the students may estimate or use a process of trial and error.

Pages 228-229

LESSON OUTCOME

Multiply fractions; multiply numbers in mixed form; express the products of fractions and numbers in mixed form in lowest terms

Materials

several sheets of paper and crayons for each student (optional)

Prerequisite Skills

Express fractions in lowest terms; express improper fractions as numbers in mixed form or as whole numbers; express a number in mixed form as an improper fraction

Checking Prerequisite Skills

Write each fraction in lowest terms.

1. $\frac{2}{8}$ $\frac{1}{4}$ **2.** $\frac{6}{9}$ $\frac{2}{3}$ **3.** $\frac{12}{15}$ $\frac{4}{5}$

Write each as a number in mixed form or as a whole number.

4. $\frac{10}{5}$ 2 **5.** $\frac{9}{4}$ $2\frac{1}{4}$ **6.** $\frac{14}{3}$ $4\frac{2}{3}$

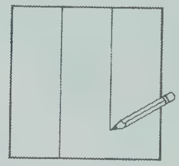
Write each number as an improper fraction.

7. $2\frac{1}{2}$ $\frac{5}{2}$ **8.** $3\frac{9}{10}$ $\frac{39}{10}$ **9.** $4\frac{3}{4}$ $\frac{19}{4}$

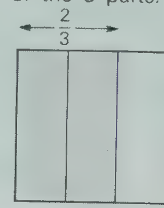
Multiplying Fractions

Melissa made these pictures to show how fractions are multiplied.

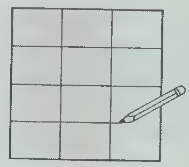
To show $\frac{2}{3}$, she divided a sheet of paper into 3 equal parts.



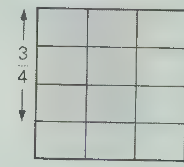
Then she colored 2 of the 3 parts.



To show $\frac{3}{4}$ of $\frac{2}{3}$, she divided the sheet of paper into 4 equal parts.



Then she colored $\frac{3}{4}$ of the $\frac{2}{3}$ darker.



$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$, or $\frac{1}{2}$

Take another look.

For $\frac{3}{4} \times \frac{2}{3}$, multiply numerators. 3×2
Then multiply denominators. 4×3

$$\frac{3}{4} \times \frac{2}{3} = \frac{3 \times 2}{4 \times 3}$$

$$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}, \text{ or } \frac{1}{2}$$

A product of two fractions may not always be in lowest terms.

Working Together

Multiply the numerators.
Then multiply the denominators.

1. $\frac{2}{5} \times \frac{1}{2}$ $\frac{2}{10}$ **2.** $\frac{1}{4} \times \frac{1}{4}$ $\frac{1}{16}$ **3.** $\frac{5}{6} \times \frac{9}{10}$ $\frac{45}{60}$ **4.** $\frac{2}{3} \times \frac{2}{9}$ $\frac{4}{27}$

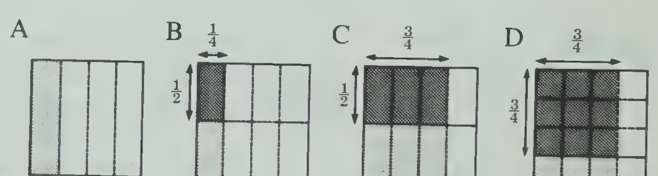
228

LESSON ACTIVITY

Before Using the Pages

- Review that multiplication of two whole numbers is related to the area of a rectangular region. For example, if a region of the display board is covered with six sheets of paper in the form of a two-by-three array, the related multiplication is $2 \times 3 = 6$.

  Demonstrate that a similar procedure can be followed for multiplication of fractions. For example, to show $\frac{1}{2} \times \frac{1}{4}$, fold one sheet of paper twice to mark it into fourths vertically (A). Color one fourth yellow. Fold the paper once to mark it into halves horizontally (B) and color one half of the yellow part blue. Ask what fraction represents the part of the paper that is green. Use a similar procedure to show $\frac{1}{2} \times \frac{3}{4}$ (C) and $\frac{3}{4} \times \frac{3}{4}$ (D), or have the students try the procedure themselves. Write the multiplication sentences on the board. For each example, relate the fractions to the edges of the region that is green.



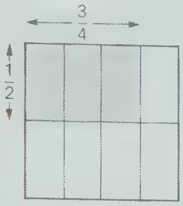
$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ $\frac{3}{4} \times \frac{3}{4} = \frac{9}{16}$

Using the Pages

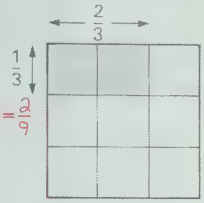
- Ask students to read the statements that accompany the diagrams. Discuss that drawing lines on the paper corresponds to folding the paper into equal parts. Draw attention to the word "of" in "$\frac{3}{4}$ of $\frac{2}{3}$" above the third diagram and relate it to the multiplication symbol in "$\frac{3}{4} \times \frac{2}{3} = \frac{6}{12}$" for the fourth diagram. For the multiplication sentence, ask the students to note the numerators 3, 2, and 6, and the denominators 4, 3, and 12. Then develop that the numerators are multiplied and the denominators are multiplied in order to multiply two fractions. Point out that the product may not be in lowest terms.

Exercises

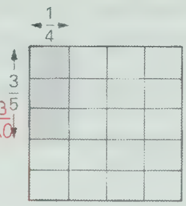
Write a multiplication sentence to match each picture.

1. 

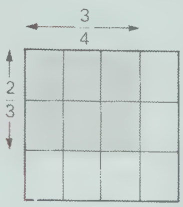
$\frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$

2. 

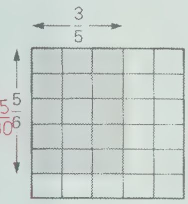
$\frac{1}{3} \times \frac{2}{3} = \frac{2}{9}$

3. 

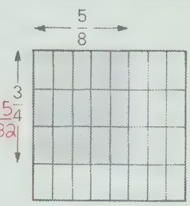
$\frac{3}{5} \times \frac{1}{4} = \frac{3}{20}$

4. 

$\frac{2}{3} \times \frac{3}{4} = \frac{6}{12}$

5. 

$\frac{5}{6} \times \frac{3}{5} = \frac{15}{30}$

6. 

$\frac{3}{4} \times \frac{5}{8} = \frac{15}{32}$

Multiply.

7. $\frac{4}{5} \times \frac{1}{2}$ $\frac{4}{10}$ $(\frac{2}{5})$ **8.** $\frac{1}{2} \times \frac{2}{7}$ $\frac{2}{14}$ $(\frac{1}{7})$ **9.** $\frac{2}{5} \times \frac{3}{8}$ $\frac{6}{40}$ $(\frac{3}{20})$ **10.** $\frac{3}{4} \times \frac{5}{6}$ $\frac{15}{24}$ $(\frac{5}{8})$ **11.** $\frac{1}{3} \times \frac{1}{3}$ $\frac{1}{9}$

12. $\frac{2}{3} \times \frac{3}{4}$ $\frac{6}{12}$ $(\frac{1}{2})$ **13.** $\frac{5}{7} \times \frac{1}{4}$ $\frac{5}{28}$ **14.** $\frac{4}{9} \times \frac{11}{12}$ $\frac{44}{108}$ $(\frac{11}{27})$ **15.** $\frac{5}{6} \times \frac{5}{6}$ $\frac{25}{36}$ **16.** $\frac{3}{5} \times \frac{2}{9}$ $\frac{6}{45}$ $(\frac{2}{15})$

17. $\frac{1}{6} \times \frac{2}{7}$ $\frac{2}{42}$ $(\frac{1}{21})$ **18.** $\frac{3}{10} \times \frac{2}{3}$ $\frac{6}{30}$ $(\frac{1}{5})$ **19.** $\frac{2}{3} \times \frac{5}{12}$ $\frac{10}{36}$ $(\frac{5}{18})$ **20.** $\frac{3}{4} \times \frac{3}{8}$ $\frac{9}{32}$ **21.** $\frac{5}{8} \times \frac{2}{5}$ $\frac{10}{40}$ $(\frac{1}{4})$

22. $\frac{6}{7} \times \frac{5}{12}$ $\frac{30}{84}$ $(\frac{5}{14})$ **23.** $\frac{7}{10} \times \frac{7}{10}$ $\frac{49}{100}$ **24.** $\frac{2}{7} \times \frac{7}{8}$ $\frac{14}{56}$ $(\frac{1}{4})$ **25.** $\frac{5}{9} \times \frac{3}{10}$ $\frac{15}{90}$ $(\frac{1}{6})$ **26.** $\frac{2}{3} \times \frac{3}{8}$ $\frac{6}{24}$ $(\frac{1}{4})$

For Exercises 7 to 26,

27. list the products in lowest terms.

To multiply numbers in mixed form,
first change them to improper fractions.
Then multiply the numerators and the denominators.

Example: For $3\frac{1}{3} \times 2\frac{1}{2}$, use $\frac{10}{3} \times \frac{5}{2} = \frac{50}{6}$, or $8\frac{1}{3}$.

Multiply.

28. $1\frac{1}{2} \times 2\frac{2}{3}$ 4 **29.** $1\frac{5}{6} \times 1\frac{1}{8}$ $2\frac{1}{16}$ **30.** $2\frac{1}{3} \times 2\frac{1}{7}$ 5 **31.** $3\frac{1}{2} \times 4\frac{2}{5}$ $15\frac{2}{5}$

32. $\frac{2}{3} \times 3\frac{9}{10}$ $2\frac{3}{5}$ **33.** $3\frac{3}{4} \times \frac{2}{5}$ $1\frac{1}{2}$ **34.** $1\frac{3}{4} \times 4\frac{2}{7}$ $7\frac{1}{2}$ **35.** $5\frac{1}{5} \times 2\frac{7}{9}$ $14\frac{4}{9}$

229

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 33-52 on page 336.
- For reinforcement, ask students to draw diagrams to illustrate some of the exercises on pages 228 and 229.
- Review that multiplication exercises can be checked by changing the order of the factors and multiplying. Have the students check some of the exercises on page 229.
- Have students play the game "King of Fractions" described on page T 381 for multiplication of fractions.
- Assign the following multiplications. When the students have completed the exercises, ask what they notice about the products. Then ask the students to write and complete two exercises similar to these.

1. $\frac{3}{4} \times \frac{4}{3}$
2. $\frac{5}{12} \times 2\frac{2}{5}$
3. $3\frac{1}{4} \times \frac{4}{13}$
4. $\frac{5}{6} \times 1\frac{1}{5}$

Return to the examples in *Before Using the Pages* and ask students to check the product of the numerators and the product of the denominators.

Working Together: Tell the students to write the exercises in horizontal form as they are shown on page 228, not in vertical form. You may wish to have the students write the products for Ex. 1 and 3 in lowest terms.

Exercises: For Ex. 1-6, the students must interpret the diagrams and write the multiplication sentences. Note that the diagrams are shaded to show only the "darker" regions. Ex. 28-35 involve numbers in mixed form. Discuss the example which shows that if the numbers in mixed form are changed to improper fractions, the improper fractions can be multiplied in the way that proper fractions are multiplied. However, remind the students that if a product is an improper fraction, it should be expressed as a number in mixed form.

Assessment

Multiply. Show the products in lowest terms.

1. $\frac{2}{9} \times \frac{3}{4}$ $\frac{1}{6}$ **2.** $\frac{7}{8} \times \frac{1}{3}$ $\frac{7}{24}$ **3.** $\frac{3}{5} \times \frac{5}{6}$ $\frac{1}{2}$

4. $1\frac{7}{8} \times 1\frac{2}{5}$ $2\frac{5}{8}$ **5.** $2\frac{1}{2} \times 1\frac{1}{3}$ $3\frac{1}{3}$ **6.** $3\frac{3}{10} \times 2\frac{2}{9}$ $7\frac{1}{3}$

Pages 230-231

LESSON OUTCOME

Multiply a fraction and a whole number; solve related word problems

Vocabulary

prime factors

Prerequisite Skills

Express a whole number as an improper fraction

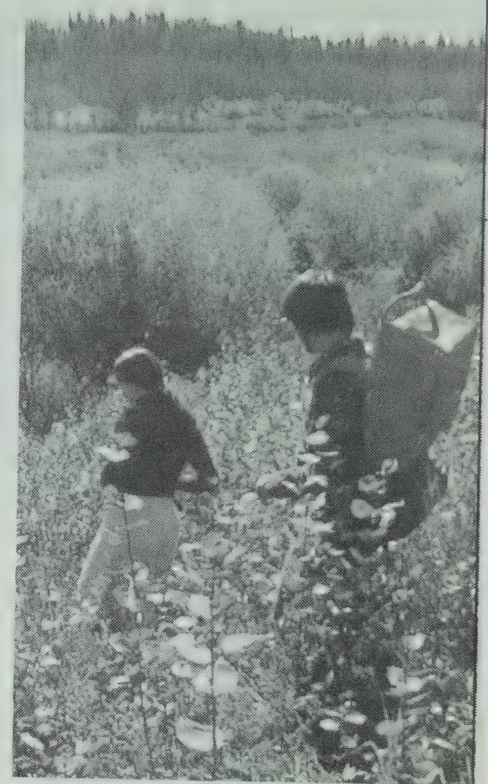
Checking Prerequisite Skills

Complete.

1. $6 = \frac{6}{■} = \frac{12}{■} = \frac{■}{3}$ 1 2 18
2. $12 = \frac{■}{1} = \frac{24}{■}$ 12 2
3. $2 = \frac{2}{■}$ 1
4. $\frac{4}{1} = ■$ 4

Multiplying Fractions and Whole Numbers

On the first day at day camp, the campers were told that they would spend about $\frac{3}{8}$ of the 14 camping days on hikes. About how many camping days would they spend on hikes?



Multiply $\frac{3}{8}$ and 14.

$\frac{3}{8} \times 14 = \frac{3}{8} \times \frac{14}{1}$, or $\frac{42}{8}$

$\frac{42}{8} = 5\frac{2}{8}$, or $5\frac{1}{4}$

About $5\frac{1}{4}$ camping days would be spent on hikes.

On one hike, the lunch ration was $\frac{2}{5}$ of a can of fruit for each hiker. How many cans of fruit would 12 hikers need?

Multiply 12 and $\frac{2}{5}$.

$12 \times \frac{2}{5} = \frac{12}{1} \times \frac{2}{5}$, or $\frac{24}{5}$

$\frac{24}{5} = 4\frac{4}{5}$

The hikers would need $4\frac{4}{5}$ cans of fruit.

The hikers would have to carry 5 cans of fruit.

Working Together

Write the whole number as a fraction with 1 as its denominator. Then multiply.

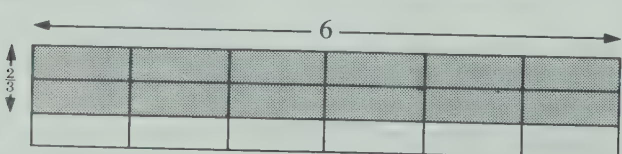
1. $\frac{2}{3} \times 4$ $2\frac{2}{3}$
2. $7 \times \frac{4}{5}$ $5\frac{3}{5}$
3. $\frac{3}{4} \times 16$ 12
4. $21 \times \frac{1}{3}$ 7

230

LESSON ACTIVITY

Before Using the Pages

- Write $\frac{2}{3} \times 6$ on the board, pointing out that one of the numbers to be multiplied is a whole number. To demonstrate the product, draw a diagram representing a rectangular region 1 unit wide and 6 units long, and shade $\frac{2}{3}$ of the region. Note that 12 thirds of the diagram is shaded, which is the same as 4 (wholes). Complete the multiplication on the board.



$\frac{2}{3} \times 6 = \frac{12}{3}$, or 4

You may wish to use other similar examples, for instance, $\frac{1}{4} \times 6$ and $\frac{3}{4} \times 2$.

Review that the product of two fractions, for example, $\frac{2}{3}$ and $\frac{6}{7}$, is found by multiplying the numerators and multiplying the denominators. Ask the students to consider how this procedure can be applied to such multiplications as $\frac{2}{3} \times 6$ and $\frac{3}{4} \times 2$.

Using the Pages

- The photograph can motivate a discussion about the students' experiences at a camp or on a picnic. Ask a student to read the first word problem. Discuss that 14 and $\frac{14}{1}$ are different names for the same number. Replacing 14 by $\frac{14}{1}$ gives $\frac{3}{8} \times \frac{14}{1}$, and the procedure of multiplying the numerators and multiplying the denominators can be applied. Note that the improper fraction $\frac{42}{8}$ is expressed as a number in mixed form and the fraction part is expressed in lowest terms. Ask a student to read the concluding statement. Have students read and explain each step of the second example.

Working Together: Remind the students to express products that are improper fractions as whole numbers or as numbers in mixed form.

Exercises: Remind the students to show their work for Ex. 18-21 and to answer with a concluding statement.

Exercises

Multiply.

1. $\frac{3}{4} \times 6$ $\frac{18}{4}$ $\left(4\frac{1}{2}\right)$ 2. $16 \times \frac{2}{3}$ $\frac{32}{3}$ $\left(10\frac{2}{3}\right)$ 3. $\frac{1}{5} \times 35$ $\frac{35}{5}$ (7) 4. $6 \times \frac{4}{9}$ $\frac{24}{9}$ $\left(2\frac{2}{3}\right)$ 5. $\frac{5}{8} \times 2$ $\frac{10}{8}$ $\left(1\frac{1}{4}\right)$

6. $7 \times \frac{3}{14}$ $\frac{21}{14}$ $\left(1\frac{1}{2}\right)$ 7. $\frac{3}{5} \times 9$ $\frac{27}{5}$ $\left(5\frac{2}{5}\right)$ 8. $22 \times \frac{1}{2}$ $\frac{22}{2}$ (11) 9. $\frac{3}{10} \times 12$ $\frac{36}{10}$ $\left(3\frac{3}{5}\right)$ 10. $20 \times \frac{7}{12}$ $\frac{140}{12}$ $\left(11\frac{2}{3}\right)$

11. $\frac{1}{4} \times 2$ $\frac{2}{4}$ $\left(\frac{1}{2}\right)$ 12. $32 \times \frac{3}{8}$ $\frac{96}{8}$ (12) 13. $\frac{5}{6} \times 24$ $\frac{120}{6}$ (20) 14. $9 \times \frac{1}{3}$ $\frac{9}{3}$ (3) 15. $\frac{11}{12} \times 10$ $\frac{110}{12}$ $\left(9\frac{1}{6}\right)$

Some products can be written as fractions, as whole numbers, or as numbers in mixed form.

16. Write any improper fraction as a number in mixed form for the products for Exercises 1 to 15.

17. List the products for Exercises 1 to 15 in lowest terms.

Solve.

18. The campers plan to stay at the camp on $\frac{3}{7}$ of the 14 camping days. On how many camping days will they stay at the camp? 6

19. The 12 hikers planned to take $\frac{2}{3}$ of a jug of water for each camper. How many jugs of water would they have to carry? 8

20. The camp kitchen had 5 loaves of bread. The hikers took $\frac{1}{4}$ of the bread. How many loaves of bread did they take? $1\frac{1}{4}$

21. $\frac{1}{2}$ of a box of raisins was packed for each of the 12 hikers. How many boxes of raisins did the hikers take? 6

Here is another way to find a product in lowest terms.

$$\frac{5}{6} \times \frac{12}{25} = \frac{5 \times 12}{6 \times 25}, \text{ or } \frac{5 \times 2 \times 2 \times 3}{2 \times 3 \times 5 \times 5}$$

the prime factors of the numerator and the denominator

Divide by the common factors. $\dfrac{\overset{1}{\cancel{5}} \times \overset{1}{\cancel{2}} \times 2 \times \overset{1}{\cancel{3}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{5}} \times 5}$ $\frac{2}{5}$ Write $\frac{5}{6} \times \frac{12}{25} = \frac{2}{5}$

Before multiplying, find the common prime factors in the numerator and the denominator. Divide by them. Write the product in lowest terms.

1. $\frac{1}{2} \times \frac{4}{5}$ $\frac{2}{5}$ 2. $\frac{4}{9} \times \frac{3}{10}$ $\frac{2}{15}$ 3. $\frac{8}{15} \times \frac{15}{20}$ $\frac{2}{5}$ 4. $\frac{6}{12} \times \frac{4}{12}$ $\frac{1}{6}$ 5. $\frac{9}{10} \times \frac{15}{18}$ $\frac{3}{4}$

try this

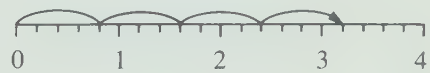
231

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 53-68 on page 336.
- Have the students use the method shown in the *Try This* feature for some of the exercises on pages 229 and 231. The results can be compared with those obtained earlier.
- Have students use repeated addition to complete multiplication exercises for which the first number is a whole number.

$$4 \times \tfrac{4}{5} = \tfrac{4}{5} + \tfrac{4}{5} + \tfrac{4}{5} + \tfrac{4}{5}$$
$$= \tfrac{16}{5}, \text{ or } 3\tfrac{1}{5}$$

They can also show such multiplications on a number line.



- Prepare exercises similar to the following. Have the students complete the exercises and then write the products in the grid. Tell the students that if their work is correct, the products form a Magic Square in which the sums of the numbers for any row, column, or diagonal are equal.

a. $\frac{1}{9} \times 270$ b. $36 \times \frac{1}{4}$ c. $48 \times \frac{1}{2}$
d. $\frac{1}{8} \times 120$ e. $14 \times \frac{3}{2}$ f. $81 \times \frac{1}{3}$
g. $\frac{1}{2} \times 36$ h. $231 \times \frac{1}{7}$ i. $\frac{3}{8} \times 32$

a 30	b 9	c 24
d 15	e 21	f 27
g 18	h 33	i 12

Try This: The concept presented in the *Try This* feature on page 213 is extended here to find a product in lowest terms. This involves writing the numerator and the denominator as products of prime factors, dividing the numerator and the denominator by their common factors, and then multiplying the remaining factors. Review what is meant by a prime number and explain that *prime factors* are factors that are prime numbers. It may be helpful to illustrate an exercise two ways on the board, for example, Ex. 2. Both methods involve multiplication and the use of common factors, but in different orders.

$$\tfrac{4}{9} \times \tfrac{3}{10} = \frac{4 \times 3}{9 \times 10} = \tfrac{12}{90} = \tfrac{4}{30} \text{ (3 is a common factor)} = \tfrac{2}{15} \text{ (2 is a common factor)}$$

$$\tfrac{4}{9} \times \tfrac{3}{10} = \frac{4 \times 3}{9 \times 10} = \frac{2 \times \overset{1}{\cancel{2}} \times \overset{1}{\cancel{3}}}{3 \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times 5} \text{ (2 and 3 are common factors)} = \tfrac{2}{15}$$

Note that if the greatest common factor, 6, were used in the first method, rather than successive steps using 3 and 2, the simplification would go directly from $\frac{12}{90}$ to $\frac{2}{15}$.

Assessment

Multiply.

1. $\frac{3}{4} \times 2$ $1\frac{1}{2}$ 2. $6 \times \frac{2}{3}$ 4 3. $\frac{1}{2} \times 4$ 2
4. $3 \times \frac{4}{5}$ $2\frac{2}{5}$ 5. $\frac{3}{10} \times 5$ $1\frac{1}{2}$ 6. $2 \times \frac{7}{8}$ $1\frac{3}{4}$

Solve.

7. $\frac{4}{5}$ of the 15 campers went on a hike. How many campers went on the hike? 12

OBJECTIVE

Demonstrate competence in multiplying fractions and in multiplying fractions and whole numbers

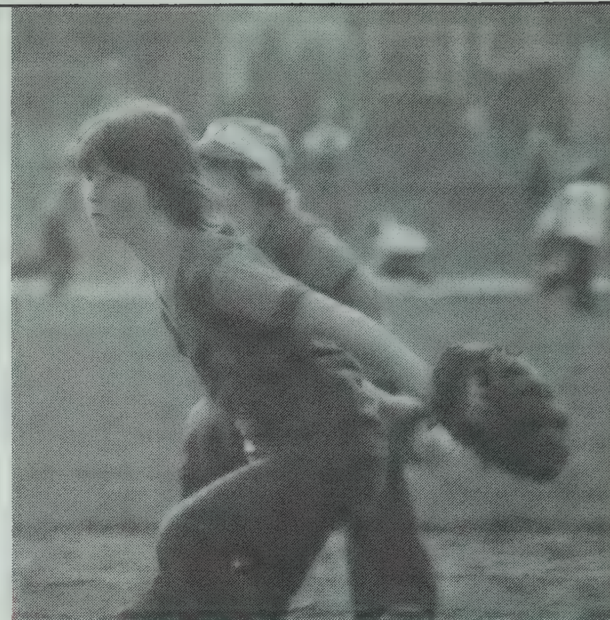
RELATED ACTIVITIES

- Have students complete number wheels for multiplication on copies of page T 391.

- Adapt the game ''Product Search'' on page T 380 for fractions. Mark the dice and the game board with such fractions as $\frac{1}{3}$, $\frac{2}{3}$, $\frac{1}{4}$, $\frac{3}{4}$, $\frac{5}{6}$, $\frac{5}{8}$, and $\frac{2}{5}$, $\frac{4}{5}$, $\frac{3}{8}$, $\frac{7}{8}$, $\frac{1}{10}$, $\frac{9}{10}$. Depending on the ability of the students, you may wish to have the players give products in lowest terms.

×	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$
$\frac{2}{5}$			
$\frac{4}{5}$			
$\frac{3}{8}$			

Practice



For each out made, the softball pitcher gets credit for pitching $\frac{1}{3}$ of an inning.

1. Early in the season, the pitcher averaged $5\frac{1}{3}$ innings for 11 games. How many innings is that in all? $58\frac{2}{3}$
2. For the whole season, the pitcher averaged $3\frac{2}{3}$ innings for 25 games. How many innings is that in all? $91\frac{2}{3}$

There are 3 outs for each inning pitched.

3. How many outs are there for $5\frac{1}{3}$ innings pitched? 16
4. How many outs are there for $3\frac{2}{3}$ innings pitched? 11

Use >, <, or = to make true statements.

5. $\frac{3}{4} \times \frac{4}{5}$ > $\frac{3}{5} \times \frac{2}{3}$
6. $\frac{1}{3} \times 5$ < $4 \times \frac{1}{2}$
7. $6 \times \frac{4}{9}$ > $\frac{1}{5} \times 13$
8. $6 \times \frac{9}{16}$ = $9 \times \frac{3}{8}$
9. $5 \times \frac{3}{4}$ > $7 \times \frac{1}{2}$
10. $\frac{1}{3} \times 2$ > $\frac{7}{8} \times \frac{2}{3}$
11. $\frac{7}{8} \times \frac{5}{7}$ < $\frac{1}{12} \times 8$
12. $\frac{3}{5} \times \frac{5}{8}$ = $\frac{1}{2} \times \frac{3}{4}$
13. $7 \times \frac{2}{3}$ > $\frac{3}{4} \times 6$

KEEPING SHARP

Add, subtract, multiply, or divide. Show your work.

1. 11 745 − 6 778 4967
2. 4967 + 634 5601
3. 5601 × 12 67 212
4. 67 212 − 11 564 55 648
5. 55 648 ÷ 148 376
6. 376 + 29 624 30 000
7. 30 000 − 24 903 5097
8. 5097 × 78 397 566
9. 397 566 ÷ 234 1699
10. 1699 + 1814 3513
11. 3513 × 24 84 312
12. 84 312 ÷ 36 2342
13. 2342 × 28 65 576
14. 65 576 ÷ 56 1171

232

LESSON ACTIVITY

Using the Page

- Before the students begin, discuss the instructions for each group of exercises to ensure that they understand what is required. The statement preceding Ex. 1 and 2 is another way of describing that there are three outs for each inning of a softball game. If two pitchers on the same team have played during a game, each pitcher gets credit for $\frac{1}{3}$ of an inning for every out he/she has made. For Ex. 1, then, the pitcher has averaged $5\frac{1}{3}$ innings for 11 games, giving $5\frac{1}{3} \times 11$, or $58\frac{2}{3}$, innings.

Keeping Sharp: These exercises provide maintenance in the four basic operations with whole numbers. If the students complete the exercises in the given sequence, they will soon notice that the result of one exercise is the same as the first number in the next exercise. They may use this as a check for their work.

Reciprocals

Two numbers whose product is 1 are **reciprocals** of each other.

$\frac{5}{8} \times \frac{8}{5} = \frac{40}{40}$, or 1.

$\frac{5}{8}$ and $\frac{8}{5}$ are reciprocals.

$4 \times \frac{1}{4} = \frac{4}{4}$, or 1.

4 and $\frac{1}{4}$ are reciprocals.

$2\frac{5}{7} \times \frac{7}{19} = \frac{19}{7} \times \frac{7}{19}$, or 1.

$2\frac{5}{7}$ and $\frac{7}{19}$ are reciprocals.

For fractions that are reciprocals, the numerator of one is the same as the denominator of the other.

Working Together

Multiply to find out whether the numbers in each pair are reciprocals.

1. $\frac{3}{4}, \frac{4}{3}$ yes
2. $\frac{1}{8}$, 8 yes
3. $1\frac{1}{5}, \frac{5}{6}$ yes
4. $\frac{9}{12}, 1\frac{1}{3}$ yes

Find the reciprocal.

Example:

For $\frac{7}{9}$, the reciprocal is $\frac{9}{7}$, or $1\frac{2}{7}$.

Check: $\frac{7}{9} \times 1\frac{2}{7} = \frac{7}{9} \times \frac{9}{7}$, or $\frac{63}{63} = 1$

5. $\frac{3}{5}$ $1\frac{2}{3}$
6. 5 $\frac{1}{5}$
7. $1\frac{1}{2}$ $\frac{2}{3}$

Exercises

Are the numbers in each pair reciprocals?

1. $\frac{2}{5}, \frac{5}{2}$ yes
2. $\frac{1}{2}, 1\frac{1}{2}$ no
3. $\frac{1}{6}$, 6 yes
4. $3\frac{1}{2}, \frac{2}{7}$ yes
5. $2\frac{2}{5}, \frac{5}{12}$ yes
6. $\frac{7}{9}, \frac{9}{7}$ yes
7. $3\frac{2}{5}, 5\frac{2}{3}$ no
8. $\frac{8}{12}, 1\frac{1}{2}$ yes
9. $1\frac{3}{4}, \frac{4}{3}$ no
10. 4, $\frac{3}{12}$ yes
11. $1\frac{1}{7}, \frac{7}{8}$ yes
12. $\frac{2}{9}, 4\frac{1}{2}$ yes

Find the reciprocal for each of these.

13. $\frac{3}{7}$ $\frac{7}{3}$ or $2\frac{1}{3}$
14. $\frac{1}{2}$ 2
15. $2\frac{2}{3}$ $\frac{3}{8}$
16. $\frac{4}{5}$ $\frac{5}{4}$ or $1\frac{1}{4}$
17. $2\frac{1}{4}$ $\frac{4}{9}$
18. $\frac{7}{12}$ $\frac{12}{7}$ or $1\frac{5}{7}$
19. $1\frac{3}{4}$ $\frac{4}{7}$
20. 3 $\frac{1}{3}$
21. $\frac{9}{10}$ $\frac{10}{9}$ or $1\frac{1}{9}$
22. $1\frac{1}{5}$ $\frac{5}{6}$

Complete.

23. $1\frac{1}{3} \times \frac{■}{■} = 1$ $\frac{3}{4}$
24. $\frac{7}{10} \times \frac{■}{■} = 1$ $\frac{10}{7}$
25. $7 \times \frac{■}{■} = 1$ $\frac{1}{7}$
26. $1\frac{4}{5} \times \frac{■}{■} = 1$ $\frac{5}{9}$
27. $5 \times \frac{■}{■} = 1$ $\frac{1}{5}$
28. $\frac{8}{9} \times \frac{■}{■} = 1$ $\frac{9}{8}$
29. $\frac{1}{9} \times \frac{■}{■} = 1$ $\frac{9}{1}$ (9)
30. $3\frac{1}{3} \times \frac{■}{■} = 1$ $\frac{3}{10}$
31. $\frac{1}{6} \times \frac{■}{■} = 1$ $\frac{6}{1}$ (6)

233

Page 233

LESSON OUTCOME

Identify reciprocals; find reciprocals of fractions, of numbers in mixed form, and of whole numbers

Vocabulary

reciprocals

Prerequisite Skills

Multiply fractions; multiply numbers in mixed form; multiply fractions and whole numbers

Checking Prerequisite Skills

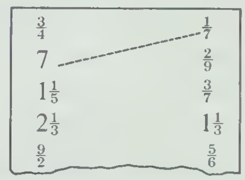
Multiply.

1. $\frac{3}{5} \times \frac{4}{9}$ $\frac{4}{15}$ **2.** $4 \times \frac{3}{8}$ $1\frac{1}{2}$

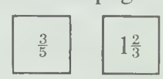
3. $2\frac{1}{2} \times 1\frac{3}{5}$ 4 **4.** $\frac{1}{4} \times \frac{4}{5}$ $\frac{1}{5}$

RELATED ACTIVITIES

- Prepare a work sheet similar to the following and have students match reciprocals.



- Prepare pairs of cards showing reciprocals for the game "Concentration" described on page T 379.



- Ask the students to name the number that is equal to its reciprocal (1).

LESSON ACTIVITY

Before Using the Page

- Assign the following exercises and direct the students to write the products in lowest terms.

$\frac{2}{3} \times \frac{3}{2}$ $6 \times \frac{1}{6}$ $2\frac{1}{2} \times \frac{2}{5}$

Ask the students what they notice about the products. To develop that there is a relationship between the two numbers, ask whether $1\frac{1}{3} \times \frac{4}{3}$ gives a product of 1, and ask a student to give the reason. Establish that the numerator of one fraction must be the same as the denominator of the other fraction if the two fractions are to have a product of 1.

Using the Page

- Introduce the term *reciprocals* to describe two numbers whose product is 1. Have students explain each example and tell how it can be determined whether two numbers are reciprocals. Then refer to the multiplication exercises assigned during the preliminary activity and have students name pairs of numbers that are reciprocals.

Working Together: Discuss the example that precedes Ex. 5-7. Pay particular attention to Ex. 6; to find the reciprocal, 5 must first be expressed as $\frac{5}{1}$. For Ex. 7, the number in mixed form must be expressed as an improper fraction before finding the reciprocal.

Exercises: Ex. 1-12 provide practice in determining whether two fractions are reciprocals. For Ex. 13-31, the students will find reciprocals for fractions, for whole numbers, and for numbers in mixed form.

Assessment

Are the numbers reciprocals?

1. $\frac{4}{5}, 1\frac{1}{4}$ yes **2.** $\frac{2}{3}, \frac{3}{4}$ no **3.** $\frac{1}{2}$, 2 yes

Find the reciprocal for each of these.

4. $\frac{7}{9}$ $\frac{9}{7}$ $\left(1\frac{2}{7}\right)$ **5.** 4 $\frac{1}{4}$ **6.** $1\frac{2}{3}$ $\frac{3}{5}$

Complete.

7. $\frac{1}{8} \times \frac{■}{■} = 1$ $\frac{8}{1}$ (8) **8.** $3 \times \frac{■}{■} = 1$ $\frac{1}{3}$ **9.** $1\frac{2}{3} \times \frac{■}{■} = 1$ $\frac{3}{5}$

LESSON OUTCOME

Divide a proper fraction by a proper fraction; divide a proper fraction by a whole number; divide a whole number by a proper fraction; express the quotients from dividing with fractions in lowest terms; multiply to check division with fractions; solve related word problems

Prerequisite Skills

Multiply fractions; multiply fractions and whole numbers; express products from multiplying with fractions in lowest terms; find reciprocals

Checking Prerequisite Skills

Multiply. Show the products in lowest terms.

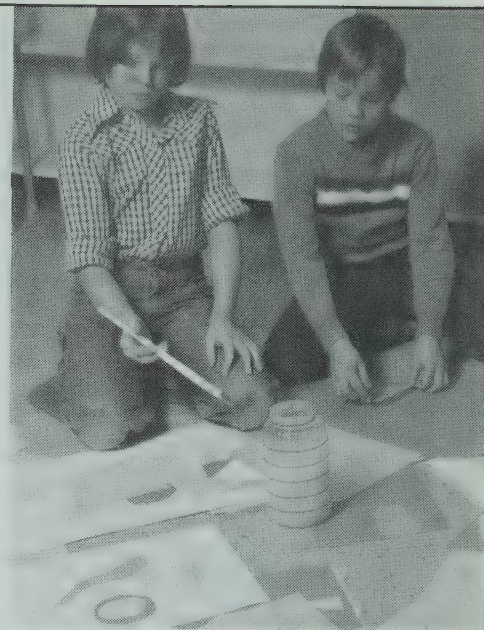
1. $\frac{3}{4} \times \frac{1}{4}$ $\frac{3}{16}$ **2.** $3 \times \frac{2}{3}$ 2

3. $5 \times \frac{6}{5}$ 6 **4.** $\frac{3}{8} \times \frac{4}{9}$ $\frac{1}{6}$

Write the reciprocal for each of these.

5. $\frac{3}{5}$ $\frac{5}{3}$ $(1\frac{2}{3})$ **6.** 2 $\frac{1}{2}$ **7.** $\frac{1}{4}$ $\frac{4}{1}$ (4) **8.** $\frac{2}{3}$ $\frac{3}{2}$ $(1\frac{1}{2})$

Dividing Fractions



The students have $\frac{2}{3}$ of a jar of paste. Each student needs $\frac{1}{6}$ of a jar of paste for art projects. How many students can make art projects?

Divide $\frac{2}{3}$ by $\frac{1}{6}$.

For $\frac{2}{3} \div \frac{1}{6}$,

multiply $\frac{2}{3} \times \frac{6}{1}$.

Dividing by a number is the same as multiplying by its reciprocal.

$\frac{2}{3} \times \frac{6}{1} = \frac{12}{3}$, or 4

4 students can make art projects.

Here are other examples of dividing with fractions.

Divide $\frac{3}{4}$ by 5.

$\frac{3}{4} \div 5 = \frac{3}{4} \div \frac{5}{1}$

$\frac{3}{4} \div \frac{5}{1} = \frac{3}{4} \times \frac{1}{5}$, or $\frac{3}{20}$

$\frac{3}{4}$ divided by 5 is $\frac{3}{20}$.

Divide 6 by $\frac{4}{5}$.

$6 \div \frac{4}{5} = \frac{6}{1} \div \frac{4}{5}$

$\frac{6}{1} \div \frac{4}{5} = \frac{6}{1} \times \frac{5}{4}$, or $\frac{30}{4}$

$\frac{30}{4} = 7\frac{2}{4}$, or $7\frac{1}{2}$

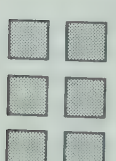
6 divided by $\frac{4}{5}$ is $7\frac{1}{2}$.

234

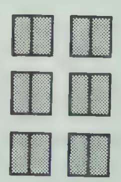
LESSON ACTIVITY

Before Using the Pages

- Draw diagrams or use models and lead the students to recall that division is related to finding the number of equal groups. After developing the division sentences for several examples (A), assign the multiplication exercises (B). Students can discover that the quotients obtained by referring to the diagrams are the same as the products obtained by multiplying by the reciprocal of the divisor. This suggests a procedure for obtaining quotients without models or diagrams.

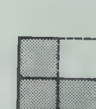


	A	B
How many groups of 3 are there in 6?	$6 \div 3 = \blacksquare$	$6 \times \frac{1}{3} = \blacksquare$



	A	B
How many groups of $\frac{1}{2}$ are there in 6?	$6 \div \frac{1}{2} = \blacksquare$	$6 \times 2 = \blacksquare$

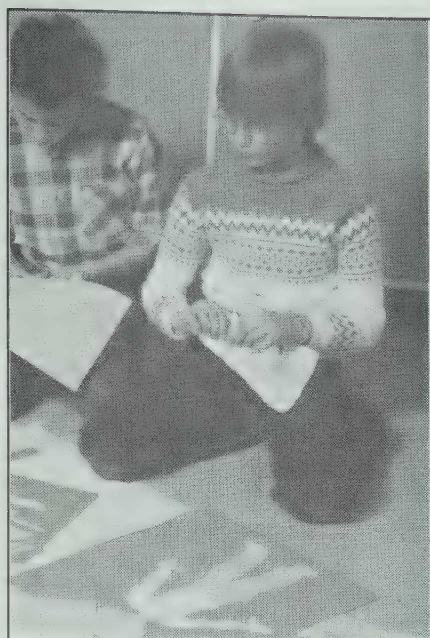
	A	B
How many groups of $\frac{2}{3}$ are there in 6?	$6 \div \frac{2}{3} = \blacksquare$	$6 \times \frac{3}{2} = \blacksquare$



	A	B
How many groups of $\frac{1}{4}$ are there in $\frac{3}{4}$?	$\frac{3}{4} \div \frac{1}{4} = \blacksquare$	$\frac{3}{4} \times 4 = \blacksquare$

Using the Pages

- The first worked example shows dividing a fraction by a fraction. The photograph can motivate a discussion about sharing materials when working on art projects. Note that



Working Together

Write the reciprocal of each divisor.

1. $\frac{7}{12} \div \frac{2}{3}$ $\frac{3}{2}$
2. $\frac{1}{2} \div \frac{1}{4}$ $\frac{4}{1}$
3. $\frac{1}{6} \div 5$ $\frac{1}{5}$
4. $6 \div \frac{3}{5}$ $\frac{5}{3}$

Replace the divisor with its reciprocal and the symbol ÷ with the symbol ×.

5. $\frac{7}{12} \div \frac{2}{3}$ $\frac{7}{12} \times \frac{3}{2}$
6. $\frac{1}{2} \div \frac{1}{4}$ $\frac{1}{2} \times \frac{4}{1}$
7. $\frac{1}{6} \div 5$ $\frac{1}{6} \times \frac{1}{5}$
8. $6 \div \frac{3}{5}$ $\frac{6}{1} \times \frac{5}{3}$

Use the reciprocal of the divisor and multiply to find the quotient.

9. $\frac{7}{12} \div \frac{2}{3}$ $\frac{7}{8}$
10. $\frac{1}{2} \div \frac{1}{4}$ 2
11. $\frac{1}{6} \div 5$ $\frac{1}{30}$
12. $6 \div \frac{3}{5}$ 10

Exercises

Divide.

1. $\frac{7}{12} \div \frac{3}{4}$ $\frac{28}{36}$ $\left(\frac{7}{9}\right)$
2. $\frac{5}{6} \div \frac{1}{2}$ $\frac{10}{6}$ $\left(1\frac{2}{3}\right)$
3. $\frac{3}{8} \div 3$ $\frac{3}{24}$ $\left(\frac{1}{8}\right)$
4. $2 \div \frac{1}{3}$ $\frac{6}{1}$ (6)
5. $\frac{5}{9} \div \frac{5}{7}$ $\frac{35}{45}$ $\left(\frac{7}{9}\right)$
6. $\frac{9}{10} \div \frac{3}{8}$ $\frac{72}{30}$ $\left(2\frac{2}{5}\right)$
7. $1 \div \frac{1}{7}$ $\frac{7}{1}$ (7)
8. $\frac{3}{4} \div 9$ $\frac{3}{36}$ $\left(\frac{1}{12}\right)$
9. $\frac{1}{2} \div 2$ $\frac{1}{4}$
10. $6 \div \frac{2}{9}$ $\frac{54}{2}$ (27)

For Exercises 1 to 10,

11. list the quotients in lowest terms.

Solve.

12. Tom and his 8 friends have $\frac{9}{10}$ of a jar of paste to share equally. How much paste will each student get? $\frac{9}{80}$
13. Ginger has 3 jars of paste. She needs $\frac{1}{4}$ of a jar of paste for each project. How many projects can she make? 12

Multiplication can be used to check division.

Example: To check $\frac{4}{5} \div \frac{2}{3} = 1\frac{1}{5}$, multiply $1\frac{1}{5}$ and $\frac{2}{3}$.

If the product does not match the first number in the division, there is a mistake in the work.

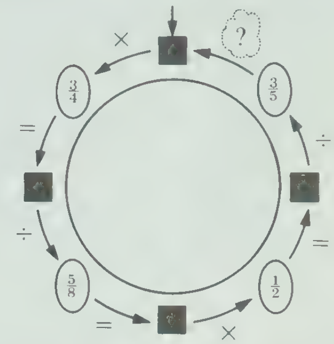
Divide. Multiply to check.

14. $\frac{1}{12} \div \frac{1}{3}$ $\frac{1}{4}$
15. $\frac{3}{4} \div 5$ $\frac{3}{20}$
16. $3 \div \frac{3}{5}$ 5
17. $\frac{3}{8} \div \frac{5}{8}$ $\frac{3}{5}$
18. $\frac{3}{4} \div \frac{9}{12}$ 1
19. $8 \div \frac{6}{[illegible]}$ $9\frac{1}{3}$

235

RELATED ACTIVITIES

- For further practice, you may wish to have students complete Ex. 1-32 on page 337.
- Students may enjoy completing the following picture using the procedure described for Ex. 1-3 on page 226.



- Have students complete exercises similar to the following. Note that the quotients increase as the divisors decrease.

$$6 \div 6 = 1$$
$$6 \div 3 = 2$$
$$6 \div 2 = 3$$
$$6 \div 1 = 6$$
$$6 \div \tfrac{1}{2} = 12$$
$$6 \div \tfrac{1}{3} = 18$$
$$6 \div \tfrac{1}{6} = 36$$

- Illustrate the following procedure for division of whole numbers.

$$6 \div 18$$
$$= \tfrac{6}{1} \times \tfrac{1}{18}$$
$$= \tfrac{6}{18}, \text{ or } \tfrac{1}{3}$$

the jar of paste in the photograph is marked into sixths and that it is $\frac{4}{6}$, or $\frac{2}{3}$, full.

Ask a student to read the word problem at the top of page 234. Establish that $\frac{2}{3}$ is divided by $\frac{1}{6}$ to find the solution, because it is necessary to find the number of groups of $\frac{1}{6}$ in $\frac{2}{3}$. Recall that in the preliminary activity it was found that dividing by a number gives the same result as multiplying by its reciprocal Ask students to explain the steps to complete the exercise.

The preceding example dealt with dividing a fraction by a fraction. The other examples on page 234 involve dividing a fraction by a whole number and dividing a whole number by a fraction. Discuss each step, emphasizing that the divisor is replaced by its reciprocal when the symbol ÷ is replaced by the symbol ×.

Working Together: The exercises are repeated in each section, Ex. 1, 5, and 9, for example, to emphasize the steps for dividing fractions. Note that for Ex. 1-4, the students are to write only the reciprocal of the divisor. Ensure that they do not write the first fraction and the division symbol with the reciprocal.

Exercises: Before the students begin, use an example involving whole numbers to recall that multiplication can be used to check division ($36 \div 9 = 4$, $9 \times 4 = 36$). Draw attention to the example for Ex. 14-19 and ask the students to multiply $1\frac{1}{5}$ and $\frac{2}{3}$. Remind them to express the product in lowest terms.

$$\tfrac{6}{5} \times \tfrac{2}{3} = \tfrac{12}{15}, \text{ or } \tfrac{4}{5}$$

Assessment

Divide. Show the quotients in lowest terms. Multiply to check.

1. $\frac{4}{9} \div \frac{2}{3}$ $\frac{2}{3}$
2. $2 \div \frac{1}{2}$ 4
3. $\frac{1}{3} \div 6$ $\frac{1}{18}$
4. $5 \div \frac{4}{5}$ $6\frac{1}{4}$
5. $\frac{3}{4} \div \frac{3}{4}$ 1
6. $3 \div \frac{2}{9}$ $13\frac{1}{2}$

Solve.

7. Marie has $\frac{4}{5}$ of a jar of paste. She needs $\frac{1}{10}$ of a jar of paste for each project. How many projects can she make? 8

OBJECTIVE

Demonstrate competence in adding, in subtracting, in multiplying, and in dividing fractions; solve related word problems

Practice

Add. Show each sum in lowest terms.

1. $3\frac{1}{5}+3\frac{2}{5}$ $6\frac{3}{5}$	2. $1\frac{3}{8}+\frac{3}{8}$ $1\frac{3}{4}$	3. $4\frac{4}{9}+2\frac{2}{9}$ $6\frac{2}{3}$	4. $\frac{2}{7}+2\frac{3}{7}$ $2\frac{5}{7}$
5. $4\frac{5}{6}+2\frac{5}{6}$ $7\frac{2}{3}$	6. $2\frac{1}{4}+3\frac{3}{4}$ 6	7. $6\frac{7}{10}+1\frac{9}{10}$ $8\frac{3}{5}$	8. $3\frac{5}{12}+\frac{7}{12}$ 4
9. $\frac{1}{4}+\frac{1}{3}$ $\frac{7}{12}$	10. $1\frac{9}{10}+\frac{3}{5}$ $2\frac{1}{2}$	11. $3\frac{3}{10}+1\frac{1}{3}$ $4\frac{19}{30}$	12. $2\frac{7}{12}+\frac{3}{8}$ $2\frac{23}{24}$
13. $4\frac{2}{3}+2\frac{3}{4}$ $7\frac{5}{12}$	14. $3\frac{3}{4}+\frac{1}{2}$ $4\frac{1}{4}$	15. $2\frac{7}{8}+7\frac{5}{6}$ $10\frac{17}{24}$	16. $\frac{2}{3}+\frac{7}{9}$ $1\frac{4}{9}$
17. $\frac{1}{4}+1\frac{5}{6}$ $2\frac{1}{12}$	18. $2\frac{2}{3}+2\frac{5}{9}$ $5\frac{2}{9}$	19. $1\frac{1}{6}+\frac{7}{12}$ $1\frac{3}{4}$	20. $5\frac{3}{4}+6\frac{5}{8}$ $12\frac{3}{8}$

Subtract. Show each difference in lowest terms. Add to check.

21. $2\frac{3}{4}-1\frac{1}{4}$ $1\frac{1}{2}$	22. $3\frac{5}{6}-3\frac{1}{6}$ $\frac{2}{3}$	23. $1\frac{7}{9}-\frac{4}{9}$ $1\frac{1}{3}$	24. $6\frac{11}{12}-3\frac{5}{12}$ $3\frac{1}{2}$
25. $4\frac{1}{3}-1\frac{2}{3}$ $2\frac{2}{3}$	26. $7-1\frac{5}{8}$ $5\frac{3}{8}$	27. $3\frac{1}{5}-2\frac{3}{5}$ $\frac{3}{5}$	28. $5\frac{3}{8}-3\frac{7}{8}$ $1\frac{1}{2}$
29. $2\frac{3}{4}-\frac{2}{3}$ $2\frac{1}{12}$	30. $1\frac{5}{6}-\frac{1}{2}$ $1\frac{1}{3}$	31. $3\frac{7}{10}-3\frac{1}{5}$ $\frac{1}{2}$	32. $4\frac{5}{12}-1\frac{2}{3}$ $2\frac{3}{4}$
33. $5\frac{1}{8}-1\frac{3}{4}$ $3\frac{3}{8}$	34. $3\frac{2}{5}-2\frac{9}{10}$ $\frac{1}{2}$	35. $9\frac{2}{3}-4\frac{5}{6}$ $4\frac{5}{6}$	36. $6\frac{1}{12}-3\frac{1}{4}$ $2\frac{5}{6}$
37. $1\frac{1}{2}-\frac{7}{8}$ $\frac{5}{8}$	38. $4\frac{2}{3}-\frac{8}{9}$ $3\frac{7}{9}$	39. $3\frac{1}{3}-2\frac{7}{12}$ $\frac{3}{4}$	40. $8\frac{3}{8}-2\frac{1}{2}$ $5\frac{7}{8}$

Multiply. Show each product in lowest terms.

41. $\frac{3}{4}\times\frac{1}{2}$ $\frac{3}{8}$	42. $\frac{5}{6}\times\frac{2}{3}$ $\frac{5}{9}$	43. $\frac{5}{8}\times\frac{4}{5}$ $\frac{1}{2}$	44. $\frac{7}{12}\times\frac{4}{7}$ $\frac{1}{3}$
45. $\frac{4}{9}\times\frac{3}{8}$ $\frac{1}{6}$	46. $\frac{7}{10}\times\frac{6}{7}$ $\frac{3}{5}$	47. $\frac{4}{5}\times\frac{5}{6}$ $\frac{2}{3}$	48. $\frac{2}{3}\times\frac{9}{10}$ $\frac{3}{5}$
49. $4\times\frac{1}{6}$ $\frac{2}{3}$	50. $\frac{3}{5}\times 7$ $4\frac{1}{5}$	51. $2\times\frac{3}{8}$ $\frac{3}{4}$	52. $\frac{1}{2}\times 3$ $1\frac{1}{2}$
53. $\frac{5}{4}\times\frac{2}{5}$ $\frac{1}{2}$	54. $\frac{4}{3}\times\frac{5}{4}$ $1\frac{2}{3}$	55. $\frac{11}{6}\times\frac{10}{7}$ $2\frac{13}{21}$	56. $\frac{10}{9}\times\frac{9}{10}$ 1
57. $5\frac{1}{3}\times\frac{1}{2}$ $2\frac{2}{3}$	58. $6\frac{1}{2}\times\frac{1}{8}$ $\frac{13}{16}$	59. $3\frac{2}{3}\times\frac{1}{6}$ $\frac{11}{18}$	60. $2\frac{5}{6}\times\frac{1}{3}$ $\frac{17}{18}$

Divide. Show each quotient in lowest terms. Multiply to check.

61. $\frac{2}{3}\div 6$ $\frac{1}{9}$	62. $\frac{3}{5}\div 2$ $\frac{3}{10}$	63. $\frac{5}{6}\div 5$ $\frac{1}{6}$	64. $\frac{9}{10}\div 3$ $\frac{3}{10}$
65. $\frac{2}{5}\div 8$ $\frac{1}{20}$	66. $\frac{1}{2}\div 9$ $\frac{1}{18}$	67. $6\div\frac{1}{7}$ 42	68. $10\div\frac{5}{8}$ 16
69. $1\div\frac{5}{6}$ $1\frac{1}{5}$	70. $7\div\frac{3}{4}$ $9\frac{1}{3}$	71. $4\div\frac{2}{3}$ 6	72. $8\div\frac{6}{7}$ $9\frac{1}{3}$
73. $\frac{1}{6}\div\frac{3}{4}$ $\frac{2}{9}$	74. $\frac{4}{5}\div\frac{2}{5}$ 2	75. $\frac{3}{8}\div\frac{1}{2}$ $\frac{3}{4}$	76. $\frac{8}{9}\div\frac{2}{3}$ $1\frac{1}{3}$
77. $\frac{1}{2}\div\frac{2}{3}$ $\frac{3}{4}$	78. $\frac{1}{7}\div\frac{3}{7}$ $\frac{1}{3}$	79. $\frac{5}{6}\div\frac{5}{8}$ $1\frac{1}{3}$	80. $\frac{2}{3}\div\frac{7}{12}$ $1\frac{1}{7}$

236

LESSON ACTIVITY

Using the Pages

- Select one or more exercises from each of the four groups on page 236. Complete them on the board with the students to review the procedure for adding, subtracting, multiplying, and dividing fractions. Note that like denominators are required for addition and subtraction. For multiplication and division, the numbers must be expressed as either proper or improper fractions. For division, the divisor is replaced by its reciprocal and the numbers are multiplied.
- You may wish to assign groups of exercises at different times, perhaps one column at a time. Note that for Ex. 81-84, the students must read carefully to determine the operation required to solve each problem.

Problem Solving: Provide time for the students to experiment with solutions. Encourage them to think of possible solutions, or partial solutions, and to work with these, perhaps with the use of diagrams. When the students are ready, discuss their solutions. Point out that because there are 3 cans of paint to be divided equally among 7 cans, each can will be $\frac{3}{7}$ full. Therefore, a can with only red, blue, or yellow paint will have $\frac{3}{7}$ of the red, blue, or yellow paint. A can with green paint, for example, will have equal amounts of blue and yellow paint. Since it will be $\frac{3}{7}$ full, it will have $\frac{3}{14}$ ($\frac{3}{7}\div 2$) of the blue paint and $\frac{3}{14}$ of the yellow paint. The cans of orange paint and purple paint may be determined in a similar way. A can with black paint will be $\frac{3}{7}$ full and will have equal amounts of red, blue, and yellow paint. Thus, it will have $\frac{1}{7}$ of the red paint, $\frac{1}{7}$ of the blue paint, and $\frac{1}{7}$ of the yellow paint.

Students may find the problem easier to understand if they first try a similar, but simpler, problem such as: Sharon has two full cans of paint, one red and one blue, and one empty can. She wants to mix the paint so that the three cans have the same amount of paint. She wants to use equal amounts of paint from two cans when she makes the purple paint. For each color, how much paint will Sharon use from each can?

$\frac{2}{3}$ of a package of beads is needed to make each bracelet.

81. How many packages of beads are needed for 7 bracelets? $4\frac{2}{3}$

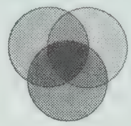
82. How many bracelets can be made with 8 packages of beads? 12

$\frac{3}{5}$ of a package of nails is needed for each birdhouse.

83. How many packages of nails are needed for 4 birdhouses? $2\frac{2}{5}$

84. How many birdhouses can be made with 6 packages of nails? 10

The primary colors are red, blue, and yellow. Mixing them in different ways gives other colors



Sharon has 3 full cans of paint, one red, one blue, and one yellow.

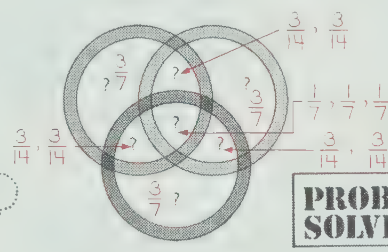
She wants to mix the paint so that

A. all 7 cans will have the same amount of paint when she is finished mixing,

B. she uses equal amounts of paint from two cans when she makes green, orange, or purple, and

C. she uses equal amounts of the red, the blue, and the yellow paint when she makes black.

1. Write fractions to show how much paint Sharon should use from each can.



Hint: The paint in 3 cans is shared equally among 7 cans. What fraction shows how full each can will be?

PROBLEM SOLVING

RELATED ACTIVITIES

- Have the students write word problems for pairs of exercises such as $3 \times \frac{1}{2}$ and $3 \div \frac{1}{2}$. Ex. 81-84 on page 237 may suggest ideas for the word problems.
- Have the students choose from the symbols $+$, $-$, $\times$, and $\div$ to complete exercises similar to the following.

$\frac{1}{4} \bigcirc \frac{3}{8} = \frac{5}{8}$ $+$	$\frac{5}{7} \bigcirc 7 = 5$ $\times$
$\frac{9}{10} \bigcirc \frac{2}{5} = \frac{1}{2}$ $-$	$\frac{2}{3} \bigcirc \frac{1}{3} = 2$ $\div$
$\frac{3}{5} \bigcirc \frac{1}{5} = 3$ $\div$	$\frac{1}{7} \bigcirc \frac{5}{7} = \frac{6}{7}$ $+$
$\frac{2}{3} \bigcirc \frac{3}{4} = \frac{1}{2}$ $\times$	$\frac{8}{9} \bigcirc \frac{1}{3} = \frac{5}{9}$ $-$

Pages 238-239

LESSON OUTCOME

Divide the numerator of a fraction by the denominator to express the fraction as a decimal, round the quotient to two or three decimal places

Prerequisite Skills

Divide whole numbers using extra zeros in the dividend; round quotients to two or three decimal places; express numbers in mixed form as improper fractions

Checking Prerequisite Skills

Divide. Round the quotients to two decimal places.

1. $6\overline{)11}$ 1.83 **2.** $8\overline{)3}$ 0.38

Divide. Round the quotients to three decimal places.

3. $9\overline{)4}$ 0.444 **4.** $7\overline{)1}$ 0.143

Write each as an improper fraction.

5. $1\frac{1}{3}$ $\frac{4}{3}$ **6.** $2\frac{7}{8}$ $\frac{23}{8}$

Changing Fractions to Decimals

In the first three games between Toronto and Montreal in the 1978 play-off series, the Montreal goalkeeper allowed 8 goals. What was the average number of goals that the Montreal goalkeeper allowed in each game?

The average number of goals allowed in each game could be shown with an improper fraction, $\frac{8}{3}$, or with a number in mixed form, $2\frac{2}{3}$.

$3\overline{)8}$ 2 R2

Often, it is shown as a decimal.

To change $\frac{8}{3}$ to a decimal, divide 8 by 3.

$$
\begin{array}{r}
2.666 \\
3\overline{)8.000} \\
6 \\
\hline
2\,0 \\
1\,8 \\
\hline
20 \\
18 \\
\hline
20
\end{array}
$$

The division could continue forever! To round the quotient to two decimal places, look at the digit in the third decimal place. The quotient rounds to 2.67.

The average number of goals that the Montreal goalkeeper allowed in each game was 2.67.

238

LESSON ACTIVITY

Before Using the Pages

- Review that a fraction for which the denominator is 10 (100) may be written as a decimal tenth (hundredth), for example, $\frac{3}{10} = 0.3$, $4\frac{7}{10} = 4.7$, and $2\frac{27}{100} = 2.27$. Ask students to find the missing terms for the following exercises. Then ask what decimals are equivalent to $\frac{1}{2}$, $\frac{4}{5}$, and $\frac{1}{4}$.

$$\frac{1}{2} = \frac{\blacksquare}{10} \qquad \frac{4}{5} = \frac{\blacksquare}{10} \qquad \frac{1}{4} = \frac{\blacksquare}{100}$$

Suggest that there is another way to find the decimal equivalents without the use of equivalent fractions. Assign the following division exercises.

$$2\overline{)1} \qquad 5\overline{)4} \qquad 4\overline{)1}$$

Point out that the quotients give the same decimal equivalents obtained by the previous method. Summarize that a fraction, for example, $\frac{4}{5}$, may be associated with division, in this case, $5\overline{)4}$. The numerator is the dividend and the denominator is the divisor. Explain that this helps to express fractions as decimals, particularly such fractions as $\frac{2}{3}$, for which the denominator is not a factor of 10 (100, 1000, . . .).

Using the Pages

- Ask a student to read the word problem. Discuss that the average number of goals is found by dividing the total number of goals by the number of games. Ask students to explain why this can be represented by $\frac{8}{3}$, or $2\frac{2}{3}$. Ask how $\frac{8}{3}$ can be expressed as a decimal. Discuss each step of the division, paying particular attention to the decimal points in the dividend and the quotient, and to the use of extra zeros in the dividend. Have the students note that the digit 6 is repeated in the decimal part of the quotient. Develop that the division could continue without end, and that the digit 6 would be in each decimal place of the quotient. Ask students to explain how the quotient is rounded to two decimal places.

Exercises

Divide the numerator by the denominator to change each fraction to a decimal. Round the quotients to two decimal places.

1. $\frac{1}{3}$ 0.33	**2.** $\frac{4}{5}$ 0.80	**3.** $\frac{5}{6}$ 0.83	**4.** $\frac{7}{12}$ 0.58	**5.** $\frac{3}{7}$ 0.43
6. $\frac{5}{8}$ 0.63	**7.** $\frac{1}{6}$ 0.17	**8.** $\frac{2}{9}$ 0.22	**9.** $\frac{11}{12}$ 0.92	**10.** $\frac{7}{10}$ 0.70
11. $\frac{23}{10}$ 2.30	**12.** $\frac{8}{5}$ 1.60	**13.** $\frac{13}{4}$ 3.25	**14.** $\frac{39}{8}$ 4.88	**15.** $\frac{23}{6}$ 3.83
16. $2\frac{1}{2}$ 2.50	**17.** $4\frac{2}{3}$ 4.67	**18.** $1\frac{3}{4}$ 1.75	**19.** $5\frac{3}{8}$ 5.38	**20.** $3\frac{5}{12}$ 3.42

Solve.

21. A goalie allowed 37 goals in 14 games. What is the average number of goals per game that the goalie allowed? 2.64

22. Who has the lesser average, a goalie who allowed 144 goals in 45 games or one who allowed 135 goals in 42 games? 144 goals in 45 games

23. A basketball team scored 1360 points in 18 games. What is the average number of points scored per game? 75.56

24. Which basketball player has the better average, one with 278 points in 12 games or one with 347 points in 15 games? 278 points in 12 games

Baseball batting averages are found by dividing the number of hits by the number of times at bat, and rounding to three decimal places. Here is how some of the Montreal Expos batted during the 1978 season.

	Hits	Times at bat	
Carter	136	533	0.255
Cash	166	658	0.252
Cromartie	180	607	0.297
Dawson	154	609	0.253
Frias	4	15	0.267
Fry	0	9	0.000
Garrett	12	69	0.174
Herman	7	40	0.175
Hutton	12	59	0.203

25. Find the batting average of each player.

For some fractions, the digits in the equivalent decimal follow a repeating pattern.

For $\frac{1}{3}$, divide 1 by 3.

$$3\overline{)1.0000\ldots}\quad 0.3333\ldots$$

Write $\frac{1}{3} = 0.\overline{3}$.

Use a bar above the repeating digits to show a decimal for each of these.

1. $\frac{5}{6}$ $0.8\overline{3}$ **2.** $\frac{2}{9}$ $0.\overline{2}$

3. $1\frac{2}{3}$ $1.\overline{6}$ **4.** $3\frac{7}{11}$ $3.\overline{63}$

5. $\frac{8}{15}$ $0.5\overline{3}$ **6.** $\frac{11}{18}$ $0.6\overline{1}$

7. $2\frac{32}{33}$ $2.\overline{96}$ **8.** $1\frac{4}{9}$ $1.\overline{4}$

try this

239

RELATED ACTIVITIES

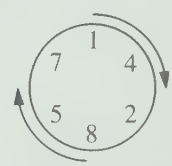
• You may wish to show that a number in mixed form need not be expressed as an improper fraction to find the decimal equivalent. For example, since $\frac{1}{2}$ is equivalent to 0.5, then $1\frac{1}{2} = 1.5$, $2\frac{1}{2} = 2.5$, and so on. For $1\frac{3}{4}$, students can complete the division $4\overline{)3}$ to obtain 0.75, and then write 1.75.

• Have students find fractions other than those in the *Try This* feature for which the equivalent decimals have repeating digits.

• Students can find the batting averages for various baseball players or averages related to other sports. For a particular sport, students can list players' averages from greatest to least. Note that in some sports such as basketball, the greatest average is the best, but in other sports such as golf, the least average is the best.

• For some sets of fractions, students can note interesting patterns in the decimal equivalents. Have them find the repeating patterns for the decimal equivalents for $\frac{1}{7}$, $\frac{2}{7}$, $\frac{3}{7}$, $\frac{4}{7}$, $\frac{5}{7}$, and $\frac{6}{7}$. If the sequence of repeating digits for one of these fractions is written around a circle, the sequences for the other fractions can be identified.

$\frac{1}{7} = 0.\overline{142857}$ $\frac{3}{7} = 0.\overline{428571}$



Exercises: Remind the students to express numbers in mixed form as improper fractions before dividing. After the students have completed the exercises, elicit the following way for completing Ex. 11.

$$\frac{23}{10} = 2\frac{3}{10}$$
$$= 2.3$$

Also, ask how they can tell whether a fraction will be equivalent to a decimal greater than one. Develop that the decimal will be greater than one if the fraction is greater than one.

Try This: The use of a bar above the repeating digits of a quotient is presented in the *Try This* feature on page 151. Now it is shown that such quotients are decimal equivalents of fractions. Remind the students that the bar is not placed over the digits that are not part of the repeating pattern; it is placed over only the first set of repeating digits.

Assessment

Divide the numerator by the denominator to change each fraction to a decimal. Round the quotients to two decimal places.

1. $\frac{2}{3}$ 0.67 **2.** $\frac{15}{4}$ 3.75 **3.** $2\frac{5}{6}$ 2.83

Divide the numerator by the denominator to change each fraction to a decimal. Round the quotients to three decimal places.

4. $\frac{8}{9}$ 0.889 **5.** $\frac{7}{3}$ 2.333 **6.** $3\frac{1}{7}$ 3.143

Pages 240-241

LESSON OUTCOME

Compare and order fractions using their decimal equivalents; add, subtract, multiply, and divide fractions and their decimal equivalents or their approximate decimal equivalents, and then compare the results

Prerequisite Skills

Order decimals; add, subtract, multiply, and divide fractions and decimals

Checking Prerequisite Skills

List in order from least to greatest.

1. 0.8, 0.81, 0.75, 0.85, 0.7
0.7, 0.75, 0.8, 0.81, 0.85

Add.

2. $1\frac{5}{6}$ + $2\frac{2}{3}$ = $4\frac{1}{2}$ **3.** 3.61 + 2.78 = 6.39

Subtract.

4. $3\frac{1}{4}$ − $1\frac{7}{8}$ = $1\frac{3}{8}$ **5.** 1.25 − 0.64 = 0.61

Multiply.

6. $3\frac{3}{4} \times \frac{1}{3}$ $1\frac{1}{4}$ **7.** 0.25×0.5 0.125

Divide.

8. $\frac{1}{6} \div 3$ $\frac{1}{18}$ **9.** $0.8 \div 4$ 0.2

Fractions and Their Equivalent Decimals

Which is a true statement,

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} > 1$, or $\frac{1}{2} + \frac{1}{3} + \frac{1}{4} < 1$?

Here is how to answer this question using fractions.

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = \frac{6}{12} + \frac{4}{12} + \frac{3}{12}$$
$$= \frac{13}{12}, \text{ or } 1\frac{1}{12}$$

Here is how to answer this question using decimals.

$\frac{1}{2} = 0.50$

$\frac{1}{3} = 0.33$ (approximately)

$\frac{1}{4} = 0.25$

$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} = 1.08$ (approximately)

$$\frac{1}{2} + \frac{1}{3} + \frac{1}{4} > 1$$

Exercises

Use decimals with up to two places to complete this chart.

Dividing and rounding to two places can help you with this chart.

	Denominator \ Numerator	1	2	3	4	5	6	7	8	9
1.	2	0.5	1	1.5	2.0	2.5	3.0	3.5	4.0	4.5
2.	3	0.33	0.67	1.0	1.33	1.67	2.0	2.33	2.67	3.0
3.	4	0.25	0.5	0.75	1.0	1.25	1.5	1.75	2.0	2.25
4.	5	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8
5.	6	0.17	0.33	0.5	0.67	0.83	1.0	1.17	1.33	1.5
6.	7	0.14	0.29	0.43	0.57	0.71	0.86	1.0	1.14	1.29
7.	8	0.13	0.25	0.38	0.5	0.63	0.75	0.88	1.0	1.13
8.	9	0.11	0.22	0.33	0.44	0.56	0.67	0.78	0.89	1.0
9.	10	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9

For this chart,

10. what patterns can you find? Answers will vary.

240

LESSON ACTIVITY

Before Using the Pages

- Write a few exercises on the board for reviewing the use of division to express a fraction as a decimal. Ask students to show and explain their work on the board.

$\frac{5}{8}$ $\frac{3}{10}$ $\frac{3}{8}$ $4\frac{2}{5}$

$$8\overline{)5.000} = 0.625$$

- Have the students recall that the sum of two fractions with unlike denominators may be found by adding equivalent fractions having like denominators. Use this procedure to add $\frac{5}{8}$ and $\frac{3}{10}$. Then ask how the results of the first activity would enable them to add $\frac{5}{8}$ and $\frac{3}{10}$ without finding equivalent fractions. Lead the students to suggest adding the decimal equivalents to obtain 0.925. Complete the addition and compare this sum with that for the previous method. Have the students describe how the results of the first activity can be used to name the sum of $4\frac{2}{5}$ and $\frac{3}{10}$, the difference of $\frac{5}{8}$ and $\frac{3}{10}$, and to determine which is greater, $\frac{3}{10}$ or $\frac{3}{8}$. You may need to review that such numerals as 0.3 and 0.30 represent the same number.

Using the Pages

- Allow the students a few moments to study the examples at the top of page 240. Ask a student to explain the procedure involving the addition of equivalent fractions with like denominators. Then ask a student to explain how decimals are used to determine the answer. Ask why the word "approximately" is used in the two instances shown. Compare the fraction $1\frac{1}{12}$ with the decimal 1.08, noting that each is greater than one. You may wish to have the students express $1\frac{1}{12}$ as a decimal using the division $12\overline{)13}$, and then compare the result, 1.083, with the sum 1.08.

Exercises: Ensure that the students understand how to complete Ex. 1-9. Point out that it will be necessary to round some quotients to the nearest hundredth, as in Ex. 2. For Ex. 10, discuss patterns in the chart, for example, the position of 1 in the chart. Also, moving downward in any column, it can

Use >, <, or = to make true statements.

Decimals can help you with most of these.

11. $\frac{1}{3}+\frac{1}{4}+\frac{1}{5}+\frac{1}{6}$ ○ 1 <

12. $\frac{1}{4}+\frac{1}{5}+\frac{1}{6}+\frac{1}{7}+\frac{1}{8}$ ○ 1 <

13. $\frac{1}{3}+\frac{1}{2}+\frac{3}{4}$ ○ $\frac{3}{8}+\frac{1}{4}+\frac{2}{3}$ >

14. $1\frac{3}{4}-\frac{1}{5}$ ○ $\frac{3}{4}+\frac{4}{5}$ =

15. $\frac{1}{2}\times\frac{4}{5}$ ○ 1 <

16. $\frac{1}{2}\times\frac{2}{3}\times\frac{3}{4}\times\frac{4}{5}\times\frac{5}{6}$ ○ $\frac{1}{6}$ =

List in order from least to greatest.

Try using decimals.

17. $\frac{1}{2}, \frac{3}{8}, \frac{2}{3}, \frac{4}{9}, \frac{3}{4}, \frac{2}{5}$ $\frac{3}{8}, \frac{2}{5}, \frac{4}{9}, \frac{1}{2}, \frac{2}{3}, \frac{3}{4}$

18. $\frac{4}{5}, \frac{7}{10}, \frac{5}{6}, \frac{5}{7}, \frac{3}{4}, \frac{7}{9}$ $\frac{7}{10}, \frac{5}{7}, \frac{3}{4}, \frac{7}{9}, \frac{4}{5}, \frac{5}{6}$

Add the fractions. Then add the equivalent decimals. Compare the results.

19. $2\frac{1}{2} + 1\frac{3}{4} = 4\frac{1}{4}$ 2.5 + 1.75 = 4.25

20. $3\frac{2}{5} + \frac{1}{2} = 3\frac{9}{10}$ 3.4 + 0.5 = 3.9

21. $4\frac{1}{4} + 2\frac{5}{8} = 6\frac{7}{8}$ 4.25 + 2.63 = 6.88

22. $3\frac{2}{3} + 3\frac{7}{9} = 7\frac{4}{9}$ 3.67 + 3.78 = 7.45

23. $1\frac{3}{10} + 2\frac{4}{5} = 4\frac{1}{10}$ 1.3 + 2.8 = 4.1

24. $1\frac{1}{2} + 2\frac{3}{8} = 3\frac{7}{8}$ 1.5 + 2.38 = 3.88

Subtract the fractions. Then subtract the equivalent decimals. Compare the results.

25. $2\frac{1}{10} - \frac{1}{2} = 1\frac{3}{5}$ 2.1 − 0.5 = 1.6

26. $7\frac{1}{4} - 5\frac{1}{8} = 2\frac{1}{8}$ 7.25 − 5.13 = 2.12

27. $3\frac{8}{9} - 2\frac{2}{3} = 1\frac{2}{9}$ 3.89 − 2.67 = 1.22

28. $1\frac{1}{2} - \frac{5}{6} = \frac{2}{3}$ 1.50 − 0.83 = 0.67

29. $4\frac{2}{5} - 2\frac{9}{10} = 1\frac{1}{2}$ 4.4 − 2.9 = 1.5

30. $7\frac{1}{6} - 3\frac{1}{2} = 3\frac{2}{3}$ 7.17 − 3.50 = 3.67

Multiply the fractions. Then multiply the equivalent decimals. Compare the results.

31. $\frac{1}{4}\times\frac{1}{2}$ $\frac{1}{8}$ **32.** $\frac{2}{3}\times\frac{3}{4}$ $\frac{1}{2}$

33. $\frac{4}{5}\times\frac{1}{4}$ $\frac{1}{5}$ **34.** $\frac{3}{8}\times\frac{2}{3}$ $\frac{1}{4}$

35. $\frac{3}{5}\times\frac{5}{9}$ $\frac{1}{3}$ **36.** $\frac{1}{2}\times\frac{4}{5}$ $\frac{2}{5}$

Add.

$\frac{1}{3}$	0.333 . . .	$0.\overline{3}$
$\frac{1}{3}$	0.333 . . .	$0.\overline{3}$
$\frac{1}{3}$	0.333 . . .	$0.\overline{3}$
$\frac{3}{3}$, or 1	0.999 . . .	$0.\overline{9}$

Since the addends are equal, the sums are equal.

$1 = 0.\overline{9}$

1. Do you agree? Answers will vary

try this

Divide the fractions. Then divide the equivalent decimals. Compare the results.

37. $\frac{3}{8}\div 3$ $\frac{1}{8}$ **38.** $\frac{4}{9}\div 2$ $\frac{2}{9}$

39. $\frac{1}{2}\div 5$ $\frac{1}{10}$ **40.** $\frac{8}{9}\div 4$ $\frac{2}{9}$

41. $\frac{2}{3}\div 6$ $\frac{1}{9}$ **42.** $\frac{4}{5}\div 8$ $\frac{1}{10}$

31. 0.5 × 0.25 = 0.125 32. 0.75 × 0.67 = 0.5025 33. 0.25 × 0.8 = 0.200
34. 0.67 × 0.38 = 0.2546 35. 0.56 × 0.6 = 0.336 36. 0.8 × 0.5 = 0.40

37. 0.38 ÷ 3 = 0.13 38. 0.44 ÷ 2 = 0.22
39. 0.5 ÷ 5 = 0.1 40. 0.89 ÷ 4 = 0.22
41. 0.67 ÷ 6 = 0.11 42. 0.8 ÷ 8 = 0.1

241

RELATED ACTIVITIES

- Students may complete some of the earlier exercises in this unit using decimals and then compare the results with those expressed as fractions.
- Have students use >, <, or = to make true statements for exercises similar to the following. Before they begin, ask why it is not necessary to perform the operations to determine each correct symbol.

$\frac{3}{10}+\frac{1}{4}+\frac{4}{5}$ ● 0.3 + 0.25 + 0.8 =

$\frac{9}{10}-\frac{1}{5}$ ● 0.8 − 0.2 >

0.75 × 0.6 ● $\frac{3}{4}\times\frac{2}{3}$ <

$\frac{3}{8}\div 3$ ● 0.38 ÷ 3 <

$\frac{7}{10}\times\frac{2}{5}$ ● 0.7 × 0.4 =

$\frac{1}{2}-\frac{4}{9}$ ● 0.5 − 0.4 <

0.75 + 0.875 ● $\frac{3}{4}+\frac{7}{8}$ =

$\frac{4}{5}\div 2$ ● 0.75 ÷ 2 >

- If the fourth activity in *Related Activities* on page T259 was not completed for that lesson, it may be assigned now.

be seen that the values of the decimals decrease and thus the corresponding fractions decrease. For Ex. 11-18 the students use only decimals; both fractions and decimals are used for Ex. 19-42. The students may refer to the completed chart from page 240 for assistance with these exercises.

Try This: Provide an opportunity for the students to discuss the example and to explain why they agree or disagree for Ex. 1. Examples such as this occur with decimals that do not terminate. The following is another similar example.

$\frac{1}{6}$	0.1666 . . .
$\frac{5}{6}$	0.8333 . . .
$\frac{6}{6}$, or 1	0.9999 . . .

Assessment

Use >, <, or = to make true statements.

1. $\frac{1}{2}+\frac{1}{3}$ ● $1\frac{1}{3}-\frac{3}{4}$ > **2.** $\frac{3}{4}\times\frac{2}{3}$ ● 1 <

List in order from least to greatest.

3. $\frac{1}{2}, \frac{3}{5}, \frac{5}{9}, \frac{3}{7}, \frac{5}{8}$ $\frac{3}{7}, \frac{1}{2}, \frac{5}{9}, \frac{3}{5}, \frac{5}{8}$ **4.** $\frac{1}{4}, \frac{2}{7}, \frac{2}{9}, \frac{1}{5}, \frac{3}{8}$ $\frac{1}{5}, \frac{2}{9}, \frac{1}{4}, \frac{2}{7}, \frac{3}{8}$

Add the fractions. Then add the equivalent decimals.

5. $3\frac{8}{9} + 1\frac{2}{3} = 5\frac{5}{9}$ 3.89 + 1.67 = 5.56 **6.** $1\frac{7}{8} + 2\frac{1}{2} = 4\frac{3}{8}$ 1.88 + 2.50 = 4.38

Subtract the fractions. Then subtract the equivalent decimals.

7. $2\frac{1}{5} - 1\frac{3}{10} = \frac{9}{10}$ 2.2 − 1.3 = 0.9 **8.** $4\frac{1}{2} - 2\frac{5}{6} = 1\frac{2}{3}$ 4.50 − 2.83 = 1.67

Multiply the fractions. Then multiply the equivalent decimals.

9. $\frac{3}{7}\times\frac{4}{5}$ $\frac{12}{35}$ **10.** $\frac{2}{9}\times\frac{1}{4}$ $\frac{1}{18}$ 9. 0.43 × 0.8 = 0.344 10. 0.22 × 0.25 = 0.0550

Divide the fractions. Then divide the equivalent decimals.

11. $\frac{4}{9}\div 4$ $\frac{1}{9}$ 0.44 ÷ 4 = 0.11 **12.** $\frac{3}{4}\div 2$ $\frac{3}{8}$ 0.750 ÷ 2 = 0.375

Pages 242-243

OBJECTIVE

Use a calculator to express fractions as decimals; use a calculator to multiply whole numbers and fractions

Fractions on a Calculator

On a calculator, a decimal can be used in place of a fraction. Divide the numerator by the denominator to find the decimal.

For $\frac{1}{2}$, press 1 ÷ 2 = . The display will show $\frac{1}{2}$ as 0.5

For a fraction, a calculator can show the *exact* equivalent decimal, or . . .

For $\frac{17}{3}$, press 17 ÷ 3 = . The display will show $\frac{17}{3}$ as 5.6666666 .

. . . the calculator shows a decimal that is *close* to the equivalent decimal.

Complete.

1. For $\frac{3}{4}$, press 3 ÷ 4 = . The display will show $\frac{3}{4}$ as ? 0.75 .
2. For $\frac{2}{5}$, press 2 ÷ 5 = . The display will show $\frac{2}{5}$ as ? 0.4 .
3. For $\frac{13}{2}$, press 13 ÷ 2 = . The display will show $\frac{13}{2}$ as ? 6.5 .
4. For $\frac{3}{8}$, press 3 ÷ 8 = . The display will show $\frac{3}{8}$ as 0.375 .
5. For $\frac{1}{3}$, press 1 ÷ 3 = . The display will show $\frac{1}{3}$ as 0.3333333 .
6. For $\frac{9}{4}$, press 9 ÷ 4 = . The display will show $\frac{9}{4}$ as ? 2.25 .
7. For $\frac{7}{10}$, press 7 ÷ 10 = . The display will show $\frac{7}{10}$ as ? 0.7 .
8. For $\frac{29}{8}$, press 29 ÷ 8 = . The display will show $\frac{29}{8}$ as 3.625 .
9. For $\frac{2}{9}$, press 2 ÷ 9 = . The display will show $\frac{2}{9}$ as 0.2222222 .

Calculator

242

LESSON ACTIVITY

Before Using the Pages

- Review that a fraction can be expressed as a decimal by dividing the numerator by the denominator. Write the fractions $\frac{14}{5}$ and $\frac{4}{9}$ on the board and have the students express each as a decimal. Note the repeating decimal in the quotient for $9\overline{)4}$.

Using the Pages

- Direct the students to read the examples at the top of page 242. To show a fraction on a calculator, note that division is used to express the fraction as a decimal. Point out that 0.5 is the exact equivalent decimal for $\frac{1}{2}$, whereas 5.6666666 is close to the equivalent decimal for $\frac{17}{3}$. Point out that, on a calculator, the repeating digits of the decimal are shown across the digit display.

  For the examples on page 243, begin by noting that $2 \times \frac{1}{6}$ and $\frac{1}{6} \times 2$ can be expressed as $\frac{1}{3}$. Then discuss the use of a calculator to multiply a fraction and a whole number. Discuss the steps of each example and compare the results showing the two ways $\frac{1}{3}$ may be represented on a calculator.

  Ex. 1-9 involve the skill presented on page 242 and Ex. 10-17 require the skills shown on page 243. You may wish to discuss both pages and then have the students complete all the exercises, or you may wish to discuss page 242, assign Ex. 1-9, and then discuss page 243 and assign Ex. 10-17.

- The repeating decimals for exercises on these pages are expressed as eight-digit numbers because the calculator shows only eight digits. Ex. 2 and 7 will show only two digits, but Ex. 5 and 9 will show eight digits. After the students have completed Ex. 10-17, have them compare corresponding exercises, for example, Ex. 10 and 11.

A calculator can display a number in different ways.

For $2 \times \frac{1}{6}$,

use		The display will show
2	[×]	2.
1	[÷]	2.
6	[=]	0.3333333

For $\frac{1}{6} \times 2$,

use		The display will show
1	[÷]	1.
6	[×]	0.1666666
2	[=]	0.3333332

On a calculator, 0.3333333 may represent $\frac{1}{3}$.

On a calculator, 0.3333332 may represent $\frac{1}{3}$.

What will the display show after each [=] ?

10. $3 \times \frac{1}{3} = 1$

Use 3 [×] 1 [÷] 3 [=] . 1

11. $\frac{1}{3} \times 3 = 1$

Use 1 [÷] 3 [×] 3 [=] . 0.9999999
(1.0000000 – if calculator rounds)

12. $3 \times \frac{2}{3}$

Use 3 [×] 2 [÷] 3 [=] . 2

13. $\frac{2}{3} \times 3$

Use 2 [÷] 3 [×] 3 [=] . 1.9999998
(2.0000000 – if calculator rounds)

14. $6 \times \frac{7}{9}$

Use 6 [×] 7 [÷] 9 [=] . 4.6666666
(4.6666667 – if calculator rounds)

15. $\frac{7}{9} \times 6$

Use 7 [÷] 9 [×] 6 [=] . 4.6666662
(4.6666667 – if calculator rounds)

16. $9 \times \frac{1}{12}$

Use 9 [×] 1 [÷] 12 [=] . 0.75

17. $\frac{1}{12} \times 9$

Use 1 [÷] 12 [×] 9 [=] . 0.7499997
(0.7500000 – if calculator rounds)

243

RELATED ACTIVITIES

- If calculators are available, have the students use them to complete several exercises in this unit. The answers may be compared with those obtained without the use of a calculator.

OBJECTIVE

Restate word problems

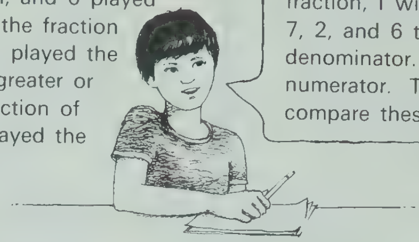
RELATED ACTIVITIES

- Choose appropriate word problems from previous lessons and have students restate them in their own words.
- Students may write word problems similar to those on page 244 for other students to restate in their own words.

Restating a Problem in Your Own Words

Sometimes stating a problem in your own words can help you understand the problem and how to solve it.

This year 9 of the 20 students who performed at the music concert played the piano. Last year 7 students played the piano, 2 played the violin, and 6 played the trumpet. Is the fraction of students who played the piano this year greater or less than the fraction of students who played the piano last year?



Write each of these problems in your own words. Then solve the problem. Answers will vary for writing the problems in their own words

1. Pam, Paul, and Pat agreed to sell 100 tickets together. After they had sold $\frac{1}{10}$ of them, they each took the same number of the remaining tickets to sell. How many tickets did each take? 30

2. Last year $\frac{5}{8}$ of the 32 students in Sally's class worked on the music concert. Half of them played an instrument. How many students in Sally's class played an instrument for the concert? 10

3. Sam bought two jars of paint of the same size, one with purple paint and the other with blue. He used $\frac{3}{4}$ of the purple paint and $\frac{2}{5}$ of the blue paint. He then poured the remainder of the blue paint into the jar of purple paint. How full was the jar? $\frac{17}{20}$

4. Four students made the background for the first part of the concert. They used $1\frac{1}{2}$ rolls of mural paper. Three other students used $2\frac{2}{3}$ rolls of mural paper for the background for the last part of the concert. How many rolls of mural paper did they have to take from the school stockroom? 5

PROBLEM SOLVING

244

LESSON ACTIVITY

Using the Page

- Ask a student to read the title and the first statement on page 244. Ask another student to read the word problem. Discuss how the problem is restated. Some students may suggest other ways to restate the problem. Have the students solve the problem. Ask how restating the problem helped in finding the solution.
- After the students have completed Ex. 1-4, they may read ways of restating each problem. Emphasize that there are different ways of restating a problem, but that each involves the same information. The reason for restating a problem is to help understand it and find a solution.

Checking Up

Add. Show each sum in lowest terms.

1. $\frac{1}{7}+\frac{4}{7}$ $\frac{5}{7}$ 2. $\frac{5}{12}+\frac{3}{12}$ $\frac{2}{3}$ 3. $3\frac{2}{9}+1\frac{4}{9}$ $4\frac{2}{3}$ 4. $2\frac{2}{5}+5\frac{2}{5}$ $7\frac{4}{5}$

5. $\frac{3}{4}+\frac{3}{4}$ $1\frac{1}{2}$ 6. $\frac{5}{8}+\frac{3}{8}$ 1 7. $1\frac{7}{10}+2\frac{9}{10}$ $4\frac{3}{5}$ 8. $4\frac{2}{3}+5\frac{2}{3}$ $10\frac{1}{3}$

9. $\frac{1}{3}+\frac{3}{8}$ $\frac{17}{24}$ 10. $\frac{1}{4}+\frac{1}{6}$ $\frac{5}{12}$ 11. $3\frac{3}{7}+1\frac{2}{5}$ $4\frac{29}{35}$ 12. $1\frac{2}{9}+2\frac{7}{12}$ $3\frac{29}{36}$

13. $\frac{1}{2}+\frac{5}{6}$ $1\frac{1}{3}$ 14. $\frac{3}{4}+\frac{5}{8}$ $1\frac{3}{8}$ 15. $3\frac{4}{5}+6\frac{2}{3}$ $10\frac{7}{15}$ 16. $5\frac{5}{12}+1\frac{7}{8}$ $7\frac{7}{24}$

Subtract. Show each difference in lowest terms.

17. $\frac{4}{5}-\frac{2}{5}$ $\frac{2}{5}$ 18. $\frac{7}{8}-\frac{3}{8}$ $\frac{1}{2}$ 19. $3\frac{5}{6}-2\frac{1}{6}$ $1\frac{2}{3}$ 20. $4\frac{7}{12}-1\frac{5}{12}$ $3\frac{1}{6}$

21. $2\frac{1}{3}-1\frac{2}{3}$ $\frac{2}{3}$ 22. $7\frac{3}{10}-2\frac{8}{10}$ $4\frac{1}{2}$ 23. $2-\frac{1}{4}$ $1\frac{3}{4}$ 24. $7-\frac{7}{9}$ $6\frac{2}{9}$

25. $\frac{1}{2}-\frac{1}{5}$ $\frac{3}{10}$ 26. $\frac{8}{9}-\frac{5}{6}$ $\frac{1}{18}$ 27. $4\frac{5}{7}-4\frac{1}{2}$ $\frac{3}{14}$ 28. $5\frac{7}{8}-3\frac{7}{12}$ $2\frac{7}{24}$

29. $8\frac{1}{3}-3\frac{7}{9}$ $4\frac{5}{9}$ 30. $3\frac{1}{4}-2\frac{5}{6}$ $\frac{5}{12}$ 31. $5\frac{3}{10}-1\frac{4}{5}$ $3\frac{1}{2}$ 32. $1\frac{1}{6}-\frac{2}{3}$ $\frac{1}{2}$

Multiply. Show each product in lowest terms.

33. $\frac{1}{2}\times\frac{1}{4}$ $\frac{1}{8}$ 34. $\frac{2}{3}\times\frac{3}{8}$ $\frac{1}{4}$ 35. $\frac{4}{5}\times\frac{1}{6}$ $\frac{2}{15}$ 36. $\frac{7}{8}\times\frac{3}{4}$ $\frac{21}{32}$

37. $\frac{4}{9}\times\frac{5}{6}$ $\frac{10}{27}$ 38. $\frac{4}{7}\times\frac{7}{12}$ $\frac{1}{3}$ 39. $\frac{3}{5}\times 7$ $4\frac{1}{5}$ 40. $2\times\frac{3}{4}$ $1\frac{1}{2}$

41. $\frac{1}{2}\times 5$ $2\frac{1}{2}$ 42. $20\times\frac{7}{10}$ 14 43. $6\times 3\frac{1}{3}$ 20 44. $3\times 1\frac{2}{7}$ $3\frac{6}{7}$

45. $2\frac{3}{8}\times 8$ 19 46. $4\frac{5}{12}\times 4$ $17\frac{2}{3}$ 47. $\frac{7}{8}\times 2\frac{2}{7}$ 2 48. $\frac{5}{9}\times 2\frac{2}{5}$ $1\frac{1}{3}$

Find the reciprocals. Complete.

49. $\frac{3}{8}$ $\frac{8}{3}$ or $2\frac{2}{3}$ 50. 4 $\frac{1}{4}$ 51. $1\frac{2}{3}$ $\frac{3}{5}$ 52. $\frac{3}{4}\times\frac{\blacksquare}{\blacksquare}=1$ $\frac{4}{3}$ 53. $7\times\frac{\blacksquare}{\blacksquare}=1$ $\frac{1}{7}$

Divide. Show each quotient in lowest terms.

54. $\frac{4}{5}\div 4$ $\frac{1}{5}$ 55. $\frac{1}{3}\div 2$ $\frac{1}{6}$ 56. $\frac{6}{7}\div 8$ $\frac{3}{28}$ 57. $\frac{3}{4}\div 5$ $\frac{3}{20}$

58. $1\div\frac{1}{9}$ 9 59. $6\div\frac{3}{8}$ 16 60. $3\div\frac{3}{4}$ 4 61. $9\div\frac{3}{5}$ 15

62. $\frac{5}{6}\div\frac{5}{8}$ $1\frac{1}{3}$ 63. $\frac{7}{10}\div\frac{3}{8}$ $1\frac{13}{15}$ 64. $\frac{4}{9}\div\frac{2}{3}$ $\frac{2}{3}$ 65. $\frac{1}{2}\div\frac{5}{6}$ $\frac{3}{5}$

Write a decimal with up to two places for each fraction.

66. $\frac{1}{2}$ 0.5 67. $\frac{3}{4}$ 0.75 68. $\frac{9}{10}$ 0.9 69. $\frac{2}{5}$ 0.4 70. $\frac{3}{12}$ 0.25 71. $\frac{6}{8}$ 0.75

72. $\frac{2}{3}$ 0.67 73. $\frac{4}{9}$ 0.44 74. $\frac{7}{8}$ 0.88 75. $\frac{5}{6}$ 0.83 76. $\frac{5}{12}$ 0.42 77. $\frac{1}{7}$ 0.14

245

Page 245

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- Adapt the game "Greatest Sum" described on page T 379 for adding, subtracting, multiplying, or dividing fractions. Use a numeral card for each digit from 1 to 9 and the appropriate format from those shown below. Have the students write a digit for each ■.

Addition: the player with the greatest sum scores one point.

$$\frac{\blacksquare}{\blacksquare}+\frac{\blacksquare}{\blacksquare}$$

Subtraction: the player with the least difference scores one point.

$$\frac{\blacksquare}{\blacksquare}-\frac{\blacksquare}{\blacksquare}$$

Multiplication: the player with the greatest product scores one point.

$$\frac{\blacksquare}{\blacksquare}\times\frac{\blacksquare}{\blacksquare},\quad \frac{\blacksquare}{\blacksquare}\times\blacksquare,\quad \blacksquare\times\frac{\blacksquare}{\blacksquare}$$

Division: the player with the least quotient scores one point.

$$\frac{\blacksquare}{\blacksquare}\div\frac{\blacksquare}{\blacksquare},\quad \frac{\blacksquare}{\blacksquare}\div\blacksquare,\quad \blacksquare\div\frac{\blacksquare}{\blacksquare}$$

You may wish to adapt the game for numbers in mixed form.

Skills	Exercises	Related Pages
Add fractions and numbers in mixed form with like denominators, no regrouping	1-4	T 236-T 237
Add fractions with unlike denominators, no regrouping	9, 10	T 238-T 239
Add fractions and numbers in mixed form, regrouping	5-8, 11-16	T 240-T 241
Subtract fractions and numbers in mixed form with like denominators, no regrouping	17-20	T 236-T 237
Subtract fractions with unlike denominators, no regrouping	25, 26	T 242-T 243
Subtract fractions, regrouping	21-24, 27-32	T 244-T 245
Multiply fractions	33-38	T 248-T 249
Multiply fractions and whole numbers	39-46	T 250-T 251
Multiply fractions and numbers in mixed form	47, 48	T 248-T 249
Find reciprocals	49-53	T 253
Divide fractions	54-65	T 254-T 255
Express fractions as decimals	66-77	T 258-T 259

Comments

Students having difficulty with this unit may require more experience with models. Also, it may be necessary to provide more practice with the prerequisite skills identified for the lessons. Some errors may be a result of confusing two operations, for example, multiplication and addition.

Encourage the students to check subtraction exercises by using addition and to check division exercises by using multiplication.

Ratio

The first lesson of this unit introduces the term *ratio* to refer to a comparison of two numbers and shows how to represent a ratio by using a colon between the numerals and by writing them in the form of a fraction. Multiplication and division are used to find equivalent ratios and the missing term in a pair of equivalent ratios in the same manner as for equivalent fractions. Cross products are also used to find missing terms and to establish whether pairs of ratios are equivalent. The concepts of a ratio are then applied to situations where constant rates apply. Finding unit rates then leads to finding unit prices and students are given experiences in determining which of two items is the better buy. The *Problem Solving* lesson introduces the role that chance plays in determining results and thereby provides an informal introduction to the topic of probability. Two other *Problem Solving* features challenge the students' abilities to think clearly. One *Keeping Sharp* feature provides exercises to maintain skills in working with decimals, which are needed in the next unit. These skills are also assessed in the *Checking Skills* exercises on page 261.

Prerequisite Skills

- write fractions
- find a common factor of two numbers

Unit Outcomes

- interpret a ratio
- write a ratio using words, using the symbol :, and using fraction notation
- use multiplication or division to find equivalent ratios
- write ratios in simplest form
- write ratio tables
- use multiplication or division to find the missing term in two equivalent ratios
- use cross products to find the missing term in two equivalent ratios
- use division to find unit rates
- choose a rate to describe a situation
- find a unit price to the nearest cent
- compare unit prices
- solve word problems involving ratios and rates
- consider the chances of various possibilities

Background

Numbers may be compared in a variety of ways. The difference of two numbers can be found by subtracting one from the other and determining which number is greater (less) than the other and by how much. Certain numbers may be compared in terms of multiplication, if one is a multiple of the other. For instance, 42 and 6 may be compared by stating that 42 is 7 times 6 ($7 \times 6 = 42$). An inverse form of comparison of these two numbers involves a fraction, such as 6 is one-seventh of 42, or $\frac{1}{7}$ of 42 is 6. Two numbers may also be compared in a *ratio* which uses them in one expression. Two forms of notation are commonly used to write a ratio; one separates the numerals by a colon, as in 3:4, and the other uses the fractional form $\frac{3}{4}$. Both of these are interpreted as comparing the number 3 with the number 4 and they are read "three to four". The numerator of $\frac{3}{4}$ and the first named number of 3:4 indicate that the 3 is being compared to the 4, not vice versa. In diagram A, there are 6 shapes — 2 squares, 1 triangle, and 3 circles. The ratio of the number of squares to the number of shapes is 2 to 6 (2:6, $\frac{2}{6}$). The ratio of the number of squares to the number of circles is 2 to 3 (2:3, $\frac{2}{3}$). The ratio of the number of circles to the number of triangles is 3 to 1 (3:1, $\frac{3}{1}$). Thus, a ratio may be used to compare the numbers of items in a set. Part of the set may be compared with the whole set, or part of the set may be compared with another part of the set.

A

Ratios may be expressed in equivalent forms using the same methods as for finding equivalent fractions. Refer to the Overview for Unit 10, paying particular attention to the parts dealing with equivalent fractions. A review of the lessons on pages 200-205 is also suggested. Tables of equivalent ratios may be made by using counting sequences, and by using multiplication and division. For example, in a package of cookies, there are 7 vanilla cookies and 5 chocolate cookies. The ratio of the number of vanilla cookies to the number of chocolate cookies (7 to 5, 7:5, $\frac{7}{5}$) may be extended in a ratio table by counting by sevens and by fives, or by multiplying 7 and 5 by the same factor. For example, the numbers for four packages may be found by counting or adding 4 sevens and 4 fives, or by multiplying 4 and 7 and 4 and 5. The ratios 7:5, 14:10, 21:15, and so on, all represent the same ratio which is 7:5 in its simplest form. Since these ratios are equivalent, they may be written in pairs with the symbol = between them.

Number of vanilla cookies	7	14	21	28
Number of chocolate cookies	5	10	15	20

7:5 = 14:10 $\quad$ 14:10 = 21:15 $\quad$ 7:5 = 21:15

$\frac{7}{5} = \frac{14}{10}$ $\quad$ $\frac{14}{10} = \frac{21}{15}$ $\quad$ $\frac{7}{5} = \frac{21}{15}$

Multiplication and division may also be used to find missing terms in equivalent ratios. In the example shown, division indicates that the factor 4 ($20 \div 5$) is to be multiplied by 3 to find the missing term 12 (3×4). These two operations are also involved in the method using cross products. In the same example, multiplication is used to find the product 60 (3×20) and division is used to find the missing term 12 ($60 \div 5$).

$$\frac{3}{5} = \frac{\blacksquare}{20}$$

There is little difference between the concepts of ratio and rate. The term *ratio* may be applied to comparison of abstract numbers, whereas the term *rate* is usually used to relate or to compare unlike quantities. Such expressions as words per minute, cost per kilometre, cans in each carton, and kilometres per hour are typical of rates in everyday life. Rates are usually expressed on a many-to-one basis, such as 80¢ for one dozen, although the unit may be more than one, as in 79¢ for 2 oranges, or 15 letters every 3 weeks. A unit rate, that is, the rate or price for one, can be established by division. It is often advisable to use data from larger samplings to establish unit rates. For example, if a factory produces 216 finished items in six hours, an average rate of 36 items per hour is found by dividing 216 by 6, whereas an actual production of that number may not have been achieved in any one hour.

Merchandising products is a complex operation. To suit the needs and desires of different customers, the same products are often packaged in different sizes, and competitive products are sometimes packaged differently to attract buyers. Customers must then determine which is the better buy, that is, the more economical. There are two ways of comparing prices: the cost for one item or for one unit of measurement may be found (69¢ per can), or the quantity of material that can be bought for one unit of money may be calculated (4.3 mL / 1¢). Unit pricing is usually calculated by finding the cost of one item or one unit of it. Many large supermarkets now display unit prices of items, but there are many smaller stores which do not. A careful shopper uses estimation and mental calculations in such circumstances. For example, if two kinds of comparable vitamin pills are available for $2.50 for 100 or $3.60 for 150, each pill of the second kind is cheaper (2.4¢ versus 2.5¢). Unit pricing is an important application of mathematics in modern life and business.

Another important mathematical element in everyday life is that of probability. The outcome of an election or of a sporting event, or the likelihood of having a fire in one's home or an accident on a holiday are not merely matters of chance. Different factors affect each type of situation. Sometimes they are relatively simple, such as in flipping a coin or in tossing a die. With a coin the chance of "heads" showing is 1 out of 2, and the chance of any one number turning up on a die is 1 out of 6. The *Problem Solving* lesson in this unit provides a simple introduction to the topic of probability. Probability — defined as the study of phenomena in which chance plays a part — became a branch of mathematics in the seventeenth century when Blaise Pascal, one of the foremost mathematicians in France, became interested in helping one of his friends who was a gambler.

Teaching Strategies

Since the first lesson on ratio shows a "standard" bicycle, it would be advisable to bring one into the classroom. By turning the bicycle upside down, it is easy to count the turns which the pedals and the rear wheel make. It may be necessary to restrain the wheel slightly, otherwise the coasting mechanism may add some unwanted revolutions. A mark or a tab of paper on the wheel should help one student to count the times the wheel turns, while another student counts the turns of the pedals. The results should be tabulated for different numbers of turns of the pedals. Then the students should count the teeth on the two gears and note whether there is any apparent relationship between the sets of numbers.

Number of turns of pedals	9	18	27	36	45
Number of turns of wheel	20	40	60	80	100

Number of teeth on back gear	18
Number of teeth on pedal gear	40

With the teacher's guidance for Ex. 1 of the *Try This* feature, the students will probably arrive at several generalizations, such as, if there are fewer teeth on the back gear, it makes the wheel turn more times and you go faster, or the difference of the numbers of teeth on the gears affects the speed — as the number of teeth decreases on the back gear you go faster and if you switch to more teeth on the pedal gear you go even faster.

The students should be watched as they write ratios lest they reverse the terms. For example, in the following diagram, there are two ratios comparing the number of triangles and the number of squares, 1 to 2, and 2 to 1, depending on which is named first. The ratio of the number of squares to the number of triangles is 1 to 2 (1:2, $\frac{1}{2}$) and the ratio of the number of triangles to the number of squares is 2 to 1 (2:1, $\frac{2}{1}$). The first number in the ratio corresponds to the first item named in the comparison.

Equivalent ratios may be found by multiplying both terms of a given ratio by any number or by dividing them by a common factor. These two operations are also used to find missing terms in pairs of equivalent ratios. Speed and accuracy in these processes depend on complete mastery of the basic facts. Many of the exercises on pages 248-255 can be completed at a glance by merely using basic facts. In this connection, knowing multiples of numbers is helpful in recognizing which factors may be used to perform the necessary operations. For example, in $\frac{27}{18} = \frac{\blacksquare}{2}$ an examination of the 2 and the 18 suggests that the factor 9 should be used, and knowledge of the basic fact $27 \div 9 = 3$ produces the missing term.

Expressing ratios in lowest terms utilizes common factors of two numbers. Using multiples of numbers is usually much faster for finding the greatest common factor than actually factoring the two numbers. At the mere mention of a single number, students should be able to recall possible factors. The number 12, for example, should immediately bring to mind 2, 6, 3, 4; and the number 32 should suggest 4 and 8. Students may, therefore, benefit from saying or writing multiples of each number from 2 to 9 to become even more aware of their factors. For example, if the multiples of 6 are well known, it would not take long to choose it as the factor required to express $\frac{18}{42}$ as $\frac{3}{7}$. Oral drill in naming two numbers at a time may also provide valuable practice in naming their greatest common factor; for example, 28 and 35 (7), 16 and 36 (4), 15 and 44 (1), and 30 and 18 (6).

The circle graph on page 255 involves ratios with money. Most students should be able to use decimal notation for showing cents in relation to a dollar, such as $\frac{0.40}{1}$, but some students may be able to complete the exercise more easily in terms of cents, with one dollar expressed as 100 cents ($\frac{40}{100}$). If any students use the latter approach, they should be directed to repeat the exercise with decimal notation for the amounts of money.

Ex. 5 on page 259 may challenge some of the more capable students to roll three dice at a time and record the results. They may also wish to read about Blaise Pascal and his work with probability.

Materials

transparent colored markers and an overhead projector; colored blocks

two similar triangles as described in *Before Using the Pages* on page T 272

centimetre ruler for each student

a die with three sides marked to show 3, two sides marked to show 2, and one side marked to show 1

Vocabulary

ratio	ratio table	unit rate
equivalent ratios	circle graph	unit price
simplest form	rate	vending machine

Pages 246-247

LESSON OUTCOME

Interpret a ratio; write a ratio using words, using the symbol : , and using fraction notation

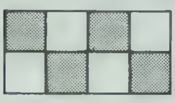
Materials
transparent colored markers and an overhead projector (optional); colored blocks

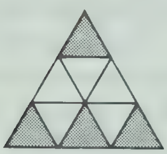
Vocabulary
ratio

Prerequisite Skills
Write fractions

Checking Prerequisite Skills
Write a fraction to show how much is shaded.

1.  $\frac{3}{7}$

2.  $\frac{4}{8}$

3.  $\frac{4}{9}$

Write the fraction

4. with numerator 2 and denominator 5. $\frac{2}{5}$
5. with numerator 11 and denominator 6. $\frac{11}{6}$

12 RATIO
Writing Ratios

For every two turns of the pedals on this bicycle, the rear wheel turns five times.

The **ratio** of turns of the pedals to turns of the rear wheel is 2 to 5

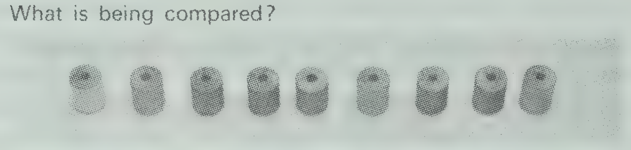
A ratio is the comparison of two like quantities.

A ratio can be shown with a colon. 2 : 5

A ratio can be shown as a fraction. $\frac{2}{5}$

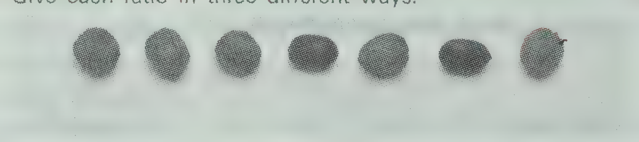
Working Together

What is being compared?



1. 1 to 3 blue thread to yellow thread
2. $\frac{3}{9}$ yellow thread to all the thread
3. 5 : 1 green thread to blue thread
4. 5 out of 9 green thread to all the thread

Give each ratio in three different ways.



5. walnuts to pecans 5 to 2, 5:2, $\frac{5}{2}$
6. pecans to walnuts 2 to 5, 2:5, $\frac{2}{5}$
7. walnuts to all the nuts 5 to 7, 5:7, $\frac{5}{7}$
8. pecans to all the nuts 2 to 7, 2:7, $\frac{2}{7}$

246

LESSON ACTIVITY

Before Using the Pages

- Use transparent markers with an overhead projector, or use colored blocks at the front of the room. Display 12 red markers and 12 blue markers. Say that there are 12 red markers *to* 12 blue markers, and that there are 24 markers in all. Write this information in a chart as indicated below. Remove 1 red marker and ask a student to describe what is shown. Record the information. Ask each of two students to remove a marker of their own color preference. Show the results in the chart. Repeat the procedure.

Red markers to blue markers			Total number of markers
12	to	12	24
11	to	12	23
11	to	10	21
9	to	9	18

Ask students to interpret different entries in the chart. For example, a student might describe the third entry by saying, "There are 11 red markers to 10 blue markers; 10 of the 21 markers are blue."

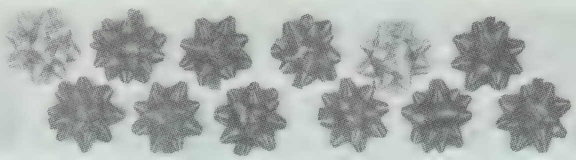
Using the Pages

- Direct the students' attention to the photograph of the bicycle on page 246. Discuss that it is a "standard" bicycle and ask how a standard bicycle differs from a five-speed or a ten-speed bicycle. Ask a student to read the title of the lesson and the statement below it. Introduce the term *ratio* and note that a ratio involves a comparison. Have students explain the three ways of showing a ratio — using the word "to", using a colon, and using fraction notation. Emphasize that each form of the given ratio is read "two to five".

Working Together: Before assigning Ex. 1-4, ask students to describe what is shown in the photograph. They may say, for example, that there are 9 spools of thread in all, of which 5 are green, 3 are yellow, and 1 is blue. These exercises demonstrate that a ratio can compare part of a group to the whole group (Ex. 2 and 4), or it can compare 2 distinct groups (Ex. 1 and 3).

Exercises

What is being compared?



1. 6 out of 12 gold bows to all the bows
2. 6 : 4 gold bows to pink bows
3. $\frac{2}{12}$ blue bows to all the bows
4. 4 to 2 pink bows to blue bows

Write each ratio in three different ways.

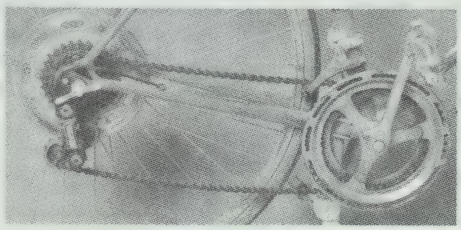
5. gold bows to blue bows 6 to 2, 6:2, $\frac{6}{2}$
6. blue bows to gold bows 2 to 6, 2:6, $\frac{2}{6}$
7. pink bows to all the bows 4 to 12, 4:12, $\frac{4}{12}$
8. pink bows to gold bows 4 to 6, 4:6, $\frac{4}{6}$

Write ratios for each of these in two other ways.

9. 2 of every 9 bicycles needed their brakes fixed. 2:9, $\frac{2}{9}$
10. 31 of every 100 bicycles sold were ten-speed bicycles. 31:100, $\frac{31}{100}$
11. The ratio of red bicycles to green bicycles was $\frac{7}{3}$. 7 to 3, 7:3
12. The ratio of new bicycles to used bicycles was 4:7. 4 to 7, $\frac{4}{7}$

On a ten-speed bicycle, there are two gear wheels at the pedals and five gear wheels on the back axle.

20 teeth



40 teeth

1. For the gear in the picture, for every 20 turns of the pedals, the rear wheel turns 40 times. Explain. Answers will vary

The other back gear wheels have 28, 24, 17, and 14 teeth. The other front gear wheel has 52 teeth.

2. Write the ratio of pedal turns to rear wheel turns for the other nine gears 28:40, 24:40, 17:40, 14:40, 28:52, 24:52, 20:52, 17:52, 14:52

PROBLEM SOLVING

247

RELATED ACTIVITIES

- If a ten-speed bicycle is available, it can be used to demonstrate the ratios for the *Problem Solving* feature. A three-speed bicycle and a five-speed bicycle can be used to find and demonstrate similar ratios.
- Students can work in pairs so that one student names a ratio. The other student uses markers to illustrate the ratio and then explains it. For example, the ratio "4 to 5" can be represented by 4 blue markers and 5 yellow markers and explained using the words "the number of blue markers to the number of yellow markers". It can also be represented by 4 blue markers and 1 yellow marker and explained using the words "the number of blue markers to the number of markers in all". To vary the activity, one student can display the markers and state what is being compared; the other student can state the ratio.
- Have students work in groups of three to prepare a list of ratios that compare items such as the following.
the number of chairs to the number of desks (32:32)
the number of textbooks to the number of students (160:32)
the number of batteries to the number of flashlights (2:1)

Ex. 5 and 6 emphasize the importance of the order of the numbers named in a ratio.

Exercises: Remind the students to pay particular attention to the order of the numbers named in a ratio. You may wish to discuss which ratios compare two distinct groups and which compare part of a group to the whole group.

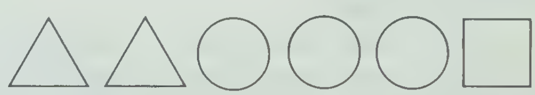
Problem Solving: Discuss the photograph and have the students share their knowledge of ten-speed bicycles. Pay particular attention to the gear wheels and the number of teeth on each gear wheel. Discuss their answers for Ex. 1. To help the students with Ex. 2, draw diagrams similar to the following on the board.

28, 24, 20, 17, 14 → 40	28, 24, 20, 17, 14 → 52

Ask students to use statements similar to the one in Ex. 1 to explain the ratios for Ex. 2.

Assessment

What is being compared?



1. 2 out of 6
2. 3 : 1
3. $\frac{1}{2}$

Write each ratio in three different ways.

4. circles to triangles
5. squares to all the shapes
6. triangles to squares
7. circles to all the shapes

1. triangles to all the shapes
2. circles to squares
3. squares to triangles
4. 3 to 2, 3:2, $\frac{3}{2}$
5. 1 to 6, 1:6, $\frac{1}{6}$
6. 2 to 1, 2:1, $\frac{2}{1}$
7. 3 to 6, 3:6, $\frac{3}{6}$

LESSON OUTCOME

Use multiplication or division to find equivalent ratios; write ratios in simplest form; write ratio tables

Materials

transparent colored markers and an overhead projector (optional); colored blocks

Vocabulary

equivalent ratios, simplest form, ratio table, vending machine

Prerequisite Skills

Write a ratio using words, using the symbol : , and using fraction notation; find a common factor of two numbers

Checking Prerequisite Skills

Write each ratio in two other ways.

1. 3 out of 7 3:7, $\frac{3}{7}$ **2.** $\frac{2}{6}$ 2 to 6, 2:6

3. 4:1 4 to 1, $\frac{4}{1}$ **4.** 5 to 8 5:8, $\frac{5}{8}$

Write a common factor for the numbers in each pair.

5. 9, 15 3 **6.** 6, 12 2, 3, or 6

Finding Equivalent Ratios

For this vending machine, past sales have shown that the ratio of the amount of white milk sold to the amount of chocolate milk sold is 3 to 2.

$$\frac{3}{2} \overset{3\cdot 2}{\underset{2\cdot 2}{=}} \frac{6}{4}$$

$$\frac{3}{2} \overset{3\cdot 3}{\underset{2\cdot 3}{=}} \frac{9}{6}$$

$$\frac{3}{2} \overset{3\cdot 4}{\underset{2\cdot 4}{=}} \frac{12}{8}$$

$\frac{3}{2}$, $\frac{6}{4}$, $\frac{9}{6}$, and $\frac{12}{8}$ are **equivalent ratios**.

Multiplying both numbers in a ratio by the same number gives an equivalent ratio.

3 out of every 6 of the cartons that fill the machine contain white milk. The ratio of the number of cartons of white milk to the total number of cartons is 3 to 6.

$$\frac{3}{6} \overset{3\div 3}{\underset{6\div 3}{=}} \frac{1}{2}$$

$\frac{3}{6}$ and $\frac{1}{2}$ are equivalent ratios.

$\frac{1}{2}$ is the **simplest form** for $\frac{3}{6}$.

Dividing both numbers in a ratio by the same number gives an equivalent ratio.

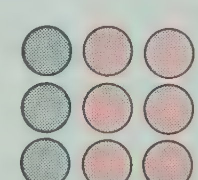
When the two numbers in a ratio are whole numbers and they have no common factor greater than 1, the ratio is in simplest form.

248

LESSON ACTIVITY

Before Using the Pages

- Use transparent colored markers with an overhead projector, or use colored blocks at the front of the room. Explain that you would like a student to place one blue marker on the overhead projector for every two red markers you display. Display 2 red markers, then 2 more red markers, and so on. For each new group of markers, ask for the ratio of the number of blue markers to the number of red markers. Also, ask if there is always 1 blue marker for every 2 red markers. Write the ratios on the board and ask how each suggests the rule "1 blue marker for every 2 red markers".



1:2, 2:4, 3:6, 4:8, . . .

- While the projector light is off, place 9 red markers and 3 blue markers on the overhead projector in any pattern. Turn on the projector light and ask for the ratio of the number of blue markers to the number of red markers (3:9). Ask if the rule for this display is the same as for the previous activity. Ask how to find the rule. Students may suggest rearranging the markers to show that there is 1 blue marker for every 3 red markers.

Using the Pages

- Ask what is shown in the photograph and elicit the term "vending machine". Note that there are six columns in the vending machine holding chocolate milk, white milk, and orange drink. Ask students to suggest why the machine holds more white milk than other drinks. Then ask a student to read the title of the lesson and the statement below it. Relate "3 to 2" to the fact that the vending machine shows cartons of white milk in three of the columns and cartons of chocolate milk in two. The photograph also shows 3 cartons of white milk for every 2 cartons of chocolate milk.

  Discuss the use of multiplication to find the equivalent

Working Together

Complete.

1. $\frac{2}{5} \overset{2 \cdot 3}{\underset{5 \cdot 3}{=}} \frac{■}{■}$ $\frac{6}{15}$

2. $\frac{8}{6} \overset{8 \cdot 2}{\underset{6 \cdot 2}{=}} \frac{■}{■}$ $\frac{16}{12}$

Multiply both numbers in each ratio by 4 to find equivalent ratios.

3. $\frac{1}{6}$ $\frac{4}{24}$ 4. 12:4 48:16

Complete.

5. $\frac{4}{14} \overset{4 \div 2}{\underset{14 \div 2}{=}} \frac{■}{■}$ $\frac{2}{7}$

6. $\frac{30}{45} \overset{30 \div 5}{\underset{45 \div 5}{=}} \frac{■}{■}$ $\frac{6}{9}$

Divide both numbers in each ratio by 3 to find equivalent ratios.

7. 6:9 2:3 8. $\frac{18}{15}$ $\frac{6}{5}$

Find three ratios equivalent to each. Other equivalent ratios are possible.

9. $\frac{1}{4}$ $\frac{2}{8}, \frac{3}{12}, \frac{4}{16}$ 10. 8:10 4:5, 12:15, 16:20

Give each of these ratios in simplest form.

11. 9:12 3:4 12. 25:100 1:4

Exercises

Write three ratios equivalent to each. Other equivalent ratios are possible

1. $\frac{1}{5}$ $\frac{2}{10}, \frac{3}{15}, \frac{4}{20}$ 2. 5:2 10:4, 15:6, 20:8 3. $\frac{3}{8}$ $\frac{6}{16}, \frac{9}{24}, \frac{12}{32}$ 4. 2:12 1:6, 3:18, 4:24 5. $\frac{15}{3}$ $\frac{5}{1}, \frac{10}{2}, \frac{20}{4}$ 6. $\frac{7}{10}$ $\frac{14}{20}, \frac{21}{30}, \frac{28}{40}$

Divide the numbers in each ratio by a common factor to get an equivalent ratio.

Ring the ratios you get that are in simplest form.

7. $\frac{8}{10}$ $\frac{4}{5}$ 8. 16:6 8:3 9. $\frac{300}{100}$ $\frac{12}{4}, \frac{6}{2}, \frac{3}{1}$ 10. 3:12 1:4 11. $\frac{35}{45}$ $\frac{7}{9}$ 12. $\frac{60}{24}$ $\frac{30}{12}, \frac{5}{2}$

Write each in simplest form.

13. $\frac{20}{25}$ $\frac{4}{5}$ 14. 14 : 2 7:1 15. $\frac{10}{6}$ $\frac{5}{3}$ 16. 27:21 9:7 17. $\frac{8}{12}$ $\frac{2}{3}$ 18. 50:100 1:2

Complete this **ratio table** for the vending machine.

		· 2	· 3	· 4	· 5	· 6
19. Cartons of white milk	3	6	9 ?	12 ?	15 ?	18 ?
Cartons of chocolate milk	2		6 ?	8 ?	10 ?	12 ?

For the vending machine, make ratio tables comparing

20. the number of cartons of each kind of drink with the total number of cartons. Tables are shown on page T372

249

RELATED ACTIVITIES

- Have the students work with attribute blocks for ratio activities as described in "Attribute Challenges" on page T381.
- Have students work with balance scales and such objects as sticks of new chalk, erasers, keys, washers, nails, scissors, corks, pennies, and so on. Have them place, for example, nails on one side of the scales and washers on the other side, to balance the two sides. Then have them write the corresponding ratio tables.

nails	3	6	9	12	15
washers	7				

The activity may be adapted for finding the mass of one object and writing the ratio table to determine the mass of several identical objects.

pennies	1	2	3	4
mass in grams				

- Ask students to read some of the completed exercises aloud to emphasize that the fraction $\frac{1}{5}$, for instance, is read "one to five" when it represents a ratio.
- The students may be interested in conducting a survey to determine the number of students who prefer white milk, chocolate milk, or orange drink. The results may be compared as ratios in simplest form.

ratios $\frac{6}{4}$, $\frac{9}{6}$, and $\frac{12}{8}$. Emphasize that each term of the ratio $\frac{3}{2}$ is multiplied by the same number to give the terms of an equivalent ratio. Point out that each of these equivalent ratios shows the ratio of the number of cartons of white milk to the number of cartons of chocolate milk; that is, it compares one group with another group.

Have students read and help to explain the second example, noting the use of division. Point out that the ratio 3:6 compares part of a group to the whole group. Introduce the term *simplest form* and have students express its meaning in their own words. Develop that equivalent ratios can be found in the same way that equivalent fractions are found, that is, by either multiplying or dividing each term by the same number.

Working Together: Ex. 1-8 guide the students by providing the multiplier or the divisor for each term of the ratio. Lead the students to realize that many answers are possible for Ex. 9 and 10. For Ex. 11, dividing by 3, the only factor common to 9 and 12, results in the simplest form; for Ex. 12, the students may divide by 5 twice or by 25 once.

Exercises: Before the students begin, discuss the term *ratio table* for Ex. 19. Ask how many ratio tables are required for Ex. 20.

Assessment

Write three ratios equivalent to each. Other equivalent ratios are possible.

1. $\frac{1}{3}$ $\frac{2}{6}, \frac{3}{9}, \frac{4}{12}$ 2. 6:5 12:10, 18:15, 24:20

Divide the numbers in each ratio by a common factor to get an equivalent ratio.

3. $\frac{20}{4}$ $\frac{10}{2}, \frac{5}{1}$ 4. 6:10 3:5

Write each in simplest form.

5. $\frac{25}{50}$ $\frac{1}{2}$ 6. 12:9 4:3

Complete the ratio table.

7. Centimetres	100	200	300	400
Metres	1	2	3	4

Pages 250-251

LESSON OUTCOME

Use multiplication or division to find the missing term in two equivalent ratios

Materials

two similar triangles as described in *Before Using the Pages*, centimetre ruler

Prerequisite Skills

Use multiplication or division to find equivalent ratios

Checking Prerequisite Skills

Write three ratios equivalent to each.

1. 5:7 **2.** $\frac{4}{3}$

Divide the numbers in each ratio by a common factor to find an equivalent ratio.

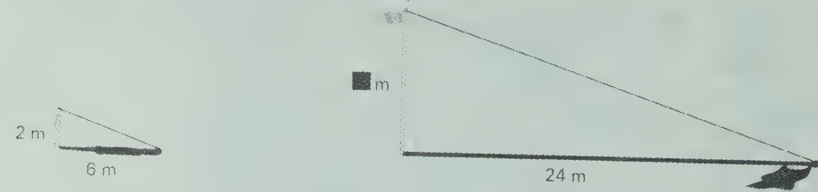
3. $\frac{8}{16}$ $\frac{4}{8}, \frac{2}{4}, \frac{1}{2}$ **4.** 15:3 5:1

Other equivalent ratios are possible.

1. 10:14, 15:21, 20:28 2. $\frac{8}{6}, \frac{12}{9}, \frac{16}{12}$

Finding the Missing Term

Geometric figures that have the same shape are similar. For similar figures, the ratios of the lengths of matching sides are equivalent.



The two triangles shown in the picture are similar.

The ratio of the length of the pole's shadow to the length of the person's shadow is $\frac{24}{6}$.

The ratio of the height of the pole to the height of the person is $\frac{■}{2}$.

The ratios

$\frac{\text{length of pole's shadow}}{\text{length of person's shadow}}$ and $\frac{\text{height of pole}}{\text{height of person}}$ are equivalent.

To find the real height of the pole,

write $\frac{24}{6} = \frac{■}{2}$.

To find the missing term,

think 6 $\xrightarrow{6 \div 3}$ 2

Then divide 24 by 3.

$$\frac{24}{6} \xrightarrow[6 \div 3]{24 \div 3} \frac{8}{2}$$

The real height of the pole is 8 m.

Here is an example of finding the missing term by multiplication.

$\frac{3}{4} = \frac{15}{■}$

To find the missing term,

think 3 $\xrightarrow{3 \cdot 5}$ 15

Then multiply 4 by 5.

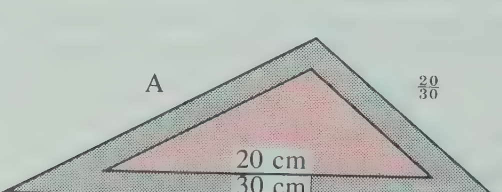
$$\frac{3}{4} \xrightarrow[4 \cdot 5]{3 \cdot 5} \frac{15}{20}$$

250

LESSON ACTIVITY

Before Using the Pages

- Cut two similar triangles from different colors of construction paper, for example, a triangle having sides 10 cm, 14 cm, and 20 cm long, and a triangle having sides 15 cm, 21 cm, and 30 cm long. Ask whether the triangles are the same size and whether they have the same shape. To help show that they have the same shape, paste the smaller triangle on top of the larger triangle (A).



Have students identify one pair of matching sides, measure their lengths, and write the ratio comparing the length of the side of the smaller triangle to the length of the side of the larger triangle (A). Ask the students to write the ratio in lowest terms (B). Repeat this for the other pairs of matching sides (C). Students can discover that the ratios for matching sides are equivalent.

B

$$\frac{20}{30} \xrightarrow[30 \div 10]{20 \div 10} \frac{2}{3}$$

C

$$\frac{10}{15} \xrightarrow[15 \div 5]{10 \div 5} \frac{2}{3} \qquad \frac{14}{21} \xrightarrow[21 \div 7]{14 \div 7} \frac{2}{3}$$

Ask what name describes figures that have the same shape but not necessarily the same size.

Using the Pages

- Students used multiplication and division in the lessons on pages 200 and 201 and pages 202 and 203 to find the missing term in two equivalent fractions. In this lesson, the same procedure is used to find the missing term in two equivalent ratios.

Read the statements at the top of page 250. Recall that similar figures have the same shape but not necessarily the same size. Have the students identify pairs of matching

Working Together

Complete each of these.

1. $\frac{3}{4} \overset{3 \cdot 2}{\underset{4 \cdot 2}{=}} \frac{6}{\blacksquare}$ 8
2. $\frac{9}{6} \overset{9 \cdot ?}{\underset{6 \cdot 3}{=}} \frac{\blacksquare}{18}$ 27 (? = 3)
3. $\frac{2}{5} \overset{2 \cdot ?}{\underset{5 \cdot ?}{=}} \frac{20}{\blacksquare}$ 50 (? = 10, 10)
4. $\frac{15}{10} = \frac{\blacksquare}{100}$ 150
5. $\frac{16}{8} \overset{16 \; 8}{\underset{8 \; 8}{=}} \frac{2}{\blacksquare}$ 1
6. $\frac{3}{21} \overset{3 \; ?}{\underset{21 \; 3}{=}} \frac{\blacksquare}{7}$ 1 (? = 3)
7. $\frac{25}{10} \overset{25 \; ?}{\underset{10 \; ?}{=}} \frac{5}{\blacksquare}$ 2 (? = 5, 5)
8. $\frac{12}{9} = \frac{\blacksquare}{3}$ 4

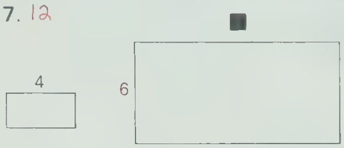
Exercises

Find the value for ■ that makes the ratios equivalent.

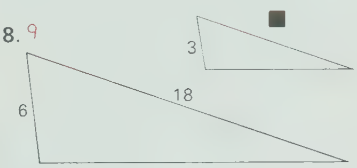
1. $\frac{7}{10} = \frac{\blacksquare}{20}$ 14
2. $\frac{8}{32} = \frac{1}{\blacksquare}$ 4
3. $\frac{2}{3} = \frac{\blacksquare}{27}$ 18
4. $\frac{42}{7} = \frac{6}{\blacksquare}$ 1
5. $\frac{12}{10} = \frac{120}{\blacksquare}$ 100
6. $\frac{27}{18} = \frac{\blacksquare}{2}$ 3
7. $\frac{6}{9} = \frac{30}{\blacksquare}$ 45
8. $\frac{20}{100} = \frac{\blacksquare}{10}$ 2
9. $\frac{33}{24} = \frac{\blacksquare}{8}$ 11
10. $\frac{3}{4} = \frac{\blacksquare}{100}$ 75
11. $\frac{9}{4} = \frac{54}{\blacksquare}$ 24
12. $\frac{24}{30} = \frac{4}{\blacksquare}$ 5
13. $\frac{75}{50} = \frac{15}{\blacksquare}$ 10
14. $\frac{3}{1} = \frac{\blacksquare}{4}$ 12
15. $\frac{3}{5} = \frac{60}{\blacksquare}$ 100
16. $\frac{20}{24} = \frac{\blacksquare}{6}$ 5

The figures in each pair are similar.
Find the value of ■ for each figure.

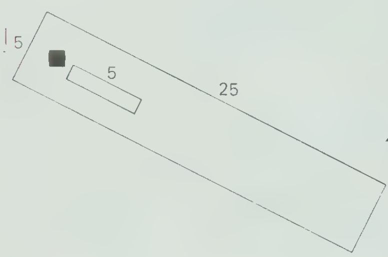
17. 12



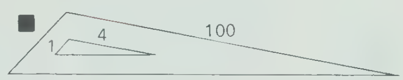
18. 9



19. 15



20. 25



RELATED ACTIVITIES

- Students may use equivalent ratios and the scale of a map to find the actual distance between two places. Have them use a centimetre ruler to measure the distance on the map between two cities, for example, 27 cm. The scale of the map, for example, 1 cm to 45 km, may be written as the ratio $\frac{1}{45}$. The actual distance in kilometres is the missing term in the following pair of equivalent ratios.

$$\frac{1}{45} = \frac{27}{\blacksquare}$$

- Adapt the third activity on page T 221 for ratios.

sides for the two triangles shown. Lead the students through the examples that develop the two equivalent ratios. Ask students to explain the use of division to find the missing term for $\frac{24}{6} = \frac{\blacksquare}{2}$. Then have students help to explain the use of multiplication to find the missing term in $\frac{3}{4} = \frac{15}{\blacksquare}$.

Working Together: Ex. 1-3 and Ex. 5-7 assist the students by indicating all or some of the steps. Emphasize that both terms of a ratio must be either multiplied or divided by the same number to find an equivalent ratio.

Exercises: Ex. 17-20 involve the concept that for similar figures the ratios of the lengths of matching sides are equivalent. The students are required to identify matching sides, to write equivalent ratios with a missing term, and then find the missing term.

An oral explanation for several completed exercises would be beneficial. For example, for Ex. 4, ask whether multiplication or division was used to find the missing term and ask what number was used as the multiplier or the divisor.

Assessment

Find the value for ■ that makes the ratios equivalent.

1. $\frac{1}{6} = \frac{\blacksquare}{12}$ 2
2. $\frac{3}{9} = \frac{1}{\blacksquare}$ 3
3. $\frac{20}{15} = \frac{\blacksquare}{3}$ 4
4. $\frac{36}{30} = \frac{6}{\blacksquare}$ 5
5. $\frac{5}{25} = \frac{\blacksquare}{100}$ 20
6. $\frac{7}{1} = \frac{28}{\blacksquare}$ 4

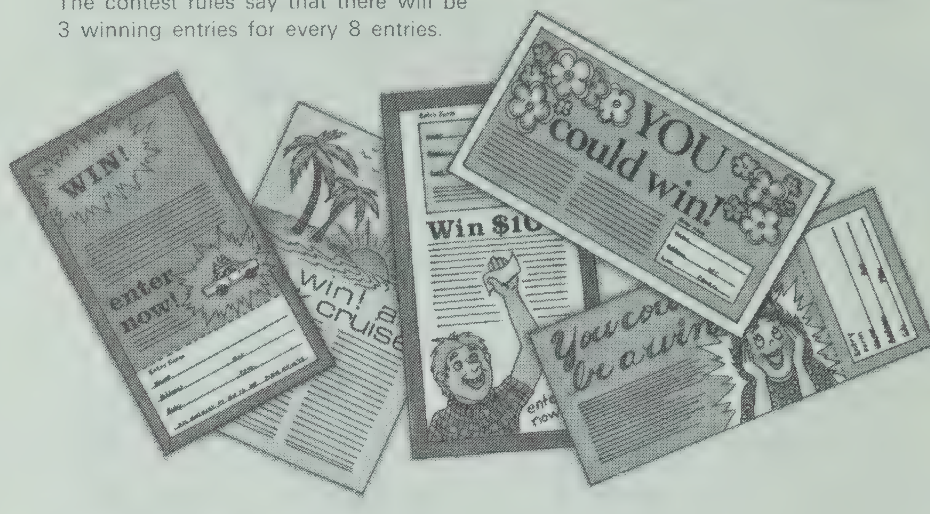
Pages 252-253

LESSON OUTCOME

Use cross products to find the missing term in two equivalent ratios; solve related word problems

Finding the Missing Term Using Cross Products

The contest rules say that there will be 3 winning entries for every 8 entries.



There were 5832 entries.
How many winning entries will there be?

The ratio of winning entries to total entries will be $\frac{3}{8}$.

To find how many of the 5832 entries will be winners, write $\frac{3}{8} = \frac{■}{5832}$

and use cross products.

$$\frac{3}{8} = \frac{■}{5832} \quad \longrightarrow \quad 8 \times ■ \qquad 3 \times 5832$$

$$8 \times ■ = 3 \times 5832$$
$$8 \times ■ = 17\,496$$
$$17\,496 \div 8 = 2187$$
$$8 \times 2187 = 17\,496$$

The number of winning entries will be 2187.

252

LESSON ACTIVITY

Before Using the Pages

- Assign two exercises to review the work of the previous lesson. Ask two students to show their work on the board and explain the procedure. Ensure that the fractions are read as ratios, for example, "two to twenty-five".

$$\frac{2}{25} \overset{2 \times ?}{=} \frac{■}{100} \quad (25 \times ?) \qquad\qquad \frac{28}{24} \overset{28 \div ?}{=} \frac{7}{■} \quad (24 \div ?)$$

Write the following exercise on the board.

$$\frac{15}{25} \overset{15 \div ?}{=} \frac{■}{15} \quad (25 \div ?)$$

Ask why the procedure of dividing each term of $\frac{15}{25}$ by a common factor is not suitable for this pair of equivalent ratios. Ask the students to suggest another procedure. If necessary, remind them of the use of cross products in finding equivalent fractions. Have them use cross products for this example and also to check the exercises completed earlier.

Write two non-equivalent ratios on the board. Have students find the cross products to emphasize that they are not equal for such ratios.

$$\frac{4}{7} \qquad \frac{2}{3}$$
$$4 \times 3 = 12$$
$$7 \times 2 = 14$$

Using the Pages

- The illustration can motivate a discussion about contests the students may have seen advertised, for example, on cereal boxes. Direct the discussion towards the word problem for the worked example. Have students explain the steps shown. Ask them to multiply 8 and 2187 to verify that the product is 17 496. Draw attention to the concluding statement.

Working Together

Write a sentence showing equal cross products.

Example: For $\frac{5}{7} = \frac{\blacksquare}{28}$, write $7 \times \blacksquare = 5 \times 28$.

1. $\frac{4}{3} = \frac{32}{\blacksquare}$ $4 \times \blacksquare = 3 \times 32$
2. $\frac{8}{10} = \frac{\blacksquare}{100}$ $10 \times \blacksquare = 8 \times 100$

Find the missing term.

Example: For $6 \times \blacksquare = 5 \times 18$, or 90, you can divide 90 by 6 to find a value for $\blacksquare$.

3. $5 \times \blacksquare = 7 \times 20$ 28
4. $\frac{9}{6} = \frac{\blacksquare}{26}$ 39

Exercises

Use cross products to find the missing term.

1. $\frac{4}{14} = \frac{6}{\blacksquare}$ 21
2. $\frac{21}{6} = \frac{\blacksquare}{10}$ 35
3. $\frac{6}{40} = \frac{15}{\blacksquare}$ 100
4. $\frac{13}{8} = \frac{\blacksquare}{24}$ 39
5. $\frac{14}{12} = \frac{35}{\blacksquare}$ 30
6. $\frac{26}{24} = \frac{\blacksquare}{36}$ 39
7. $\frac{45}{75} = \frac{60}{\blacksquare}$ 100
8. $\frac{26}{8} = \frac{\blacksquare}{36}$ 117
9. $\frac{18}{30} = \frac{21}{\blacksquare}$ 35
10. $\frac{72}{80} = \frac{\blacksquare}{100}$ 90
11. $\frac{5}{15} = \frac{6}{\blacksquare}$ 18
12. $\frac{24}{9} = \frac{\blacksquare}{12}$ 32
13. $\frac{9}{12} = \frac{30}{\blacksquare}$ 40
14. $\frac{66}{60} = \frac{\blacksquare}{100}$ 110
15. $\frac{2}{16} = \frac{7}{\blacksquare}$ 56

Solve.

16. There were 5832 entries in the contest. If there had been 168 more, how many winning entries would there have been? 2250
17. How many more entries were needed for there to have been 2400 winning entries? 568
18. 5 of every 9 entries were from women. How many of the 5832 entries were from women? 3240
19. 7 of every 12 entries came from Ontario. How many entries came from outside Ontario? 2430

Add, subtract, multiply, or divide.

1. 9.7 + 0.825 10.525
2. 2.9 × 5.4 15.66
3. 6.575 − 3.8 2.775
4. 69.56 ÷ 4 17.39
5. 6.09 + 16 22.09
6. 107.8 ÷ 35 3.08
7. 11.4 − 9.47 1.93
8. 4.7 × 0.38 1.786
9. 5 − 1.2 3.8
10. 5.582 + 5.93 11.512
11. 45.5 ÷ 28 1.625
12. 2.06 × 3.86 7.9516
13. 0.07 × 0.19 0.0133
14. 1.96 ÷ 7 0.28
15. 10 − 3.05 6.95
16. 1.49 + 0.5717 2.0617

KEEPING SHARP

253

RELATED ACTIVITIES

- Encourage the students to notice and report ratios indicated in contest rules. For example, the rules may state that there will be one prize for every 6 entries. For such contests, have the students write and solve word problems similar to those on page 253.
- Prepare a work sheet for which students must use cross products to match equivalent ratios.

$\frac{9}{6}$	$\frac{14}{10}$
$\frac{18}{16}$	$\frac{6}{4}$
$\frac{21}{15}$	$\frac{45}{40}$

- By questioning the students, you may lead them to discover other ways of finding the missing term for exercises on page 253. In Ex. 1, for example, $\frac{4}{14}$ may be replaced by the equivalent ratio $\frac{2}{7}$. Then, because 6 is equal to 2×3, the missing term for $\frac{6}{\blacksquare}$ may be found by multiplying 7 by 3.

$$\frac{4}{14} = \frac{6}{\blacksquare}, \qquad \frac{2}{7} \underset{7 \times 3}{\overset{2 \times 3}{=}} \frac{6}{\blacksquare}$$

Students may be interested in trying this procedure for some of the exercises that have been completed by using cross products.

Working Together: The example above Ex. 1 and 2 shows how cross products are written for equivalent ratios. The example above Ex. 3 shows how to find the missing term after writing the cross products. For Ex. 4, the students are required to write the cross products and to find the missing term.

Exercises: To emphasize that cross products are needed, ask why the missing term for these exercises cannot be found by either multiplying or dividing each term of a ratio by a common factor.

The information in the word problems of Ex. 16-19 relate to the worked example on page 252. To help students write ratios for Ex. 16 and 17 in the correct order, write the following on the board.

$$\frac{\text{winning entries}}{\text{total entries}} = \frac{3}{8}$$

Note that there are different ways to solve Ex. 16, 17, and 19, and students who suggest alternative methods should show and explain them on the board. More capable students should be encouraged to find two ways to solve the same problem. For Ex. 19, for example, students may use the ratio $\frac{7}{12}$ to find the number of entries from Ontario, and then subtract this number from 5832. Alternatively, they may use the ratio $\frac{5}{12}$ and find the number of entries from outside Ontario directly.

Keeping Sharp: These exercises help to maintain skills in addition, subtraction, multiplication, and division with decimals. They also help to prepare the students for *Checking Skills* on page 261. Note that Ex. 39-61 on page 255 will involve multiplying and dividing decimals.

Assessment

Use cross products to find the missing term.

1. $\frac{36}{27} = \frac{8}{\blacksquare}$ 6
2. $\frac{12}{30} = \frac{\blacksquare}{100}$ 40
3. $\frac{15}{6} = \frac{10}{\blacksquare}$ 4
4. $\frac{10}{8} = \frac{25}{\blacksquare}$ 20
5. $\frac{6}{9} = \frac{\blacksquare}{12}$ 8
6. $\frac{16}{26} = \frac{\blacksquare}{13}$ 8

Solve.

7. Prizes were given to 4 out of every 16 entries for the contest. 386 entries received prizes. How many entries were there? 1544

Pages 254-255

OBJECTIVE

Demonstrate competence in writing ratios, finding equivalent ratios, expressing a ratio in simplest form, and finding the missing term in two equivalent ratios; solve related word problems

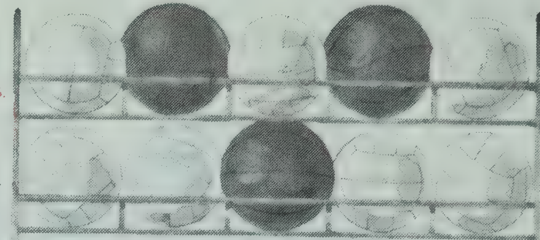
Materials

centimetre ruler for each student

Vocabulary

circle graph

Practice



Give a ratio for
The ratios can be written in other ways.

7:3 **1.** volleyballs to basketballs.

3:7 **2.** basketballs to volleyballs.

3:10 **3.** basketballs to all the balls.

7:10 **4.** volleyballs to all the balls.

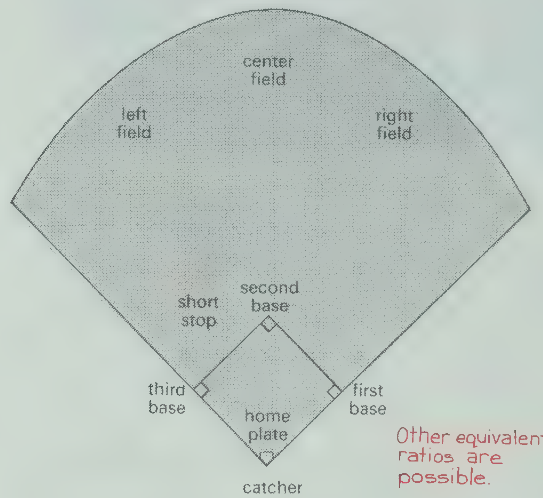
Write five ratios that are equivalent to each of these.

5. $\frac{15}{25}$ **6.** 6:4 **7.** $\frac{3}{18}$

Write each of these ratios in simplest form.

8. 8:12 2:3 **9.** $\frac{75}{100}$ $\frac{3}{4}$ **10.** 24:8 3:1

There are 9 positions on a baseball field.



3 are outfield positions.

6 are infield positions.

Write a ratio for each of these. The ratios can be written in other ways.

11. outfield positions to infield positions 3:6

12. infield positions to outfield positions 6:3

13. outfield positions to all the positions 3:9

14. infield positions to all the positions 6:9

Other equivalent ratios are possible.
5. $\frac{3}{5}, \frac{6}{10}, \frac{9}{15}, \frac{12}{20}, \frac{18}{30}$
6. 3:2, 9:6, 12:8, 15:10, 18:12
7. $\frac{1}{6}, \frac{2}{12}, \frac{4}{24}, \frac{5}{30}, \frac{6}{36}$

The baseball diamond above is drawn with a scale ratio of 1 cm to 15 m.

15. About how far is it from home plate to second base on the real diamond? about 45 m

16. About how far is it from home plate to first base on the real diamond? about 30 m

17. About how far is it around the real diamond? about 120 m

***18.** About how far is it from home plate to the outfield fence on the real field? Answers will depend on which part of the fence is used. Shortest distance is about 105 m. Longest distance is about 130 m.

254

LESSON ACTIVITY

Using the Pages

- Before the students begin, draw attention to the diagram of the baseball field. Point out the scale in the statement below the diagram. Discuss that it will be necessary to measure distances on the diagram in centimetres and then use equivalent ratios to solve Ex. 15-18. Tell the students to measure to the nearest centimetre. Ex. 18 is starred because different answers are possible. On the diagram, the shortest distance from home plate to the outfield fence is about 7 cm and the longest distance is about 9 cm.

  Students will encounter decimals as terms in ratios for the first time in Ex. 39-58. It is advisable to complete the given example and one or two exercises on the board with the students before assigning the exercises. Cross products are used to find the missing terms. Emphasize that the procedure is the same as for whole numbers. Skill with such exercises as these will be required for Unit 15.

  Introduce the term *circle graph* and have a few students explain what Milly's graph shows. For example, it shows that she plans to put \$0.40 of each \$1.00 in the bank. Develop that this may be expressed as the ratio $\frac{0.40}{1}$. Similarly, develop the ratios $\frac{0.25}{1}$ and $\frac{0.35}{1}$. These will be used in solving Ex. 59 and 60. Note that for Ex. 61, it is not necessary to write equivalent ratios and to find missing terms. Milly's total amount of money includes her allowance of \$2.80 (Ex. 59) and her earnings (Ex. 60). Students may use addition to find the total amount for the bank, for example, because the individual amounts have been found in Ex. 59 and 60 as indicated below.

From Ex. 59,
$\frac{0.4}{1} = \frac{■}{2.80}$
gives \$1.12.
From Ex. 60,
$\frac{0.4}{1} = \frac{■}{4.00}$ gives \$1.60. $\frac{0.4}{1} = \frac{■}{2.60}$ gives \$1.04. $\frac{0.4}{1} = \frac{■}{2.60}$ gives \$0.24.
For Ex. 61, the total amount for the bank can be found by addition.
$$\$1.12 + \$1.60 + \$1.04 + \$0.24 = \$4.00$$

Find the value for $\blacksquare$ that makes the ratios equivalent.

19. $\frac{25}{10} = \frac{\blacksquare}{14}$ 35 20. $\frac{3}{6} = \frac{50}{\blacksquare}$ 100 21. $\frac{27}{21} = \frac{\blacksquare}{49}$ 63 22. $\frac{5}{30} = \frac{4}{\blacksquare}$ 24 23. $\frac{54}{18} = \frac{\blacksquare}{27}$ 81

24. $\frac{14}{8} = \frac{\blacksquare}{12}$ 21 25. $\frac{25}{100} = \frac{\blacksquare}{4}$ 1 26. $\frac{15}{40} = \frac{9}{\blacksquare}$ 24 27. $\frac{45}{40} = \frac{\blacksquare}{48}$ 54 28. $\frac{49}{70} = \frac{\blacksquare}{100}$ 70

29. $\frac{33}{30} = \frac{\blacksquare}{40}$ 44 30. $\frac{6}{9} = \frac{14}{\blacksquare}$ 21 31. $\frac{38}{24} = \frac{\blacksquare}{36}$ 57 32. $\frac{36}{60} = \frac{\blacksquare}{100}$ 60 33. $\frac{21}{9} = \frac{\blacksquare}{15}$ 35

34. $\frac{10}{25} = \frac{14}{\blacksquare}$ 35 35. $\frac{24}{12} = \frac{\blacksquare}{18}$ 36 36. $\frac{33}{15} = \frac{22}{\blacksquare}$ 10 37. $\frac{10}{14} = \frac{\blacksquare}{63}$ 45 38. $\frac{9}{30} = \frac{\blacksquare}{100}$ 30

Sometimes ratios will use decimals.

Example: For $\frac{1.2}{3} = \frac{\blacksquare}{2}$, write $3 \times \blacksquare = 1.2 \times 2$, or 2.4.

Then divide 2.4 by 3 to find the value for $\blacksquare$.

Find the value for $\blacksquare$ that makes the ratios equivalent.

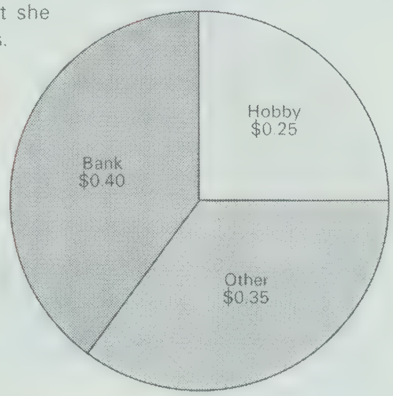
39. $\frac{0.6}{4} = \frac{\blacksquare}{2}$ 0.3 40. $\frac{5}{3} = \frac{1.5}{\blacksquare}$ 0.9 41. $\frac{2}{7} = \frac{\blacksquare}{4.2}$ 1.2 42. $\frac{9}{4.5} = \frac{5}{\blacksquare}$ 2.5 43. $\frac{1.2}{3} = \frac{\blacksquare}{5}$ 2.0

44. $\frac{8}{1.6} = \frac{7}{\blacksquare}$ 1.4 45. $\frac{7}{2} = \frac{\blacksquare}{0.4}$ 1.4 46. $\frac{6}{3} = \frac{1.8}{\blacksquare}$ 0.9 47. $\frac{0.7}{3} = \frac{\blacksquare}{9}$ 2.1 48. $\frac{5}{1.5} = \frac{4}{\blacksquare}$ 1.2

49. $\frac{5}{6} = \frac{\blacksquare}{4.2}$ 3.5 50. $\frac{9}{2.7} = \frac{5}{\blacksquare}$ 1.5 51. $\frac{2.1}{18} = \frac{\blacksquare}{6}$ 0.7 52. $\frac{6}{2} = \frac{1.5}{\blacksquare}$ 0.5 53. $\frac{2.1}{7} = \frac{\blacksquare}{9}$ 2.7

54. $\frac{5}{1.5} = \frac{6}{\blacksquare}$ 1.8 55. $\frac{1.2}{4} = \frac{\blacksquare}{3}$ 0.9 56. $\frac{21}{6.3} = \frac{7}{\blacksquare}$ 2.1 57. $\frac{0.7}{2} = \frac{\blacksquare}{6}$ 2.1 58. $\frac{10}{0.5} = \frac{4}{\blacksquare}$ 0.2

Milly drew a **circle graph** to show what she plans to do with each dollar she receives.



59. Milly is given \$2.80 each week as an allowance. What would she do with the money according to her circle graph?

Bank \$1.12
Hobby \$0.70
Other \$0.98

This chart shows how much Milly earned last week.

Babysitting	\$4.00
Gardening	\$2.60
Errands	\$0.60

60. According to the circle graph, how will she spend the money from each activity?

Babysitting: Bank \$1.60, Hobby \$1.00, Other \$1.40
Gardening: Bank \$1.04, Hobby \$0.65, Other \$0.91
Errands: Bank \$0.24, Hobby \$0.15, Other \$0.21

*61. According to the circle graph, how will Milly spend the total amount of money?

Bank \$4.00
Hobby \$2.50
Other \$3.50

255

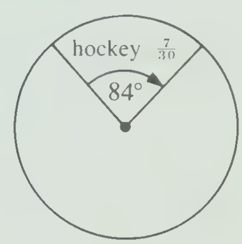
Some students may find the sum of Milly's earnings and allowance and use equivalent ratios. The two solutions should be shown on the board and discussed.

RELATED ACTIVITIES

- Have students work in small groups to make a scale drawing of a hockey field or a football field and show the scale used. They may use reference books to find the measurements of the fields. The scale drawings may generate word problems similar to Ex. 11-18 on page 254. Ask students to write the problems on cards and place them with the drawings for others to answer.
- Students can search newspapers and magazines for examples of circle graphs. These may be displayed and discussed.
- As enrichment, you may wish to help the students draw circle graphs to display information obtained by a survey. For example, a survey of 30 students may be conducted using a tally sheet to find the favorite sport of each student. The results are expressed as ratios, for example, $\frac{7}{30}$, if 7 students chose hockey as their favorite sport, and then as equivalent ratios for which the second term is 360.

$$\frac{7}{30} = \frac{\blacksquare}{360}, \quad \frac{7}{30} = \frac{84}{360}$$

The first number of the second ratio (84) indicates the number of degrees for the angle at the center of the circle.



LESSON OUTCOME

Use division to find unit rates; choose a rate to describe a situation

Vocabulary

rate, unit rate

Prerequisite Skills

Use cross products to find the missing term in two equivalent ratios

Checking Prerequisite Skills

Find the missing term.

1. $\frac{36}{27} = \frac{■}{9}$ 12 **2.** $\frac{12}{15} = \frac{■}{10}$ 8

3. $\frac{30}{10} = \frac{■}{1}$ 3 **4.** $\frac{14}{7} = \frac{■}{1}$ 2

Finding Unit Rates

Linda can type 240 words in 4 min. This **rate** can be shown like this → $\frac{240 \text{ words}}{4 \text{ min}}$. What is this rate in words per minute?

A rate is the comparison of two unlike quantities.

To find the rate in words per minute, write and find the missing term. → $\frac{240}{4} = \frac{■}{1}$

$4 \times ■ = 240 \times 1$, or 240

$240 \div 4 = 60$

$4 \times 60 = 240$

Linda can type at the rate of

$\frac{60 \text{ words}}{1 \text{ min}}$,

or 60 words per minute.

This is an example of a **unit rate**. A unit rate is usually written like this.

Take another look:
To find the missing term in

$\frac{240}{4} = \frac{■}{1}$,

you could divide 240 by 4.

To find any unit rate, you divide one quantity by the other.

256

LESSON ACTIVITY

Before Using the Pages

- Write 60 km/h on the board. Ask students where they have seen this expression and ask them to explain what it means. Develop that 60 km/h means 60 km per hour, or 60 km in 1 h. Ask the students what distance they would travel in 2 h, 3 h, and so on, if they traveled at 60 km/h. Use other examples such as 15 m/min and 20 cm/s.

Using the Pages

- Ask a student to read the title at the top of page 256 and the word problem below it. Introduce the term *rate* and note that the rate compares a number of words to a number of minutes. Relate this to the information in the ''thought cloud''. Discuss that the rate ''240 words in 4 min'' can be written using the fraction $\frac{240}{4}$. Discuss the use of equivalent rates to find the rate in words per minute, in other words, the number of words typed in 1 min. Ask students to explain how cross products are used to find the missing number. Introduce the term *unit rate* to describe a rate for which the second number is 1.

  Students may suggest that the cross products for $\frac{240}{4} = \frac{■}{1}$ are easily found because the factor 1 is involved. Thus, the only computation required is the division $240 \div 4$. This suggests the procedure shown at the bottom of page 256. Thus, to find unit rates, students may begin by using division rather than by writing equivalent rates and their cross products.

Working Together: Because a rate deals with two unlike quantities, it is necessary to determine the unit for expressing a unit rate. Ex. 1-3 deal with this concept. Discuss the given example, emphasizing that the symbols m and s are used for the unit rate, not the words ''metres'' and ''second''.

Ex. 4-6 deal with finding unit rates. Remind the students to include the units from the answers for Ex. 1-3.

Working Together

What unit will be used with the unit rate?

Example: For 100 m in 16 s, the unit will be metres per second (m/s).

1. 270 km in 3 h km/h 2. 0.5 L in 4 s L/s 3. 12 g for 10 cm^3 g/cm^3

Divide the first number by the second number to find the unit rate.

Be sure to give the correct unit for the unit rate.

4. 270 km in 3 h 90 km/h 5. 0.5 L in 4 s 0.125 L/s 6. 12 g for 10 cm^3 1.2 g/cm^3

Exercises

Find the unit rate.

1. 100 m in 16 s 6.25 m/s 2. 2880 kg in 12 h 240 kg/h 3. 1628 km in 8 d 203.5 km/d
4. 35 mL in 10 s 3.5 mL/s 5. 7.5°C in 4 d 1.875 °C/d 6. 640 m in 100 s 6.4 m/s
7. 4.8°C in 24 h 0.2°C/h 8. 760 km in 8 h 95 km/h 9. 4 L for 10 m^2 0.4 L/m^2
10. 1962 m^3 in 18 h 109 m^3/h 11. 3 kg for 24 L 0.125 kg/L 12. 7875 L in 5 h 1575 L/h

Choose one rate to describe each of these.

mL/s	m^3/min	g/s	°C/min	cm/s	g/cm	cm^3/s
°C/h	cm/min	L/m^2	g/d	L/min	kg/L	km/h

13. how fast a snail crawls cm/min
14. how fast the temperature falls °C/h
15. how fast the bathtub empties
16. how fast a balloon inflates cm^3/s
17. how fast hot chocolate cools
18. how fast an airplane flies km/h
19. how paint will cover a surface
20. how fast your mass increases g/d

15 L/min 17 °C/min 19 L/m^2

PROBLEM SOLVING

1. If 6 hens can lay 6 eggs in 6 min, how long will it take 8 hens to lay 8 eggs? 6 min

257

RELATED ACTIVITIES

- Have students work in pairs timing each other to determine the number of words written in 10 s or the number of metres walked in 3 min. Then have them calculate the unit rate for each situation.
- Have the students list examples of unit rates that they have seen or that they are able to find in newspapers.
- Students may be assigned to two teams for the following game. Prepare cards showing rates such as 32 m in 2 s. Include only rates that can be converted to unit rates without a pencil and paper. Each player in turn draws a card and states the unit rate, for example, 16 m/s (sixteen metres per second) for the rate given above. If the unit rate is correct, the player's team scores one point. The team with the most points after several rounds is the winner.

Exercises: For Ex. 1-12, the students are required to find various unit rates. You may wish to discuss possible uses for these unit rates. For example, Ex. 5 may describe the rise or fall in temperature each day for several days. After the students have completed Ex. 13-20, provide an opportunity for them to explain their answers. For example, L/min is a better choice for Ex. 15 than mL/s because the latter rate is usually associated with a small quantity.

Problem Solving: Many students will likely enjoy expressing their opinions on the answer to this problem. You may wish to delay a discussion of their answers for a day or two and then have them list their answers on the board. Tallies can be marked to show which answer is chosen most frequently. Then ask students to explain the different answers. The concept of rate is inherent in the problem, and the rate might be thought of as 1 egg per hen in 6 min. Thus, 8 hens can lay 8 eggs in 6 min.

Assessment

Find the unit rate.

1. 50 m in 4 s 12.5 m/s 2. 8 L in 10 h 0.8 L/h 3. 15°C in 6 d 2.5 °C/d

Choose one rate to describe each of these.

g/s	km/h	cm^3/s	km/s	°C/h	kg/L

4. how fast the temperature increases °C/h
5. how fast you walk km/h

LESSON OUTCOME

Find a unit price to the nearest cent; compare unit prices

Vocabulary
unit price

Prerequisite Skills
Use division to find unit rates

Checking Prerequisite Skills
Find the unit rate.

1. 27 km in 6 min 4.5 km/min
2. 70 mL in 8 s 8.75 mL/s
3. 2 L for 40 m² 20 m²/L

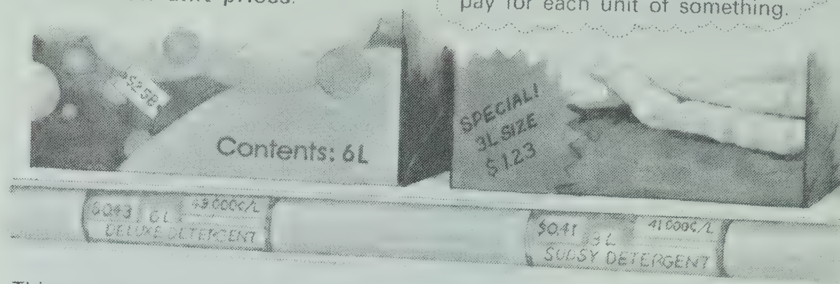
RELATED ACTIVITIES

- Have students bring empty containers from home. Display these, and have the students find the unit price for each item. If there are two or more brands for one item, the students can find the better or the best buy. Newspaper advertisements and catalogs are useful for this activity.

Unit Pricing

The tags along grocery store shelves show **unit prices**.

A unit price shows how much you pay for each unit of something.



This tag shows that 1 L of detergent in the larger box costs $0.43.

This tag shows that 1 L of detergent in the smaller box costs $0.41.

To find the cost of 1 L, divide the cost by the number of litres.

For the 6 L box that costs $2.58, divide $2.58 by 6.

$$6\overline{)\$2.58}\quad \$0.43$$

1 L of detergent in this box costs $0.43.

Exercises

Find the unit price to the nearest cent.

1. 3 heads of lettuce for $2.07 $0.69
2. 52¢ for 2 pears 26¢
3. 6 cans of soup for $1.86 $0.31
4. $1.29 for 10 kg of potatoes $0.13
5. 5 bars of soap for $2.45 $0.49
6. $1.71 for 8 cans of juice $0.21
7. 3 lemons for 89¢ 30¢
8. $3.98 for 4 jars of jam $1.00
9. 6 tomatoes for 96¢ 16¢
10. $1.45 for 10 tomatoes $0.15

Which is the better buy,

11. 2 grapefruit for 76¢ or 5 grapefruit for $1.79?
12. $3.99 for 2 kg of nuts or $2.15 for 1 kg of nuts?
13. 4 kg of sugar for $2.16 or 10 kg of sugar for $4.98?
14. 63¢ for 1 L of milk or $1.65 for 3 L of milk?
15. 30 eggs for $2.25 or 12 eggs for 92¢?

258

LESSON ACTIVITY

Using the Page

- Begin with a brief discussion of the need to shop for items with care to ensure that the best buys are obtained. Introduce the term *unit pricing* and ask students to read the statements at the top of page 258. For the illustration, ask questions such as "How much detergent does the first box contain?" "What is the price for the first box?" Draw attention to the tags below the boxes. Some students may have noticed unit prices displayed on tags in grocery stores. Ask students to explain what is shown on the tags in the illustration. For example, the first tag shows that each box of Deluxe Detergent contains 6 L of detergent, the price of each litre is $0.43, and the unit price is 43.000¢/L. Note that three decimal places are shown in the unit prices.

  Ask how to find the cost of 1 L of Sudsy Detergent. Note that finding a unit price is similar to finding a unit rate.

Exercises: For Ex. 1, 3, 5, 7, and 9, the number of items is given first and then the cost. For Ex. 2, 4, 6, 8, and 10, the cost is given first and then the number of items. It may be necessary to remind some students that the cost is divided by the number of items. Point out that the unit price is to be expressed to the nearest cent. For example, for Ex. 4, $0.129 for 1 kg of potatoes is expressed as $0.13. However, for Ex. 15, it will be necessary to express the two unit prices to the nearest tenth of a cent and compare them.

Assessment

Find the unit price to the nearest cent.

1. 9 peaches for $1.95 $0.22
2. $8.36 for 2 kg of cheese $4.18

Which is the better buy,

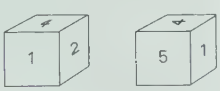
3. 78¢ for 1 L of juice or $2.30 for 3 L of juice?
4. 5 jars of jelly for $6.05 or 3 jars of jelly for $3.65?

Considering the Chances

Knowing the chance for something happening can help you know what to expect. It can also help you make a choice. What plans would you make if you heard this weather report?

Answer each of these. Explain your answers. Explanations will vary

1. There are 4 brown hats and 1 blue hat in a box. What chances do you have of choosing a brown hat in the dark? 4 out of 5
2. When you flip a penny, how often would you expect it to show "heads"? half of the times the penny is flipped
3. Suppose each hour of the day (01 : 00, 02 : 00, 03 : 00, and so on) is written on a piece of paper and the papers are placed in a hat. If you draw one piece of paper from the hat, what are the chances that you would draw 09 : 00, 10 : 00, or 11 : 00? 3 out of 24 or 1 out of 8
4. Which would you have the best chance of guessing: the time of the day, the day of the week, the day of the month, or the month of the year in which a person was born? the day of the week
5. Suppose each of two cubes has the numbers 1 to 6 on its faces. The cubes are rolled and the sum for their top faces is found. What sum would you have the best chance of rolling? 7



PROBLEM SOLVING

259

Page 259

OBJECTIVE

Consider the chances of various possibilities

Materials

a die with three sides marked to show 3, two sides marked to show 2, and one side marked to show 1

RELATED ACTIVITIES

- The students may be interested in conducting the experiments suggested by Ex. 1-5. They may work in small groups using the necessary materials, adapting markers for the hats suggested in Ex. 1. Tally charts will be useful for recording the results. Suggest that they perform at least 50 trials for each experiment. They may write ratios comparing the number of times each possibility occurred with the number of times the experiment was conducted. Have them compare the results obtained with their previous answers for Ex. 1-5. You may wish to have the students draw and display bar graphs to show the results of the experiments.

LESSON ACTIVITY

Before Using the Page

- Display a die with three sides marked to show 3, two sides marked to show 2, and one side marked to show 1. Ask the students what number is most (least) likely to be on the top face when the die is tossed. Have students help to toss the die 30 times, for example, and record the results in a tally chart. Then compare the results with the students' predictions. Discuss that the results for a small number of trials will likely be less accurate than the results for a large number of trials.

Number of trials	30
Number of times die shows 1	||||

Using the Page

- Ask a student to read the title of the lesson and to explain what it means. Note that the illustration suggests that a picnic is being planned. Have the students read the statements in the example given. Discuss the plans they would make and have them give their reasons. Ask for examples other than weather predictions for which the outcomes are considered in advance, for example, the results of an election.
- You may wish to have the students write brief explanations for their answers. These may be explained in a class discussion or in a discussion with a small group of students. For example, for Ex. 4, they may explain that there is 1 chance in 7 of naming the correct day of the week, whereas the chance of naming the correct month of the year is 1 in 12. For Ex. 5, students may find it helpful to write an addition table in which the numbers 1 to 6 are the addends.

second cube

	+	1	2	3	4	5	6
first cube	1						
	2						
	3						
	4						
	5						
	6						

Page 260

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

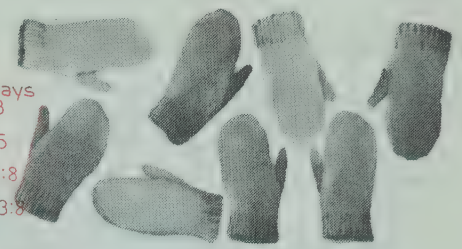
- Students may work in pairs outdoors on a sunny day for the following activity. Have them measure their heights and the lengths of their shadows. Then have them measure the shadow of a tall object such as a tree. The procedure outlined in the worked example on page 250 may then be used to find the height of the tree.
- For practice with equivalent ratios, adapt the game "Name the Fraction" described on page T 381.
- For enrichment, have students prepare a rate table, for example, for the cost of gasoline.

1 L	2 L	3 L	10 L
26.2¢	52.4¢	78.6¢	$2.62

Have them use the table to determine the cost, for instance, of 57 L of gasoline. The price for 10 L is multiplied by 5 and this is added to the price for 7 L.

50 L: $5 \times \$2.62$ =	$13.10
7 L:	1.83
57 L:	$14.93

Checking Up



Give a ratio for
The ratios can be written in other ways

1. red mittens to blue mittens. 5:3
2. blue mittens to red mittens. 3:5
3. red mittens to all the mittens. 5:8
4. blue mittens to all the mittens. 3:8

Write three ratios that are equivalent to each of these. Other equivalent ratios are possible

Write each of these ratios in simplest form.

5. $\frac{1}{8}$ $\frac{2}{16}, \frac{3}{24}, \frac{4}{32}$
6. 3:6 1:2, 2:4, 4:8
7. $\frac{27}{9}$ $\frac{3}{1}, \frac{6}{2}, \frac{54}{18}$
8. $\frac{10}{15}$ $\frac{2}{3}$
9. 21:15 7:5
10. $\frac{18}{8}$ $\frac{9}{4}$

Find the value for ■ that makes the ratios equivalent.

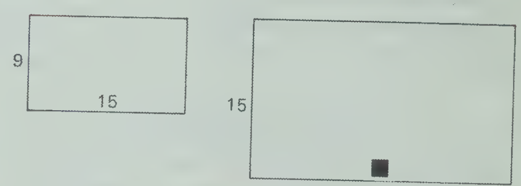
11. $\frac{1}{8} = \frac{4}{■}$ 32
12. $\frac{12}{18} = \frac{■}{3}$ 2
13. $\frac{45}{20} = \frac{9}{■}$ 4
14. $\frac{7}{6} = \frac{■}{18}$ 21
15. $\frac{32}{10} = \frac{■}{15}$ 48
16. $\frac{5}{10} = \frac{14}{■}$ 28
17. $\frac{20}{14} = \frac{■}{49}$ 70
18. $\frac{35}{45} = \frac{42}{■}$ 54
19. $\frac{4}{15} = \frac{2}{■}$ 7.5
20. $\frac{4.9}{7} = \frac{■}{2}$ 1.4
21. $\frac{5}{13} = \frac{1.5}{■}$ 3.9
22. $\frac{0.5}{13} = \frac{■}{39}$ 1.5

Solve.

23. 56 boys had their teeth checked. 5 out of every 8 had no new cavities. How many boys had no new cavities? 35
24. The ratio of "yes" votes to "no" votes was 7 to 3. There were 63 "yes" votes. How many "no" votes were there? 27

The figures at the right are similar.

25. Find the value for ■. 25



Find the unit rate.

26. 1032 km in 12 h 86 km/h
27. 6 kg in 8 m³ 0.75 kg/m³
28. 150 m in 20 s 7.5 m/s

Find the unit price to the nearest cent.

29. 75¢ for 6 pencils 13¢
30. 12 boxes of berries for $4.56 $0.38

260

Skills	Exercises	Related Pages
Write a ratio	1-4	T 268-T 269
Write equivalent ratios	5-7	T 270-T 271
Write ratios in simplest form	8-10	T 270-T 271
Find the missing term in two equivalent ratios or rates	11-22, 25	T 272-T 277
Find unit rates	26-28	T 278-T 279
Find unit prices	29, 30	T 280
Solve related word problems	23, 24	

Comments

Determine whether errors result from not understanding the concept of ratio, from being unable to find equivalent ratios, or from having difficulty in multiplying and dividing with decimals. Then provide remedial assistance for the appropriate concepts. Refer to the teaching suggestions and the related activities. Objects may be used to demonstrate some of the exercises on page 260 and other similar exercises.

For Ex. 11-14, the students can use cross products or they can multiply or divide to find the missing term. For Ex. 15-22, it is necessary to use cross products. Ex. 19-22 involve decimals.

Checking Skills

Multiply.

1. 2.3 6 13.8	2. 9.2 55 506.0	3. 3.65 10 36.5
4. 6.49 58 367.42	5. 3.108 70 217.560	6. 8.698 72 626.256
7. 52.8 100 5280	8. 0.93 717 666.81	9. 9.285 600 5571.000
10. 4.3 0.1 0.43	11. 2.4 1.3 3.12	12. 9.4 3.4 31.96
13. 2.77 3.8 10.526	14. 3.15 0.7 2.205	15. 7.06 0.01 0.0706
16. 8.324 3.7 30.7988	17. 4.747 4.8 22.7856	18. 4.856 2.4 11.6544
19. 16.9 2.91 49.179	20. 65.3 0.01 0.653	21. 7.89 39.6 312.444
22. 0.4 0.2 0.08	23. 0.28 0.1 0.028	24. 0.16 0.5 0.080
25. 0.039 0.7 0.0273	26. 0.34 0.19 0.0646	27. 0.08 0.01 0.0008
28. \$7.29 2 \$14.58	29. \$5.78 10 \$57.80	30. \$4.15 2.6 \$10.79

Solve.

31. How many square metres are in a rectangle that is 3.5 m long and 1.8 m wide? 6.30 m²

32. How much will 1.25 kg of hamburger cost if each kilogram costs \$3.52? \$4.40

Add, subtract, multiply, or divide.

1. 4.6 + 2.5 7.1	2. 29 × 4.586 132.994
3. 13.72 ÷ 100 0.1372	4. 11.4 − 7 4.4
5. 15 × 0.1 1.5	6. 7.4 ÷ 2 3.7
7. 5.36 − 3 2.36	8. 3.159 + 5.878 9.037
9. 2.9 ÷ 5 0.58	10. 4.03 × 100 403
11. 270 ÷ 75 3.6	12. 6.53 − 2.756 3.774
13. 8.3 + 2.85 11.15	14. 20.048 ÷ 10 2.0048
15. 0.35 × 0.1 0.035	16. 8.392 − 4.7 3.692
17. 5.75 + 5 10.75	18. 3.76 × 100 376
19. 66.7 ÷ 23 2.9	20. 7.486 + 4.55 12.036
21. 65 − 17.5 47.5	22. 5.7 × 0.58 3.306
23. 1.77 + 8.58 10.35	24. 1.428 ÷ 3 0.476
25. 0.42 ÷ 28 0.015	26. 70.5 − 29.58 40.92
27. 3.2 × 5.029 16.0928	28. 5.9589 + 8.1 14.0589
29. 25.6 − 6.7 18.9	30. 2.3 × 3.4 7.82
31. 98 ÷ 100 0.98	32. 9.5 − 6.877 2.623
33. 8.291 + 16 24.291	34. 4.268 × 0.1 0.4268
35. 8 − 2.3 5.7	36. 12.3 ÷ 6 2.05
37. 29 − 9.52 19.48	38. 8.3 + 46 54.3
39. 0.019 × 0.3 0.0057	40. 7 − 5.469 1.531
41. 3.638 + 7.6 11.238	42. 189.15 ÷ 39 4.85
43. 62.96 ÷ 8 7.87	44. 1.97 × 0.01 0.0197
45. 8.2 − 5.64 2.56	46. 9 + 7.26 16.26
47. 0.1 × 0.2 0.02	48. 4.023 − 2.848 1.175
49. 10.45 − 3.99 6.46	50. 0.37 × 4.92 1.8204
51. 7.97 + 2.474 10.444	52. 318.5 ÷ 52 6.125
53. 0.74 × 0.05 0.037	54. 7.48 ÷ 10 0.748
55. \$9.27 + \$3.86 \$13.13	56. \$8.82 − \$3.98 \$4.84
57. 10 × \$2.36 \$23.60	58. \$37.66 ÷ 7 \$5.38
59. 1.862 + 5.7 + 2.97 10.532	
60. \$7.95 + \$6.89 + \$8.98 \$23.82	

261

Page 261

OBJECTIVE

Demonstrate competence in addition, subtraction, multiplication, and division with decimals

RELATED ACTIVITIES

- Students may be asked to complete one or more of the following for extra practice and review.
  1. Use addition (multiplication) to check the answers for several subtraction (division) exercises.
  2. Estimate the answers for a selection of exercises.
  3. Write the word name for each number in a selection of exercises.
  4. Tell the place value of each digit in each numeral of an exercise.
  5. Write the answers for five or six exercises in order from least to greatest.
- For enrichment, assign exercises similar to the following. To find the missing numbers, students may need to use the inverse of the given operation.
  1. $68.21 + \boxed{3}.\boxed{9}\boxed{4}\boxed{6} = 72.156$
  2. $7.26 - 1.99 = \boxed{5}.\boxed{2}\boxed{7}$
  3. $637 \times \boxed{0}.\boxed{2}\boxed{8} = 178.36$
  4. $58.88 \div \boxed{8} = 7.36$
  5. $\boxed{4}.\boxed{8}\boxed{3}\boxed{1} + 7 = 11.831$
  6. $9.6 \times 4.03 = \boxed{3}\boxed{8}.\boxed{6}\boxed{8}\boxed{8}$
  7. $\boxed{2} - 0.18 = 1.82$
  8. $110 \div 16 = \boxed{6}.\boxed{8}\boxed{7}\boxed{5}$
  9. $\boxed{3}.\boxed{0}\boxed{7} - 1 = 2.07$

LESSON ACTIVITY

Using the Page

- The exercises are presented in two sections. The first section deals only with multiplication involving decimals in one or both factors. This reviews the work presented in Unit 7 and prepares for the concept of division by a decimal in Unit 15.

  The exercises in the second section include addition, subtraction, multiplication, and division with decimals. These concepts were reviewed earlier in the *Keeping Sharp* feature on page 253. For the division exercises, it may be necessary to use extra zeros in the dividends, but quotients will terminate by the third decimal place. Encourage the students to write only the result when multiplying or dividing by 10, 100, 0.1, or 0.01. For exercises such as Ex. 4, remind the students to show the same number of decimal places in each number when writing the exercises in vertical form.

- It would likely be best to have the students complete a few exercises each day for several days. They may be assigned in different ways, according to the needs of the students. For example, for the second section, you may wish to assign only the addition exercises to review the skills with that operation. Then continue in a similar manner with the other operations. You may prefer to assign a few exercises for one operation along with a few for the inverse operation, for example, for addition and subtraction. Skill with one can reinforce work with the inverse operation. A third possibility is to assign a few exercises at a time in the given sequence so that skills in the four operations are practiced together.

  Review concepts and skills with which students experience difficulty. Suggestions for the lessons in Units 5 and 7 may be adapted for this purpose. Other practice exercises may be selected from pages 330-334.

Unit 13 Overview

Percent

The content of this unit follows the work with ratios in Unit 12. The first lesson introduces percent as a way of expressing any ratio in which a number is compared with one hundred. Equivalent ratios expressed in words, with colon notation, as decimals, as fractions, and with the percent symbol are converted from one form to another. Then one-place and two-place decimals and proper fractions are converted to percents, and vice versa. The decimal equivalents of percents are used as multipliers to find percents of numbers. Related word problems are solved using these skills before applications of percent in the business world are presented. Students calculate interest, discount, and net prices. The *Problem Solving* lesson provides experiences in solving word problems which require more than one step. One *Problem Solving* feature applies percent in considering probability and another provides a brief look at compound interest. Percents which cannot be expressed exactly as whole numbers are shown as decimals and as fractions in mixed form.

Prerequisite Skills

- write decimal hundredths
- write a ratio in three ways
- write an equivalent fraction with a denominator of 100 for a given fraction
- divide a one-digit number by a one-digit number, rounding the quotient to two decimal places
- write a fraction with a denominator of 100 in lowest terms
- multiply a whole number by a decimal

Unit Outcomes

- write percents
- convert among ratios out of 100, fractions with denominators of 100, decimal hundredths, and percents
- express a fraction as a percent
- express a decimal as a percent (for tenths and hundredths)
- express a percent as a decimal, as a fraction with a denominator of 100, and as a fraction in lowest terms
- find a percent of a number
- use percent to calculate interest
- use percent to calculate discount
- solve word problems involving percents
- solve problems in more than one step

Background

In daily life the most frequently encountered ratio is the one we call *percent*. The word ''percent'' comes from the Latin words *per centum* meaning ''out of a hundred''. Thus, any percent compares a number with one hundred. For example, 63 percent means that 63 is compared with 100. As discussed in the previous unit, such a ratio may be written in several forms: 63 to 100, 63 : 100, or $\frac{63}{100}$. Any ratio in which 100 is the second term may be called a percent and the symbol % is then used instead of the numeral 100, as in 63%.

Although the other forms of ratios are sometimes used, there are several reasons why percents are used so widely. Percents are used not only to compare two numbers, but they are also used to make comparisons which are easily comprehended. The results of public opinion polls based on different numbers of samplings are difficult to compare. For instance, if two surveys were taken and the results tabulated as shown, the ratios 64 : 91 and 74 : 114 compare two groups of persons in favor, but it is difficult to know which group gave the more favorable response. However, if these ratios are converted to percents (A: 70%, B: 65%), it is easy to make the comparison.

	Opinion Survey			
	For	Undecided	Against	Total
A	64	20	7	91
B	74	25	15	114

It is interesting to note that the familiar symbol for percent, %, actually came from the business arithmetics written by the Italians in the fifteenth century. Modern business uses percent in a wide variety of transactions. Interest, discount, and commission are based on percent, as well as markup, profit, loss, and taxes. Because a rate expressed as a percent can be applied easily to both small and large numbers it is used extensively, whereas ratios would be difficult to use. Although they are useful in making comparisons, percents can also be used to misrepresent conditions. For example, if 4 out of 5 persons report that they have a four-door car, the generalization might be drawn that 80% of the people in a region have four-door cars. But the sampling is too small to draw such a conclusion. Percents must, therefore, be treated with discretion.

It was stated previously that a percent is one of several ways for showing a ratio and that conversions can be made from one form to another. The notation for percent using the symbol % is not a form which can be used in computation. It is necessary to convert a percent to one of its related forms, the most common of which are the fraction form and the decimal form. Of these, the decimal form is usually the more convenient, especially if a calculator is used. For example, to find 80% of 356, either the fraction $\frac{80}{100}$ (or $\frac{4}{5}$) or the decimal 0.80 (or 0.8) may be used as shown. Of these two, the decimal calculation is faster and shorter.

Find 80% of 356.

$$\frac{4}{5} \times 356 = \frac{1424}{5}$$
$$= 284\tfrac{4}{5}$$
$$0.8 \times 356 = 284.8$$

If, instead of 356, the number were a multiple of 5, such as 450, it might be easier to use the fraction $\frac{4}{5}$, but this decision requires a sophisticated analysis of both the percent and the other number. In most instances, it is as easy, or easier, to use a decimal equivalent.

$$\frac{4}{5} \times 450 = 4 \times 90$$
$$= 360$$

On page 267 the *Try This* feature demonstrates that percents involving numbers in mixed form are more precise than percents which have repeating decimals. The latter are never-ending, and hence, never exact. Even rounding a repeating decimal does not make it exactly equivalent to the fraction. On the other hand, when the remainder is shown as a fraction of the divisor, the change is complete, and the percent is exactly equivalent to the fraction.

$\frac{1}{3} = 0.333...$

$$\begin{array}{r} 0.333... \\ 3\overline{)1.000} \\ \underline{9} \\ 10 \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

$\frac{1}{3} = \frac{\blacksquare}{100}$

$$\begin{array}{r} 33\frac{1}{3} \\ 3\overline{)100} \\ \underline{9} \\ 10 \\ \underline{9} \\ 1 \end{array}$$

$\frac{1}{3} = 33\frac{1}{3}\%$

Success in problem solving is achieved through a combination of careful reading, identification of mathematical relationships, selection of appropriate operations and sequences of steps, and accurate calculations. Many programs have been devised to improve students' abilities to solve word problems. Some of them use a set of basic questions such as "What is given?" "What is to be found?" "Which numbers can be used?" "What operation(s) can be used?" Others structure the solution by a type of flow chart, particularly if more than one operation is required. One of the more recent approaches uses open number sentences by which problem situations are translated into mathematical relationships. If a solution involves only one operation, a single number sentence is used (A), but if there are several steps in a solution, more number sentences are usually required (B). It requires even more mathematical insight to use just one number sentence to structure a solution requiring several steps (C).

A

Find the average mass of 6 stones, if their total mass is 15 kg.

$$15 \div 6 = n$$

B

Find the cost of a jacket, regularly priced at \$19.95, offered at a discount of 25%. Include a sales tax of 7%.

$$0.25 \times 19.95 = \blacksquare$$

$$19.95 - \blacksquare = \bullet$$

$$\bullet \times 0.07 = \blacktriangle$$

$$\bullet + \blacktriangle = n$$

C

Find the cost of 4 m of cloth at \$3.59 for one metre and 3 spools of thread at 39¢ each.

$$(4 \times 3.59) + (3 \times 0.39) = n$$

To solve problems involving more than one step, students at this level will probably use several successive steps as shown in B. Whether or not they use open sentences to structure the steps, they will consciously or subconsciously think "What must be done first, with what numbers, and with what operation?" and "What must be done next, and how?" Students need to experience a variety of problem situations to develop the ability to solve problems, particularly problems requiring several steps.

Teaching Strategies

The sequence of fractions and operations with them, followed by ratios, in Units 10 to 12 leads directly to percent in this unit. Many of the concepts and skills in the previous units are reviewed and extended in working with percent. For example, it would be desirable to review the method of changing a fraction to an equivalent decimal presented on pages 238-240 prior to the lesson on page 264 where fractions are converted to percents. Also, a review of procedures to use in expressing fractions in lowest terms should prove valuable before students reach page 266, where percents are to be converted to fractions in lowest terms.

Students may be helped in the first of the *Related Activities* suggested on pages T 288 and T 289 if they follow a sequence of changes to find the percents. A suggested sequence includes

a. write the numbers in the sentence as a ratio,
b. write the ratio as a fraction,
c. express the fraction as an equivalent decimal,
d. express the decimal as a percent.

Prior to the lessons on pages 270-273, have students collect advertisements from newspapers, magazines, and pamphlets showing percents. Interest rates are frequently advertised by banks, trust companies, and loan corporations, and discounts are often listed in sales advertisements. These may be displayed for several days for students to see how the work with percent is related to everyday experiences.

In connection with the lesson on interest, it should be emphasized that the amount calculated is received at the end of one year and only if the amount of money remains unchanged for that period of time. The interest for a shorter period of time is related to the interest for one year in the same ratio as the period of time is related to one year. This aspect concerning interest does not occur in the lessons, but any misconceptions concerning interest may be easily avoided by a brief discussion of this aspect. Some students may have had little experience with the concepts of interest and discount and it may be difficult for them to differentiate between their effects; namely, that interest is added and discount is subtracted. Many students will have had experiences with sales tax and are probably aware that it is added to the price of an item.

Problems require careful reading and interpretation to select the proper operations and numbers for their solution. Sometimes it is necessary to read a problem several times. For the lesson on page 244, the students were encouraged to restate problems in their own words, and this problem-solving skill should be emphasized as they attempt to solve complex problems which require several steps in the solution.

Materials

models for 0.72 and 0.08 prepared from a copy of page T 394 or two copies of page T 382 as described on page T 89
a ten-by-ten grid cut from a copy of page T 382 or page T 394
step-on scales for the students to find their masses

Vocabulary

percent (%)	profit
interest	discount
deposit	marked down
withdrawal	regular price
savings account	sale price

Pages 262-263

LESSON OUTCOME

Write percents; convert among ratios out of 100, fractions with denominators of 100, decimal hundredths, and percents

Materials

models for 0.72 and 0.08 prepared from a copy of page T 394 or from two copies of page T 382 as described on page T 89

Vocabulary

percent (%)

Prerequisite Skills

Write decimal hundredths; write a ratio in three ways

Checking Prerequisite Skills

Write the decimal.

1. three-hundredths 0.03
2. twenty-hundredths 0.20
3. ninety-two hundredths 0.92

Complete the chart to show each ratio in three ways.

4.	1 to 100	1:100	$\frac{1}{100}$
5.	24 to 100	24:100	$\frac{24}{100}$
6.	37 to 100	37:100	$\frac{37}{100}$

Background

Reference to *percent* meaning "out of a hundred" will help to emphasize that expressions such as 42%, 0.42, $\frac{42}{100}$, and 42:100 are equivalent.

13 PERCENT

Writing Percents

In a poll, 48 out of 100 students knew all the words for "O Canada".

This result can be shown as a ratio.

48 : 100 $\quad \frac{48}{100}$

This result can be shown as a decimal.

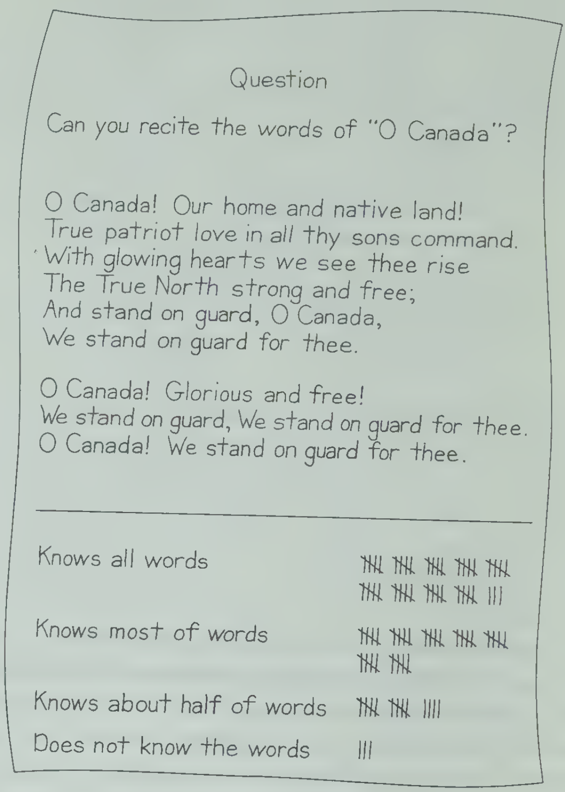
0.48

Of the students asked, 0.48 knew all the words for "O Canada".

This result can be shown as a **percent**.

48%

Of the students asked, 48% knew all the words for "O Canada".



Percent shows how many out of 100.

For the symbol %, you would say "percent".

Working Together

How many students knew most of the words for "O Canada"? Show the result in five ways.

1. 35 ■ out of 100   2. 35 ■ : 100   3. $\frac{■\ 35}{100}$ 4. 0. ■ 35   5. ■ 35 %

Show the other results of the poll.

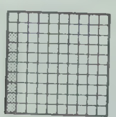
6. 14 ■ % of the students knew about half of the words.
7. 3 ■ % of the students did not know the words.

262

LESSON ACTIVITY

Before Using the Pages

- Display a model for 0.72 and ask what number is represented. Ask students to write the fraction and the decimal for "seventy-two hundredths" on the board. Point out that 72 of 100 parts are blue and ask what ratio can be written to express this. Have a student write the ratio using colon notation. Emphasize that 72:100, $\frac{72}{100}$, and 0.72 are different ways to represent the part of the model that is blue. Use a similar procedure for a model for 0.08, paying particular attention to the zero in the tenths' place of the decimal.



eight-hundredths
8:100 $\quad \frac{8}{100} \quad$ 0.08

Tell the students that there is another way to express "72 out of 100" and "8 out of 100". If no student suggests a percent, have the students turn to page 262.

Using the Pages

- The photograph on page 263 can motivate a discussion about making a survey and using a tally chart to record the information. Direct the students' attention to the tally chart on page 262. Ask students to count the tallies and to explain the results of the survey. Ask how many students in all were questioned for the survey.

  Ask a student to read the introductory statement at the top of page 262. Note that "48 out of 100" is shown as a ratio using the symbol : , as a fraction, and as a decimal. Introduce the term *percent* and the symbol %. If you wish, tell the students that the word comes originally from the Latin *per centum* meaning "out of a hundred". Thus, 48% means 48 out of 100. Ask in what way the symbol % suggests 100. The students will likely say that the oblique line and the two small 0's suggest the digits 1, 0, and 0 for 100. Return to the examples presented in *Before Using the Pages* and ask students to write each number as a percent.

Exercises

Complete.

1.	75 out of 100	■ :100 (75)	$\frac{■}{100}$ 75	0.■ (75)	■ % (75)
2.	9 out of ? 100	9:100	$\frac{9}{100}$?	0.09 ?	9% ?
3.	50 out of ? 100	? 50:100	$\frac{50}{100}$	0.50 ?	50% ?
4.	1 out of ? 100	? 1:100	$\frac{1}{100}$?	0.01	1% ?
5.	95 out of ? 100	? 95:100	$\frac{95}{100}$?	0.95 ?	95%

For each of these, write a sentence using %. Answers for Ex. 6–9 are given below.

6. In the poll, 83 out of 100 students knew most or all of the words for "O Canada".
7. 70 out of the 100 students in the poll had to use the tune to help them remember the words.

For each of these, write a sentence using a ratio.

8. 7% of the students said that they had the song on a record at home.
9. 90% of the students knew the tune for "O Canada".

263

RELATED ACTIVITIES

- Have the students prepare a display for the topic of percent. Draw attention to the photograph on the cover of *Starting Points in Mathematics 6*. Students can find examples of percents on coupons, on food containers, and so on. Some banks or stores may provide signs no longer needed, showing examples of percents. The display can provide a reference, and it can be used to generate word problems during this unit.
- For reinforcement, have students color copies of page T 394 to represent some of the percents shown on pages 262 and 263.
- For enrichment, provide students with copies of page T 394. Have them color part of a model blue, write the percent to show what part is blue, write the percent to show what part is not blue, and note that the sum of the two percents is 100%.
- Have students prepare pairs of cards such as the following for the game "Concentration" described on page T 379.

48%	0.48	$\frac{30}{100}$	30%

- To review graphing and to relate percents and graphs, have students draw a pictograph or a bar graph to show the results of the survey on page 262.

Working Together: The students are required to refer to the tally chart to complete Ex. 1-7. Point out that Ex. 1-5 show different ways of representing the same result.

Exercises: For Ex. 1, the students are guided as they show "75 out of 100" in different ways. Pay particular attention to the use of zeros in Ex. 2-4. For example, 9:100 for Ex. 2 is changed to 0.09 and to 9%, whereas $\frac{50}{100}$ for Ex. 3 is changed to 0.50 and to 50%.

Assessment

Complete.

1.	22 out of 100	22 : 100	$\frac{22}{100}$	0.22	22%
2.	31 out of 100	31 : 100	$\frac{31}{100}$	0.31	31%
3.	78 out of 100	78 : 100	$\frac{78}{100}$	0.78	78%
4.	5 out of 100	5 : 100	$\frac{5}{100}$	0.05	5%
5.	8 out of 100	8 : 100	$\frac{8}{100}$	0.08	8%

6. In the poll, 83% of the students knew most or all of the words for "O Canada".
7. 70% of the students in the poll had to use the tune to help them remember the words.
8. 7 out of 100 students said that they had the song on a record at home.
9. 90 out of 100 students knew the tune for "O Canada".

Pages 264-265

LESSON OUTCOME

Express a fraction as a percent; express a decimal as a percent (for tenths and hundredths)

Prerequisite Skills

Write an equivalent fraction with a denominator of 100 for a given fraction; express a fraction with a denominator of 100 as a percent; express a decimal showing hundredths as a percent; divide a one-digit number by a one-digit number, rounding the quotient to two decimal places

Checking Prerequisite Skills

For each of these, write an equivalent fraction with a denominator of 100.

1. $\frac{1}{4}$ $\frac{25}{100}$ **2.** $\frac{3}{5}$ $\frac{60}{100}$

Show each fraction as a percent.

3. $\frac{1}{100}$ 1% **4.** $\frac{80}{100}$ 80%

Show each decimal as a percent.

5. 0.92 92% **6.** 0.06 6%

Divide. Round the quotient to two decimal places.

7. $8\overline{)9}$ 1.13 **8.** $3\overline{)2}$ 0.67

Fractions and Decimals as Percents

Jerry estimated that $\frac{3}{4}$ of the students were wearing running shoes.

The fraction $\frac{3}{4}$ is equivalent to $\frac{75}{100}$.

3 × 25

$$\frac{3}{4} = \frac{75}{100}$$

4 × 25

$\frac{75}{100}$ can be shown as 0.75 or as 75%.

Jerry estimated that 0.75 of the students were wearing running shoes.

Jerry estimated that 75% of the students were wearing running shoes.

Working Together

Show each fraction as a decimal.

Example: For $\frac{3}{4}$, use $\frac{3}{4} = \frac{75}{100}$, or $4\overline{)3.00}$ with quotient 0.75.

To change a fraction to a decimal, divide the numerator by the denominator.

1. $\frac{1}{2}$ 0.5 **2.** $\frac{9}{10}$ 0.9 **3.** $\frac{2}{5}$ 0.4 **4.** $\frac{6}{8}$ 0.75

Show each decimal as a percent.

5. 0.25 25% **6.** 0.9 90% **7.** 0.06 6%

Show each fraction as a percent.

8. $\frac{37}{100}$ 37% **9.** $\frac{2}{5}$ 40% **10.** $\frac{6}{8}$ 75%

264

LESSON ACTIVITY

Before Using the Pages

- Briefly review the work of the previous lesson. Ask the students to express a test mark of 72 out of 100 as a fraction, as a decimal, and as a percent. Then point out that tests are not necessarily marked out of 100. Ask the students if a test mark such as 8 out of 10 can be expressed as a percent, emphasizing that a percent shows how many out of 100. Write the fraction $\frac{8}{10}$ on the board. Lead the students to suggest that $\frac{8}{10}$ can be expressed as an equivalent fraction with a denominator of 100 and then written as a percent.

$$\frac{8}{10} = \frac{80}{100}, \text{ or } 80\%$$

Using the Pages

- Draw attention to the photograph. Ask how many of the four students are wearing running shoes. Then ask a student to read the statement at the top of page 264. Ask how $\frac{3}{4}$ can be expressed as a fraction having a denominator of 100. Ask why it is necessary for the denominator to be 100. Have students read the last two statements of the worked example and compare them with the first statement.

Working Together: Discuss the example to review that the numerator may be divided by the denominator to express a fraction as a decimal. Note the extra zeros used in the dividend. For other similar exercises, students may use either equivalent fractions or division.

Ex. 1-4 deal with expressing a fraction as a decimal; Ex. 5-7 involve expressing decimals as percents. For Ex. 6, emphasize that 0.9 is equivalent to 90%, not 9%; similarly, in Ex. 7, 0.06 is equivalent to 6%. For Ex. 8-10, each fraction is to be expressed as a percent.

Exercises: Before the students begin, draw attention to Ex. 19 and lead the students to recall that for $\frac{1}{3}$, the division $3\overline{)1.000}$ with quotient 0.333 can continue forever. Point out that it will be necessary to round such quotients to two decimal places. The rounded quotient, 0.33, will be used to express an

Exercises

Show each decimal as a percent.

1. 0.45 45% **2.** 0.2 20% **3.** 0.02 2% **4.** 0.8 80% **5.** 0.91 91% **6.** 0.05 5%

Show each fraction as a percent.

7. $\frac{71}{100}$ 71% **8.** $\frac{3}{5}$ 60% **9.** $\frac{7}{10}$ 70% **10.** $\frac{9}{12}$ 75% **11.** $\frac{5}{5}$ 100% **12.** $\frac{1}{2}$ 50%

13. $\frac{3}{10}$ 30% **14.** $\frac{1}{20}$ 5% **15.** $\frac{4}{5}$ 80% **16.** $\frac{12}{16}$ 75% **17.** $\frac{4}{4}$ 100% **18.** $\frac{16}{20}$ 80%

Sometimes more than two decimal places are needed when a fraction is changed to a decimal. Change each of these to a decimal by dividing the numerator by the denominator. Then round the quotient to two decimal places.

19. $\frac{1}{3}$ 0.33 **20.** $\frac{5}{8}$ 0.63 **21.** $\frac{2}{9}$ 0.22 **22.** $\frac{6}{7}$ 0.86 **23.** $\frac{1}{12}$ 0.08 **24.** $\frac{1}{6}$ 0.17

Use your results from Exercises 19 to 24 to write a percent for each of these fractions.

25. $\frac{1}{3}$ 33% **26.** $\frac{5}{8}$ 63% **27.** $\frac{2}{9}$ 22% **28.** $\frac{6}{7}$ 86% **29.** $\frac{1}{12}$ 8% **30.** $\frac{1}{6}$ 17%

For each of these, write a sentence using percent.

31. 5 out of every 10 apples were bruised in the hailstorm.

32. 1 out of every 8 entries received a prize. 13% of the entries received a prize

33. 3 out of every 4 students knew the name of the premier.

34. Mrs. Tull works 5 of the 7 d of the week. Mrs Tull works on 71% of the days of the week

35. We can expect warm weather in 5 of the 12 months.

36. The team won 2 out of every 3 games it played. The team won 67% of the games played

37. We work 8 of the 16 h that we are awake.

38. We work 8 of the 24 h in each day. We work 33% of each day

31. 50% of the apples were bruised in the hailstorm
33 75% of the students knew the name of the premier
35 We can expect warm weather in 42% of the months
37 We work 50% of the time that we are awake

265

RELATED ACTIVITIES

• Students may enjoy completing statements similar to the following and expressing the information as percents.

1. _2_ of the _5_ children in our family are boys.
   $\frac{2}{5}$, 0.4, 40%
2. ____ of the ____ letters in my name are vowels.

• Students may conduct surveys similar to the one described on pages 262 and 263. Suggest questions such as "Who is the premier of our province?" or "What is the capital of Nova Scotia?" Fractions may be written to show the number of students who know the answers to all the questions, to three out of four questions, and so on, for the survey. Have the students express the fractions as percents and, if you wish, show the results of the survey in the form of a graph.

• The students may be interested in expressing their test marks as percents. Discuss how percents can help them to compare the marks they receive for various assignments.

approximate percent for $\frac{1}{3}$ in Ex. 25 (33%). Similarly, the rounded quotients in Ex. 20-24 will be used to express approximate percents for Ex. 26-30.

Note that Ex. 17 presents the fraction $\frac{4}{4}$ which is equivalent to $\frac{100}{100}$, thus, the percent is 100%.

Assessment

Show each decimal as a percent.

1. 0.62 62% **2.** 0.09 9% **3.** 0.1 10%

Show each fraction as a percent.

4. $\frac{4}{5}$ 80% **5.** $\frac{3}{3}$ 100% **6.** $\frac{1}{4}$ 25%

Pages 266-267

LESSON OUTCOME

Express a percent as a decimal, as a fraction with a denominator of 100, and as a fraction in lowest terms

Materials

a ten-by-ten grid cut from a copy of page T 382 or page T 394

Prerequisite Skills

Express a fraction or a decimal as a percent; write a fraction with a denominator of 100 in lowest terms

Checking Prerequisite Skills

Show each fraction as a percent.

1. $\frac{3}{4}$ 75% **2.** $\frac{9}{10}$ 90%

Show each decimal as a percent.

3. 0.81 81% **4.** 0.4 40%

Write each fraction in lowest terms.

5. $\frac{80}{100}$ $\frac{4}{5}$ **6.** $\frac{75}{100}$ $\frac{3}{4}$

Percents as Decimals and Fractions

Some people spend 33% of their lifetimes asleep.

$$33\% = 0.33,$$
$$\text{or } \frac{33}{100}$$

$\frac{33}{100}$ is almost equal to $\frac{1}{3}$

33% is about $\frac{1}{3}$

Some people spend about $\frac{1}{3}$ of their lifetimes asleep.

Working Together

Show each percent as a decimal.

1. 62% 0.62 2. 7% 0.07 3. 70% 0.70

Show each decimal as a fraction with a denominator of 100.

4. 0.50 $\frac{50}{100}$ 5. 0.80 $\frac{80}{100}$ 6. 0.75 $\frac{75}{100}$

Show each percent as a fraction with a denominator of 100.

7. 70% $\frac{70}{100}$ 8. 53% $\frac{53}{100}$ 9. 5% $\frac{5}{100}$

Show each percent as a fraction in lowest terms.

10. 40% $\frac{2}{5}$ 11. 25% $\frac{1}{4}$ 12. 8% $\frac{2}{25}$

Exercises

Show each percent as a decimal.

1. 95% 0.95 2. 1% 0.01 3. 30% 0.30 4. 80% 0.80 5. 8% 0.08 6. 50% 0.50

Show each percent as a fraction with a denominator of 100.

7. 3% $\frac{3}{100}$ 8. 99% $\frac{99}{100}$ 9. 13% $\frac{13}{100}$ 10. 7% $\frac{7}{100}$ 11. 21% $\frac{21}{100}$ 12. 60% $\frac{60}{100}$

Show each percent as a fraction in lowest terms.

13. 10% $\frac{1}{10}$ 14. 75% $\frac{3}{4}$ 15. 50% $\frac{1}{2}$ 16. 35% $\frac{7}{20}$ 17. 4% $\frac{1}{25}$ 18. 80% $\frac{4}{5}$

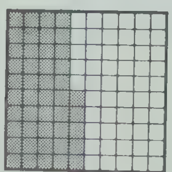
266

LESSON ACTIVITY

Before Using the Pages

- Write 45% on the board and ask a student to explain the meaning of the numeral. Display a ten-by-ten grid cut from a copy of page T 382 or page T 394. Ask what part of the grid should be colored blue to illustrate 45% and have students help to do this. Then ask for the decimal and the fraction that describe the same diagram. Tape the diagram to the board and write beside it the numerals shown.

45 out of 100



45% 0.45 $\frac{45}{100}$

45 percent 45 hundredths

Ask whether the fraction $\frac{45}{100}$ is in lowest terms. Have a student name the common factor by which 45 and 100 can be divided. Write $\frac{9}{20}$ beside $\frac{45}{100}$ on the board. Emphasize that the four numerals (45%, 0.45, $\frac{45}{100}$, $\frac{9}{20}$) are different ways of describing the same diagram.

Using the Pages

- Ask a student to read the title of the lesson and the statement at the top of page 266. Discuss that 33% can be expressed as 0.33 or as $\frac{33}{100}$. Refer to Ex. 19 on page 265. Recall that for $\frac{1}{3}$, the division $3\overline{)1.000}$ with quotient 0.333 can continue forever and that the rounded quotient 0.33 is used to obtain 33%. This will help them to understand the statement in the worked example, "33% is almost equal to $\frac{1}{3}$". Ask a student to read the concluding statement.

Working Together: It is advisable at this time to have students express percents similar to Ex. 3 as two-place decimals. In other words, 70% is expressed as 0.70, not as 0.7. This will help students to understand that 7% (Ex. 2) is equivalent to 0.07. For Ex. 10-12, the students can express each percent as a fraction with a denominator of 100 and then show the fraction in lowest terms.

For each of these, write a sentence using a fraction in lowest terms.

19. Some people spend 67% of their lifetimes awake. Some people spend $\frac{67}{100}$ of their lifetimes awake.

20. An average 11-year-old boy has grown to about 81% of his adult height. An average 11-year-old boy has grown to about $\frac{81}{100}$ of his adult height

21. An average 11-year-old girl has grown to about 93% of her adult height. An average 11-year-old girl has grown to about $\frac{93}{100}$ of her adult height

22. An average 12-year-old boy has grown to about 84% of his adult height. An average 12-year-old boy has grown to about $\frac{21}{25}$ of his adult height

23. About 20% of the air we breathe in is oxygen. About $\frac{1}{5}$ of the air we breathe in is oxygen

24. About 80% of the air we breathe in is nitrogen. About $\frac{4}{5}$ of the air we breathe in is nitrogen

25. About 17% of the air we breathe out is oxygen. About $\frac{17}{100}$ of the air we breathe out is oxygen.

26. About 78% of the air we breathe out is nitrogen. About $\frac{39}{50}$ of the air we breathe out is nitrogen

27. About 5% of the air we breathe out is carbon dioxide. About $\frac{1}{20}$ of the air we breathe out is carbon dioxide.

28. The mass of oxygen in the human body is about 63% of the total mass of the body. The mass of oxygen in the human body is about $\frac{63}{100}$ of the total mass of the body

Match one of these fractions with the percent closest in value to it.

$\frac{1}{2}$	$\frac{1}{3}$	$\frac{1}{4}$	$\frac{1}{5}$	$\frac{1}{6}$	$\frac{1}{7}$	$\frac{1}{8}$	$\frac{1}{9}$

29. 25% $\frac{1}{4}$ 30. 14% $\frac{1}{7}$ 31. 50% $\frac{1}{2}$ 32. 11% $\frac{1}{9}$
33. 20% $\frac{1}{5}$ 34. 33% $\frac{1}{3}$ 35. 13% $\frac{1}{8}$ 36. 17% $\frac{1}{6}$

Sometimes percents will be decimals or numbers in mixed form.

Examples: 33.3% is 0.333.
It is closer to $\frac{1}{3}$ than 33% is.
$33\frac{1}{3}$% is exactly $\frac{1}{3}$.

Use a one-place decimal to show a percent for each of these.

1. $\frac{2}{3}$ 66.7% 2. $\frac{5}{6}$ 83.3% 3. $\frac{1}{8}$ 12.5% 4. $\frac{8}{9}$ 88.9% 5. $\frac{7}{12}$ 58.3%

Show a percent with a fraction for each of these.

6. $\frac{2}{3}$ $66\frac{2}{3}$% 7. $\frac{5}{6}$ $83\frac{1}{3}$% 8. $\frac{1}{8}$ $12\frac{1}{2}$% 9. $\frac{8}{9}$ $88\frac{8}{9}$% 10. $\frac{7}{12}$ $58\frac{1}{3}$%

try this

267

RELATED ACTIVITIES

- Have students research other facts similar to the ones in Ex. 20-28 on page 267 and express the percents as fractions or vice versa.
- As an extension of the *Try This* feature, you may wish to have the students continue some of the divisions. They may use a three-place decimal or a four-place decimal to show a percent for some of the fractions in Ex. 1-5 or for some of the fractions in Ex. 1-20 on page 239. Note that some of the percents will be greater than 100% because some of the numbers on page 239 are greater than one.
- Students can prepare sets of cards similar to the following for the game "Match Up" described on page T 381.

20%	20:100	$\frac{1}{5}$

- Have students complete charts similar to the following.

10%	0.10	$\frac{10}{100}$	$\frac{1}{10}$	
20%	0.20	$\frac{20}{100}$	$\frac{2}{10}$	$\frac{1}{5}$
30%				
90%				
25%				

Exercises: Note that for Ex. 19-21, 25, and 28, the fractions with a denominator of 100 are in lowest terms. For Ex. 22-24, 26, and 27, the fractions with a denominator of 100 are not in lowest terms.

For Ex. 29-36, it would likely be best to express each fraction in the chart as a decimal and then as a percent, rounding quotients with repeating digits to two decimal places where necessary. Emphasize that some of the given percents are almost equal — not exactly equal — to the fractions in the chart. For example, for Ex. 32, 11% is almost equal to $\frac{1}{9}$ because 0.111. . ., the decimal equivalent for $\frac{1}{9}$, is rounded down to 0.11, which is then expressed as 11%.

Try This: Refer to Ex. 29 and 34 on page 267. Review that $\frac{1}{4}$ and 25% are equivalent, whereas $\frac{1}{3}$ and 33% are close in value but not equivalent. $\frac{33}{100}$ is equivalent to 33% and $\frac{1}{3}$ is equivalent to $\frac{33}{99}$ which is close to $\frac{33}{100}$, or 33%. Explain that it is possible to write a percent closer to $\frac{1}{3}$ than 33%, and that a percent equivalent to $\frac{1}{3}$ can also be written. This is best demonstrated on the board using cross products and division as indicated below. Point out that the percent shows a decimal or a fraction.

$$\frac{1}{3} = \frac{■}{100}$$

$$3 \times ■ = 1 \times 100 \qquad 100 \div 3 = 33\tfrac{1}{3} \text{ or } 33.3\ldots$$

$$\frac{1}{3} = \frac{33\frac{1}{3}}{100} \ \left(\text{about } \frac{33.3}{100}\right) \qquad \frac{1}{3} = 33\tfrac{1}{3}\% \text{ (about } 33.3\%)$$

Emphasize that $\frac{1}{3}$ is closer in value to 33.3% than to 33%, and that $\frac{1}{3}$ and $33\frac{1}{3}$% are equivalent.

Assessment

Show each percent as a decimal.

1. 70% 0.70 **2.** 43% 0.43 **3.** 2% 0.02

Show each percent as a fraction with a denominator of 100.

4. 26% $\frac{26}{100}$ **5.** 50% $\frac{50}{100}$ **6.** 4% $\frac{4}{100}$

Show each percent as a fraction in lowest terms.

7. 8% $\frac{2}{25}$ **8.** 81% $\frac{81}{100}$ **9.** 30% $\frac{3}{10}$

LESSON OUTCOME

Find a percent of a number; solve related word problems

Materials

step-on scales for the students to find their masses

Prerequisite Skills

Express a percent as a decimal; multiply a whole number by a decimal

Checking Prerequisite Skills

Show each percent as a decimal.

1. 4% 0.04 **2.** 20% 0.20

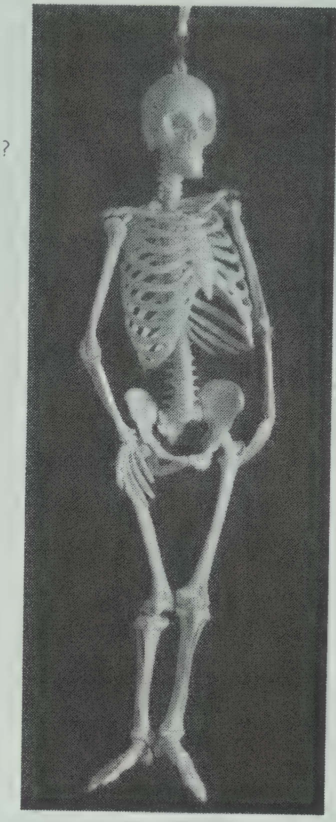
Multiply.

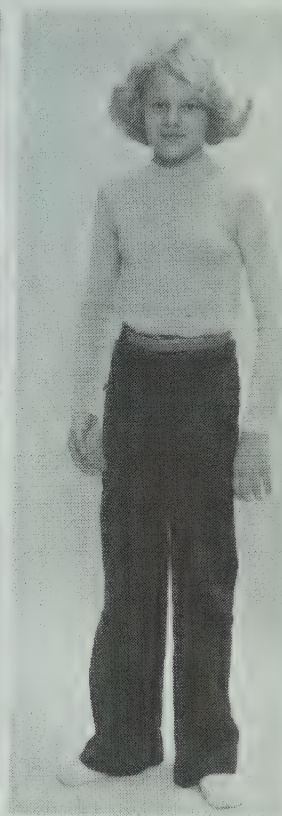
3. 0.75×26 19.5

4. 0.9×782 703.8

Finding a Percent of a Number

The mass of bone in the human body is about 18% of the total mass of the body. Stephanie has a mass of 38 kg. How heavy are her bones?





To find

18% of 38,

find

0.18 of 38,

or

0.18×38.

$$\begin{array}{r} 38 \\ \underline{0.18} \\ 3\,04 \\ \underline{3\,80} \\ 6.84 \end{array}$$

To find a percent of a number, change the percent to a decimal and multiply.

Stephanie's bones have a mass of 6.84 kg.

Working Together

What two numbers are multiplied to find each of these?

1. 85% of 67 0.85 and 67 **2.** 6% of 120 0.06 and 120 **3.** 70% of 2379 0.70 and 2379

Find each of these.

4. 85% of 67 56.95 **5.** 6% of 120 7.2 **6.** 70% of 2379 1665.3

268

LESSON ACTIVITY

Before Using the Pages

- Write the following on the board.

  1. $\frac{1}{2}$ of 14 2. $\frac{1}{2} \times 14$ 3. 50% of 14

  Review that Ex. 1 and 2 give the same result, 7, because the word "of" indicates multiplication. Ask what can replace the word "of" in Ex. 3. Write $50\% \times 14$ on the board. Develop that the answer to Ex. 3 must also be 7, because 50% is another way of representing $\frac{50}{100}$, or $\frac{1}{2}$. However, it may seem unusual to show $50\% \times 14$ for Ex. 3 because it appears to suggest the whole-number multiplication $50 \times 14 = 700$. Let the students consider this dilemma for a few moments. Explain that in previous lessons they have learned how to multiply with whole numbers and with decimals, and that in this lesson, they will learn how to multiply with percents. Then have them turn to page 268.

Using the Pages

- Provide an opportunity for the students to discuss the photographs on page 268. Then direct the discussion to the word problem. Discuss that 18% is equivalent to 0.18, and therefore, 18% of 38 is equal to 0.18 of 38, or 0.18×38. Have students explain the multiplication and the information in the "thought cloud". Read the concluding statement.

  Have the students multiply 0.50 and 14 to check the third exercise from *Before Using the Pages*.

Working Together: The numbers in Ex. 1-3 are repeated in Ex. 4-6. Ex. 1-3 deal with expressing a percent as a decimal. Ex. 4-6 deal with multiplying by the decimal. Pay particular attention to the zeros for expressing 6% as 0.06 and for expressing 70% as 0.70.

Exercises: Provide step-on scales so that the students can find their masses to the nearest kilogram for Ex. 15.

Exercises

Find each of these.

1. 40% of 375 150
2. 5% of 60 3
3. 50% of 172 86
4. 95% of 780 741
5. 67% of 48 32.16
6. 75% of 16 000 12 000
7. 1% of 35 0.35
8. 10% of 350 35
9. 30% of 4570 1371
10. 25% of 9724 2431

This chart shows information about the mass of the human body.

	Female	Male
Water	54%	60%
Muscle	36%	42%
Fat	28%	18%
Bone	18%	18%

Use the chart to help you answer these questions.

11. How much of Stephanie is water? 20.52 kg
12. How much of Stephanie is muscle? 13.68 kg
13. How much of Stephanie is fat? 10.64 kg

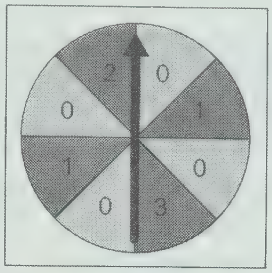
Todd has a mass of 34 kg.

14. How much of Todd is water? 20.40 kg muscle? fat? bone? 14.28 kg, 6.12 kg, 6.12 kg
*15. How much of you is water? muscle? fat? bone? Answers will vary

Percents you observe can help you make estimates.

1. A baseball pitcher has struck out 40% of the batters in past games. In the next game, what would be a reasonable number to expect the pitcher to strike out in facing 27 batters? 11
2. 33% of the nuts in a can are cashews. If you take 35 nuts from the can without looking, what would be a reasonable number of cashews to expect? 12

This spinner is divided into 8 sections, all the same size.



What percent of the spaces

3. show 0? 50%
4. show 1? 25%

In 200 spins, how many times could you expect to

5. score 0? 100
6. score 1? 50

What total score could you reasonably expect in

7. 200 spins? 175
8. 1000 spins? 875

PROBLEM SOLVING

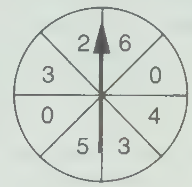
269

RELATED ACTIVITIES

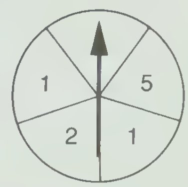
• For more practice, you may wish to have the students complete Ex. 33-48 on page 337.

• Have students use copies of the circle on page T 385 to make spinners like the one in the *Problem Solving* feature. Have them estimate how many times they would expect to score 0 (1, 2, 3) in 50 spins and then estimate the total score they would expect in 50 spins. Have them test their estimates by spinning the arrow and recording the results in tally charts. This activity can be varied by changing the numerals on the sections (A), the number of sections on the spinner (B), or the sizes of the sections (C).

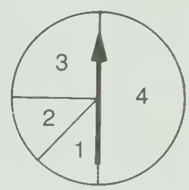
A



B



C



Problem Solving: These exercises show how percent can be used to estimate probability. Ex. 1 and 2 involve finding a percent of a number. For Ex. 3 and 4, the students write a ratio or a fraction and then express it as a percent. The percents found for Ex. 3 and 4 are used to make estimates for Ex. 5-8.

Assessment

Find each of these.

1. 42% of 78 32.76 **2.** 60% of 364 218.4

3. 7% of 920 64.4 **4.** 98% of 5708 5593.84

Solve.

5. 75% of the 40 students in Donna's class saw the skeleton at the museum. How many students saw the skeleton? 30

LESSON OUTCOME

Use percent to calculate interest; solve related word problems

Vocabulary

interest, deposit, withdrawal, savings account, profit

Prerequisite Skills

Find a percent of a number

Checking Prerequisite Skills

Find each of these.

1. 10% of 42 4.2
2. 25% of 738 184.5
3. 8% of 6490 519.2

Interest

Interest is the amount paid for using money that belongs to someone else.

Banks pay interest at different times during the year.

Sandra has $50 in her savings account. Each year the bank pays Sandra interest that is equal to 5% of the $50.

For 5% of $50, multiply 0.05 and 50.

$$0.05 \times 50 = 2.50$$

Each year the bank pays Sandra $2.50 interest.

The bank uses Sandra's $50 and other money to make loans. Each year the bank receives interest that is equal to 9% of its loans.

For 9% of $50, multiply 0.09 and 50.

$$0.09 \times 50 = 4.50$$

Each year that the bank is able to have Sandra's $50 out on loan, it receives $4.50 for the $50.

When it is able to have Sandra's $50 out on loan, the bank earns 4% of the $50, or $2.00, in profit.

$$0.04 \times 50 = 2.00$$

This is one way that a bank is able to stay in business.

270

LESSON ACTIVITY

Using the Pages

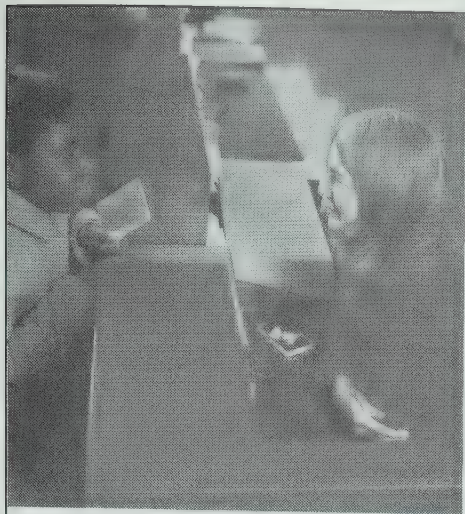
- Use the photographs to motivate a discussion about banks. Discuss such aspects as depositing money in a bank account, writing cheques, and borrowing money from a bank. Relate the discussion to the students' knowledge and experience with banks. They may refer to stories they have read about banks or programs they have watched on television. Some students may have savings accounts or may have visited banks with their parents. During the discussion, ask the students about receiving *interest* for money that has been deposited in a bank, and paying interest for money that has been borrowed from a bank.

  Ask one student to read the statement below the title of the lesson. Ask another student to read the information describing Sandra's savings account. Ask how 5% of $50 is found, recalling the procedure of the lesson on pages 268 and 269.

  Have students read the example describing how a bank uses the money that people deposit. Compare the interest the bank receives for $50 that is loaned and the interest Sandra receives for $50 that is deposited. Develop that the difference between the amounts is the difference between 9% of $50 and 5% of $50, which is 4% of $50, or $2.00. Note that the difference, $2.00, could also be found by subtracting $2.50, the interest received by Sandra, from $4.50, the interest received by the bank.

Exercises: Remind the students to include the symbol $ in each answer. Ensure that they understand how to complete the chart for Ex. 14-16.

Problem Solving: Explain that interest left in an account at the end of one year provides more money on which to receive interest for the following year. Provide an opportunity for the students to share their solutions.



Exercises

Find each of these.

1. 4% of \$350 \$14
2. 5% of \$350 \$17.50
3. 6% of \$350 \$21
4. 7% of \$350 \$24.50
5. 8% of \$350 \$28
6. 9% of \$350 \$31.50
7. 10% of \$350 \$35
8. 12% of \$350 \$42
9. 13% of \$350 \$45.50
10. 14% of \$350 \$49
11. 15% of \$350 \$52.50

Solve.

12. You deposit \$75.50 in a bank. In a year the bank will pay you 6% of \$75.50 in interest. How much will the interest be? \$4.53

13. You borrow \$25 from your parents and offer to pay 2% of \$25 as interest. How much will the interest be? \$0.50

Complete this chart.

	Deposit	Each year the bank pays interest that is this percent of the deposit.	In one year the interest will be
14.	\$10 000	9%	\$900 ?
15.	\$ 2 500	7%	\$175 ?
16.	\$ 39.50	6%	\$2.37 ?

At the start of each year for 5 years, Emlyn deposited \$50 in his savings account. He received 9% interest on his savings at the end of each year. Each year he added the interest to his account.

Interest on a savings account is rounded to the nearest cent.

PROBLEM SOLVING

1. If Emlyn made no other deposits or withdrawals, how much would he have in his account at the end of 5 years? \$326.17

271

RELATED ACTIVITIES

- Have students complete exercises similar to the following on copies of page T 391.

	3%
427	12.81
94	
6	
1920	

	346
2%	6.92
15%	
64%	
28%	

- Provide information about the interest received from banks and trust companies for various kinds of accounts. Have the students find the interest that would be received in one year for such amounts as \$200 and \$3000. They can compare the amounts of interest for different kinds of accounts. If signs from banks or trust companies are on display as examples of percents, these can be used as reference.
- Adapt the preceding activity for interest paid for loans.

Assessment

Find each of these.

1. 14% of \$56 \$7.84 **2.** 9% of \$730 \$65.70

3. 2% of \$4200 \$84 **4.** 20% of \$940 \$188

Solve.

5. Jamie had \$25 in a bank account for a year. The bank paid interest equal to 8% of the money deposited. How much interest did Jamie receive? \$2.00

Pages 272-273

LESSON OUTCOME

Use percent to calculate discount

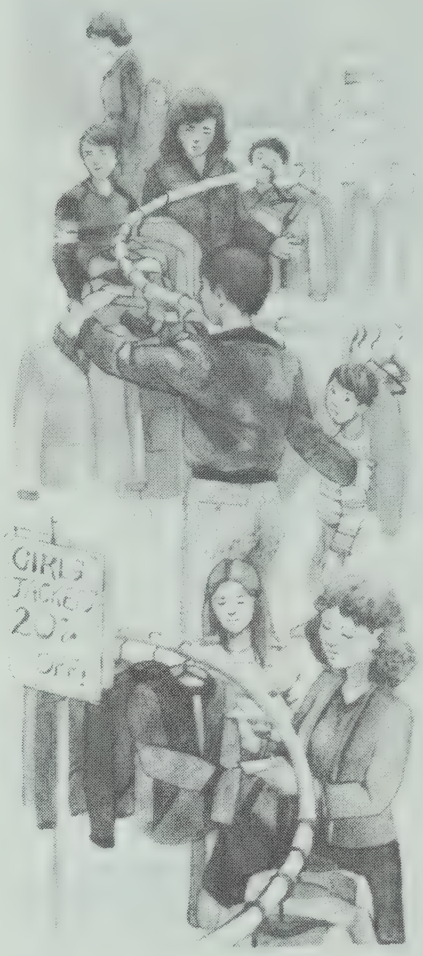
Vocabulary
discount, marked down, regular price, sale price

Prerequisite Skills
Find a percent of an amount of money

Checking Prerequisite Skills
Find each of these to the nearest cent.

1. 33% of $62 $20.46
2. 50% of $74.50 $37.25
3. 9% of $8 $0.72

Discount



A **discount** is the amount that is taken away from a given price or from a given amount of money.

For the sale on Wednesday, there will be a discount on each item that is 20% off the regular price.

The regular price for a jacket is $8.48. What will the discount be? What will the sale price of the jacket be on Wednesday?

For 20% of $8.48, multiply 0.20 and 8.48.

$$\begin{array}{r} 8.48 \\ 0.20 \\ \hline 1.6960 \end{array}$$

1.6960 rounds to 1.70.

The discount on the jacket on Wednesday will be $1.70

For the sale price on Wednesday, subtract 1.70 from 8.48.

$$\begin{array}{r} 8.48 \\ 1.70 \\ \hline 6.78 \end{array}$$

The sale price for the jacket on Wednesday will be $6.78.

272

LESSON ACTIVITY

Before Using the Pages

- Begin with a brief discussion about buying items that are on sale. Ask for ways in which stores can indicate that the price of an item is less than the regular selling price. For example, the price tag may show the original price and the reduced price (~~$18.95~~, $15.50). Stores may advertise that two of a particular item will be sold for the price of one.

Using the Pages

- Draw attention to the announcement illustrated at the top of page 273. Point out that percents are used to indicate sale prices. Ask a student to explain what is meant by "marked down 20%", for Wednesday's bargain sale. Then direct the discussion to the worked example on page 272.

  Ask a student to read the statement at the top of page 272 to introduce the term *discount*. Discuss the example showing how to find 20% of $8.48. Emphasize that $1.70 is the discount, which means that this amount is subtracted from the regular price to find the sale price of the jacket.

  Refer again to the information on page 273 about the sale. Review that the price on Wednesday for the jacket is $6.78. Ask how to find the price that would be charged on Thursday and on Friday.

Exercises: Ensure that the students understand that the discount on Thursday (Friday) is applied to the price charged the previous day, not to the original price. You may wish to discuss how to complete the first line of the chart. Point out that to find the discount on Friday, $33\frac{1}{3}\%$ may be expressed as the fraction $\frac{1}{3}$ or as the decimal 0.33. This option will result in slight differences in answers. For example, for Ex. 1, the sale price on Thursday is $2.40. The discount on Friday can be found in the following ways.

$$\begin{aligned} &\tfrac{1}{3} \text{ of } \$2.40 \\ &= \tfrac{1}{3} \times \$2.40 \\ &= \$0.80 \end{aligned}$$

$$\begin{array}{r} \$2.40 \\ \times\ \ 0.33 \\ \hline 720 \\ 720 \\ \hline \$0.7920 \end{array} \text{ (about \$0.79)}$$

Whatever you do this week, don't miss our

THE BARGAIN MART

GOING OUT OF BUSINESS

SALE ! ! !

THE BARGAIN MART

Wednesday: All merchandise will be marked down ⟶ 20%

Thursday: Remaining merchandise will be marked down another ⟶ 25%

Friday: Remaining merchandise will be marked down another ⟶ $33\frac{1}{3}\%$

After that, our doors will be closed forever!

HURRY! HURRY! HURRY!

Exercises

Complete.

Answers may vary.

			Wednesday		Thursday		Friday	
		Regular price	Discount	Sale price	Discount	Sale price	Discount	Sale price
1.	Shirts	$4.00	$0.80?	$3?20	$0.80?	$2?40	$0.80?	$1?60
2.	Jeans	$7.60	$1.52?	$6?08	$1.52?	$4?56	$1.52?	$3?04
3.	Hats	$3.75	$0.75?	$3?00	$0.75?	$2?25	$0.75?	$1?50
4.	Belts	$3.26	$0.65?	$2?61	$0.65?	$1?96	$0.65?	$1?31
5.	Shoes	$10.50	$2.10?	$8?40	$2.10?	$6?30	$2.10?	$4?20
6.	Socks	69¢	14¢ ?	?55¢	14¢ ?	?41¢	14¢ ?	?27¢

7. Give a price of something that you bought recently.
What would be its price in this store on Friday of this sale? Answers will vary

273

Some students may express $\frac{1}{3}$ as 0.333 and use $0.333 \times \$2.40$ to obtain a discount of $0.79920, which is rounded up to $0.80.

When the students have completed the chart, discuss that the chosen percents allow items to be marked down by approximately the same amount each day.

Assessment

Complete the chart. Prices will be marked down by 25%.

	Item	Regular price	Discount	Sale price
1.	Jackets	$36.00	$9.00	$27.00
2.	Hats	$5.50	$1.38	$4.12
3.	Sunglasses	98¢	25¢	73¢

RELATED ACTIVITIES

- For more practice, have the students complete a work sheet similar to the following.

Item	Regular price	Percent off regular price	Sale price
Sweater	$12	15%	
Gloves	$8.95	20%	

- Provide newspaper clippings of items on sale for which the discount is indicated by a percent and have the students find the sale price of each item.
- For an art lesson, have the students make signs advertising a sale. Display the signs and have the students write and solve word problems related to them.

OBJECTIVE

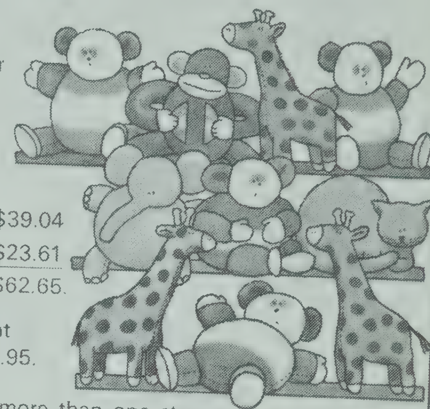
Solve problems in more than one step

RELATED ACTIVITIES

- As a challenge, have students write word problems for which solutions require several steps. Have other students solve the word problems. The pictures in this unit or a story which the students have read may suggest a topic for the problems.
- If your province has a sales tax, have the students find the total cost for items shown in catalogs and newspapers.

Solving Problems in More Than One Step

Mrs. Cotter bought a stuffed panda for each of four nieces and a stuffed giraffe for each of three other nieces. Each stuffed panda cost \$9.76. Each stuffed giraffe cost \$7.87. What was the average amount she spent for each niece?



4 stuffed pandas cost 4 × \$9.76, or \$39.04
3 stuffed giraffes cost 3 × \$7.87, or \$23.61
The 7 stuffed animals cost \$62.65.

The average amount Mrs. Cotter spent for each niece was \$62.65 ÷ 7, or \$8.95.

Solve these problems. Each requires more than one step.

1. A pencil costs 19¢. An eraser costs 15¢. Lloyd bought 3 pencils for himself and a pencil and an eraser for each of his 6 friends. How much did he spend? \$2.61

2. Sheila bought 6 trays, each a different size. The smallest cost \$7.89. Each increase in size increased the price of a tray by \$2.74. What was the average price for each tray? \$14.74

3. In Manitoba an amount equal to 5% of the regular price is added to the regular price as a sales tax. In Nova Scotia the sales tax is 8% of the regular price. A game costs \$24 in Manitoba. In Nova Scotia, it costs 25¢ less. In which province would you pay more for the game? Nova Scotia

4. The regular mail-order price was \$11.95. The sale price was 20% off the regular price. The sales tax is 8%. There is also a fee of \$2.25 for postage. How much must be paid for the mail order? \$12.57

5. Mrs. Cotter bought 5 cans of engine oil at \$1.32 each. At another store she bought 6 cans of the same oil for 25% off \$1.32. What was the average price that Mrs. Cotter paid for a can of oil? \$1.14

PROBLEM SOLVING

274

LESSON ACTIVITY

Using the Page

- Allow a few moments for the students to read the worked example independently. Then ask questions such as "What did Mrs. Cotter buy?" "What are you required to find?" "Why is more than one step needed to find the average?" Develop that the following steps are necessary to solve the problem.
  1. Multiply to find the cost of the stuffed pandas.
  2. Multiply to find the cost of the stuffed giraffes.
  3. Add to find the total cost for the stuffed pandas and the stuffed giraffes.
  4. Divide to find the average amount spent for each niece.

  Discuss each step and have students explain the reason for each operation that is used. You may wish to have the students perform the indicated operations to check the amounts shown in the example.
- After the students have completed the exercises, have several of them tell the steps they used to solve the word problems.

Checking Up

Complete.

1.	58 out of 100	58?100	$\frac{58}{100}$?	0.58 ?	58%?
2.	30 out? of 100	30:100	? $\frac{30}{100}$	0.30 ?	30%?
3.	7 out? of 100	7:?100	$\frac{7}{100}$	0.07 ?	7%?
4.	94 out? of 100	94?100	$\frac{94}{100}$?	0.94	94%?
5.	2 out? of 100	2:?100	? $\frac{2}{100}$	0.02 ?	2%

Write a percent for each decimal.

6. 0.82 82% **7.** 0.25 25% **8.** 0.08 8% **9.** 0.6 60% **10.** 0.33 33%

Write a percent for each fraction.

11. $\frac{23}{100}$ 23% **12.** $\frac{3}{4}$ 75% **13.** $\frac{7}{10}$ 70% **14.** $\frac{2}{5}$ 40% **15.** $\frac{3}{12}$ 25%

16. $\frac{1}{3}$ 33% **17.** $\frac{4}{9}$ 44% **18.** $\frac{3}{8}$ 38% **19.** $\frac{1}{6}$ 17% **20.** $\frac{5}{7}$ 71%

Write a decimal for each percent.

21. 49% 0.49 **22.** 7% 0.07 **23.** 40% 0.40 **24.** 75% 0.75 **25.** 1% 0.01

Write a fraction with a denominator of 100 for each percent.

26. 81% $\frac{81}{100}$ **27.** 9% $\frac{9}{100}$ **28.** 54% $\frac{54}{100}$ **29.** 70% $\frac{70}{100}$ **30.** 75% $\frac{75}{100}$

Write a fraction in lowest terms for each percent.

31. 30% $\frac{3}{10}$ **32.** 60% $\frac{3}{5}$ **33.** 50% $\frac{1}{2}$ **34.** 25% $\frac{1}{4}$ **35.** 80% $\frac{4}{5}$

Find each of these.

36. 38% of 75 28.5 **37.** 67% of 300 201 **38.** 40% of 95 38

Solve.

39. How heavy are the sunflower seeds in 25 kg of bird seed when 40% of the mass is sunflower seeds? 10 kg

40. The discount is 20% off the original price. The original price was \$2.80. How much is the discount? \$0.56

41. Each year the bank pays interest equal to 9% of the money in a savings account. How much interest will it pay on savings of \$178? \$16.02

42. Each year the bank charges interest equal to 12% of the amount it has loaned. What will be the interest for a loan of \$650? \$78

275

Skills	Exercises	Related Pages
Convert among ratios, fractions, decimals, and percents	1-5	T 286-T 287
Express a decimal as a percent	6-10	T 288-T 289
Express a fraction as a percent	11-20	T 288-T 289
Express a percent as a decimal	21-25	T 290-T 291
Express a percent as a fraction with a denominator of 100	26-30	T 290-T 291
Express a percent as a fraction in lowest terms	31-35	T 290-T 291
Find a percent of a number	36-38	T 292-T 293
Solve word problems	39-42	

Page 275

OBJECTIVE

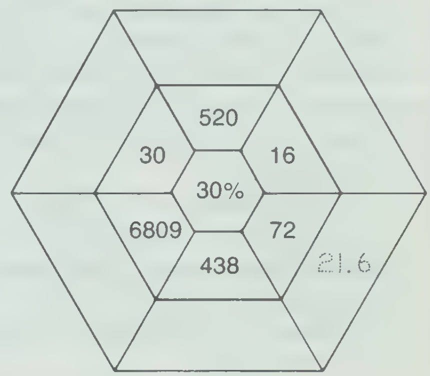
Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- For more practice, provide a work sheet similar to the following. Have students ring equivalent fractions, decimals, and ratios for each percent given.

25%	$\frac{1}{4}$,	75:100,	0.25,	$\frac{25}{100}$,	3:12
60%	60:10,	0.6,	$\frac{3}{5}$,	$\frac{60}{100}$,	0.60

- Have the students complete diagrams similar to the following on copies of page T 391.



- Have students play the game "Inventory" described on page T 380 for examples of percent.

Comments

Use the results of the students' work to determine which concepts need to be reviewed. Some difficulties may occur because students have not mastered a previously taught skill such as dividing with extra zeros in the dividend. Group the students for review according to their needs. Adapt the teaching suggestions and related activities for the appropriate pages to review or to reteach the necessary skills.

Unit 14 Overview

Motion Geometry

Congruency of shapes is studied in this unit by means of motions. In Unit 9, congruent figures are determined by tracings. In this unit, tracings are also used to identify three kinds of motions: the slide, the flip, and the turn. Students use tracings to draw images for these three kinds of motions, and they also learn to use methods based on counting for identifying and drawing slide and flip images on grids. Turns are used to determine whether shapes have rotational symmetry. Tiling is examined to identify by what motions two or more shapes may be used to create patterns. Students also examine tessellations, patterns which use only one shape. The value of models in solving certain kinds of problems is pointed out in the *Problem Solving* lesson of the unit.

Unit Outcomes

- identify one figure as the slide image of another figure for a given slide arrow
- draw the slide image of a given figure for a given slide arrow, with a grid and without a grid
- draw the slide image of a given figure on a grid by using a rule
- write the rule for a given slide arrow
- identify one figure as the flip image of another figure for a given flip line
- draw the flip image of a given figure for a given flip line, with a grid and without a grid
- draw the flip image of a given figure on a grid for a given flip line by counting units
- identify one figure as the turn image of another figure for a given turn angle
- draw the turn image of a given figure for a given turn angle, with a grid and without a grid
- identify figures having rotational symmetry (turn symmetry)
- identify the number of different turns less than a full turn for which a shape has rotational symmetry
- make a tiling pattern using more than one shape
- identify shapes that can be used to make tessellations
- draw tessellations
- find the sum of the measurements of the angles at a corner in a tessellation
- identify slides, flips, and turns in tiling patterns
- identify slide, flip, and turn images in tiling patterns
- use models to help solve problems

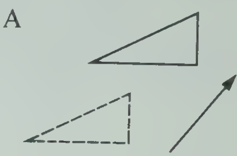
Background

In previous work, students may have had informal experiences related to the topic of congruency. They may have superimposed cutouts of polygons on others to see if they had the same sizes and shapes. Under such circumstances, they may have acquired misconceptions about congruency by paying attention to the region inside each polygon, rather than to only the size and shape of its outline. It is only the points in the line segments, rather than the points in the enclosed space, that constitute the polygon. Superimposing a tracing of a polygon on another polygon is somewhat better in this regard because the line segments, their lengths, and their contained angles are emphasized.

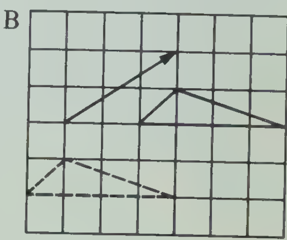
In this unit, another approach is used to determine congruency by noting whether a tracing of a figure can be moved to match another figure in the same plane. This is known as *motion geometry*, or more formally, as *transformation geometry*. Again, tracings are used, but more emphasis is given to the matching of points, and to observing whether the points in one figure match the corresponding points in another figure.

Three basic transformations are studied: the *translation* (slide), the *reflection* (flip), and the *rotation* (turn). The less formal terms in parentheses are used to describe the motions at this level. A slide can occur along a straight line and the orientation of the shape is not changed. Both the direction and the length of a slide may be indicated by a slide arrow (A). Every point of the shape moves the same distance and in the same direction. Therefore, the image and the original shape must be congruent. On a grid, a slide arrow can be described by a move of a number of units to the right (R) or to the left (L), and then a number of units up (U) or down (D). For example, the slide arrow shown in B is described by the rule (3R, 2U). If the same rule is applied to each vertex of a triangle on a grid, the vertices of the slide image are located.

A

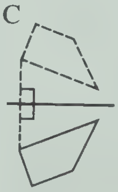


B

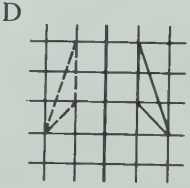


A shape can be flipped about a line, and the image is the reverse of the original shape. Corresponding points of a figure and its image are equal distances from the flip line, and the line joining them is perpendicular to the flip line, that is, it intersects the flip line at right angles (C). If a figure and a flip line appear on a grid with the flip line along one line of the grid, the vertices of the image may be plotted by counting the units from each vertex to the flip line (D). Flip lines can occur outside a shape (C, D), along one side (E), at a vertex (F), or inside the shape (G). In every case, there is a reversal, and corresponding points are the same distance from the flip line and on opposite sides of it.

C

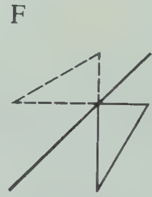


D



E

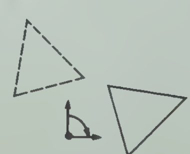
F



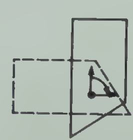
G

A shape can be turned using any point as a turn centre. The turn centre may be outside the shape (H), inside the shape (J), or on the shape (K, L). The direction and amount of rotation is indicated by a turn arrow which shows both the turn centre and the turn angle.

H

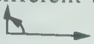


J



K

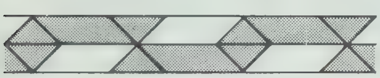
L

The orientation of the shape is altered according to the amount of rotation. Every point in the shape turns the same amount, hence the original shape and its image are congruent. Since a turn may be in either a counterclockwise or a clockwise direction, the same image may be obtained by two different turn angles. For example, a $\frac{1}{4}$ turn counterclockwise, , and a $\frac{3}{4}$ turn clockwise, , give the same result for the same turn center.

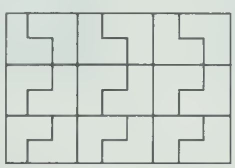
In previous books of *Starting Points in Mathematics*, flip lines are used to discover line symmetry in polygons. It was found that regular polygons have more than one line of symmetry and that the number is related to the number of sides. In this book, turns are used to discover that some polygons have *rotational symmetry*, or *turn symmetry*. It is noted that several different turns less than a full turn may be used to fit a regular polygon onto itself. For example, a square will fit onto itself after a rotation of a $\frac{1}{4}$ turn, a $\frac{1}{2}$ turn, or a $\frac{3}{4}$ turn. It is easily seen that the number of ways is related to the number of sides of equal length for the regular polygon: an equilateral triangle (3 sides) two ways, a square (4 sides) three ways, a regular pentagon (5 sides) four ways, and a regular hexagon (6 sides) five ways. In each case, the number of images which match the original regular polygon is one less than the number of sides of equal length. An irregular polygon, however, has no rotational symmetry and only a full turn can be used to fit the shape onto itself.

Interesting tiling patterns may be made by using shapes which have some of their sides congruent. In forming a pattern, the shapes must not overlap, neither must they leave any spaces. Sometimes as few as two or three shapes may be used, as shown (M). If, however, only one shape is used to tile a region, the pattern is known as a *tessellation* (N). In this book only polygons are used to make tessellations, although curved shapes are frequently found in artistic decorations.

M



N

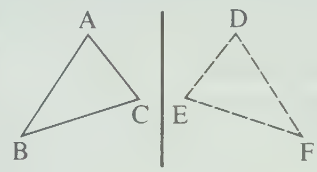


Teaching Strategies

The lessons in this unit involve considerable student activity, utilizing several different kinds of materials, such as tracing paper, graph paper, and various shapes to represent polygons. The teacher's role in developing the concepts of the unit is particularly important in guiding the pre-book activities. Since these form a foundation from which the textbook activities can be carried out, they need to be directed carefully by the teacher. It is important for the teacher to work with and watch the students as they proceed through the steps outlined in *Working Together* and *Exercises* of the lessons. It may be advisable, therefore, to divide the class into several groups. Students' previous achievements in number operations can be ignored in structuring groups, since the concepts of motion geometry are unrelated to them. It is suggested that while the teacher and one group of students are engaged in a lesson, other groups may be assigned other work. Some of them may repeat exercises in earlier lessons of the unit or they may become involved in several of the *Related Activities*. Still others may be assigned exercises from *Checking Skills* on page 297. The exercises on this page may be divided into several sets and assigned over four or five days.

In the exercises for the lesson on pages 286 and 287 it is necessary to find turn centres for the polygons. Trial-and-error methods may be used, but students may reach wrong conclusions if the turn centres are not located accurately. It is suggested that a more formal method be used to locate turn centres. If a polygon has an even number of sides, the turn centre is the intersection of two diagonals that join opposite vertices. For a polygon with an odd number of sides, lines are drawn from two vertices to the midpoints of the sides opposite them. The intersection of these lines may be used as a turn centre.

In the *Background* section it is pointed out that, for a flip, each point of an image is the same perpendicular distance from a flip line as the corresponding point of the original figure. Thus, if a figure and its image are labeled, the order of naming the vertices can indicate the corresponding vertices. For instance, for triangle ABC, the flip image is triangle DFE. The name of the image matches the name of the original triangle point for point, A and D, B and F, C and E. Other names of the image, such as FED, EFD, and EDF may be used in other circumstances, but for an image of triangle ABC the correct order of the vertices for the given flip image is DFE. On page 293 this style is used in identifying the triangles in Ex. 1-4, but it may be too much to expect students to use this precision in naming the images in Ex. 5-10 since they have been accustomed to naming the vertices of a triangle in any order. The teacher should be prepared to accept alternative answers in these exercises.



If students have any difficulties with the set of division exercises on page 297, they should be directed to review the lessons on pages 146-149 which introduce the use of extra zeros in the dividend.

Materials

a copy of the equilateral triangle on page T 383, a red pencil, and a blue pencil for each student

large sheets of graph paper and tracing paper or an overhead projector and a transparency marked with a grid

tracing paper, plain paper, construction paper, copies of page T 397, copies of page T 400, a straight edge, a sharp pencil, a protractor, scissors, centimetre ruler (optional) for each student

pins (optional)

triangular shapes prepared for the activity suggested in *Before Using the Pages* on page T 302

a square box and a rectangular box with covers

pattern blocks such as parquetry blocks

a copy of the parallelogram on page T 384 for each student

models for rectangular prisms, triangular prisms, and pentagonal prisms (optional)

semitransparent plexiglass mirrors (optional)

Vocabulary

slide	turn	half turn
slide arrow	turn image	$\frac{1}{4}$ turn, $\frac{1}{2}$ turn
slide image	turn angle	$\frac{3}{4}$ turn, full turn
flip	turn centre	tiling
flip image	rotational symmetry	tessellations
flip line	turn symmetry	finger wheel

Pages 276-277

LESSON OUTCOME

Identify one figure as the slide image of another figure for a given slide arrow; draw the slide image of a given figure for a given slide arrow, with a grid and without a grid

Materials

a copy of the equilateral triangle on page T 383, a red pencil, and a blue pencil for each student; large sheets of graph paper; tracing paper, plain paper, copies of page T 397, a straight edge and a sharp pencil for each student; pins (optional)

Vocabulary

slide, slide arrow, slide image

14 MOTION GEOMETRY

Slides

The ski tow is like a **slide arrow**. A skier at the top of the hill is like a **slide image** of a skier at the bottom of the hill.

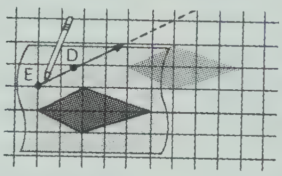
The slide arrow shows the distance and the direction of the slide.

Working Together

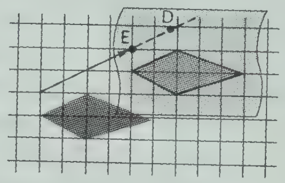
Is the blue shape the slide image of the red shape for the slide arrow shown?

When a tracing matches a shape, the tracing fits exactly onto the shape.

Extend the arrow. Trace the red shape. Mark points E (end point) and D on the tracing.



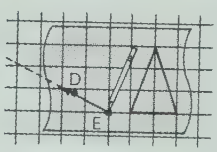
Slide the tracing so that E and D move along the slide arrow until E is over the tip of the arrow.



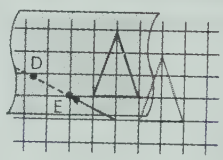
If the tracing matches the blue shape, the blue shape is the slide image of the red shape for the slide arrow shown.

Draw the slide image of the triangle for the slide arrow shown.

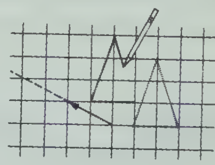
Extend the arrow. Trace the triangle. Mark points E (end point) and D on the tracing.



Slide the tracing so that E and D move along the slide arrow until E is over the tip of the arrow.



Mark the vertices. Remove the tracing. Draw the slide image.

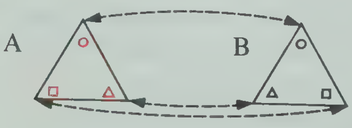


276

LESSON ACTIVITY

Before Using the Pages

- Give each student a copy of the equilateral triangle from page T 383 and ask the students to mark a red circle, a red square, and a red triangle at the vertices as shown (A). On the opposite side, have them mark in blue the same shape at each vertex as shown (B).



Direct the students to centre the shapes on their desks and then move them so that the vertex showing the red circle always points to the front of the classroom, that is, the shapes must neither turn nor flip. Elicit the word *slide* to describe the motion and give directions such as ''slide left'', ''slide down'', and ''slide diagonally''. Develop that a slide involves direction and distance.

Using the Pages

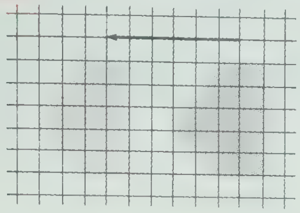
- Read the title of the unit. Then draw the students' attention to the photograph. Point out the slide arrow on the photograph, and discuss how it shows the distance and the direction of the slide. Have a student read the statements that introduce the terms *slide arrow* and *slide image*. Ask for other examples of slides, such as a toboggan sliding on the snow and certain doors sliding open or shut.

Working Together: The first example shows how to use tracing paper to test whether one shape is the slide image of another shape for a given slide arrow. Before the lesson, copy the red shape, the blue shape, and the slide arrow on a large sheet of graph paper and tape the paper to the chalkboard. Emphasize that the line containing the slide arrow is to be extended far enough so that at least one more identical slide arrow can be shown. Note that marking points E and D and extending the slide arrow ensures that students do not turn the tracing, and that they slide the tracing for the given direction and distance. Demonstrate the procedure of

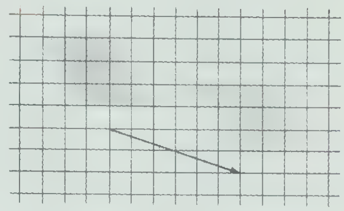
Exercises

Copy each picture on graph paper. Use tracing paper to test whether the blue shape is the slide image of the red shape for the slide arrow shown.

1. yes

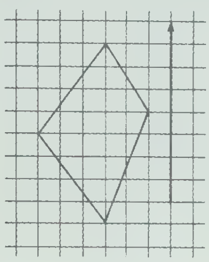


2. no

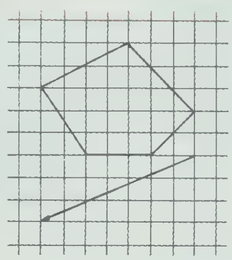


Copy each shape and the slide arrow on graph paper. Use tracing paper to help you draw the slide image of the shape for the slide arrow shown. Answers are shown on page T373

3.



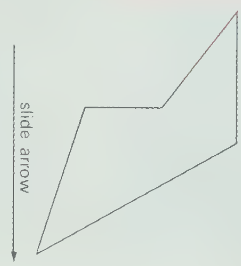
4.



Trace each shape and the slide arrow on plain paper. Use tracing paper to help you draw the slide image. Answers are shown on page T373

5.

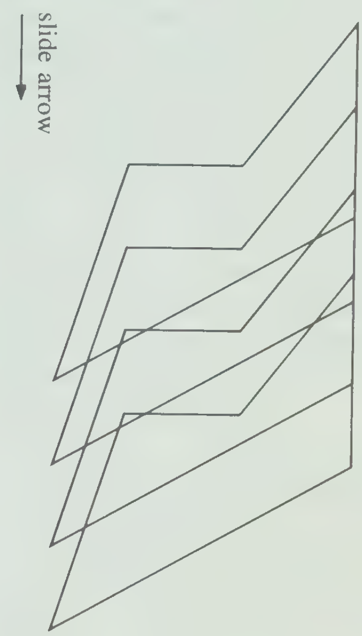
6.



277

RELATED ACTIVITIES

- Have students cut pictures from magazines of examples of slides, for example, children on a playground slide and children pulling a sled. The pictures can be displayed and labeled "Examples of Slides".
- Students can create interesting patterns based on slides for which a figure and its image overlap. Ex. 3-6 on page 277 may be adapted for this by using a slide arrow shorter than the one given. Students may also draw a shape of their own choice and a slide arrow to produce an overlapping design. The following example is adapted from Ex. 6. Students may wish to color their designs.



tracing the red shape and sliding the tracing to match the blue shape. Then ask a few students to demonstrate the procedure. Point out that the blue shape is the *slide image* of the red shape because the tracing of the red shape matches the blue shape. Discuss that the slide image would be in a different position if the slide arrow were of a different length and/or in a different direction.

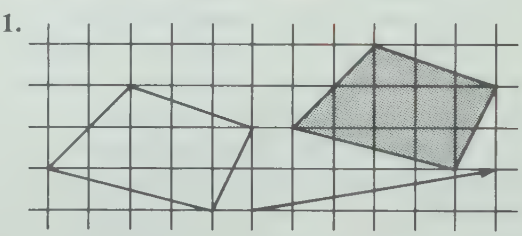
The second example shows how to use tracing paper to draw the slide image of a shape for a given slide arrow. As before, demonstrate the procedure on a large sheet of graph paper prepared in advance and have several students repeat the steps on the board. Note that the step of marking the vertices can be shown by using either pins or sharp pencils and pressing firmly so that the indentations show on the graph paper under the tracing paper.

Exercises: It is essential that the students work on a flat surface and that they extend the slide arrows as shown in *Working Together*. For these reasons, they are directed to copy the diagrams. Provide them with copies of page T 397 for Ex. 1-4 and plain paper for Ex. 5 and 6.

Assessment

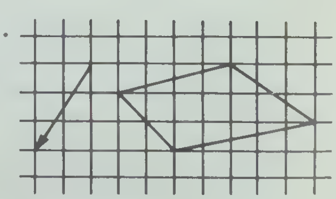
Copy the picture on graph paper. Use tracing paper to test whether the grey shape is the slide image of the white shape for the slide arrow shown. yes

1.



Copy the shape and the slide arrow on graph paper. Use tracing paper to help you draw the slide image of the shape for the slide arrow shown. Answer is shown on page T 373.

2.



Draw a shape and a slide arrow on plain paper.

3. Use tracing paper to help you draw the slide image.

Pages 278-279

LESSON OUTCOME

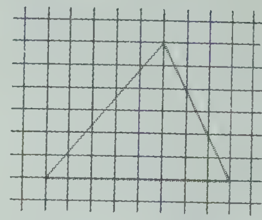
Draw the slide image of a given figure on a grid by using a rule; write the rule for a given slide arrow

Materials

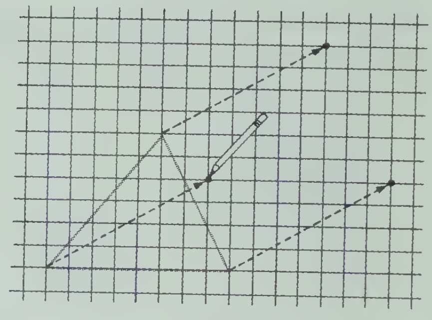
a large sheet of graph paper and tracing paper, or an overhead projector and a transparency marked with a grid, copies of page T 397 and a straight edge for each student

Drawing a Slide Image Using a Rule

Draw the slide image of the red shape for the rule (7R, 4U).

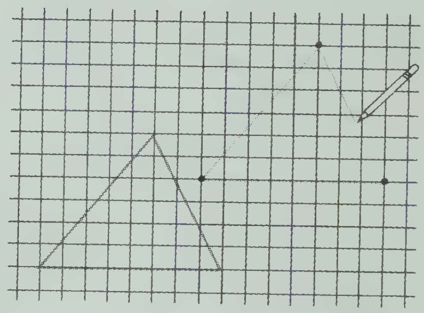


From each vertex, count 7 units to the *Right* and 4 units *Up*. Draw the image point for each vertex.



Join the points.

The blue shape is the slide image of the red shape for the rule (7R, 4U).



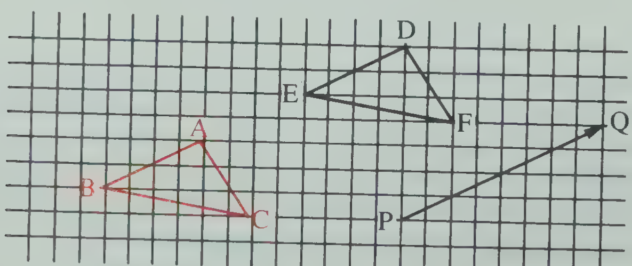
(7L, 4D) would mean 7 units to the *Left* and 4 units *Down*.

278

LESSON ACTIVITY

Before Using the Pages

- Use a large sheet of graph paper showing a triangle and a given slide arrow. Review the procedure of tracing a shape and sliding the tracing for the given slide arrow to obtain the slide image. You may prefer to use the overhead projector and a transparency marked with a grid. The triangle may be traced on a sheet of acetate placed over the grid. After the image is drawn, label the triangles and the slide arrow as shown.



Have students identify matching vertices of the two triangles, for example, A ⟶ D, B ⟶ E, and C ⟶ F. Indicate the path from point A to point D that follows grid lines for a move of 2 units to the right and a move of 5 units up. Ask the students what point is reached by starting at point B and moving 2 units to the right and 5 units up. Ask a similar question for starting from point C. Then ask students to explain why the rule "2 units to the right and 5 units up" from any vertex gives the corresponding vertex of the slide image. Lead them to suggest that the length and the direction of the slide arrow PQ are determined by that rule. Ask if there is a way to draw the slide image of triangle ABC for the given slide arrow without using tracing paper.

Using the Pages

- Ask a student to read the title of the lesson and the instruction at the top of page 278. Ask the students what they think is meant by (7R, 4U). Have a student read the instructions beside the second diagram. For the third diagram, have the

Working Together

Draw a point on graph paper as shown. Draw its image for each rule.

1. (5R, 2D) 2. (3L, 0)

Write the rule for each slide arrow.

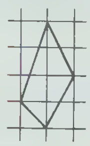
5.  (2L,2U) 6.  (1R, 3D)

Answers are shown on page T373

Copy the shape on graph paper. Draw its slide image for each rule

3. (0, 5U)
4. (3R, 3D)

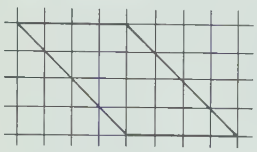
Copy the shape on graph paper. For each slide arrow, write the rule and draw the slide image.



7.  (4R, 0)
8.  (2L, 1D)

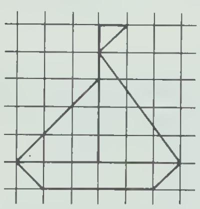
Exercises

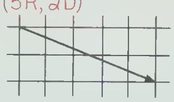
Copy the shape on graph paper.
Draw its slide image for each rule. Answers are shown on pages T373 and T374

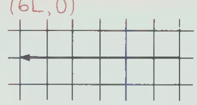


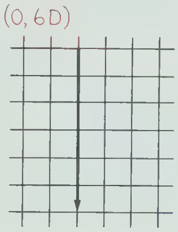
1. (5R, 2U) 2. (0, 4D)
3. (3L, 4U) 4. (7L, 7D)
5. (4L, 0) 6. (6R, 8D)

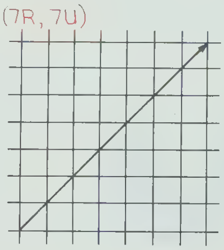
Copy the shape on graph paper. For each slide arrow, write the rule and draw the slide image. Answers are shown on page T374



7. (5R, 2D) 

8. (6L, 0) 

9. (0, 6D) 

10. (7R, 7U) 

279

RELATED ACTIVITIES

- For more practice, you may wish to have the students use the method introduced on page 278 to identify slide images for Ex. 1 and 2 on page 277 and to draw slide images for Ex. 3 and 4 on page 277.
- Have students work in pairs using a geoboard and rubber bands. One student makes a shape and writes the rule for a slide. The other student makes the slide image for the rule. As a variation, one student can make a shape and a slide image of the shape. The other student can write the rule.
- For enrichment, provide students with copies of page T 397. Have them label number lines for graphing ordered pairs of numbers. (See page 42.) Name three ordered pairs such as (2,1), (5,4), and (3,5). Have the students plot the points and join them to form a triangle. Have the students draw the slide image for a given rule such as (3R, 2U). Ensure that the rule does not result in a slide image that is off the grid. Have the students write the ordered pairs of numbers for the vertices of the slide image and compare these ordered pairs with those for the original triangle, to note a pattern, as indicated below.

+ 3

(2,1) ⟶ (5,3) Rule: (3R, 2U)
(5,4) ⟶ (8,6)
(3,5) ⟶ (6,7)

+ 2

students count 7 units to the right and 4 units up from each vertex of the red shape to check the vertices of the blue shape. Establish that the blue shape is the slide image of the red shape for the rule (7R, 4U). Point out that the rule is written in parentheses in the same way as for an ordered pair. Discuss what is meant by (7L, 4U). Summarize that a rule shows a move to the left or to the right first, and then a move up or down.

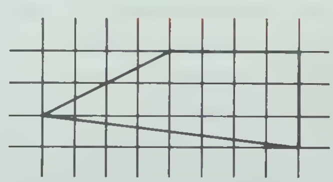
Working Together: Provide copies of page T 397 for Ex. 1-4, 7, and 8. For Ex. 2 and 3, pay particular attention to the 0 in each rule. In Ex. 2, the 0 indicates that there is no vertical move; in Ex. 3, the 0 indicates that there is no horizontal move.

Exercises: For Ex. 1-6, the slide images are drawn for one shape for six different rules. Direct the students to draw the parallelogram in the center of one copy of page T 397. The six slide images may be drawn on the same sheet. Provide another copy of page T 397 for showing Ex. 7-10.

Assessment

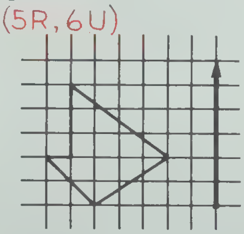
Copy the shape on graph paper. Draw its slide image for the rule (3L, 4U). Answer is shown on page T374.

1.



Copy the shape on graph paper. Write the rule and draw the slide image for the slide arrow. Answer is shown on page T374.

2. (5R, 6U)



Pages 280-281

LESSON OUTCOME

Identify one figure as the flip image of another figure for a given flip line; draw the flip image of a given figure for a given flip line, with a grid and without a grid

Materials

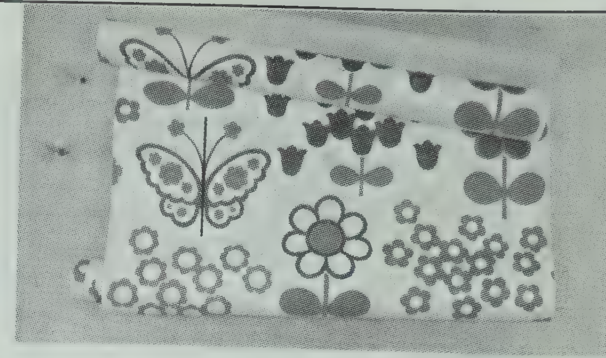
the triangular shapes suggested in *Before Using the Pages* on page T 302; large sheets of graph paper or an overhead projector; tracing paper, plain paper, copies of page T 397, and a straight edge for each student; pins (optional)

Vocabulary

flip, flip image, flip line

Flips

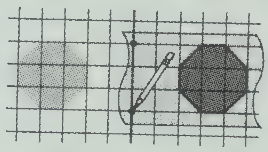
One side of the butterfly is like a **flip image** of the other side. The black line represents the **flip line**.



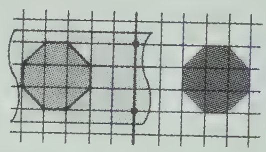
Working Together

Is the blue shape the flip image of the red shape for the flip line shown?

Mark two points on the flip line. Trace the red shape and the two points.



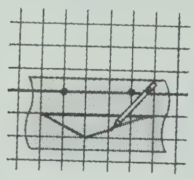
Flip the tracing over. Match the two points on the tracing with those on the flip line.



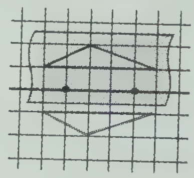
If the tracing matches the blue shape, the blue shape is the flip image of the red shape for the flip line shown.

Draw the flip image of the triangle for the flip line shown.

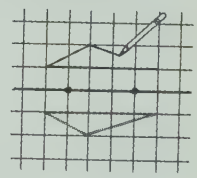
Mark two points on the flip line. Trace the triangle and the two points.



Flip the tracing over. Match the two points on the tracing with those on the flip line.



Mark the vertices. Remove the tracing. Draw the flip image.



280

LESSON ACTIVITY

Before Using the Pages

- Have the students use the triangular shapes they prepared as suggested on page T 302 to review the concept of a slide. Ask them to centre the triangles on their desks so that the vertex showing the red circle points to the *back* of the classroom. After they have performed motions for directions such as "slide right", ask if the shapes can slide so that the vertex showing the blue circle points to the *front* of the classroom. Then elicit the term *flip* to describe the motion that would make this possible. Emphasize that the shape does not slide. Have the students demonstrate other flips using the shapes.

Using the Pages

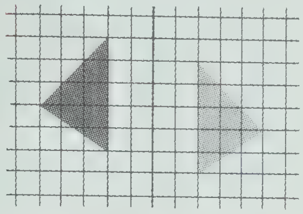
- Draw attention to the wallpaper pattern in the photograph. Ask how the black line marks the butterfly in a special way. Ask a student to read the statements at the top of page 280 to introduce the terms *flip image* and *flip line*. Fold a piece of paper in half and draw the outline of half a butterfly around the fold. Cut out the drawing and unfold it, demonstrating that one part is the flip image of the other, and that the fold represents the flip line. Emphasize that a shape and its flip image, unlike a slide, point in opposite directions, that is, their orientation is reversed. Ask for other examples of flips on the wallpaper, such as the leaf on one side of a stem and the leaf on the other side of the stem.

Working Together: The first example shows how to use tracing paper to test whether one shape is the flip image of another shape for the flip line shown. Demonstrate each step using an overhead projector or a large sheet of graph paper. Establish the need to mark two reference points on the flip line to ensure that the image will be the same distance from the flip line as the original shape. They also ensure that the image is directly opposite the original shape, with no sliding upward or downward during the flip motion. Emphasize that there are many flip images of the red shape, but the blue shape is its only flip image for the flip line shown.

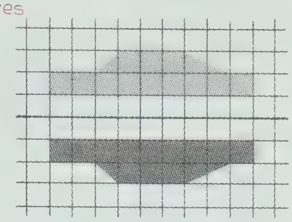
Exercises

Copy each picture on graph paper. Use tracing paper to test whether the blue shape is the flip image of the red shape for the flip line shown.

1. no

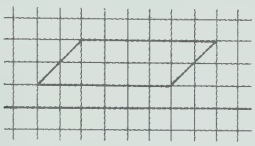


2. yes

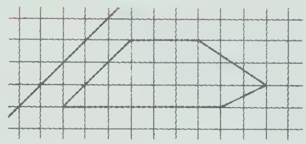


Copy each shape and the flip line on graph paper. Use tracing paper to help you draw the flip image of the shape for the flip line shown. Answers are shown on page T374

3.

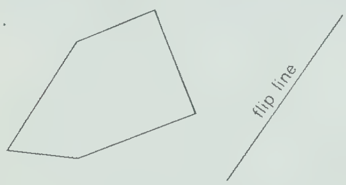


4.

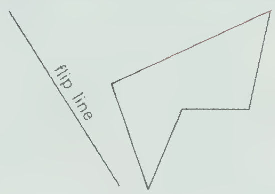


Trace each shape and the flip line on plain paper. Use tracing paper to help you draw the flip image. Answers are shown on page T375.

5.



6.



Divide. Use extra zeros if needed.

1. 10)$64 $6.40	2. 26)$444.34 $17.09	3. 45)$160.65 $3.57	4. 84)$21 $0.25
5. 77)191.73 2.49	6. 36)291.78 8.105	7. 930)2864.4 3.08	8. 10)396.7 39.67
9. 657)571.59 0.87	10. 14)126.63 9.045	11. 100)6539 65.39	
12. 600)2826 4.71	13. 875)945 1.08	14. 55)3874.2 70.44	

KEEPING SHARP

281

RELATED ACTIVITIES

- Adapt the first activity on page T 303 for flips. For example, in some apartment buildings, the floor plan of one apartment is a flip image of the floor plan of another apartment. The motion of turning a light switch on or off is an example of a flip.
- Have students draw a shape and a flip line on a copy of page T 397 and place a semitransparent plexiglass mirror on the flip line. They can trace the reflection to draw the flip image of the shape. Students may also use such mirrors to check their results for Ex. 1-6 on page 281. Students may enjoy designing wallpaper patterns that are based on slides and/or flips.
- Discuss the relationship between a flip and line symmetry. For example, the line of symmetry for the picture on page 178 is a flip line; that is, one side of the picture is a flip image of the other side. Students can observe that the flip line is inside the shape.

The second example shows how to use tracing paper to draw the flip image of a shape for a given flip line. As before, demonstrate the procedure as students read the corresponding steps in *Working Together*. Use a pin or a sharp pencil to mark the vertices of the flip image. Point out that reference points are marked and that the tracing paper is flipped in the same way as for the first example.

You may wish to have the students try these examples using copies of page T 397 and tracing paper.

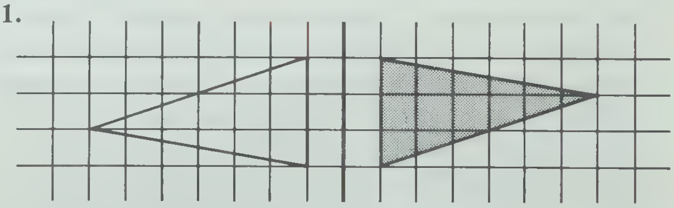
Exercises: Provide the students with tracing paper, plain paper, and copies of page T 397 to complete these exercises. You may wish to provide pins for marking the vertices of the flip images.

Keeping Sharp: Direct the students to record each quotient as a decimal, using extra zeros in the dividend as required to terminate the quotient. Encourage the students to write only the quotient for a division when the divisor is 10 or 100 (Ex. 1, 8, and 11).

Assessment

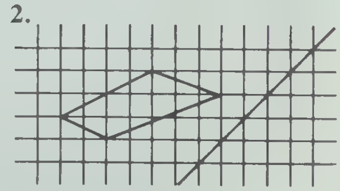
Copy the picture on graph paper. Use tracing paper to test whether the grey shape is the flip image of the white shape for the flip line shown. no

1.



Copy the shape and the flip line on graph paper. Use tracing paper to help you draw the flip image of the shape for the flip line shown. Answer is shown on page T375.

2.



Draw a shape and a flip line on plain paper.

3. Use tracing paper to help you draw the flip image.

Pages 282-283

LESSON OUTCOME

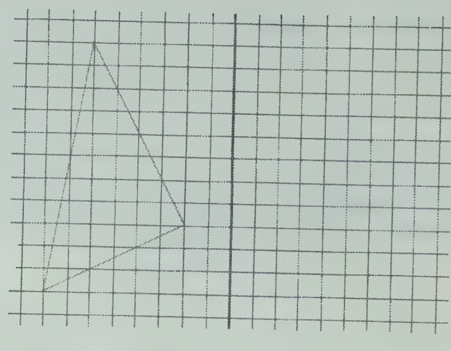
Draw the flip image of a given figure on a grid for a given flip line by counting units

Materials

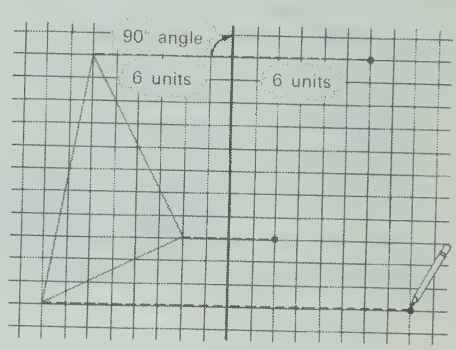
copies of page T 397 and a straight edge for each student, pins

Drawing a Flip Image by Counting

Draw the flip image of the red shape for the flip line shown.



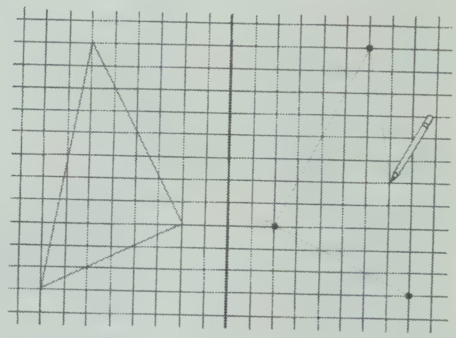
From each vertex, count the units from the vertex to the flip line. Then count that many units on the other side of the flip line. Draw the image point.



Join the points.

Each point of the red shape is the same distance from the flip line as its flip image.

The blue shape is the flip image of the red shape for the flip line shown.

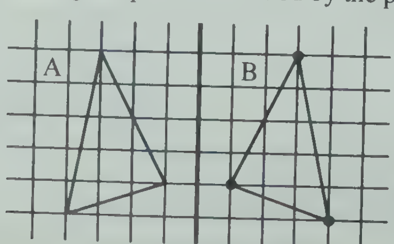


282

LESSON ACTIVITY

Before Using the Pages

- Cut copies of page T 397 into four equal parts and give one part to each student. Tell the students to draw a flip line along one of the grid lines near the centre of the paper. Ask them to use a straight edge to draw a triangle on one side of the flip line, with vertices at intersections of grid lines (A). Have them fold their papers carefully along the flip line, diagram on the outside, and use a pin to mark a hole at each vertex. Then have them unfold the papers and draw the flip image by joining the points indicated by the pin marks (B).



Have the students fold their papers, diagram on the inside, to check that one triangle is the flip image of the other. Then ask them to place one finger at one vertex of the original triangle and another finger at the corresponding vertex of the flip image. Have them move each finger along the grid line toward the flip line, counting the units from each vertex to the flip line. Note that there is the same number of units for each. Repeat the procedure for the other pairs of matching vertices. The students may also exchange papers and count the units for matching vertices.

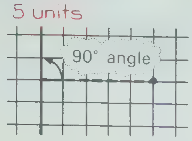
Recall that earlier in this unit, a method was developed for drawing a slide image on grid paper without the use of tracing paper. Ask the students if there is a way to locate the flip image of a figure on grid paper without the use of tracing paper or pins.

Using the Pages

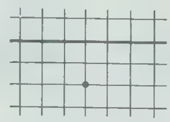
- The worked example demonstrates a procedure for finding the flip image of a figure on a grid. The procedure is based on the fact that corresponding points of a figure and its flip

Working Together

Find the distance between the point and the line.

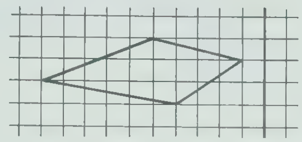
1\. 

Copy the picture on graph paper. Draw the flip image of the point for the flip line shown.

2\. 

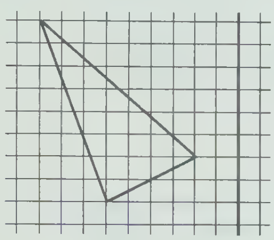
Answer is shown on page T375

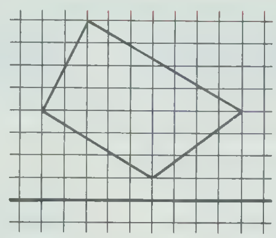
Copy the picture on graph paper. Draw the flip image of the shape for the flip line shown.
Answer is shown on page T375

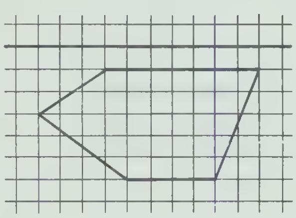
3\. 

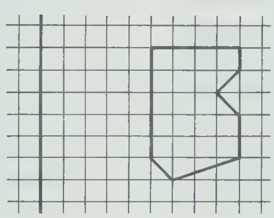
Exercises

Copy each picture on graph paper. Draw each flip image of the shape for the flip line shown. Answers are shown on page T375.

1\. 

2\. 

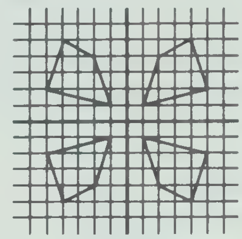
3\. 

4\. 

283

RELATED ACTIVITIES

- You may wish to have the students use the method introduced in this lesson for Ex. 1-3 on page 281.
- Each student can make a design on a copy of page T 397 as follows. Two perpendicular line segments are drawn as flip lines. A shape is drawn in one of the four sections and flip images are drawn in turn, moving clockwise or counterclockwise, until four shapes are shown.



- Have the students draw a triangle and a flip line on plain paper. Have them fold the paper along the flip line and use a pin to locate the flip image as described in *Before Using the Pages*. Corresponding vertices are joined and labeled as shown. Direct the students to measure the following line segments and angles: $\overline{AG}$ and $\overline{GD}$; $\overline{BH}$ and $\overline{HE}$; $\overline{CI}$ and $\overline{IF}$; $\angle$ AGH, $\angle$ BHI, and $\angle$ CIH; $\angle$ ABC and $\angle$ DEF; $\angle$ BCA and $\angle$ DFE; $\angle$ BAC and $\angle$ EDF.

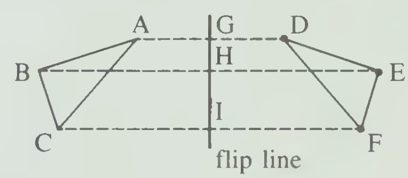


image are the same (perpendicular) distance from the flip line. For the exercises on these pages, the flip line is a grid line, and the vertices of the figures are at the intersections of grid lines.

Lead the students through the example. Ask them to count the units along the grid lines from corresponding vertices to the flip line. Point out that the dotted line segments joining corresponding vertices are perpendicular to the flip line.

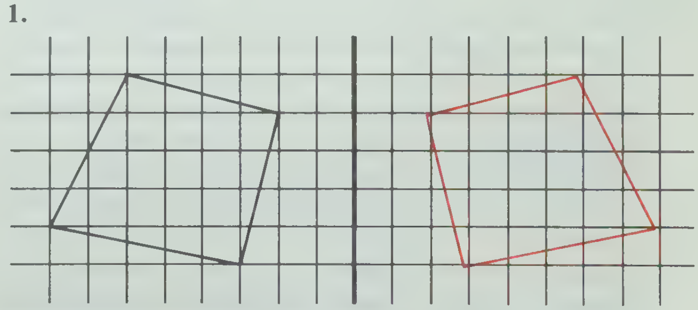
Working Together: Point out the angle of 90° for Ex. 1. Ex. 3 involves drawing four image points and then joining them to form the flip image. Provide a copy of page T 397 and a straight edge for each student to complete Ex. 2 and 3.

Exercises: Have the students copy each picture on a copy of page T 397 and then use a straight edge to draw the flip image.

Assessment

Copy the picture on graph paper. Draw the flip image of the shape for the flip line shown.

1.



Pages 284-285

LESSON OUTCOME

Identify one figure as the turn image of another figure for a given turn angle; draw the turn image of a given figure for a given turn angle, with a grid and without a grid

Materials

the triangular shapes suggested in *Before Using the Pages* on page T 302; large sheets of graph paper or an overhead projector; tracing paper, plain paper, copies of page T 397, and a straight edge for each student; pins (optional)

Vocabulary

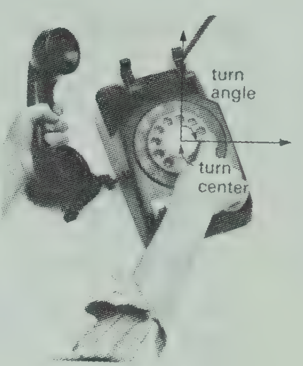
turn, turn image, turn angle, turn centre, finger wheel

Turns

When a number is dialed, the finger wheel is like a **turn image** of itself at rest.

The **turn angle** shows the amount and the direction of the turn.

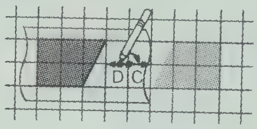
Turns can be clockwise or counterclockwise.



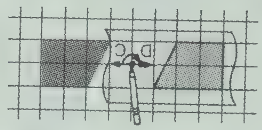
Working Together

Is the blue shape the turn image of the red shape for the turn angle shown?

Trace the red shape. Mark points C (turn center) and D on the tracing.



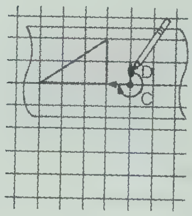
Turn the tracing about C until D is on the other ray of the turn angle.



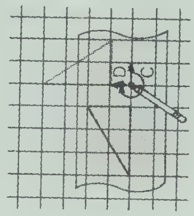
If the tracing matches the blue shape, the blue shape is the turn image of the red shape for the turn angle shown.

Draw the turn image of the triangle for the turn angle shown.

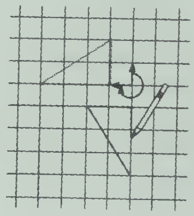
Trace the triangle. Mark points C (turn center) and D on the tracing.



Turn the tracing about C until D is on the other ray of the turn angle.



Mark the vertices. Remove the tracing. Draw the turn image.



284

LESSON ACTIVITY

Before Using the Pages

- Have the students use the triangular shapes they prepared as suggested on page T 302 to review the concepts of a slide and a flip. Have them centre the triangles on their desks so that the vertex showing the red circle points to the front of the classroom. Have them perform the motions for directions such as ''slide down'' and ''use the right side as a flip line''. Ask questions such as ''Which vertex points down (up)?''

  Hold one of the students' triangular shapes on the chalkboard, or use a larger similar shape, so that the vertex showing the red circle points up. Ask the students to shut their eyes as you slide or flip the shape. When they open their eyes, ask if the motion performed was a slide or a flip and ask how they can tell. Repeat for other slides and flips. Then turn the shape about one vertex so that the vertex showing the red circle points to the left. Develop that the motion performed was neither a slide nor a flip. Elicit the word *turn* to describe the motion performed and demonstrate it for the students. Then have them demonstrate turns using their triangular shapes on their desks.

Using the Pages

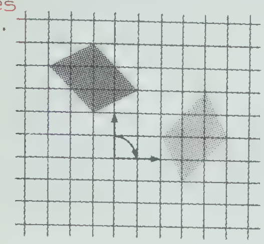
- Ask how a telephone can be used to demonstrate a turn motion. Ask students to read the three introductory statements to introduce the terms *turn image* and *turn centre*. Ask if the turn angle is greater when dialing 2 or when dialing 7. Point out the arrow that shows the direction of the turn. Discuss whether turns demonstrated by the finger wheel are clockwise or counterclockwise. Ask for other objects that demonstrate turns, for example, a door knob and the arms of a windmill.

Working Together: The first example shows how tracing paper can be used to test whether one shape is the turn image of another shape for a given turn angle. Ask students to read the instructions in *Working Together* as you demonstrate

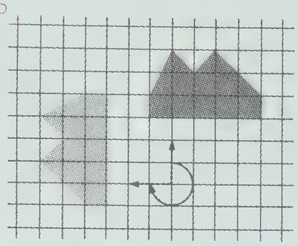
Exercises

Copy each picture on graph paper. Use tracing paper to test whether the blue shape is the turn image of the red shape for the turn angle shown.

yes
1.

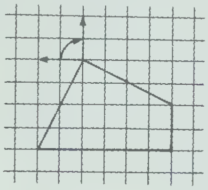


no
2.

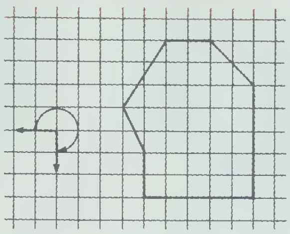


Copy each shape and the turn angle on graph paper. Use tracing paper to help you draw the turn image of the shape for the turn angle shown. Answers are shown on page T375.

3.

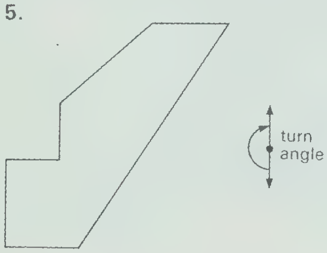


4.

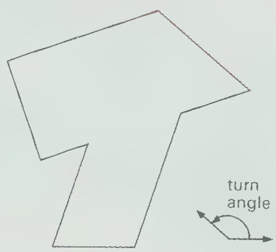


Trace each shape and the turn angle on plain paper. Use tracing paper to help you draw the turn image. Answers are shown on page T375.

5.



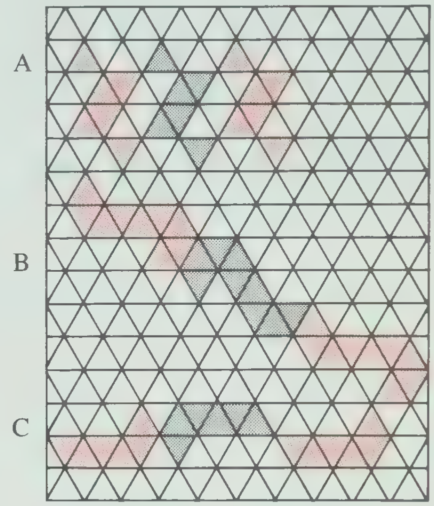
6.



285

RELATED ACTIVITIES

- Adapt the first activity on page T 303 for turns. Three examples are a bicycle wheel, a ferris wheel, and the hands of a clock.
- Provide the students with copies of the triangular grid on page T 399, on which to draw designs based on slides (A), flips (B), and turns (C).



the procedure using an overhead projector or a large sheet of graph paper. Use a pin or a sharp pencil to hold the turn centre in place. Pay particular attention to the direction of the turn. Have the students recall that it was necessary to mark two reference points for a slide arrow and for a flip line. For a turn, the reference points are the turn centre and another point on the appropriate ray. Repeat the turn motion several times to familiarize the students with the procedure. Then ask a few students to demonstrate the procedure.

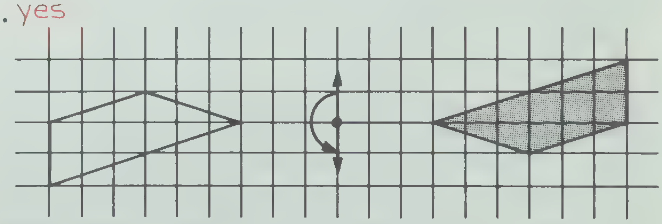
The second example shows how to use tracing paper to draw the turn image of a shape for a given turn angle. Demonstrate these steps in a similar manner.

Exercises: Allow the students sufficient time to copy the pictures and to exercise care in tracing the shapes. A sharp pencil can be used to hold the turn center in place as the paper is turned. Provide the students with plain paper, copies of page T 397, tracing paper, straight edges, and, if you wish, pins.

Assessment

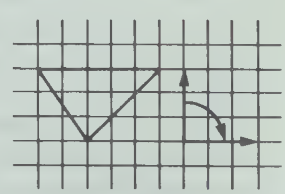
Copy the picture on graph paper. Use tracing paper to test whether the grey shape is the turn image of the white shape for the turn angle shown.

1. yes



Copy the shape and the turn angle on graph paper. Use tracing paper to help you draw the turn image of the shape for the turn angle shown. Answer is shown on page T375.

2.



Draw a shape and a turn angle on plain paper.

3. Use tracing paper to help you draw the turn image.

LESSON OUTCOME

Identify figures having rotational symmetry (turn symmetry); identify the number of different turns less than a full turn for which a shape has rotational symmetry

Materials

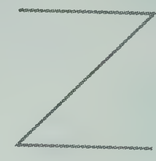
a square box and a rectangular box with covers (shoe boxes, gift boxes); tracing paper, and a straight edge for each student; plain paper (optional)

Vocabulary

rotational symmetry, turn symmetry, half turn

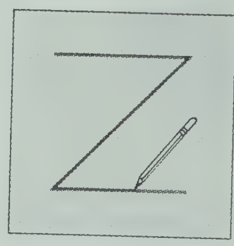
Rotational Symmetry

The letter Z has **rotational symmetry**, or **turn symmetry**.

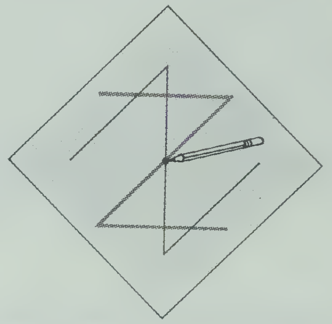


A shape that fits onto itself after a turn less than a full turn has rotational symmetry, or turn symmetry.

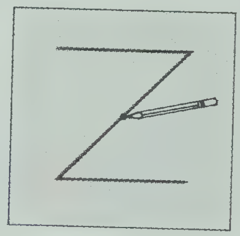
Trace the letter Z.



Turn the tracing ...



... until it fits onto the original letter Z.



After a half turn, the tracing fits onto the original letter Z.

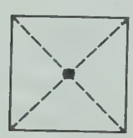
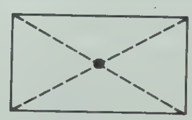
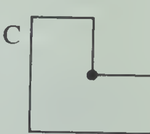
286

LESSON ACTIVITY

Before Using the Pages

- Display a square box, a rectangular box, and the cover for each box. Ask students to demonstrate different ways a cover can be placed on the corresponding box. For the rectangular cover there are two ways. For the square cover there are four ways. Note that a turn motion is applied to a cover to show the different ways it may fit the box. Ask why a slide and a flip are not useful in this regard.

  Discuss the position of the turn centre for each cover. You may wish to draw the two diagonals on the square cover (A) and on the rectangular cover (B) to locate the turn center at their intersection. Ask students to describe "how far around" a cover is turned for each new position to fit the box; this corresponds to the turn angle for a turn motion. Draw diagram C on the board and ask the students to imagine that a box and its cover have that shape. Ask how many different ways would be possible for the cover to fit the box. Trace the shape on tracing paper and turn the tracing to show that the "cover" fits the "box" only in the original position or when it is turned all the way around to the original position. The turn centre is indicated by the dot on the diagram.

A  B  C 

Using the Pages

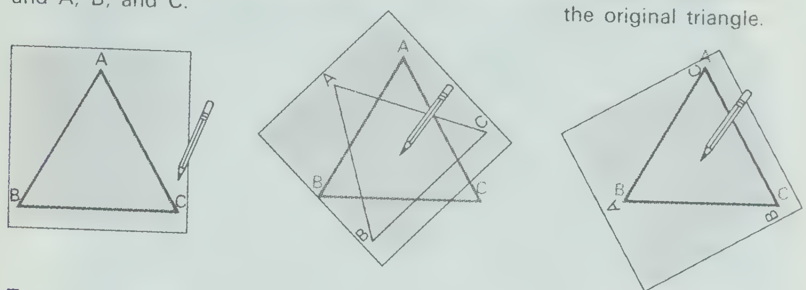
- Ask a student to read the title of the lesson. Remind the students of their earlier work with line symmetry on pages 178 and 179. Recall that a tracing of a figure is folded in half to test for line symmetry. If the two halves match, the shape has line symmetry. Explain that for *rotational symmetry*, or *turn symmetry*, a tracing of the figure is turned until the tracing matches the figure.

  Have the students try the procedure by using tracing paper on the first diagram on page 286. Introduce the term *half turn* to describe the motion. Point out that the turn

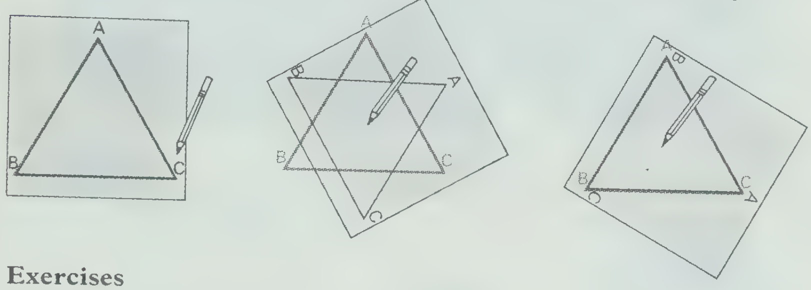
Working Together

Does $\triangle ABC$ have turn symmetry?

Trace the triangle and A, B, and C. | Turn the tracing . . . | . . . until it fits onto the original triangle.



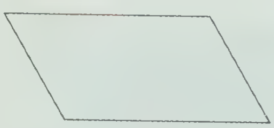
Trace the triangle and A, B, and C. | Turn the tracing in the other direction . . . | . . . until it fits onto the original triangle.



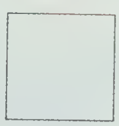
Exercises

Use tracing paper to test each of these for turn symmetry. For those with turn symmetry, give the number of different turns less than a full turn for which the shape fits onto itself.

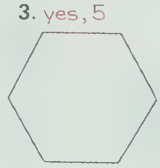
1. yes, 1



2. yes, 3



3. yes, 5



4. the letters A to Y H, 1
I, 1
N, 1
O, 1
S, 1
X, 1

287

RELATED ACTIVITIES

- Have the students test some of the shapes on copies of pages T 383 - T 385 for rotational symmetry.
- Students may test numerals such as 88 and 8968 for rotational symmetry. Afterward, have them write other numerals that have rotational symmetry.
- Ask students to identify letters of the alphabet that have line symmetry and those that have rotational symmetry. Adapt the procedure for different polygons. Ask questions such as "If a figure has line symmetry, does it also have rotational symmetry?" Have them identify figures that have both kinds of symmetry.

center is *on* the letter Z and that the half turn may be performed in a clockwise or in a counterclockwise direction. Lead the students to realize that if the tracing is turned past the half-turn position, it will fit onto the letter Z again. This represents a full turn from the original position. Emphasize that *if a tracing fits the figure only after a full turn, the figure does not have rotational symmetry.* For example, shape C in *Before Using the Pages* does not have rotational symmetry.

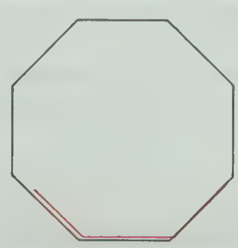
Working Together: For these examples, the original triangle and the letters A, B, and C are shown in red; the tracing is shown in black. Explain that the letters A, B, and C are traced so that the positions of the vertices can be identified after the tracing is turned. Have the students try the procedure using tracing paper on the first diagram of each of the two examples. The turn centre may be located by trial and error, by folding the tracing two ways to obtain the intersection of two lines of symmetry, or by finding the midpoints of two sides and joining them to opposite vertices.

Exercises: Provide each student with tracing paper and a straight edge for Ex. 1-4. You may wish to give the students plain paper for drawing large letters for Ex. 4. For Ex. 4, the letter O may fit onto itself for an unlimited number of turns if the letter is circular in shape; if the letter is oval in shape, the number of different turns less than a full turn is 1.

Assessment

Use tracing paper to test this shape for turn symmetry. Give the number of different turns less than a full turn for which the shape fits onto itself. 7

1.



LESSON OUTCOME

Make a tiling pattern using more than one shape

Materials

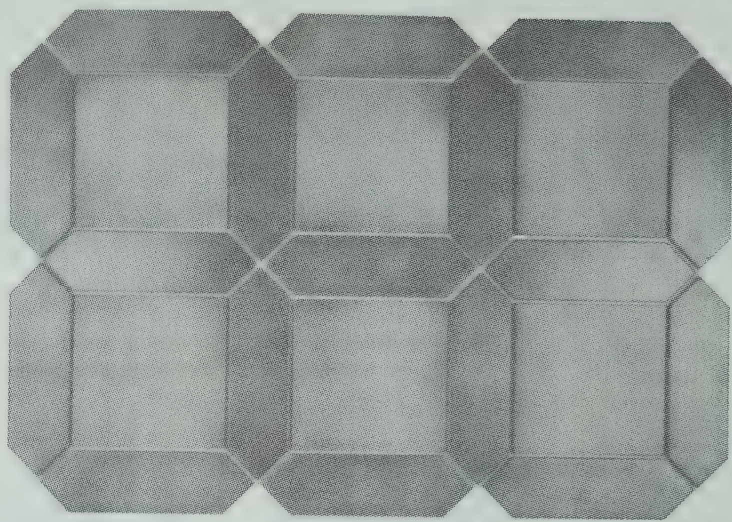
pattern blocks such as parquetry blocks; tracing paper or a copy of page T 400 for each student; construction paper and plain paper for each student

Vocabulary

tiling, $\frac{1}{4}$ turn, $\frac{1}{2}$ turn, $\frac{3}{4}$ turn, full turn

Tiling Patterns

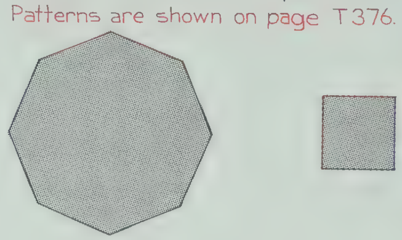
The tiles are geometric shapes that form a pattern.



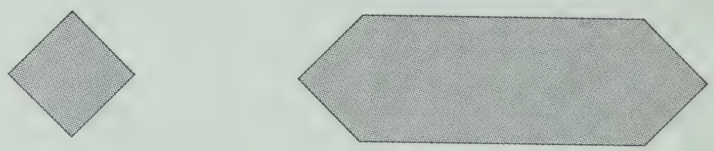
Exercises

Use these shapes to make a pattern without spaces.

1. Patterns are shown on page T376.



2.



288

LESSON ACTIVITY

Before Using the Pages

- Several days before the lesson provide the students with pattern blocks such as parquetry blocks. Have them arrange several blocks to form a design. Discuss which designs use one shape, two shapes, or more than two shapes. Have them note which designs have spaces and which do not.

  If possible, refer to examples in the classroom to develop the concept of tiling. For instance, draw attention to ceiling tiles and/or floor tiles which are likely square or rectangular. Discuss that identical shapes can be placed together so that they do not overlap, nor do they leave any spaces, and this process is called *tiling*. Squares and rectangles are used frequently in practical instances of tiling.

Using the Pages

- Ask a student to read the title of the lesson and the statement at the top of page 288. Ask how many different shapes are used to form the pattern and have students identify the shapes (square and hexagon). Point out that a tiling pattern shows no spaces and that the shapes do not overlap. Ask the students where they have seen tiling patterns made with more than one geometric shape. They may mention floors or walls in kitchens and bathrooms, and in commercial and government buildings.

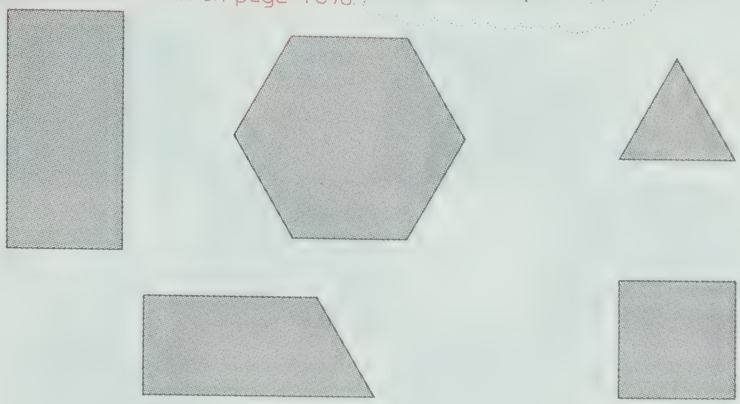
Exercises: To complete these exercises, the students will need to prepare cutouts of the shapes shown. Provide copies of page T 400 for the students to cut out the appropriate shapes, or have them use tracing paper to trace a shape on pages 288 and 289, paste the tracing paper on heavy construction paper, and cut around the shape. Each student can make one cutout for each shape and trace each shape several times on a piece of plain paper to form a pattern. The students can color the completed patterns. Another possibility is to have each student make several cutouts for each shape by tracing around the first cutout on construction paper of different colors. The cutouts may be pasted on a sheet of white paper to emphasize the tiling pattern. You

Use at least three of these shapes together to make a pattern.

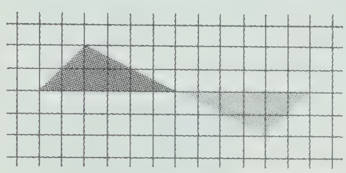
The shapes should touch, but not overlap.

Six patterns are shown on page T376.

3.



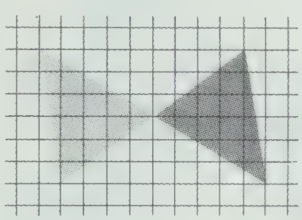
This is an example of a $\frac{1}{2}$ turn.



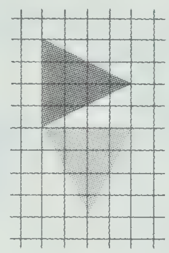
Here are examples of different turns.

The blue shape is a turn image of the red shape. Use tracing paper to find out whether the turn is a $\frac{1}{4}$, a $\frac{1}{2}$, or a $\frac{3}{4}$ turn.

1. $\frac{1}{2}$



2. $\frac{1}{4}$ or $\frac{3}{4}$



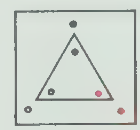
try this

289

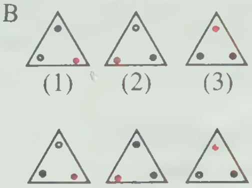
RELATED ACTIVITIES

• Have each student cut an equilateral triangle from a sheet of Bristol board and mark the vertices with colored stickers as shown (A). The same color should be marked on both sides of each vertex. Have the students trace the triangle on a sheet of paper and place stickers of the same color at each vertex. Direct the students to flip or turn the tracing so that it matches the original triangle in as many different ways as possible. Have them draw diagrams to show the six different positions (B).

A



B



Ask the students to explain the motion required from the starting position (1) to obtain each of the other positions. For each motion, have them describe the turn centre or the flip line.

• You may wish to introduce the concept of a *reflex angle*, an angle for which the measure is greater than 180° but less than 360°. The $\frac{3}{4}$ turn described in the *Try This* feature is an example of a reflex angle. For that example, students can derive the measure by thinking of three consecutive clockwise turns of 90°, resulting in a turn of 270°.

may wish to have the students use one of these methods for one exercise and another method for the other exercises. The tiling patterns can be displayed for several days.

Two shapes are used for each of Ex. 1 and 2. Three or more shapes are used for Ex. 3. Ensure that the students understand that the patterns should have no spaces and that the shapes should not overlap.

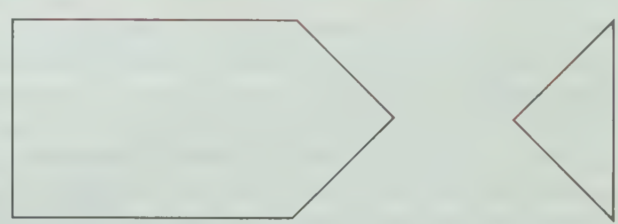
Try This: For the example, have the students use tracing paper to show that the blue shape is a turn image of the red shape, in a clockwise direction and in a counterclockwise direction. Tell the students that the turn centre is the common vertex of the two shapes. Note that the turn angle is not shown and develop that the motion is a $\frac{1}{2}$ turn in a clockwise direction or in a counterclockwise direction. Discuss each of the other examples of turn angles, noting that each is a clockwise turn that begins at the red ray and ends at the blue ray.

For Ex. 1 and 2, the common vertex is the turn centre. For Ex. 2, the turn is a $\frac{1}{4}$ turn in a clockwise direction, or a $\frac{3}{4}$ turn in a counterclockwise direction.

Assessment

Use these shapes to make a pattern without spaces.

1. Two patterns are shown on page T376.



Pages 290-291

LESSON OUTCOME

Identify shapes that can be used to make tessellations; draw tessellations; find the sum of the measurements of the angles at a corner in a tessellation

Materials

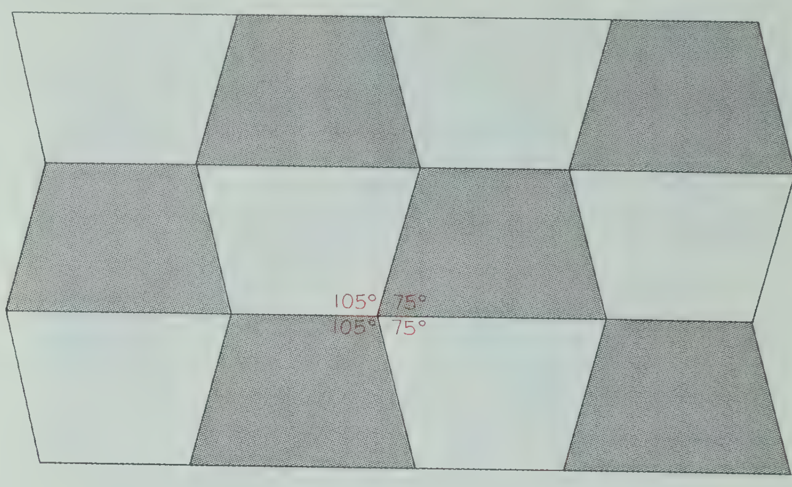
a copy of the parallelogram on page T 384 for each student; a large sheet of construction paper; centimetre rulers (optional); a protractor, a straight edge, and construction paper for each student; tracing paper or a copy of page T 400 for each student; models for rectangular prisms, triangular prisms, and pentagonal prisms (optional)

Vocabulary

tessellations

Tiling the Plane

Anita made this tiling pattern with a trapezoid.



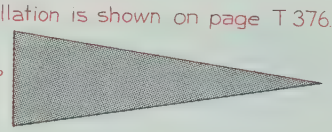
Patterns like this that can be made using one shape are called **tessellations**.

Working Together

For the pattern shown above,

1. measure each angle at a corner. 105°, 75°, 75°, 105°
2. find the sum of the measurements of the angles at a corner. 360°

Use tracing paper to help you make a cutout in the shape of this triangle.



3. Use the cutout and draw a tessellation. A tessellation is shown on page T 376.
4. Measure each angle at a corner. 18°, 81°, 81°, 18°, 81°, 81°
5. Find the sum of the measurements of the angles at a corner. 360°

290

LESSON ACTIVITY

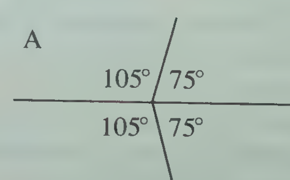
Before Using the Pages

- Give each student a copy of the parallelogram on page T 384. Demonstrate that these identical parallelograms can form a tiling pattern by having each student in turn paste her/his parallelogram on the same large sheet of construction paper. The activity may be adapted for small groups of students, if you prefer. When the students have finished, ask them to turn to page 191. The grids on this page can now be interpreted as tiling patterns based on a square (Ex. 1), on a rectangle (Ex. 2 and 3), and on a parallelogram (Ex. 4-6). Tell the students that a tiling pattern that is made using one shape has a special name.

Using the Pages

- Review that a *trapezoid* is a quadrilateral having one pair of parallel sides. For the trapezoid used to form this pattern, the non-parallel sides are the same length. Students can use their centimetre rulers to check this. Ask how many times the same trapezoid is used to make the pattern shown. Introduce the term *tessellation* to describe a tiling pattern that is made with only one shape. Explain that some shapes will form tessellations and others will not. Tell the students that in this lesson they will encounter examples of each kind.

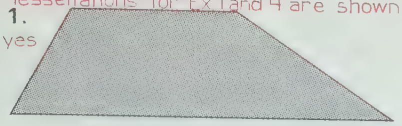
Working Together: For Ex. 1, the students are to use a protractor to measure the four angles at one corner within the pattern. You may need to review how to use a protractor to measure an angle. Ask the students to write the measurements on the diagram (A). For Ex. 2, have several students state the sum and indicate the corner at which the angles were measured. They will note that the sum is 360° at any corner.



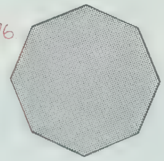
Exercises

Which of these shapes can be used to make tiling patterns that are tessellations?
Use cutouts and draw the tessellations.
Tessellations for Ex. 1 and 4 are shown on page T 376.

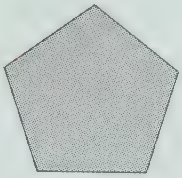
1. yes



2. no



3. no

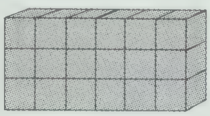


4. yes

For each tessellation that you drew,

5. find the sum of the measurements of the angles at a corner. 360°

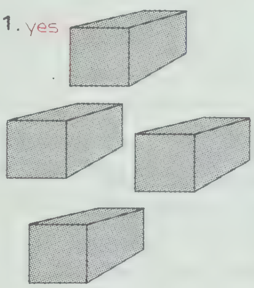
Matching cubes can be stacked to fill a space.



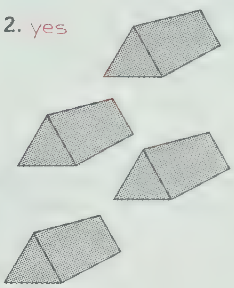
If the bases of prisms can be used to make a tessellation, the prisms can be stacked to fill a space.

Which of these solid shapes can be stacked to fill a space?

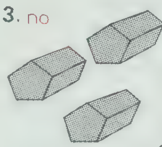
1. yes



2. yes



3. no

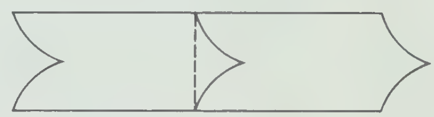


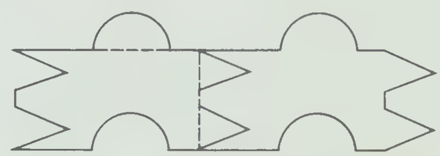
try this

291

RELATED ACTIVITIES

- Have the students work in small groups using copies of page T 387 to prepare several models of each shape shown in Ex. 1-3 in the *Try This* feature. Then ask them to stack and paste identical models together to demonstrate which leave spaces and which do not. The ends of the stacked prisms illustrate tiling patterns. The patterns which have no spaces are tessellations. This activity may be adapted for other prisms, particularly hexagonal prisms (see page T 388), for which an example is found in a honeycomb.
- If possible, display a small section of a honeycomb as an example of a hexagonal prism stacked to fill a space.
- Students can create interesting tessellations based on a shape that is derived from a rectangle. A rectangular piece of cardboard is cut into two or more parts. A new shape is formed by sliding the cut part to the opposite side and taping it in place. Two examples are shown. Students can use color to enhance the tessellations.





To prepare the cutout for Ex. 3 and the cutouts for Ex. 1-4 on page 291, have the students use copies of page T 400 or use tracing paper as described in *Exercises* on pages T 314 and T 315. Ex. 4 and 5 will enable the students to verify that the sum of the measurements of the angles at a corner is 360°.

Exercises: Before the students begin, remind them that some shapes can be used to make tessellations, but others cannot. Draw attention to the fact that Ex. 1 shows a trapezoid for which there are no sides of equal length, in contrast to the trapezoid used to form the pattern shown on page 290. This may lead some students to think that the trapezoid for Ex. 1 will not make a tessellation. They will be surprised to learn that it can.

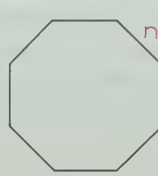
After the students have completed Ex. 5, compare the answers to note that for each tessellation, the sum of the measurements of the angles is 360°.

Try This: The two-dimensional concept of tiling a plane is extended to three dimensions in considering solid shapes that can be stacked to fill a space. The students are familiar with stacking cubes from a study of volume in Unit 6. It is recommended that students be given an opportunity to manipulate solid shapes similar to those illustrated on page 291, to discover which shapes can be stacked. These can be related to the information in the "thought cloud". For example, a pentagon cannot be used to form a tessellation (Ex. 3 of *Exercises*) and a pentagonal prism cannot be stacked to fill a space (Ex. 3 of *Try This*).

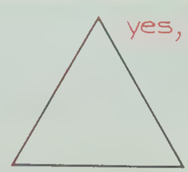
Assessment

Which of these shapes can be used to make tiling patterns that are tessellations? Use cutouts and draw the tessellations. Then measure each angle at a corner and find the sum of the measurements. A tessellation is shown on page T 377.

1. no



2. yes, 360°



LESSON OUTCOME

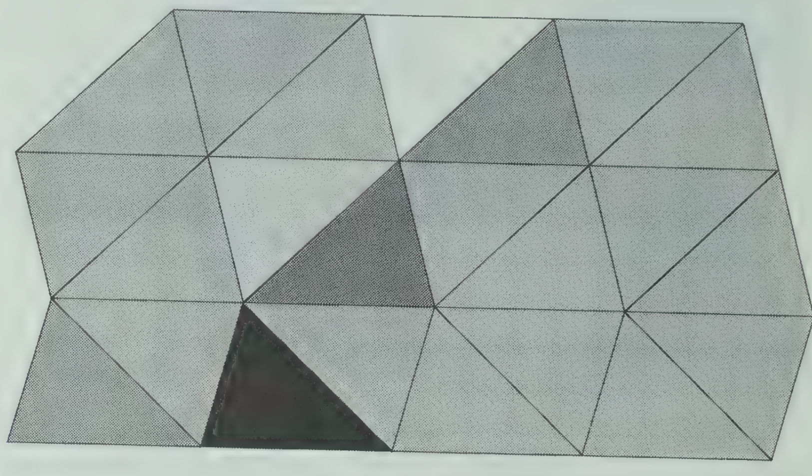
Identify slides, flips, and turns in tiling patterns; identify slide, flip, and turn images in tiling patterns

Materials

tracing paper, construction paper, scissors, and a straight edge for each student; semitransparent plexiglass mirrors (optional)

Identifying Slide, Flip, and Turn Images in Tiling Patterns

A cutout congruent to the red shape was used to make this tiling pattern.



The blue shape is a slide image of the red shape.

The green shape is a flip image of the red shape.

The yellow shape is a turn image of the red shape.

Working Together

For the tiling pattern above,

1. under what motion is the purple shape an image of the yellow shape? a slide
2. under what motion is the blue shape an image of the purple shape? a turn
3. what shape is a flip image of the blue shape? the black shape
4. what shape is a turn image of the white shape? the orange shape

292

LESSON ACTIVITY

Using the Pages

- Ask a student to read the title of the lesson and the introductory statement. Have the students use the method described in *Exercises* on pages T 314 and T 315 to prepare a cutout to match the red shape. Then have them place the cutout on the appropriate shape in the pattern to illustrate each motion described below the pattern. For example, have them place the cutout on the red shape and then slide the cutout to the blue shape.

Working Together: Direct the students to try to complete Ex. 1-4 without using their cutouts; then have them use their cutouts to check their answers. Ex. 1 and 2 involve determining whether the motion that relates two shapes is a slide, a flip, or a turn. For Ex. 3 and 4, the motion is given, and the students are required to identify the image. You may wish to ask other similar questions.

Exercises: Have the students make a cutout or use tracing paper to help them complete the exercises and/or check their answers. Review that a triangle can be named in different ways; for example, ABF, BFA, and FAB are names for the same triangle. Remind the students that the direction of a slide may be horizontal, vertical, or diagonal in the diagram. Note that answers will vary according to the point used as a turn centre or according to the line used as the flip line. For this reason it is important to discuss the students' answers and have them explain their results. For example, Ex. 1 suggests a flip with JK as the flip line, or a half turn with the midpoint of JK as the turn centre. Similarly, Ex. 4 suggests either a flip or a half turn. Ex. 2 suggests a slide, or a flip for which the flip line is the line through points B, J, and U. To reduce the number of possibilities, you may wish to tell the students that a turn centre must be a vertex of a triangle. If semitransparent plexiglass mirrors are available, students may use these on the diagram to check for flip images.

Exercises

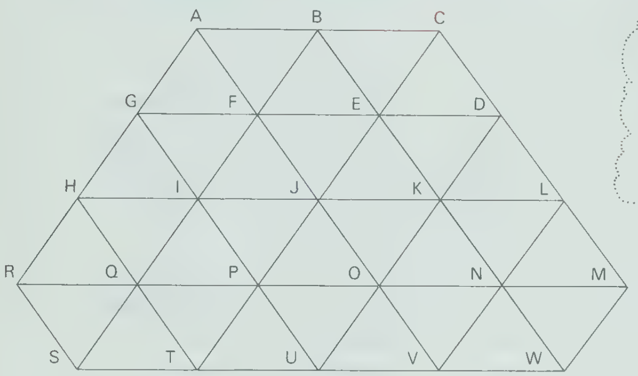
For the tiling pattern below, under what motion is

1. △OKJ an image of △EKJ? a flip
2. △IQH an image of △LNK? a slide
3. △IGF an image of △BEF? a turn
4. △JFI an image of △JPI? a flip

In the tiling pattern, name

Answers for Ex. 5-7 are given below

5. eight triangles that are slide images of △JPI.
6. four triangles that are flip images of △KNO. Other answers are possible for Ex. 8, 9, and 10
7. three triangles that are turn images of △IPQ.
8. two triangles that are slide images of each other. △NWV and △OVU
9. two triangles that are flip images of each other. △CDE and △DKE
10. two triangles that are turn images of each other. △OKJ and △OVU



You could use a cutout that is congruent to the triangles in the pattern to help you.

Find the result.

Remember to work inside the parentheses first. Then work from left to right.

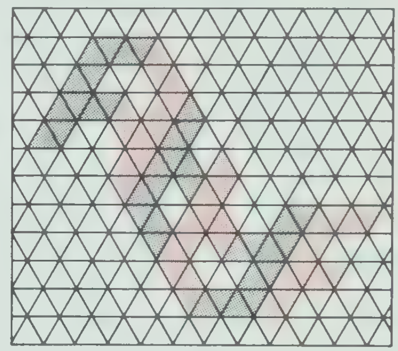
1. 7002 − (3981 + 1932) 1089
2. 371.52 ÷ (36 × 24) 0.43
3. (2642 ÷ 10) + 493.38 757.58
4. (922 × 173) − 63 299 96 207
5. 0.87 + (0.83 × 9.1) 8.423
6. 6.02 × (3 + 1.25) 25.585
7. 700 − (25 370 ÷ 43) + 648 758
8. 4838 ÷ (400 − 195) 23.6
9. (938 + 487.5) ÷ (4.72 + 5.28) 142.55
10. (3.71 × 0.92) + 15 − (369 ÷ 36) 8.1632

KEEPING SHARP

293

RELATED ACTIVITIES

- For the pattern on page 293, students may identify slide, flip, and turn images of a large triangle, for example, triangle AHJ.
- Ex. 1-10 on page 293 may be adapted for tiling patterns prepared during the lesson on pages 290 and 291.
- Have the students color patterns on copies of the triangular grid on page T399. Ask them to form a pattern based on a repetition of flip, turn, and slide motions. The pattern may be based on one or more of their initials. Students may exchange completed patterns and describe them.

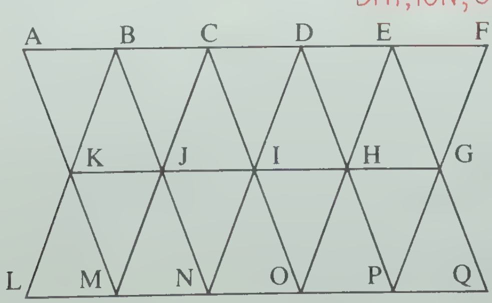


Keeping Sharp: These exercises provide practice in addition, subtraction, multiplication, and division with whole numbers and with decimals. They also provide practice in simplifying expressions involving parentheses.

Assessment

For the tiling pattern below,

1. under what motion is △ BJK an image of △ NJI? a turn
2. under what motion is △ DIH an image of △ OIH? a flip
3. name three triangles that are slide images of △ HPO. DHI, ION, GQP



5. The possible slide images are △BFA, △CEB, △FIG, △EJF, △DKE, △IQH, △KOJ, △LNK, △QSR, △PTQ, △OUP, △NVO, △MWN.
6. △KDE, △LNM, △JPO, and △VNO, if the flip line is vertical or horizontal and goes through a vertex.
7. △IGF, △UPO, and △SRQ, if the turn center is a vertex of △IPQ.

Page 294

OBJECTIVE
Use models to help solve problems

Materials
several pieces of paper, a straight edge, and a protractor for each student

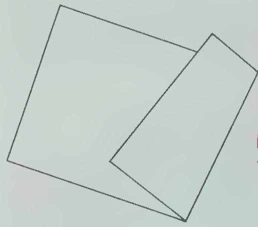
RELATED ACTIVITIES
- Ex. 5 may be adapted for other polygons, either regular or irregular. Have the students complete the following chart.

Polygon	Sum of angle measurements
triangle	180°
square	360°
pentagon	540°
hexagon	720°
octagon	900°
decagon	1080°

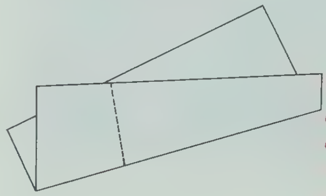
- Provide the students with copies of polygons from pages T383-T385. Have them adapt the procedure outlined in Ex. 1 to bisect at right angles each side of a polygon. Also, have them adapt the procedure outlined in Ex. 3 to bisect each angle of a polygon.

Working with Models

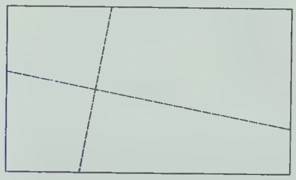
Fold a piece of paper.



Open it. Fold it again so that one half of the first fold line matches the other half.



Open the paper.



The two fold lines form right angles. The two fold lines are perpendicular.

Try these by folding paper.

1. Draw a line segment. Fold one end point of the line segment onto the other end point. What is special about the fold line and the line segment?
   The fold divides the line segment into two equal parts. The fold is perpendicular to the line segment.
2. Make two folds that cross each other, but do not form right angles. Are any of the angles formed by the two fold lines congruent?
   yes

Fold your paper again to check.

3. Draw an angle. Fold one ray of the angle onto the other ray. What is special about the angles formed by the first angle and the fold line?
   The angles on each side of the fold line are congruent.
4. How can you fold paper to make two parallel fold lines?
5. Fold a piece of paper to make a quadrilateral. Measure the four angles. Add the measurements. Repeat with another quadrilateral. What is special about the sums of the angle measurements?
   They are 360°.

4. (1) Fold the paper so that one edge is along the opposite edge.
(2) Unfold the paper.
(3) Fold the paper so that one of the two edges that met for the first step is along the fold line. The two fold lines are parallel.

PROBLEM SOLVING

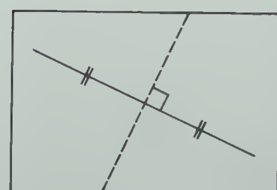
294

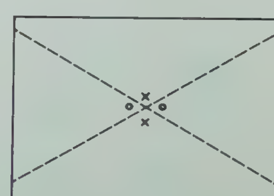
LESSON ACTIVITY

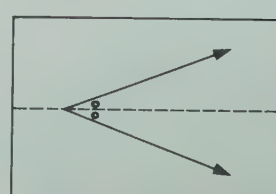
Using the Page

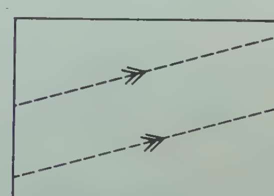
- Provide each student with a piece of paper to complete the steps described in the example. Ask a few students to demonstrate the procedure. Draw attention to the fact that the fold lines are not lines of symmetry in the given example. Ask what kind of angles the two fold lines appear to form. Have the students use their protractors to measure the angles. Review that an angle of 90° is called a right angle. Summarize that the two fold lines form four right angles.
- Provide several pieces of paper for each student to complete Ex. 1-5. You may wish to have the students repeat Ex. 1-3 several times to note whether the results are consistent. For example, Ex. 3 can be demonstrated for an acute angle, for a right angle, and for an obtuse angle. It may be necessary to guide some students as they complete the exercises and to review such terms as *end point* and *ray*. When they have completed the exercises, discuss the results and ask several students to demonstrate the procedure they used.

The exercises on this page provide an opportunity to show the students how to indicate congruent line segments (A), congruent angles (B and C), and parallel lines (D) on diagrams.

A


B


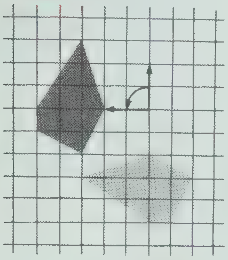
C


D


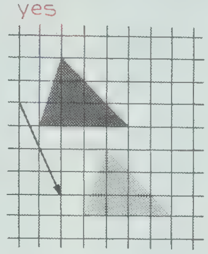
Checking Up

Use tracing paper to test whether

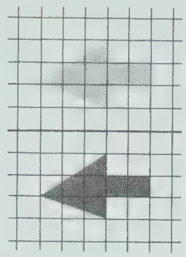
1. the blue shape is the turn image of the red shape for the turn angle shown. yes



2. the blue shape is the slide image of the red shape for the slide arrow shown. yes

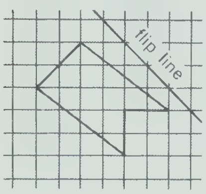


3. the blue shape is the flip image of the red shape for the flip line shown. no

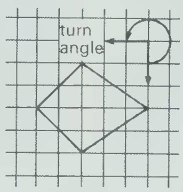


Copy each picture on graph paper.
Use tracing paper to help you draw Answers are shown on page T377

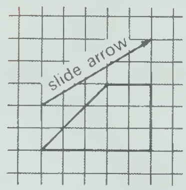
4. the flip image.



5. the turn image.

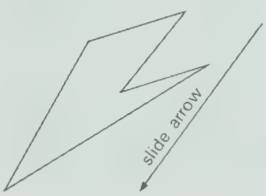


6. the slide image.

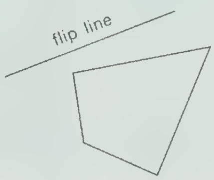


Trace each picture on plain paper.
Use tracing paper to help you draw Answers are shown on page T377

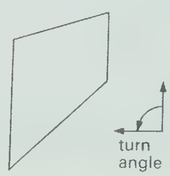
7. the slide image.



8. the flip image.



9. the turn image.



295

Pages 295-296

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Materials

copies of page T 397, tracing paper, plain paper, construction paper, a straight edge, scissors, a sharp pencil, and a protractor for each student

Skills	Exercises	Related Pages
Identify one figure as the turn image of another figure for a given turn angle	1	T 310-T 311
Identify one figure as the slide image of another figure for a given slide arrow	2	T 302-T 303
Identify one figure as the flip image of another figure for a given flip line	3	T 306-T 307
Draw a flip image using tracing paper	4, 8	T 306-T 307
Draw a turn image using tracing paper	5, 9	T 310-T 311
Draw a slide image using tracing paper	6, 7	T 302-T 303
Draw a slide image by using a rule	10, 11	T 304-T 305
Draw a flip image by counting units	12	T 308-T 309
Identify rotational symmetry	13	T 312-T 313
Make a tiling pattern using more than one shape	14	T 314-T 315
Draw a tessellation and measure the angles at a corner	15	T 316-T 317
Identify motions in a tiling pattern	16	T 318-T 319
Identify flip and slide images in a tiling pattern	17, 18	T 318-T 319

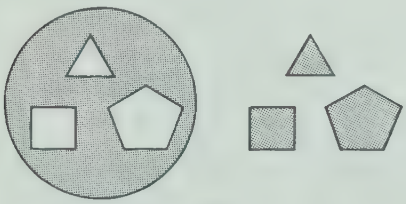
Comments

Remind the students of the need to take care in copying and tracing the pictures to obtain accurate results.

Students having difficulty with this work may benefit from using objects such as parquetry blocks to show slides, flips, and turns, and tracing around the blocks to show the shapes and their images. Plastic stencils are useful for drawing tiling patterns based on slides, flips, and turns. Congruent cutouts can be glued on a sheet of paper to prepare tiling patterns. The students should not be rushed in these exercises. Allow them ample time to work carefully at tracing shapes and manipulating the tracings.

RELATED ACTIVITIES

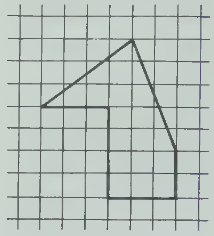
- Provide samples of wallpaper, fabric, or gift wrapping paper, and have the students describe the patterns in terms of slides, flips, and turns. Then give the students large sheets of paper on which to design their own patterns for wallpaper or gift wrapping paper.
- Plastic stencils are useful for drawing tiling patterns. Such stencils are available commercially or you may, with care, prepare your own. Collect the plastic covers from food containers. Draw shapes on the covers and use a sharp knife to cut them out. The covers form the stencils and the cutouts may be used to investigate tiling patterns.



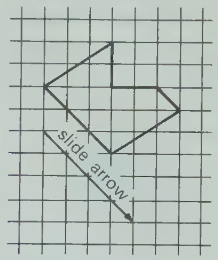
- The first of the *Related Activities* on page T315 may be adapted for a square, a rhombus, a rectangle, and an isosceles triangle. In some instances, the move from the starting position to another position will require a flip and a turn.

Copy each picture on graph paper.
Without using tracing paper, Answers are shown on page T377.

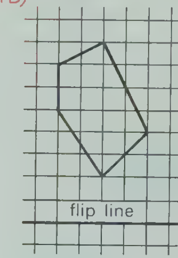
10. draw the slide image for the rule (5L, 4D).



11. write the rule and draw the slide image. (4R, 4D)

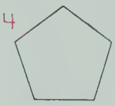


12. draw the flip image.



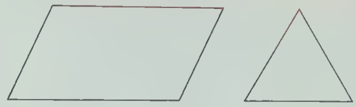
Use tracing paper to test this shape for turn symmetry. Then give the number of different turns less than a full turn for which the shape fits onto itself.

13. 4



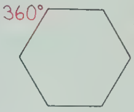
Use both of these shapes and draw a pattern without spaces.

14. One pattern is shown on page T377.



Use a cutout and draw a tiling pattern that is a tessellation. Find the sum of the measurements of the angles at a corner.

15. 360° A tiling pattern is shown on page T377.

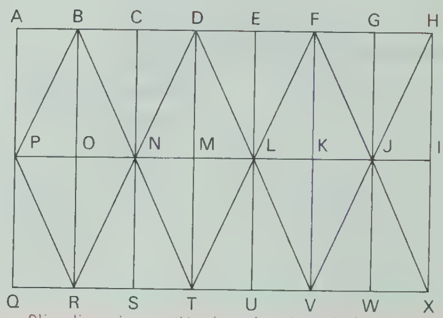


For this tiling pattern,

16. under what motion is △JVK an image of △LFK? a turn

17. name three triangles that are flip images of △MTN.

18. name two triangles that are slide images of each other.



17. △DMN, △MLT, △NRO, if the flip line is vertical or horizontal and goes through a vertex of △MTN.
18. △BPO, △DNM Other answers are possible.

296

Checking Skills

Divide. Use extra zeros if needed.

1. 61)$298.29 $4.89
2. 50)$3530 $70.60
3. 10)$597.30 $59.73
4. 220)$1859 $8.45
5. 84)$671.16 $7.99
6. 30)$1201.50 $40.05
7. 10)$7892 $789.20
8. 49)$522.83 $10.67
9. 756)$393.12 $0.52
10. 63)$635.04 $10.08
11. 200)$1670 $8.35
12. 842)$791.48 $0.94
13. 35)$310.80 $8.88
14. 100)$9234 $92.34
15. 78)7371 94.5
16. 96)46.608 0.4855
17. 40)12.76 0.319
18. 100)9.31 0.0931
19. 18)73.17 4.065
20. 10)8.927 0.8927
21. 943)264.04 0.28
22. 70)4.326 0.0618
23. 10)4302 430.2
24. 66)366.3 5.55
25. 54)14.85 0.275
26. 300)24 024 80.08
27. 725)101.5 0.14
28. 100)6090 60.9
29. 43)124.7 2.9
30. 880)3828 4.35
31. $560 ÷ 100 $5.60
32. 3412 ÷ 10 341.2
33. $24.49 ÷ 79 $0.31
34. 83.08 ÷ 67 1.24
35. $20 ÷ 400 $0.05
36. 61.56 ÷ 90 0.684
37. $6.90 ÷ 10 $0.69
38. 81 ÷ 20 4.5
39. $52.80 ÷ 60 $0.88
40. 99.3 ÷ 331 0.3
41. $10.45 ÷ 95 $0.11
42. 740.6 ÷ 92 8.05
43. $205.20 ÷ 27 $7.60
44. 231 ÷ 462 0.5
45. $624 ÷ 100 $6.24
46. 304 ÷ 320 0.95
47. $2058 ÷ 600 $3.43
48. 2.4 ÷ 48 0.05

Find the result. Work inside the parentheses first. Then work from left to right.

1. 30 004 − (17 843 + 12 048) 113
2. 527.25 ÷ (2.5 × 38) 5.55
3. (36 480 ÷ 76) − 89 391
4. 20 − (343.9 ÷ 38) 10.95
5. 400.4 ÷ (37.62 + 12.38) 8.008
6. (346 × 84) − 28 777 287
7. 5028 + (94 564 ÷ 94) 6034
8. 7.31 × (17 − 6.8) 74.562
9. (3.92 − 3) × 100 92
10. 100 − (1403 ÷ 46) 69.5
11. 52 722 ÷ (1000 − 394) 87
12. 10 × (604 + 209) 8130
13. 30 000 − (3496 + 7258) 19 246
14. 27 + (602.58 ÷ 83) 34.26
15. 2.2 − (0.9 × 0.71) 1.561
16. (672 − 589) × (20 736 ÷ 32) 53 784
17. (63.19 ÷ 71) + (0.72 × 1.28) 1.8116
18. (1458 × 100) ÷ (100 × 2.43) 600
19. (3.24 + 6.76) − (300.8 ÷ 47) 3.6
20. (100 × 7920) ÷ (2098 − 1999) 8000
21. (136 ÷ 16) + 19 + (70 ÷ 100) 28.2
22. (106 × 48) − 4989 − (27 + 30) 42
23. (345.18 + 28.41) ÷ (39.6 − 27.6) 31.1325
24. (211 ÷ 25) × 6.3 − (3.42 × 10) 18.972
25. (100 + 61) × 73 ÷ (82 + 79) 73
26. 64.07 + 32 − (17.1 + 36.428) 42.542
27. 707 × 30 ÷ (129.63 − 59.63) 303
28. 12 − 8.276 − (142.12 ÷ 68) 1.634
29. 58 400 ÷ 73 + (204 × 1000) 204 800

297

Page 297

OBJECTIVE

Demonstrate competence in dividing whole numbers and decimals and in simplifying expressions involving parentheses

RELATED ACTIVITIES

- Have the students use parentheses in the following exercises to make true statements.

(2 × 8) + 7 = 23
30 ÷ (5 + 5) = 3
(6 + 15) ÷ 3 = 7
8 × (9 + 2) = 88
(81 ÷ 9) + 17 = 26
(56 ÷ 8) + 14 = 21
50 − (6 × 6) = 14
9 × (25 − 17) = 72
(39 + 6) ÷ 9 = 5
22 − (12 ÷ 2) = 16
(8 + 7) × (5 − 2) = 45

LESSON ACTIVITY

Using the Page

- The exercises in the first set (Ex. 1-48) involve decimal quotients resulting from dividends that are whole numbers or decimals and divisors that are whole numbers. This reviews the aspects of division presented in Unit 7 and prepares for Unit 15 which involves decimals as divisors. Direct the students to use extra zeros in the dividends to terminate the quotients. For divisors of 10 or 100, ask them to perform the calculations mentally and to write only the quotients.

  The exercises in the second set (Ex. 1-29) involve operations with whole numbers and decimals in expressions having parentheses. These concepts were reviewed earlier in the *Keeping Sharp* feature on page 293.

- It would be preferable to have the students complete a few exercises each day for several days. Review concepts that students find difficult by adapting the suggestions for the lessons and for the *Related Activities* on the appropriate pages.

Unit 15 Overview

Dividing with Decimals

This unit begins with a review of place value in decimals to thousandths since place value is so important in developing meaningful procedures in dividing with decimals. Multiplying both the divisor and the dividend by the same power of ten (10, 100, or 1000) is the method used to obtain whole-number divisors in the earlier lessons of the unit. The students soon adopt the simple procedure of moving the decimal points to the right the same number of places for both numbers. Rounding quotients is achieved by dividing to one more place and examining the digit in that place. Rounding is carried out to the nearest tenth, to the nearest hundredth, and to the nearest thousandth. The students are shown a method for estimating quotients using an adjusted divisor of only one digit and a similarly adjusted and rounded dividend. This unit continues and completes a strand which began in Unit 7 concerning the shift of digits in decimals when they are multiplied or divided by 10, 100, 1000, 0.1, 0.01, and 0.001. Relationships are established between the inverse operations and pairs of factors and divisors which give the same results. The skills of dividing with decimals are applied in solving word problems at each stage of development. The *Problem Solving* lesson shows how the distributive property of multiplication over addition can often simplify calculations to a non-paper-and-pencil level.

Prerequisite Skills

- write decimals
- interpret place value in decimals to thousandths
- multiply by 10, 100, and 1000
- divide a whole number or a decimal by a whole number
- regroup among the places for numbers to 99.9
- multiply a decimal with up to three decimal places and a whole number with up to three digits
- multiply a decimal with up to three decimal places and a one-place decimal or a two-place decimal

Unit Outcomes

- interpret place value and regroup for ones, tenths, and hundredths
- change the place values in a ratio by multiplying each term by 10, 100, or 1000
- divide a whole number or a one-place decimal by a one-place decimal less than one, whole-number quotient and remainder zero; multiply to check division
- divide a whole number or a decimal with up to three decimal places by a one-place decimal, using extra zeros in the dividend; multiply to check division
- divide by 0.1, writing only the quotient
- round a decimal quotient to the nearest tenth, to the nearest hundredth, or to the nearest thousandth when dividing by decimal tenths
- divide a whole number or a decimal with one or two decimal places by a two-place decimal, whole-number quotient
- divide a whole number or a decimal by a decimal with up to three decimal places
- round a decimal quotient to the nearest tenth, to the nearest hundredth, or to the nearest thousandth when dividing by decimal hundredths and by decimal thousandths
- estimate a quotient
- solve word problems involving division with decimals
- use the distributive property of multiplication over addition to solve problems without pencil and paper

Background

There are a number of ways in which division with numbers can be shown: $28 \div 14$, $14\overline{)28}$, and $\frac{28}{14}$. Each of these indicates a division relationship between the numbers 14 and 28. In some instances, this relationship may be interpreted as a ratio of 28 to 14 ($\frac{28}{14}$), which, by division, can be equated to a ratio of 2 to 1 in lowest terms. This relationship, or ratio, remains the same if both terms are multiplied or divided by the same number. In each case, the ratio is equivalent to $\frac{2}{1}$, and if this relationship is shown as division, the quotient is also the same in each case.

$$\frac{28}{14} = \frac{2.8}{1.4} = \frac{0.28}{0.14} = \frac{0.028}{0.014}$$

A $\begin{array}{r} 2 \\ 14\overline{)28} \end{array}$ B $\begin{array}{r} 2 \\ 1.4\overline{)2.8} \end{array}$

C $\begin{array}{r} 2 \\ 0.14\overline{)0.28} \end{array}$ D $\begin{array}{r} 2 \\ 0.014\overline{)0.028} \end{array}$

It is seen that if a divisor is a decimal, the example may be changed into one in which the divisor is a whole number by multiplying both the divisor and the dividend by the required power of ten. For example, in C the divisor shows hundredths (2 decimal places) and needs to be multiplied by the second power of 10 (10^2, or 100), and in D the factor needed is 10^3, or 1000, to obtain the whole-number divisor shown in A.

There is a two-fold effect when a number is multiplied by a power of ten. The strand introduced in blue panels in Unit 7 is continued in this unit. In the first instance, on page 135, it is pointed out that "when a number is multiplied by 10 each digit moves one place to the left". This principle was extended to multiplication by 100 and by 1000 on page 137. In a decimal, the positions of the digits and the decimal point are relative, and if the digits move to the left, the decimal point appears to move to the right. Thus, although it is more accurate mathematically to think of ones becoming tens and of tenths becoming ones if a decimal such as 4.28 is multiplied by 10, the same effect is obtained by moving the decimal point one place to the right. Therefore, if it is necessary to move the decimal point in a divisor two places to the right, signifying multiplication by 100, the same must be done in the dividend to maintain the same relationship and to obtain the correct quotient (E). If the number of decimal places in the dividend is the same as, or greater than, the number of decimal places in the divisor, the new dividend is either a whole number (F) or a decimal (G). If, on the other hand, the number of decimal places in the dividend is less than the number of decimal places in the divisor, extra zeros are necessary in the dividend (H, K). The need for zeros in these cases is obvious if the dividends, 17.6 and 17, are actually multiplied by 100; $17.6 \times 100 = 176\underline{0}$ and $17 \times 100 = 17\underline{00}$.

$$10 \times 4.28 = 42.8$$

4.28

42.8

E $2.45\overline{)16.285}$ F $2.3\overline{)6.9}$ G $2.3\overline{)6.95}$

H $2.35\overline{)17.60}$ K $0.34\overline{)17.00}$

The same algorithm is used for dividing decimals as for whole numbers. Renaming a remainder and the next place value to the right proceeds in the same manner, as far as necessary to terminate the division, or as far as necessary to obtain a satisfactory quotient. If a quotient is desired to the nearest tenth, the process is continued to one more decimal place, to hundredths, and the digit in that place determines whether the digit in the tenths' place should remain unchanged or be increased by one. Division is always carried to one more place than that to which a quotient is to be rounded. In some cases, the size of the remainder makes it obvious what to do without actually performing the division step, but students should not rely on this as a sure method.

The strand which began on page 135 is concluded on page 311 of this unit. The generalizations are given below.

When a number is →	multiplied by 10, 100, or 1000,	divided by 10, 100, or 1000,
each digit moves 1, 2, or 3 places to →	the left.	the right.

When a number is →	multiplied by 0.1, 0.01, or 0.001,	divided by 0.1, 0.01, or 0.001,
each digit moves 1, 2, or 3 places to →	the right.	the left.

As stated earlier, the apparent movement of a decimal point is opposite to that of the digits when a number is either multiplied or divided by a power of ten. A brief examination of the charts also reveals the inverse relationship between multiplication and division. For example, multiplication by 10 and division by 0.1 produce the same result, and division by 10 and multiplication by 0.1 produce the same result. Similar inverse relationships are operative for two-place and three-place decimals and corresponding whole numbers (0.01 and 100, 0.001 and 1000). Exponential notation of the powers in a place-value structure helps to emphasize this. For instance, multiplication by 10^3 (1000) and division by 10^{-3} (0.001) have the same effect.

10^3	10^2	10^1	10^0	10^{-1}	10^{-2}	10^{-3}
1000	100	10	1	0.1	0.01	0.001

For the number sentences in the *Problem Solving* lesson on page 314, parentheses are used to group numbers in certain ways. This particular lesson emphasizes the use of the *distributive property of multiplication over addition* to simplify calculations. This property, often called simply the *distributive property*, is discussed in the Overview for Unit 4 to explain how and why the algorithm for multiplication works. Stated briefly, if one factor in multiplication is expressed as the sum of two or more numbers, each of the numbers is multiplied by the other factor and these products are then added (L). The converse is also true; namely, if two or more factors are to be multiplied by the same factor and their products then added, the sum of the original factors may be used as a single factor. This is illustrated in the following example. If 6 children go to the swimming pool and each pays 25¢ admission and 15¢ for a locker, the total cost may be found by first finding the total cost for each child (25 + 15 = 40) and then multiplying this sum by 6 (M). This is shorter than finding separate products and then adding them (N).

L
$$\begin{aligned} 7 \times 9 &= 7 \times (5 + 4) \\ &= (7 \times 5) + (7 \times 4) \\ &= 35 + 28 \\ &= 63 \end{aligned}$$

M
$$\begin{aligned} & 6 \times (25 + 15) \\ &= 6 \times 40 \\ &= 240 \end{aligned}$$

The total cost is \$2.40.

N
$$\begin{array}{r} 6 \times 25 = 150 \\ 6 \times 15 = \underline{\ \ 90} \\ 240 \end{array}$$

The total cost is \$2.40.

Whenever multiplication is used to check division by a decimal divisor, it should be performed with the quotient obtained and the original divisor. Their product must match the original dividend. Using the original number serves as a double check on the accuracy of shifting the decimal points in the divisor and in the dividend. Any error of this type can be identified quickly if the product is the same except for the position of the decimal point. It should be pointed out that if there is a remainder to be added in the checking procedure, its place value must be watched carefully. For instance, in 1.5)5.58, the quotient is 3.7 and the remainder appears to be 3, but in relation to the original positions of the decimal point and the digits in the dividend, it is in reality 3 hundredths, or 0.03, which must be added.

$$\begin{array}{r} 3.7 \\ 1.5\overline{)5.58} \\ \underline{4\ 5} \\ 1\ 08 \\ \underline{1\ 05} \\ 3 \end{array} \qquad \text{Check: } \begin{array}{r} 3.7 \\ \underline{\times\ 1.5} \\ 185 \\ \underline{37} \\ 5.55 \\ \underline{+\ 0.03} \\ 5.58 \end{array}$$

Teaching Strategies

Division with decimal divisors uses the same algorithm as division with whole-number divisors and it is suggested that a preliminary assessment of skills in long division be made before the first lesson using decimal divisors. The division exercises on page 297 can be used for this purpose. Any significant weaknesses should be diagnosed and corrected. It may be necessary to structure different groups so that students with similar needs may work together. Some may require more time with the activities outlined in *Before Using the Pages* in preparation for the lessons presented in the textbook. For instance, prior to changing decimal divisors to whole numbers, some students may need more practice in converting terms in ratios so that the decimal portion of one or both terms is eliminated. Finding missing terms in equivalent ratios, such as $\frac{3.4}{1.2} = \frac{■}{12}$, $\frac{27}{0.9} = \frac{■}{9}$, and $\frac{1.25}{0.5} = \frac{■}{5}$, can provide worthwhile preparation for similar changes in division.

A brief review of the principles in rounding decimals would be beneficial before the students encounter rounding of decimal quotients. (See pages 98 and 99.) It should be emphasized that it is the digit in the next place to the right of the digit that is being rounded that must be considered.

Materials

models for ones, tenths, and hundredths (copies of pages T 392 - T 394)

models for 1.2, 1.6, and 2.0 prepared from copies of page T 393

a copy of page T 395 and a copy of the triangular grid on page T 399 for each student (optional)

three nickels and three pennies or three markers of one color and three markers of another color for each student

LESSON OUTCOME

Interpret place value and regroup for ones, tenths, and hundredths

Materials
models for ones, tenths, and hundredths (copies of pages T392-T394)

Prerequisite Skills
Write decimals

Checking Prerequisite Skills
Write the decimals.

1. three and nine-tenths 3.9
2. one and six-hundredths 1.06
3. four-tenths 0.4
4. five and twenty-one hundredths 5.21
5. sixty-hundredths 0.60

RELATED ACTIVITIES

- Provide students with copies of pages T392-T394. Have them color appropriate models to show the different ways of representing the decimal for one of the exercises on page 298.
- Have students name an amount of money less than $10.00 and express the amount three ways as shown below.

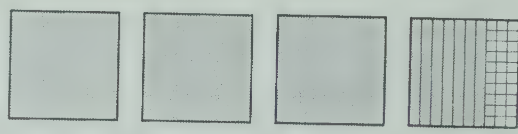
$2.37		
2 dollars	3 dimes	7 pennies
	23 dimes	7 pennies
		237 pennies

15 DIVIDING WITH DECIMALS

Reviewing Place Value

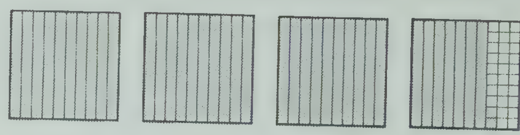
The number 3.72 can represent

ones	tenths	hundredths
3	7	2



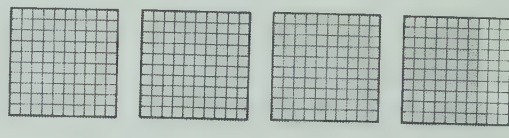
or 3.72 can represent

ones	tenths	hundredths
0	37	2



or 3.72 can represent

ones	tenths	hundredths
0	0	372



Exercises

Copy and complete.

1. 5.89 can represent 5 ones 8 tenths 9 hundredths,
   or 0 ones 58 tenths 9 hundredths,
   or 0 ones 0 tenths 589 hundredths.
2. 4.8 can represent 4 ones 8 tenths,
   or 0 ones 48 tenths.
3. 0.26 can represent 0 ones 2 tenths 6 hundredths,
   or 0 ones 0 tenths 26 hundredths.
4. 3.76 can represent 3 ones 7 tenths 6 hundredths,
   or 0 ones 37 tenths 6 hundredths,
   or 0 ones 0 tenths 376 hundredths.

298

LESSON ACTIVITY

Before Using the Page

- Draw a place-value chart on the board for ones, tenths, and hundredths. Name decimals such as "two and three-tenths" and "one and sixty-four hundredths". Ask students to write the standard numerals and show the numbers in the place-value chart. Also, show decimals in the place-value chart and ask students to write and then read the standard numerals.

	ones	tenths	hundredths
2.3	2	3	
1.64	1	6	4

- Display models for ones, tenths, and hundredths. Review the three relationships 1 one = 10 tenths, 1 tenth = 10 hundredths, and 1 one = 100 hundredths.

Using the Page

- Draw attention to the three groups of models. Note that the same amount is shown in blue in each group; thus, each group represents the same number, 3.72, in a different way. Discuss that the first group shows ones, tenths, and hundredths, the second shows tenths and hundredths, and the third shows hundredths.

Relate each group to the corresponding place-value chart. Ask students to explain the regrouping indicated by the models. For example, 3 ones are regrouped as 30 tenths for 37 tenths in all. Then, 37 tenths are regrouped as 370 hundredths for 372 hundredths in all.

Draw attention to the title of this unit and explain that regrouping decimals as discussed will be helpful in learning to divide with decimals.

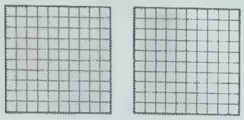
Exercises: Each exercise involves different regroupings for a given number.

Assessment
Complete.

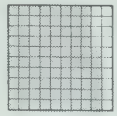
1. 2.35 can represent 2 ones ___ tenths 5 hundredths, 3
   or 0 ones ___ tenths 5 hundredths, 23
   or 0 ones 0 tenths ___ hundredths. 235

Changing Place Values in Ratios

Give the ratio that compares the blue regions.



1.53 : 0.86
or
15.3 : 8.6
or
153 : 86



> If both terms in a ratio are multiplied by the same number to change the place values, the ratios are equivalent.

Multiply by 10 to change to tenths.

$\frac{1.53}{0.86} = \frac{15.3 \text{ tenths}}{8.6 \text{ tenths}}$, or $\frac{15.3}{8.6}$

$\frac{1.53}{0.86} = \frac{15.3}{8.6}$ Multiply both terms of the ratio by 10.

Multiply by 100 to change to hundredths.

$\frac{1.53}{0.86} = \frac{153 \text{ hundredths}}{86 \text{ hundredths}}$, or $\frac{153}{86}$

$\frac{1.53}{0.86} = \frac{153}{86}$ Multiply both terms of the ratio by 100.

Exercises

In both terms of the ratio, change the place values in the same way to get an equivalent ratio.

1. $\frac{2.17}{3.68} = \frac{21.7}{■}$ 36.8
2. $\frac{7.63}{4.18} = \frac{■}{418}$ 763
3. $\frac{2.9}{3.2} = \frac{29}{■}$ 32
4. $\frac{0.64}{0.82} = \frac{■}{82}$ 64
5. $\frac{3.8}{1.75} = \frac{■}{17.5}$ 38
6. $\frac{0.95}{1.2} = \frac{95}{■}$ 120
7. $\frac{7.641}{1.8} = \frac{■}{180}$ 764.1
8. $\frac{3}{4.5} = \frac{30}{■}$ 45
9. $\frac{0.07}{0.82} = \frac{7}{■}$ 82
10. $\frac{36}{0.18} = \frac{■}{1.8}$ 360
11. $\frac{4}{1.75} = \frac{400}{■}$ 175
12. $\frac{3.282}{7.64} = \frac{■}{764}$ 328.2
13. $\frac{95.4}{36.2} = \frac{■}{362}$ 954
14. $\frac{0.182}{0.006} = \frac{18.2}{■}$ 0.6
15. $\frac{5.6}{1.37} = \frac{■}{13.7}$ 56
16. $\frac{5.308}{2} = \frac{5308}{■}$ 2000

299

Page 299

LESSON OUTCOME

Change the place values in a ratio by multiplying each term by 10, 100, or 1000

Prerequisite Skills

Interpret place value in decimals to thousandths; multiply by 10, 100, and 1000

Checking Prerequisite Skills

What does each 6 mean?

1. 7.6436 6 tenths
2. 6.257 6 ones
3. 1.56 6 hundredths
4. 3.986 6 thousandths

Multiply each number by 10, 100, and 1000.

5. 3 30, 300, 3000
6. 2.55 25.5, 255, 2550
7. 6.138 61.38, 613.8, 6138
8. 1.7 17, 170, 1700

RELATED ACTIVITIES

- Have the students prepare sets of cards and play the game "Match Up" described on page T381. To make the game more challenging, have the students use the same digits for different sets so that recognizing equivalent ratios requires comparing place values.

One set:

$\frac{3.6}{0.15}$ $\frac{36}{1.5}$ $\frac{360}{15}$

Another set:

$\frac{0.036}{1.5}$ $\frac{0.36}{15}$ $\frac{3.6}{150}$

LESSON ACTIVITY

Before Using the Page

- Review the procedure for finding equivalent ratios and for finding the missing term in two equivalent ratios. Emphasize multiplying each term of a ratio by the same number (rather than dividing), in preparation for division by a decimal.

$\frac{3}{4} \overset{3 \times ?}{\underset{4 \times ?}{=}} \frac{■}{■}$ $\frac{7}{8} \overset{7 \times ?}{\underset{8 \times ?}{=}} \frac{■}{80}$ $\frac{11}{2} \overset{11 \times ?}{\underset{2 \times ?}{=}} \frac{1100}{■}$

- Write the following on the board and ask the students to determine whether the missing factor is 10, 100, or 1000.

$3.5 \times ▒ = 350$ $0.26 \times ▒ = 260$

Using the Page

- Establish that the models at the left represent 1 one and 53 hundredths, or 1.53, and that the model on the right represents 86 hundredths, or 0.86. Remind the students that if both terms in a ratio are multiplied by the same number, the two ratios will be equivalent. Discuss that multiplying each term of 1.53:0.86 by 10 gives 15.3:8.6, and that multiplying each term of 1.53:0.86 by 100 gives 153:86. Emphasize that the three ratios are equivalent. Ask what the blue arrows indicate.

Exercises: For each exercise, the students must first determine whether the first (second) term of the original ratio is multiplied by 10, by 100, or by 1000 to result in the first (second) term of the equivalent ratio. Then they must multiply the other term in the original ratio by the same number to find the missing term.

Assessment

In both terms of the ratio, change the place values in the same way to get an equivalent ratio.

1. $\frac{4.126}{3.5} = \frac{■}{35}$ 41.26
2. $\frac{1.6}{8.83} = \frac{160}{■}$ 883
3. $\frac{0.02}{0.796} = \frac{■}{796}$ 20

Pages 300-301

LESSON OUTCOME

Divide a whole number or a one-place decimal by a one-place decimal less than one, whole-number quotient and remainder zero; multiply to check division; solve related word problems

Materials

models for 1.2, 1.6, and 2.0 prepared from copies of page T393

Prerequisite Skills

Change the place values in a ratio by multiplying each term by 10; divide whole numbers; regroup among the places for numbers to 99.9; multiply whole numbers by decimal tenths less than one

Checking Prerequisite Skills

Multiply each term of the ratio by 10 to show an equivalent ratio.

1. $\frac{1.2}{0.3}$ $\frac{12}{3}$ 2. $\frac{24}{0.6}$ $\frac{240}{6}$

3. $\frac{31.6}{0.4}$ $\frac{316}{4}$ 4. $\frac{15}{0.2}$ $\frac{150}{2}$

Divide.

5. $6\overline{)126}$ 21 6. $8\overline{)112}$ 14

Complete.

7. 7 ones 8 tenths = 78 tenths

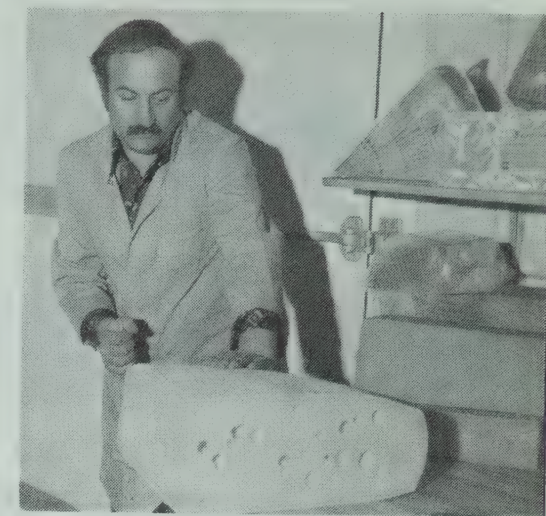
8. 2 ones = 20 tenths

Multiply.

9. 0.6 × 9 5.4 10. 0.4 × 12 4.8

Dividing by Numbers Less Than 1

There are 15 kg of cheese in the large block. It will be cut into smaller blocks that are about 0.5 kg each. About how many smaller blocks will there be?



Divide 15 by 0.5.

For $0.5\overline{)15}$,

think $5 \text{ tenths}\overline{)150 \text{ tenths}}$

Use $5\overline{)150}$ 30.

Both the divisor and the dividend are multiplied by 10.

150 tenths ÷ 5 tenths = 30

15 ÷ 0.5 = 30

Check by multiplying:
0.5 × 30 = 15.0, or 15

There will be about 30 smaller blocks of cheese.

> If the divisor and the dividend are multiplied by the same number to change the place values, the quotient remains the same.

You can use this rule to replace decimal divisors with whole numbers.

Take another look:

For $0.5\overline{)3.5}$, think $5 \text{ tenths}\overline{)35 \text{ tenths}}$

Use $5\overline{)35}$ 7.

$0.5\overline{)3.5}$ 7

3.5 ÷ 0.5 = 7

Check: 0.5 × 7 = 3.5

300

LESSON ACTIVITY

Before Using the Pages

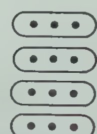
- Recall the use of multiplication to find quotients in division. Assign the following exercises. Discuss that the missing factor in the multiplication is the quotient in the corresponding division.

1. $\begin{array}{c|c} \times & ? \\ \hline 3 & 12 \end{array} \rightarrow 3\overline{)12}$? 2. $\begin{array}{c|c} \times & ? \\ \hline 4 & 20 \end{array} \rightarrow 4\overline{)20}$?

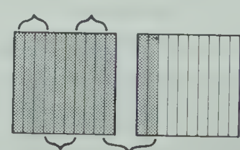
Tell the students that the above procedure can help to find the quotient when the divisor is a decimal. Assign the following exercises.

3. $\begin{array}{c|c} \times & ? \\ \hline 0.3 & 1.2 \end{array} \rightarrow 0.3\overline{)1.2}$? 4. $\begin{array}{c|c} \times & ? \\ \hline 0.4 & 2.0 \end{array} \rightarrow 0.4\overline{)2.0}$?

The students will likely be puzzled that whole numbers are obtained as quotients. They will likely note that the quotients in Ex. 1 and 3 are the same, and that the quotients in Ex. 2 and 4 are the same. Demonstrate the similarity for such divisions. For example, for Ex. 1 and 3, draw 12 dots on the board and ask how many groups of 3 there are in 12. Then draw or display a model for 1.2, recall that 1.2 is equivalent to 12 tenths, and ask how many groups of 3 tenths there are in 1.2.



$3\overline{)12}$ 4



$3 \text{ tenths}\overline{)12 \text{ tenths}}$ 4

Explain that for every division in which the divisor is a decimal, there is a corresponding division in which the divisor is a whole number such that the quotients are the same. This suggests a procedure for performing such divisions. Ask students to suggest how to obtain the division involving a whole number as the divisor which corresponds to $0.6\overline{)7.2}$.

Working Together

Copy each of these. Change place values in the dividend to match the change in the divisor.

1. $0.4\overline{)1.6}$ 2. $0.2\overline{)5.0}$ 3. $0.9\overline{)34.2}$ 4. $0.5\overline{)7.0}$

For each of these, change both numbers so that the divisor is a whole number.

Change place values in the divisor first to get a whole number. Then make a matching change in the dividend.

5. $0.3\overline{)2.1}$ 6. $0.8\overline{)28.0}$ 7. $0.6\overline{)16.2}$ 8. $0.7\overline{)35.0}$

Change place values in the divisor and dividend in the same way. Divide. Then check by multiplying.

9. $0.4\overline{)3.6}$ 9 10. $0.9\overline{)28.8}$ 32 11. $0.8\overline{)36.0}$ 45 12. $0.7\overline{)42.0}$ 60

Exercises

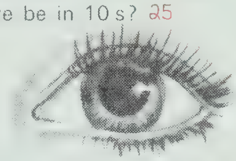
Divide. Check by multiplying.

1. $0.2\overline{)2.8}$ 14 2. $0.5\overline{)16}$ 32 3. $0.3\overline{)9.6}$ 32 4. $0.4\overline{)10}$ 25
5. $0.6\overline{)15.6}$ 26 6. $0.1\overline{)4.3}$ 43 7. $0.2\overline{)13}$ 65 8. $0.9\overline{)16.2}$ 18
9. $0.8\overline{)12}$ 15 10. $0.3\overline{)12}$ 40 11. $0.6\overline{)3}$ 5 12. $0.7\overline{)1.4}$ 2

Solve.

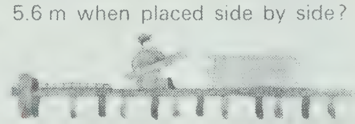
13. The capacity of each glass is 0.6 L. How many glasses can be filled from the pitcher? 8

14. A blink takes 0.4 s. How many blinks could there be in 10 s? 25



15. How many pieces of crepe paper 0.5 m long could be cut from a piece 14 m long? 28

16. Each board is 0.1 m wide. How many boards are needed to cover 5.6 m when placed side by side? 56



301

RELATED ACTIVITIES

- Some students may benefit from showing some of Ex. 1-12 in the following way.

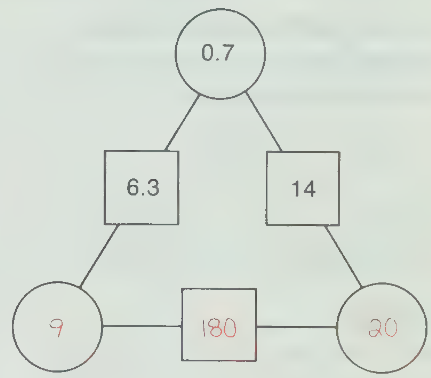
Ex. 1 2 tenths$\overline{)28}$ tenths

Ex. 2 5 tenths$\overline{)160}$ tenths

- Have the students complete sets of exercises similar to the following. Ask them to find the quotient for one division in the set and then use the result to write the quotients for the other divisions in the set.

$6\overline{)84}$ $6\overline{)8.4}$
$0.6\overline{)8.4}$ $0.6\overline{)84}$

- Have students complete diagrams similar to the following on copies of page T391. Factors are shown in the circles. Products are shown in the squares.



Using the Pages

- The worked example demonstrates that to divide by a decimal, a corresponding division with a whole number as the divisor is used. This is illustrated by expressing the division $0.5\overline{)15}$ as 5 tenths$\overline{)150}$ tenths. This helps to show that the division $5\overline{)150}$ may be used to find the quotient for $0.5\overline{)15}$. Emphasize the use of multiplication to check the given division. The quotient for $0.5\overline{)15}$ is obtained from $5\overline{)150}$, but it is checked by using the multiplication $0.5 \times 30 = 15$. Draw attention to the blue arrows in the "thought cloud". Relate the example to changing place values in ratios (page 299).

Working Together: Establish that place values are changed in Ex. 1-4 to obtain divisors that are whole numbers. Point out that for each exercise, the divisor is a decimal tenth, thus, the divisor and the dividend are multiplied by 10. For Ex. 2, the zero in the dividend is dotted to show that 5 is thought of as 5.0 before changing place value in the dividend. This skill is applied in Ex. 4. Since the quotients for the exercises in this lesson are whole numbers, decimal points are not needed in the quotients. However, emphasize that the decimal point in the quotient is above the final position for the decimal point in the dividend. This can be pointed out for the example $\overset{7}{0.5\overline{)3.5}}$ on page 300.

Exercises: Note that the students are to check their answers by multiplying the quotient obtained and the original divisor. For each of Ex. 13-16, have the students show their work and then answer with a concluding statement. After the students have completed the exercises, discuss Ex. 6 and 16 which have 0.1 as the divisor.

Assessment

Divide. Check by multiplying.

1. $0.7\overline{)4.9}$ 7 2. $0.4\overline{)5.6}$ 14 3. $0.6\overline{)3}$ 5

Solve.

4. How many pieces of rope 0.5 m long can be cut from a piece 10 m long? 20

Pages 302-303

LESSON OUTCOME

Divide a whole number or a decimal with up to three decimal places by a one-place decimal, using extra zeros in the dividend; multiply to check division; divide by 0.1, writing only the quotient

Materials

a copy of page T395 and a copy of the triangular grid on page T399 for each student (optional)

Prerequisite Skills

Change the place values in a ratio by multiplying each term by 10; divide a whole number or a decimal by a two-digit whole number, remainder zero; multiply a whole number or a decimal with one or two decimal places by a decimal tenth

Checking Prerequisite Skills

Multiply each term of the ratio by 10 to show an equivalent ratio.

1. $\frac{3.52}{6.9}$ $\frac{35.2}{69}$ **2.** $\frac{0.002}{5.2}$ $\frac{0.02}{52}$ **3.** $\frac{7}{1.5}$ $\frac{70}{15}$

Divide.

4. $23\overline{)1058}$ 46 **5.** $51\overline{)16.32}$ 0.32

Multiply.

6. 2.5×76 190.0 **7.** 6.3×1.59 10.017

Dividing with Tenths

The gasoline tank in the car holds 55.2 L. The car uses 4.8 L of gasoline for each round trip to school.
How many round trips are possible on a full tank?

Divide 55.2 by 4.8.

For $4.8\overline{)55.2}$,

think 48 tenths$\overline{)552 \text{ tenths}}$

$$\begin{array}{r} 11.5 \\ \text{Use } 4.8\overline{)55.20} \\ 48 \\ \hline 7\,2 \\ 4\,8 \\ \hline 2\,4\,0 \\ 2\,4\,0 \\ \hline 0 \end{array}$$

Check by multiplying:

$$\begin{array}{r} 11.5 \\ 4.8 \\ \hline 9\,20 \\ 46\,00 \\ \hline 55.20 \end{array}$$

11 round trips are possible on a full tank.

Working Together

Copy each of these. Change place values in the dividend to match the change in the divisor.

1. $1.4\overline{)4.9}$ **2.** $3.6\overline{)7.74}$ **3.** $2.4\overline{)1.8}$ **4.** $1.5\overline{)54}$

Change place values in the divisor and dividend in the same way. Divide. Then check by multiplying.

5. $3.8\overline{)25.65}$ 6.75 **6.** $7.1\overline{)639}$ 90 **7.** $1.2\overline{)0.78}$ 0.65 **8.** $4.5\overline{)0.126}$ 0.028

302

LESSON ACTIVITY

Before Using the Pages

- Assign the divisions $0.3\overline{)2.4}$, $0.8\overline{)20}$, and $0.6\overline{)7.2}$ to review dividing by a decimal tenth less than 1. Emphasize in each example that the divisor and the dividend are multiplied by 10 because the divisor is a decimal tenth. This gives a corresponding division in which the divisor is a whole number. You may wish to have the students use the following written format for the exercises. The quotient is written in the original division exercise after it is found by using a whole number as the divisor.

$$\begin{array}{r} 25 \\ 0.8\overline{)20} \end{array} \longrightarrow \begin{array}{r} 25 \\ 8\overline{)200} \\ 16 \\ \hline 40 \\ 40 \\ \hline 0 \end{array}$$

Using the Pages

- Ask a student to read the word problem at the top of page 302. For the division $4.8\overline{)55.2}$, point out that the divisor is a decimal tenth greater than one. The division in the "thought cloud" suggests that the place values for $4.8\overline{)55.2}$ may be changed to show $48\overline{)552}$. In other words, as in the lesson on pages 300 and 301, the divisor and the dividend are multiplied by 10 to obtain a corresponding division with a divisor that is a whole number. Draw attention to the blue arrows and to the position of the decimal point in the quotient. Write $48\overline{)552}$ on the board and have students complete and explain the division. Compare the quotient with that of the example on page 302, noting the extra zero in the dividend. Then discuss how multiplication is used to check the division. Read and discuss the concluding statement.

Working Together: For Ex. 2, the dividend becomes 77.4, which is not a whole number as for the exercises on page 301. For Ex. 4, a decimal point and extra zeros are required in the dividend. For Ex. 5-8, you may wish to have the

Exercises

Divide. Check by multiplying.

1. 3.2)56.32 17.6	2. 1.7)1.02 0.6	3. 2.5)0.2 0.08	4. 9.6)336 35
5. 5.4)5.535 1.025	6. 0.8)1.3 1.625	7. 3.9)12.012 3.08	8. 1.4)2464 1760
9. 2.1)84 40	10. 1.6)0.12 0.075	11. 0.1)0.53 5.3	12. 3.3)0.264 0.08
13. 4.2)86.1 20.5	14. 7.9)23.7 3	15. 3.5)637 182	16. 1.8)5.13 2.85

When a number is divided by 0.1,
each digit moves one place to the left.

82.9
0.1)8.2.9

Example:

tens	ones	tenths	hundredths
	8	2	9

÷ 0.1 =

tens	ones	tenths	hundredths
8	2	9	

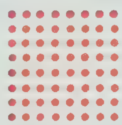
Divide each number by 0.1.
Write only the quotients.

17. 9.2 92 **18.** 6.21 62.1 **19.** 0.08 0.8 **20.** 7 70 **21.** 64.05 640.5 **22.** 8.716 87.16

Show the first 6 members of each pattern.

1. 

2.	1	4	9	16 ?	25?	36?

5. 

3. 

4.	1	3	6	10 ?	15 ?	21 ?

5. Draw the 8th member of the patterns in Exercises 1 and 3.
6. Find the 12th member of the patterns in Exercises 2 and 4. 144, 78

try this

303

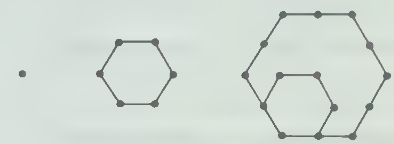
RELATED ACTIVITIES

- For more practice in dividing decimals, you may wish to assign Ex. 45-68 on page 332. To provide more practice in dividing by 0.1, have the students complete Ex. 17-32 on page 333.
- For enrichment, the students may investigate pentagonal numbers (A) and hexagonal numbers (B). Also, ask them to add consecutive triangular numbers and to note the pattern (C).

A

1 5 12

B



1 6 15

C

1 3 6 10

1, 1 + 3, 3 + 6, 6 + 10, . . .
gives
1, 4, 9, 16, . . . (square numbers)

students use the format described in *Before Using the Pages*. The quotients for Ex. 7 and 8 will require a zero to the left of the decimal point; the quotient for Ex. 8 will also require a zero to the right of the decimal point.

Exercises: Remind the students that the divisor becomes a whole number when place values are changed, whereas the dividend does not necessarily become a whole number.

Ex. 17-22 are of particular importance because they concern division by 0.1. Discuss the example preceding these exercises. Compare the place-value chart showing the dividend and the place-value chart showing the quotient. Ask students to explain how such exercises can be completed without written computation. It is important to compare the procedure for these exercises with that for Ex. 17-22 on page 135.

Try This: For Ex. 2, the pattern 1, 4, 9,... can be thought of as 1 × 1, 2 × 2, 3 × 3, ... Because the dot patterns form square arrays, the numbers in the sequence are known as *square numbers*. By discovering the pattern, students can show the array of dots and write the number for the 8th member and for the 12th member (Ex. 5 and 6) without determining other numbers of the pattern.

For Ex. 4, the pattern 1, 3, 6, ... can be thought of as 1, 1 + 2, 1 + 2 + 3, ... The dot patterns in Ex. 3 suggest equilateral triangles, and thus the numbers in the sequence are known as *triangular numbers*. The third number in the sequence relates to a triangular pattern of dots with 3 dots on each side. The twelfth number would relate to a triangular pattern with 12 dots on each side. Copies of page T395 can help students show the dot patterns for the square numbers. Copies of the triangular grid on page T399 may be helpful in showing dot patterns for the triangular numbers.

Assessment

Divide. Check by multiplying.

1. 6.2)39.68 6.4 **2.** 9.5)342 36 **3.** 5.1)4.794 0.94

Divide each number by 0.1. Write only the quotients.

4. 3.6 36 **5.** 5 50 **6.** 2.89 28.9

LESSON OUTCOME

Round a decimal quotient to the nearest tenth, to the nearest hundredth, or to the nearest thousandth when dividing by decimal tenths; solve related word problems

Prerequisite Skills

Divide a whole number or a decimal by a whole number or a one-place decimal

Checking Prerequisite Skills

Divide.

1. $0.7\overline{)44.8}$ 64
2. $2.8\overline{)8.82}$ 3.15
3. $4.6\overline{)299}$ 65
4. $9.3\overline{)68.82}$ 7.4
5. $364\overline{)6552}$ 18
6. $405\overline{)9720}$ 24

RELATED ACTIVITIES

- Students who have difficulty finding trial digits for the quotients may benefit from the suggestions for the lessons and the *Related Activities* on pages T78 and T79. Also, a review of how to determine the place value of the first digit in the quotient may be necessary. Exercises similar to the following may be assigned.
- Draw a frame to show the place value of the first digit in the quotient.

1. $7\overline{)83.6}$
2. $7\overline{)1.36}$
3. $24\overline{)31.06}$
4. $24\overline{)11.76}$

Rounding Quotients

Casey's mother bought 3.2 m of cloth for \$10.18. How much did each metre of cloth cost?

To find the price of each metre, divide 10.18 by 3.2.

For $3.2\overline{)10.18}$, use $3.2\overline{)10.1800}$

$$\begin{array}{r} 3.181 \\ 3.2\overline{)10.1800} \\ 9\,6 \\ \hline 5\,8 \\ 3\,2 \\ \hline 2\,60 \\ 2\,56 \\ \hline 40 \\ 32 \\ \hline 8 \end{array}$$

Since the quotient means an amount of money, round it to two decimal places.

3.181 rounds to 3.18.

Multiply 3.18 by 3.2 to check.

If the product is not close to the dividend, there is a mistake in your work.

Each metre of cloth cost \$3.18.

Exercises

Divide. Round each quotient to the nearest tenth.

1. $0.4\overline{)0.547}$ 1.4
2. $2.3\overline{)795}$ 345.7
3. $0.7\overline{)0.622}$ 0.9
4. $1.9\overline{)122}$ 64.2
5. $3.9\overline{)7.606}$ 2.0
6. $0.3\overline{)1475}$ 4916.7
7. $2.8\overline{)0.26}$ 0.1
8. $0.5\overline{)1.0234}$ 2.0

Divide. Round each quotient to the nearest hundredth.

9. $0.4\overline{)0.547}$ 1.37
10. $2.3\overline{)795}$ 345.65
11. $0.7\overline{)0.622}$ 0.89
12. $1.9\overline{)122}$ 64.21
13. $4.5\overline{)1758}$ 390.67
14. $0.6\overline{)0.05}$ 0.08
15. $3.2\overline{)35}$ 10.94
16. $0.9\overline{)4.17}$ 4.63

Divide. Round each quotient to the nearest thousandth.

17. $0.4\overline{)0.547}$ 1.368
18. $2.3\overline{)795}$ 345.652
19. $0.7\overline{)0.622}$ 0.889
20. $1.9\overline{)122}$ 64.211
21. $1.7\overline{)6.53}$ 3.841
22. $0.3\overline{)22.7}$ 75.667
23. $3.4\overline{)1.01}$ 0.297
24. $0.7\overline{)487}$ 695.714

Solve.

25. Craig needs 0.9 m of cloth to make each pair of overalls. He has 3.5 m of cloth. How many pairs of overalls can he make? 3

*26. Jenny used 0.8 m of denim to make a skirt. She had 7.5 m of denim left. How many skirts could she make in all? 10

304

LESSON ACTIVITY

Before Using the Page

- Write such decimals as 4.1926 and 3.9588 on the board to review the procedure for rounding a decimal to the nearest tenth, to the nearest hundredth, and to the nearest thousandth. Emphasize that it is necessary to examine only the digit to the right of the one being rounded. Discuss, for example, how it is known whether the digit in the tenths' place remains the same or increases by one, when rounding to the nearest tenth. Include such examples as rounding 2.795 to the nearest hundredth (2.80).

Using the Page

- For the word problem establish that 10.18 is divided by 3.2 to find the solution. Have students explain why and how place values are changed in the divisor and in the dividend. Ask why the quotient should be rounded to two decimal places.
  Have the students complete the multiplication 3.2×3.18 to check the division. Develop that the product of the quotient and the divisor will not be the exact dividend because the quotient is not the exact quotient.

Exercises: Draw attention to the instructions for each group of exercises from Ex. 1-24. Then have the students observe that the first four exercises in each group show the same divisions.

For Ex. 25, ask why the quotient, 3.8, is rounded down to 3 for the solution. For Ex. 26, the quotient 9.375, is rounded down to the nearest one, thus, Jenny can make 9 skirts with the denim that is left. There would be 10 skirts in all.

Assessment

Divide and round the quotient to the nearest

1. tenth. $0.5\overline{)2.43}$ 4.9
2. hundredth. $8.6\overline{)9.827}$ 1.14
3. thousandth. $3.7\overline{)741}$ 200.270

Solve.

4. Each art project needs 0.8 m of crepe paper. How many art projects can be made from 4.3 m of crepe paper? 5

Practice

Divide. Check by multiplying.

1. 0.2)1.2 6
2. 0.4)14 35
3. 0.5)2.56 5.12
4. 0.6)0.09 0.15
5. 4.7)8.46 1.8
6. 1.2)0.726 0.605
7. 2.8)9.45 3.375
8. 3.5)9.59 2.74

Divide. Show each quotient rounded to the nearest thousandth, the nearest hundredth, and the nearest tenth.

9. 3.2)4.13 1.291 1.29 1.3
10. 0.8)4.4444 5.556 5.56 5.6
11. 1.4)0.64 0.457 0.46 0.5
12. 0.6)3.7 6.167 6.17 6.2
13. 0.3)3.605 12.017 12.02 12.0
14. 2.7)10 3.704 3.70 3.7
15. 0.5)0.0373 0.075 0.07 0.1
16. 4.9)4.8 0.980 0.98 1.0

When a number is →	multiplied by 10,	divided by 10,	multiplied by 0.1,	divided by 0.1,
each digit moves one place to →	the left.	the right.	the right.	the left.

Complete this sentence in two ways. Other answers are possible.

17. Multiplying a number by 10 gives the same result as dividing the number by 0.1.

Dividing a number by 10 gives the same result as multiplying the number by 0.1.

Divide or multiply.
Write only the results.

18. 4.62 × 10 46.2
19. 3.7 ÷ 10 0.37
20. 75 × 0.1 7.5
21. 0.9 ÷ 0.1 9
22. 6.29 ÷ 10 0.629
23. 10 ÷ 0.1 100
24. 0.1 × 10 1
25. 5.03 × 0.1 0.503

The pages of this book are numbered from 2 to 346.

If the pages of this book were numbered to 578, tell whether each of these pages would be on the left side or on the right side of the book.

1. 524 left
2. 420 left
3. 397 right
4. 477 right
5. Are more of the page numbers odd or are more even? even
6. How many digits are used to show the page numbers from 2 to 346? 929
7. Are more of the digits odd or are more even? odd

PROBLEM SOLVING

305

Page 305

OBJECTIVE

Demonstrate competence in dividing decimals, in multiplying to check division, in rounding quotients, and in multiplying and dividing by 10 and 0.1

RELATED ACTIVITIES

- You may wish to have students complete some of the following exercises: Ex. 1-16 on page 333 for multiplying by 10; Ex. 1-16 on page 334 for dividing by 10; Ex. 17-32 on page 334 for multiplying by 0.1; and Ex. 17-32 on page 333 for dividing by 0.1.
- Give the students copies of the following charts. Have them complete the charts to show the conditions necessary for the sum, the difference, or the product of two whole numbers to be even or odd.

addend	addend	sum
even	even	
odd	even	
odd	odd	

minuend	subtrahend	difference
even	even	
even	odd	
odd	even	
odd	odd	

factor	factor	product
even	even	
odd	even	
odd	odd	

LESSON ACTIVITY

Before Using the Page

- To prepare the students for the concepts presented in Ex. 17-25, you may wish to assign the following exercises. Ask the students to write only the results. Leave the completed exercises on the board for reference during a discussion of Ex. 17.

1. 6.24 ÷ 10
2. 6.24 × 0.1
3. 73 × 10
4. 73 ÷ 0.1
5. 368 × 0.1
6. 368 ÷ 10
7. 1.23 ÷ 0.1
8. 1.23 × 10

Using the Page

- Begin with a discussion of Ex. 17 and the example preceding it. Ask students to explain different ways to complete Ex. 17.

Draw attention to the fact that Ex. 1-8 are to be checked by using multiplication, and that each quotient for Ex. 9-16 is to be rounded three ways.

Problem Solving: Ask what is meant by an even number and by an odd number. Ask how to tell quickly whether a whole number is even or odd. Elicit that the number is even if the ones' digit is 0, 2, 4, 6, or 8, and odd if the ones' digit is 1, 3, 5, 7, or 9. Ask the students to describe the numbers associated with the left-hand pages of a book, and those associated with the right-hand pages. They will realize that all the numbers on the left are even numbers, and that all those on the right are odd numbers.

Exercises such as Ex. 5-7 can, of course, be solved by listing all the numbers from 2 to 346 and then referring to the list. However, an efficient approach would be to look for and to use patterns.

Pages 306-307

LESSON OUTCOME

Divide a whole number or a decimal with one or two decimal places by a two-place decimal, whole-number quotient; solve related word problems

Prerequisite Skills

Change the place values in a ratio by multiplying each term by 100; regroup whole numbers and decimals; divide by a three-digit whole number; multiply a whole number and a decimal hundredth

Checking Prerequisite Skills

Multiply each term of the ratio by 100 to show an equivalent ratio.

1. $\frac{12.46}{0.25}$ $\frac{1246}{25}$ **2.** $\frac{14.3}{1.29}$ $\frac{1430}{129}$ **3.** $\frac{63}{4.02}$ $\frac{6300}{402}$

Complete.

4. 6 tenths 9 hundredths = 69 hundredths

5. 3 ones 4 tenths 2 hundredths = 342 hundredths

6. 8 ones = 800 hundredths

Divide.

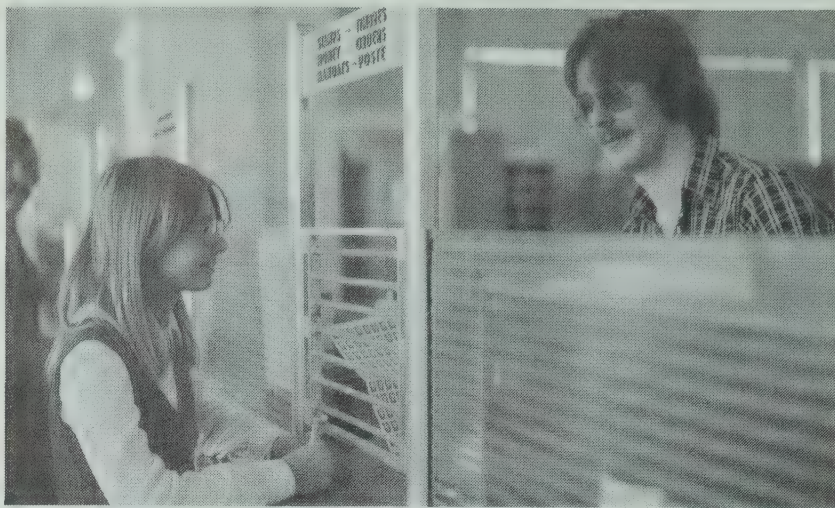
7. $304\overline{)7904}$ 26 **8.** $216\overline{)7344}$ 34

Multiply.

9. 0.23 × 32 7.36 **10.** 1.16 × 230 266.80

Dividing with Hundredths

Mona spent \$7 on 14¢ stamps.
How many stamps did she buy?



Divide 7 by 0.14.

For $0.14\overline{)7}$,

think 14 hundredths$\overline{)700 \text{ hundredths}}$

Use $0.14\overline{)7.00}$ → 50; 70; 00

Multiply 50 by 0.14 to check.

Mona bought 50 stamps.

Working Together

Copy each of these. Change place values in the dividend to match the change in the divisor.

1. $0.23\overline{)1.38}$ **2.** $1.46\overline{)21.90}$ **3.** $0.08\overline{)0.40}$ **4.** $2.25\overline{)81.00}$

Change place values in the divisor and dividend in the same way. Divide. Then check by multiplying.

5. $9.37\overline{)37.48}$ 4 **6.** $0.54\overline{)13.50}$ 25 **7.** $0.01\overline{)3.20}$ 320 **8.** $5.65\overline{)452.00}$ 80

306

LESSON ACTIVITY

Before Using the Pages

- Assign one or two exercises to review the procedure of dividing by a decimal tenth for quotients that are whole numbers. Have the students multiply to check their answers. Recall that place values are changed so that the divisor becomes a whole number. This is achieved by multiplying the divisor and the dividend by 10. For the following format, the quotient is written in the original division after it is found by using a divisor that is a whole number.

$$3.6\overline{)262.8} \longrightarrow 36\overline{)2628}$$

```
      73            73
3.6)262.8 ──► 36)2628
                 252
                  108
                  108
                    0
```

Check:

```
   73
 × 3.6
  438
  219
 262.8
```

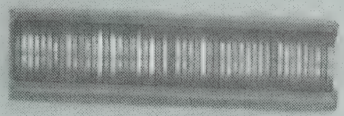
Write $0.36\overline{)26.28}$ on the board and ask the students to compare it with the division $3.6\overline{)262.8}$. Note that the divisor, 0.36, is a two-place decimal. Some students may suggest that the quotients for $0.36\overline{)26.28}$, $3.6\overline{)262.8}$, and $36\overline{)2628}$ are the same number, 73. If so, ask them to multiply 0.36 and 73 to see if the product is equal to the dividend 26.28. Then ask students to suggest why the three divisions have the same quotient.

Using the Pages

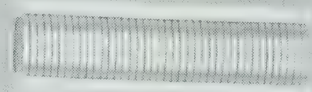
- Use the photograph to motivate a discussion about buying stamps and to introduce the word problem for the worked example. Discuss that the solution is found by dividing 7 by 0.14. Have students explain the form in the first "thought cloud", emphasizing that the 7 shows 7 ones, or 700 hundredths. Then have students explain each step in the division. Pay particular attention to the corresponding changes in the divisor and the dividend, to the extra zeros used in the dividend, and to the place values of the digits in the dividend and the quotient. Ask a few students to

Exercises

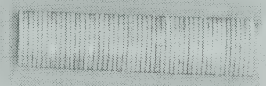
Divide



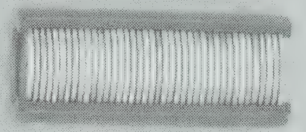
1. 0.5 by 0.01 to find how many pennies are in the \$0.50 roll of pennies. 50



2. 2 by 0.05 to find how many nickels are in the \$2 roll of nickels. 40



3. to find how many dimes are in the \$5 roll. 50



4. to find how many quarters are in the \$10 roll. 40

Divide. Check by multiplying.

5. $0.75\overline{)28.5}$ 38
6. $0.09\overline{)5.85}$ 65
7. $1.86\overline{)102.3}$ 55
8. $3.18\overline{)31.8}$ 10
9. $1.46\overline{)73}$ 50
10. $0.95\overline{)85.5}$ 90
11. $2.79\overline{)19.53}$ 7
12. $6.25\overline{)100}$ 16
13. $2.03\overline{)223.3}$ 110
14. $0.25\overline{)7}$ 28
15. $0.33\overline{)4.95}$ 15
16. $4.62\overline{)207.9}$ 45
17. $0.07\overline{)35}$ 500
18. $3.83\overline{)72.77}$ 19
19. $3.35\overline{)67}$ 20
20. $0.99\overline{)98.01}$ 99

Solve.

21. Each metre of wire cost \$0.30. Mr. Granville paid \$2.10 for a roll of wire. How many metres of wire were in the roll? 7
22. The oranges cost \$1.45 for each kilogram. The tag on the bag showed \$5.80. How many kilograms of oranges were in the bag? 4
23. Mrs. Bankes bought \$8.16 worth of gasoline. Each litre cost \$0.24. How many litres did she buy? 34
24. Sandy charges \$0.75 for each hour that she babysits. Last Saturday she earned \$4.50. How many hours did she work? 6

307

perform the multiplication on the board to check the division. Read the concluding statement.

Working Together: Ex. 2-4 involve using extra zeros in the dividends when place values are changed. Emphasize that the divisor and the dividend are multiplied by 100 because the divisor is a decimal hundredth. For Ex. 5-8, you may wish to have the students use the format suggested in *Before Using the Pages*.

Exercises: Students may have seen coins enclosed as shown in the photograph or rolled in papers provided by banks. These facilitate counting amounts of money in coins and exchanging them for bills. For Ex. 1 and 2, ask why the divisors are 0.01 and 0.05 respectively. Then ask what the divisors will be for Ex. 3 and 4. Explain that the quotient is exact for each of Ex. 5-20; thus, the product of the divisor and the quotient is expected to be the exact dividend.

RELATED ACTIVITIES

• Name two numbers such as 0.12 and 6. Have the students write and complete the two divisions, in this case, $0.12\overline{)6}$ and $6\overline{)0.12}$. Then ask the students to find the product of the quotients.

$$\begin{array}{r}50\\0.12\overline{)6.00}\end{array} \qquad \begin{array}{r}0.02\\6\overline{)0.12}\end{array}$$

$$50 \times 0.02 = 1$$

The following pairs of numbers may be used.

1. 0.5 and 8
2. 0.7 and 14
3. 0.4 and 0.25
4. 0.6 and 0.24

Assessment

Divide. Check by multiplying.

1. $0.86\overline{)38.7}$ 45
2. $3.17\overline{)82.42}$ 26
3. $2.75\overline{)121}$ 44

Solve.

4. Gus paid \$8.40 for stamps. Each stamp cost \$0.35. How many stamps did he buy? 24

Pages 308-309

LESSON OUTCOME

Divide a whole number or a decimal by a decimal with up to three decimal places; solve related word problems

Prerequisite Skills

Change the place values in a ratio by multiplying each term by 1000; multiply a decimal thousandth and a whole number or a decimal

Checking Prerequisite Skills

Multiply each term of the ratio by 1000 to show an equivalent ratio.

1. $\frac{3.913}{0.043}$ **2.** $\frac{7.38}{0.126}$ **3.** $\frac{16}{0.008}$

1. $\frac{3913}{43}$ 2. $\frac{7380}{126}$ 3. $\frac{16\,000}{8}$

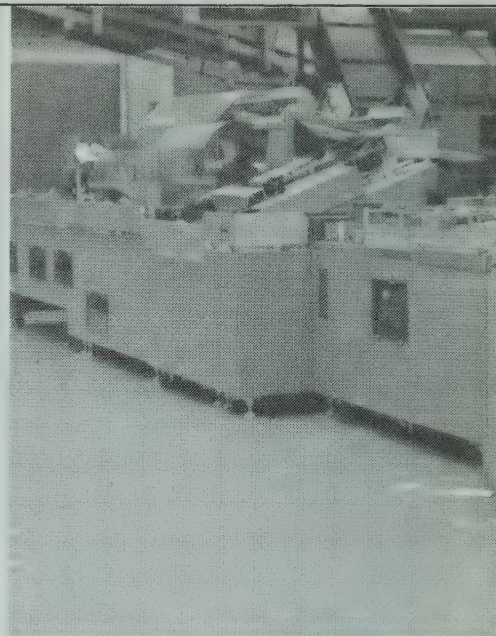
Multiply.

4. 0.145×24 3.480

5. 0.036×1.4 0.0504

6. 0.007×0.42 0.00294

7. 0.212×21 4.452



Dividing with Tenths, Hundredths, Thousandths

Five letter-sorting machines sort letters at the post office. They take about 0.033 s to handle each letter. About how many letters can be sorted in 1 min?

There are 60 s in 1 min.

Divide 60 by 0.033.

For $0.033\overline{)60}$,

```
              1 818.1
use 0.033 )60.000 0
             33
             27 0
             26 4
                60
                33
               270
               264
                 6 0
                 3 3
                 2 7
```

1818.1 rounds to 1818.

About 1818 letters can be sorted in 1 min.

Working Together

Copy each of these. Change place values in the dividend to match the change in the divisor.

1. $0.009\overline{)0.0603}$ 2. $2.4\overline{)9.3}$ 3. $3.75\overline{)60.00}$ 4. $0.068\overline{)30.600}$

Change place values in the divisor and dividend in the same way. Divide. Then check by multiplying.

5. $7.6\overline{)15.58}$ 2.05 6. $0.04\overline{)7.00}$ 175 7. $4.18\overline{)2.717}$ 0.65 8. $0.125\overline{)0.100}$ 0.8

308

LESSON ACTIVITY

Before Using the Pages

- Write exercises on the board and ask the students to change the place values so that the divisor in each example is a whole number. Ask students to explain their work on the board. For example, for $1.3\overline{)4.16}$, the divisor is a decimal tenth, thus, the divisor and the dividend are multiplied by 10 to give $13\overline{)41.6}$. Similarly, for $2.03\overline{)243.6}$, the divisor and the dividend are multiplied by 100 because the divisor, 2.03, is a decimal hundredth.

$$1.3\overline{)4.16} \qquad 2.03\overline{)243.60}$$

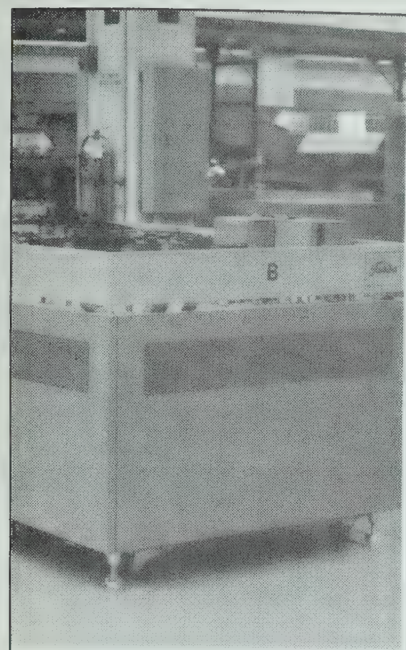
Write an exercise on the board for which the divisor has three decimal places, for example, $0.006\overline{)0.15}$. Ask how many decimal places are shown in the divisor and ask what multiplier would be used to give a divisor that is a whole number. Emphasize that the divisor determines the multiplier, but both the divisor and the dividend are multiplied by the same number.

Using the Pages

- Briefly discuss the photograph and the use of machines to sort letters at the post office. Ask a student to read the word problem for the worked example, noting the use of the word ''about''. Develop that 1 min must be expressed as seconds because the approximate time required to handle one letter is given as 0.033 s. Then, 60 is divided by 0.033 to find the approximate number of letters that can be sorted.

  Have students explain each step in the division exercise. Emphasize the place-value changes in the divisor and the dividend, and the position of the decimal point in the quotient. Establish that the quotient is rounded to the nearest whole number to show about how many letters can be sorted in 1 min. Ask a student to show the multiplication 0.033×1818.1 on the board to check the division. Ask why the product, 59.9973, is close to but not equal to the dividend, 60.

Working Together: For each of Ex. 3, 4, 6, and 8, note the use of extra zeros in the dividend when it is multiplied to match



Divide. Check by multiplying.

4. 0.7)0.84 1.2
5. 2.6)7.28 2.8
6. 0.08)0.2 2.5
7. 3.2)3952 1235
8. 6.6)21.78 3.3
9. 0.04)0.05 1.25
10. 0.8)2.4 3
11. 6.73)2.692 0.4
12. 5.08)1.778 0.35
13. 0.75)21 28
14. 0.027)0.0594 2.2
15. 3.68)176.64 48
16. 0.4)426 1065
17. 0.04)120.4 3010
18. 6.5)24.96 3.84
19. 0.006)0.003 0.5
20. 0.39)2.73 7
21. 17.4)2784 160
22. 0.065)0.2132 3.28
23. 12.5)177.5 14.2
24. 4.6)18.308 3.98
25. 0.25)22 88

Exercises

Solve.

1. The stamps on letters are canceled with a machine at the post office. It takes about 0.11 s to cancel each letter. About how many letters can be canceled in 1 min? 545
2. Each box costs $1.48 to mail. The total postage bill was $23.68. How many boxes were mailed? 16
3. A store mailed letters to its customers. The mass of each letter was about 20.9 g. The mass of all the letters was about 19.6 kg. About how many letters were there? 938

When a number is divided by 0.01, each digit moves 2 places to the left. When it is divided by 0.001, each digit moves 3 places to the left.

Examples:

2.5 ÷ 0.01 = 250

0.0075 ÷ 0.001 = 7.5

Divide each number by 0.1, 0.01, and 0.001. Write only the quotients.

26. 1.8 18 180 1800
27. 7 70 700 7000
28. 0.9 9 90 900
29. 3.06 30.6 306 3060
30. 4.928 49.28 492.8 4928
31. 0.065 0.65 6.5 65

309

RELATED ACTIVITIES

- For more practice in dividing decimals, you may wish to assign Ex. 69-88 on page 332. To provide practice in dividing by 0.01 and by 0.001, have the students complete Ex. 49-64 and Ex. 81-96 on page 333.
- As an extension of multiplying by 100 and dividing by 0.001, have the students complete Ex. 97-120 on page 333. Compare the results for multiplying by 500 and for dividing by 0.002, for example, for Ex. 97 and 103. Also, compare the results for multiplying by 200 and for dividing by 0.005.
- Have students find the quotient for one of the following exercises and use the result to write only the quotient for as many other exercises as possible.

67)1541	67)154.1
67)15.41	67)1.541
67)0.1541	67)0.01541
6.7)1541	6.7)154.1
6.7)15.41	6.7)1.541
6.7)0.1541	6.7)0.01541
0.67)1541	0.67)154.1
0.67)15.41	0.67)1.541
0.67)0.1541	0.67)0.01541

the place-value change in the divisor. Ask students to explain the place-value changes in terms of the number of decimal places in the divisor. Summarize that for every division in which the divisor is a decimal, there is an equivalent or corresponding division in which the divisor is a whole number such that the quotients are equal.

Exercises: For Ex. 1 and 3, the students should round the quotients to the nearest whole number to find the approximate number of letters. Also, for Ex. 3, 19.6 kg must be expressed as grams before division can be performed.

Ex. 26-31 are of particular importance because they concern division by 0.1, 0.01, and 0.001. You may wish to develop a few examples in a place-value chart on the board.

hundreds	tens	ones	tenths		hundreds	tens	ones	tenths
	6	4		÷ 0.01 =	6	4	0	
	0	3		÷ 0.001 =	3	0	0	

0.01)6.40 0.001)0.300

Relate these examples to the examples and the statements preceding Ex. 26-31 on the page.

Assessment

Divide. Check by multiplying.

1. 6.8)23.46 3.45
2. 9.25)3.145 0.34
3. 0.375)12 32

Solve.

4. Each parcel has a mass of 2.6 kg. The mass of all the parcels was 65 kg. How many parcels were there? 25

Page 310

LESSON OUTCOME

Round a decimal quotient to the nearest tenth, to the nearest hundredth, or to the nearest thousandth when dividing by decimal hundredths and by decimal thousandths; solve related word problems

Prerequisite Skills

Divide a whole number or a decimal by a decimal with up to three decimal places

Checking Prerequisite Skills

Divide.

1. 2.38)15.232 6.4
2. 0.125)20 160
3. 0.7)98 140
4. 4.6)3.726 0.81

RELATED ACTIVITIES

- Prepare price tags similar to the one on page 310, omitting any one of the three numbers. The students must decide whether multiplication or division is required, and find the missing number. Several price tags can be drawn on a work sheet or each price tag can be drawn on a separate card.

Rounding Quotients

The price per kilogram is blurred on the price tag. To find the price per kilogram, divide 5.65 by 4.38.

Delicious Apples

Price per kilogram	Total price	Net mass in kilograms
[illegible]	$5.65	4.38

Canada No. 1 Grade

For 4.38)5.65, use 4.38)5.65 000

```
         1.289
4.38 )5.65 000
      4 38
      1 27 0
        87 6
        39 40
        35 04
         4 360
         3 942
           418
```

Since the quotient means an amount of money, round it to two decimal places.

1.289 rounds to 1.29.

The price per kilogram for the apples is $1.29.

Multiply 1.29 by 4.38 to check.

If the product is not close to the dividend, there is a mistake in your work.

Exercises

Divide. Round each quotient to the nearest tenth.

1. 0.08)0.237 3.0
2. 3.14)8.639 2.8
3. 0.007)65 9285.7
4. 0.471)1.83 3.9

Divide. Round each quotient to the nearest hundredth.

5. 0.08)0.237 2.96
6. 3.14)8.639 2.75
7. 1.95)76 38.97
8. 0.7)9.26 13.23

Divide. Round each quotient to the nearest thousandth.

9. 0.08)0.237 2.963
10. 3.14)8.639 2.751
11. 6.7)29.308 4.374
12. 0.085)12 141.176

Solve.

13. 2.38 kg of meat cost $5.88. To the nearest cent, how much did each kilogram cost? $2.47

14. Each kilogram of grapes costs $3.50. To the nearest kilogram, what mass of grapes can be bought for $7.55? 2 kg

15. 3.6 kg of peanuts cost $7.67. To the nearest cent, how much did each kilogram of peanuts cost? $2.13

*16. Each 500 g of walnuts costs $2.20. To the nearest kilogram, what mass of walnuts can be bought for $4.66? 1 kg

310

LESSON ACTIVITY

Using the Page

- Draw attention to the price tag illustrated at the top of the page. Note that the price per kilogram is not clearly indicated. Discuss that the total price is the product of the missing number and the number of kilograms. Ask what operation can be used to find the missing number, that is, the price per kilogram. Have students explain the place-value changes in the divisor and in the dividend. Then ask students to explain the steps in the division. Emphasize the position of the decimal point in the quotient.

  Ask why the quotient is rounded to two decimal places. Review that when the quotient is rounded, the product of the divisor and the quotient is close to the dividend.

Exercises: Encourage the students to multiply to check any divisions that seem difficult. Have the students observe that the first four exercises in each group for Ex. 1-24 are the same. This allows the comparison of quotients rounded to different decimal places. After the students have completed Ex. 13-16, ask them to explain how they rounded each quotient and to give reasons for their choice. Ex. 16 is starred because it involves multiplying the cost of 500 g of walnuts by 2, to find the cost of 1000 g, or 1 kg, of walnuts; then, 4.66 is divided by the cost of 1 kg.

Assessment

Divide. Round each quotient to the nearest

1. tenth. 0.07)2.983 42.6
2. hundredth. 8.3)56 6.75
3. thousandth. 0.724)7.58 10.470

Solve.

4. 2.4 kg of peaches cost $4.38. To the nearest cent, how much did each kilogram of peaches cost? $1.83

Practice

Divide. Check by multiplying.

1. 7.64)133.7 17.5	2. 6.6)16.863 2.555	3. 0.725)6.09 8.4	4. 6.5)36.4 5.6
5. 0.24)6 25	6. 0.375)18 48	7. 2.16)83.16 38.5	8. 0.008)0.1 12.5

Divide. Show each quotient rounded to the nearest tenth, the nearest hundredth, and the nearest thousandth.

9. 0.9)26 28.9 28.89 28.889	10. 6.37)26.2 4.1 4.11 4.113	11. 0.54)7.3 13.5 13.52 13.519	12. 5.6)18.75 3.3 3.35 3.348

When a number is →	multiplied by 10, 100, or 1000,	divided by 10, 100, or 1000,	multiplied by 0.1, 0.01, or 0.001,	divided by 0.1, 0.01, or 0.001,
each digit moves 1, 2, or 3 places to →	the left.	the right.	the right.	the left.

Complete this sentence in two ways. Other answers are possible.

13. _?_ a number by _?_ gives the same result as _?_ the number by _?_, or
Multiplying 1000 dividing 0.001
Dividing a number by 0.001 gives the same result as multiplying the number by 1000.

Divide or multiply.
Write only the results.

14. 3.8 × 0.1 0.38 15. 3.8 ÷ 10 0.38 16. 6.2 × 100 620 17. 6.2 ÷ 0.01 620
18. 0.075 ÷ 0.1 0.75 19. 0.075 × 10 0.75 20. 4160 ÷ 1000 4.16 21. 4160 × 0.001 4.16
22. 84.5 × 0.01 0.845 23. 84.5 ÷ 100 0.845 24. 2.63 × 1000 2630 25. 2.63 ÷ 0.001 2630

In this game,

a penny may move to the right only;
a nickel may move to the left only;
a penny may jump over a single nickel;
a nickel may jump over a single penny.
Each may only move into an empty space.

A B C D E F

1 2 3 4 5 6 7

1. Start with pennies and nickels as shown. Follow the rules and move them so that they change places.

PROBLEM SOLVING

C to 4, D to 3, E to 5, C to 6, B to 4, A to 2, D to 1, E to 3, F to 5, C to 7, B to 6, A to 4, E to 2, F to 3, A to 5
Other answers are possible.

311

Page 311

OBJECTIVE

Demonstrate competence in dividing by a decimal, in multiplying to check division, in rounding quotients, and in multiplying and dividing by 10, 100, 1000, 0.1, 0.01, and 0.001

Materials

three nickels and three pennies or three markers of one color and three markers of another color for each student

RELATED ACTIVITIES

- For more practice in multiplying by 0.001, have the students complete Ex. 81-96 on page 334. As an extension of dividing by 100 and multiplying by 0.001, have the students complete Ex. 97-120 on page 334. Compare the results for dividing by 200 and for multiplying by 0.005, for example, for Ex. 97 and 103. Also, compare the results for dividing by 500 and for multiplying by 0.002.
- Assign oral exercises by naming a decimal and then saying, "Divide by 100. Multiply by 0.1. Divide by 0.01. What is the result?" The number of operations required can vary according to the ability of the students.
- Students may enjoy adapting the game described in the *Problem Solving* feature for two pennies and two nickels or for four pennies and four nickels.

LESSON ACTIVITY

Before Using the Page

- To prepare the students for the concepts presented in Ex. 13-25, you may wish to assign the following exercises. Ask the students to write only the results. Leave the completed exercises on the board for reference.

1. 1.4 × 10	2. 1.4 ÷ 0.1
3. 2 × 100	4. 2 ÷ 0.01
5. 3.5 × 1000	6. 3.5 ÷ 0.001
7. 6.79 ÷ 10	8. 6.79 × 0.1
9. 4.58 ÷ 100	10. 4.58 × 0.01
11. 93 ÷ 1000	12. 93 × 0.001

Using the Page

- Begin with a discussion of Ex. 13 and the example preceding it. Ask students to explain different ways to complete Ex. 13. Note that the results for Ex. 14 and 15 suggest one way to complete Ex. 13. Pairs of exercises from Ex. 16-25 suggest other ways.

Note that Ex. 1-8 are to be checked by using multiplication, and that each quotient for Ex. 9-12 is to be rounded three ways. If students perform each division once to four decimal places for Ex. 9-12, the quotient may then be rounded according to the three ways directed.

Problem Solving: Provide each student with three nickels and three pennies or three markers of one color and three markers of another color to play the game. Have each student copy the game board shown. Ensure that the students understand the rules for the game. At the conclusion of the game, the game board should show nickels in positions 1, 2, and 3 and pennies in positions 5, 6, and 7.

Encourage the students to experiment to find how to move the coins so that they change places. The letters and the numbers are given with the chart to enable the students to record their solutions as shown in the answer given. The letters refer to the coins and the numbers refer to the spaces. You may wish, however, to have the students play the game without recording their solutions.

LESSON OUTCOME

Estimate a quotient

Prerequisite Skills

Divide a whole number or a decimal by a whole number or a decimal

Checking Prerequisite Skills

Divide.

1. $0.48\overline{)1.68}$ 3.5
2. $62\overline{)4185}$ 67.5
3. $7.3\overline{)66.43}$ 9.1
4. $0.125\overline{)0.6}$ 4.8

Estimating Quotients

When estimating a quotient, first change the divisor to a one-digit number. Then change the dividend to a multiple of that number.

If the divisor and the dividend are multiplied by the same number, the quotient remains the same.

For $0.29\overline{)1.827}$,

$0.2\,9\overline{)1.8\,27}$

estimate $3\overline{)18}$

18 is a multiple of 3.

$18 \div 3 = 6$

$1.827 \div 0.29$ is about 6.

$$\begin{array}{r} 6.3 \\ 0.29\overline{)1.82\,7} \\ 1\,74 \\ \hline 8\,7 \\ 8\,7 \\ \hline 0 \end{array}$$

$1.827 \div 0.29 = 6.3$

For $38.5\overline{)207.9}$,

$3\,8.5\overline{)20\,7.9}$

estimate $4\overline{)20}$

20 is a multiple of 4.

$20 \div 4 = 5$

$207.9 \div 38.5$ is about 5.

$$\begin{array}{r} 5.4 \\ 38.5\overline{)207.9\,0} \\ 192\,5 \\ \hline 15\,4\,0 \\ 15\,4\,0 \\ \hline 0 \end{array}$$

$207.9 \div 38.5 = 5.4$

For $62\overline{)2511}$,

$6\,2.\overline{)251\,1.}$

estimate $6\overline{)240}$

240 is a multiple of 6.

$240 \div 6 = 40$

$2511 \div 62$ is about 40.

$$\begin{array}{r} 40.5 \\ 62\overline{)2511.0} \\ 248 \\ \hline 31\,0 \\ 31\,0 \\ \hline 0 \end{array}$$

$2511 \div 62 = 40.5$

312

LESSON ACTIVITY

Before Using the Pages

- Ask a student to write the multiples of 4 that are less than 40 on the board. Ask which multiple is closest to 7. Develop that the division $4\overline{)8}$ may be used to estimate the quotient for $4\overline{)7}$. Have students refer to the multiples of 4 to suggest divisions that might be useful for estimating the quotients for the following.

  $4\overline{)15}$ $\quad 4\overline{)27.7}$ $\quad 4\overline{)12.98}$ $\quad 4\overline{)35.2}$

  Use a similar procedure for multiples of 8 and the following divisions.

  $8\overline{)7}$ $\quad 8\overline{)8.6}$ $\quad 8\overline{)56.8}$ $\quad 8\overline{)63}$ $\quad 8\overline{)31.72}$

Using the Pages

- Read the title of the lesson and review that estimates are approximate results, not exact results.

  Recall that if the divisor and the dividend in a division are multiplied by the same number, the quotient remains the same. For example, the divisor and the dividend for division exercises in this unit were multiplied by 10, 100, or 1000. Read the method for estimating a quotient described at the top of page 312. For each example on page 312, develop how this method is applied. For example, for $0.29\overline{)1.827}$, the divisor is multiplied by 10 to give 2.9, which is then rounded up to 3. The dividend is also multiplied by 10 to give 18.27. Then 18.27 is rounded to 18, the closest multiple of 3. The division $3\overline{)18}$ gives 6 as an estimate for $0.29\overline{)1.827}$. For the exact division shown, the quotient, 6.3, is close to the estimate, 6.

  The second example is similar, except that the divisor, 38.5, is multiplied by 0.1 to give 3.85, which is rounded up to 4. The dividend is also multiplied by 0.1.

  In the third example, a decimal point may be written to the right of the ones' place in the divisor and in the dividend because they are whole numbers. The divisor is multiplied by 0.1 to give 6.2, which is rounded to 6. The dividend is also multiplied by 0.1 to give 251. The number 251 has 25

Working Together

Round each divisor to a whole number between 1 and 10. Then change each dividend to show a multiple of the divisor.

Example: For $7.6\overline{)0.479}$,

estimate $8\overline{)0.48}$ 48 is a multiple of 8.

1. $2.4\overline{)7.95}$ $2\overline{)8}$ 2. $5.8\overline{)3.427}$ $6\overline{)3.6}$ 3. $7.15\overline{)342}$ $7\overline{)350}$

Change place values in the divisor and in the dividend to get a divisor between 1 and 10. Then round the divisor to a whole number and change the dividend to show a multiple of the divisor.

Example: $0.06\,8\overline{)41.40}$ $\xrightarrow{\text{becomes}}$ $7\overline{)4200}$

4. $0.52\overline{)3.6}$ $5\overline{)35}$ 5. $38.5\overline{)7.72}$ $4\overline{)0.8}$ 6. $62\overline{)1.7}$ $6\overline{)0.18}$

Estimate each quotient.

7. $0.43\overline{)1.11}$ 3 8. $68\overline{)22.5}$ 0.3 9. $83.7\overline{)49.9}$ 0.6

Exercises

Estimate each quotient.

1. $2.1\overline{)15.9}$ 8 2. $61\overline{)4100}$ 70 3. $0.482\overline{)3.14}$ 6

4. $9.2\overline{)250}$ 30 5. $0.72\overline{)13.5}$ 20 6. $48\overline{)9.81}$ 0.2

7. $19\overline{)997}$ 50 8. $0.88\overline{)0.71}$ 0.8 9. $38\overline{)27.1}$ 0.7

10. $0.081\overline{)0.43}$ 5 11. $6.75\overline{)1.493}$ 0.2 12. $26\overline{)107}$ 4

13. $0.79\overline{)5.7}$ 7 14. $0.29\overline{)0.273}$ 0.9 15. $5.7\overline{)4.6}$ 0.8

16. $0.038\overline{)0.015}$ 0.4 17. $4.08\overline{)35.75}$ 9 18. $0.07\overline{)0.062}$ 0.9

19. $0.058\overline{)0.288}$ 5 20. $76\overline{)4767}$ 60 21. $6.4\overline{)5.478}$ 0.9

22. $4.9\overline{)2.49}$ 0.5 23. $31.7\overline{)187.2}$ 6 24. $78\overline{)632}$ 8

313

RELATED ACTIVITIES

• You may wish to have the students divide to find the quotients for Ex. 1-24. For quotients that do not terminate at the hundredths' place, have the students round the quotients to the nearest tenth. Then have them compare the results with the estimates obtained earlier.

tens and the nearest multiple of 6 is 24. Therefore, the dividend is rounded to 24 tens, or 240.

Working Together: Ex. 1-3 deal with rounding each divisor to a whole number and changing the dividend to show a multiple of the rounded divisor. Place-value changes are not involved. Discuss the example above Ex. 1, emphasizing that it is not necessary for the dividend to be a whole number.

Ex. 4-6 involve the following steps: change the place value in the divisor to result in a divisor between 1 and 10; round the divisor to a one-digit whole number; change place values in the dividend to match the change in the divisor; change the dividend to show a multiple of the divisor. These skills are applied in estimating the quotients for Ex. 7-9.

Exercises: The exercises deal with estimating the quotient for the following: a decimal divided by a decimal, such as Ex. 1; a decimal divided by a whole number, such as Ex. 6; a whole number divided by a decimal, such as Ex. 4; and a whole number divided by a whole number, such as Ex. 2.

Assessment

Estimate each quotient.

1. $0.07\overline{)6.2}$ 90 2. $91\overline{)743}$ 8 3. $29.5\overline{)180.2}$ 6

Page 314

OBJECTIVE

Use the distributive property of multiplication over addition to solve problems without pencil and paper

RELATED ACTIVITIES

- Assign a work sheet similar to the following for students to complete by matching each expression in the left column with the corresponding expression in the right column.

$(2 \times 8) + (3 \times 8)$	2×50
$(4 \times 3) + (7 \times 3)$	4×20
$(2 \times 41) + (2 \times 9)$	5×8
$(4 \times 17) + (4 \times 3)$	11×3

Solving Problems Without Pencil and Paper

The 30 students in Hugh's class sold tickets for the school concert. The average number of tickets that each of them sold was 23 for the afternoon performance and 27 for the evening performance. How many tickets did they sell in all?

To find the number of tickets, think

$(30 \times 23) + (30 \times 27)$

$= 30 \times (23 + 27)$, or

30×50, or 1500

30 groups of 23 and 30 groups of 27 are the same as 30 groups of 50.

$(30 \times 23) + (30 \times 27)$

$= 690 + 810$, or 1500

They sold 1500 tickets in all.

Write only the concluding statements.

1. Barry's class used 12 boxes of 64 lead pencils and 12 boxes of 36 colored pencils this year. How many pencils did they use this year?
They used 1200 pencils this year.

2. Meg's class of 32 students went to a play. It cost \$1.25 each for admission and \$0.75 each for transportation. What was the total cost of the trip?
The total cost of the trip was \$64.00.

3. The 31 students in Sara's class and the 29 students in Carl's class worked on a bulletin board from 13:45 to 15:45. How many "student-hours" were spent on the bulletin board?
The students spent 120 "student-hours" on the bulletin board.

4. Nina's class of 25 students worked on a mural from 09:00 to 10:25 and from 14:00 to 15:35. How many "student-hours" were spent on the mural?
Nina's class spent 75 "student-hours" on the mural.

5. Four students carried books from their classroom to the library. They carried 17 novels each having a mass of 0.425 kg and 17 science books each having a mass of 0.575 kg. What was the mass of all the books they carried?
The mass of all the books they carried was 17 kg.

6. Alan's class bought 12 cans of orange juice, 17 cans of grape juice, and 11 cans of apple juice to sell at the field day. Each can contained 1.5 L of juice. How many millilitres of juice did they buy?
They bought 60 000 mL of juice.

PROBLEM SOLVING

314

LESSON ACTIVITY

Before Using the Page

- Write the following on the board and ask the students to find the total number of dots.

  13 rows of 4 dots
  17 rows of 4 dots

  Elicit the following two methods for finding the answer.

  A 13 rows of 4 dots: $13 \times 4 = 52$
  17 rows of 4 dots: $17 \times 4 = 68$

  $$\begin{array}{r} 52 \\ +\ 68 \\ \hline 120 \end{array}$$

  B 13 rows of 4 dots
  17 rows of 4 dots
  30 rows of 4 dots $\quad 30 \times 4 = 120$

  Ask students to describe the two methods. Point out that each method involves the operations of addition and multiplication but in different orders. Ask which of the two methods is likely easier to compute mentally.

  Use a similar procedure for the following example.

  A 2 columns of 7 dots
  2 columns of 13 dots
  2 columns of 20 dots
  $2 \times 20 = 40$

  B $2 \times 7 = 14$
  $2 \times 13 = 26$

  $$\begin{array}{r} 14 \\ +\ 26 \\ \hline 40 \end{array}$$

Using the Page

- Ask a student to read the title of the lesson and the word problem at the top of the page. Discuss each of the ways of solving the problem and ask the students which is easier to perform without using pencil and paper.
- The problems involve computing the answers mentally and then writing the concluding statements. For each problem, the distributive property of multiplication over addition may be applied to facilitate finding the solution without pencil and paper.

Checking Up

Divide.

1. $0.3\overline{)0.21}$ 0.7
2. $0.8\overline{)49.3}$ 61.625
3. $0.4\overline{)3}$ 7.5
4. $8.3\overline{)39.01}$ 4.7
5. $2.7\overline{)9.882}$ 3.66
6. $0.007\overline{)0.644}$ 92
7. $0.425\overline{)5.78}$ 13.6
8. $0.02\overline{)0.759}$ 37.95
9. $0.05\overline{)0.8}$ 16
10. $0.38\overline{)0.171}$ 0.45
11. $9.4\overline{)32.9}$ 3.5
12. $0.075\overline{)2.4}$ 32
13. $0.75\overline{)39}$ 52
14. $0.608\overline{)1.52}$ 2.5
15. $0.6\overline{)0.072}$ 0.12
16. $0.49\overline{)24.5}$ 50
17. $5.42\overline{)81.3}$ 15
18. $0.7\overline{)3.948}$ 5.64
19. $8.27\overline{)30.599}$ 3.7
20. $0.62\overline{)1.55}$ 2.5
21. $0.08\overline{)0.68}$ 8.5
22. $9.35\overline{)59.84}$ 6.4
23. $0.09\overline{)0.063}$ 0.7
24. $7.92\overline{)396}$ 50

Divide. Round each quotient to the nearest tenth.

25. $0.6\overline{)8}$ 13.3
26. $5.3\overline{)28.4}$ 5.4
27. $2.55\overline{)7.1}$ 2.8
28. $0.22\overline{)7.97}$ 36.2

Divide. Round each quotient to the nearest hundredth.

29. $0.3\overline{)7.61}$ 25.37
30. $9.7\overline{)6}$ 0.62
31. $4.17\overline{)75}$ 17.99
32. $0.96\overline{)0.65}$ 0.68

Divide. Round each quotient to the nearest thousandth.

33. $0.8\overline{)3.627}$ 4.534
34. $7.4\overline{)3.9}$ 0.527
35. $8.94\overline{)0.495}$ 0.055
36. $0.68\overline{)3.8}$ 5.588

Divide or multiply. Write only the results.

37. 2.6×100 260
38. 0.06×0.1 0.006
39. $38 \div 0.01$ 3800
40. $1.63 \div 100$ 0.0163
41. $750 \div 1000$ 0.75
42. 24.6×10 246
43. $4 \div 0.1$ 40
44. 4.1×1000 4100
45. $60.02 \div 10$ 6.002
46. 0.4×0.01 0.004
47. $25.48 \div 0.001$ 25 480
48. 16×0.001 0.016
49. 0.007×1000 7
50. 8×0.001 0.008
51. $3.5 \div 10$ 0.35

Solve.

52. Each kilogram of plums costs \$2.24. How many kilograms of plums can be bought for \$3.36? 1.5

53. Lia needs 0.18 m of ribbon to make one bookmark. How many bookmarks can she make with 0.3 m of ribbon? 1

315

Page 315

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

RELATED ACTIVITIES

- If it is known that a quotient is to be rounded to the nearest tenth, for example, some students may realize that the only trial digit required for the hundredths' place is 5. Two examples are shown below.

$$\begin{array}{r} ? \\ 2.85 \\ 8\overline{)23.10} \\ \underline{16} \\ 7\,1 \\ \underline{6\,4} \\ 70 \\ \underline{40} \end{array} \qquad \begin{array}{r} ? \\ 3.25 \\ 7\overline{)22.60} \\ \underline{21} \\ 1\,6 \\ \underline{1\,4} \\ 20 \\ \underline{35} \end{array}$$

Using 5 hundredths in the first example indicates that the quotient will be rounded up to 2.9. In the second example, 5 is too great; thus, the quotient 3.2 is correct to the nearest tenth. Discuss the procedure with the more capable students and have them use it for exercises similar to Ex. 25-36.

Skills	Exercises	Related Pages
Divide by a one-place decimal	1-5, 11, 15, 18	T330-T331, T336-T337
Divide by a two-place decimal	8-10, 13, 16, 17, 19-24	T334-T335, T336-T337
Divide by a three-place decimal	6, 7, 12, 14	T336-T337
Round quotients when dividing by decimal tenths	25, 26, 29, 30, 33, 34	T332
Round quotients when dividing by decimal hundredths	27, 28, 31, 32, 35, 36	T338
Multiply by 10, 100, or 1000	37, 42, 44, 49	T333, T339
Multiply by 0.1, 0.01, or 0.001	38, 46, 48, 50	T333, T339
Divide by 10, 100, or 1000	40, 41, 43, 45, 51	T330-T331, T333, T336-T337, T339
Divide by 0,1, 0.01, or 0.001	39, 47	T333, T336-T337, T339

Comments

Encourage the students to use multiplication to check their work for some of these exercises. Note that for Ex. 1-24 the product for the divisor and the quotient should be the exact dividend, and that for Ex. 25-36 it should be close to the dividend. Estimation as presented on pages 312 and 313 may be used to check the quotients, especially the place value of the first digit in each quotient. For Ex. 53, rounding the quotient upward to the nearest one is not reasonable, because only the whole number obtained in the actual quotient can be accepted in this case.

Unit 16 Overview

Integers

This is the first unit in which negative numbers are presented formally. In Unit 8 they were encountered in temperatures below zero, but only in social and environmental settings. The students now discover a set of numbers that are less than zero. They learn to distinguish between positive and negative numbers and to refer to them as *integers*. They also learn that each integer has an opposite integer which is equidistant from, and on the opposite side of, zero on the number line. Number lines are used in a variety of ways: to represent integers, to compare integers, and to add integers. Subtraction of integers is not presented as such, but students do find differences between temperatures by referring to thermometers. The *Problem Solving* lesson in this unit presents typical, real-life situations in which possible solutions are influenced by various factors which may need to be taken into consideration. Two *Try This* features in this unit show how exponents may be used with factors to name numbers.

Unit Outcomes

- identify positive integers and negative integers
- read and write positive and negative integers
- write opposite integers
- match integers with points on a number line
- compare and order integers
- add integers using a number line
- find the difference between two temperatures and state whether the temperature rose or fell
- solve word problems involving integers
- find different solutions for word problems

Background

The history of the human race traces the use of numbers from shortly after the Stone Age to the present time. As long ago as 3000 B.C., both the Egyptians and the Sumerians of Iran had usable systems of numbers and numerals. Different civilizations developed their own number bases and symbols. Some of them even had systems for recording fractional numbers. Numbers had always been associated with real objects from counting on a one-to-one basis, and centuries passed during which there were symbols for only the counting numbers. One great step forward was taken when the idea of zero and the symbol 0 were invented, making it possible to write numerals for very large numbers with a set of only ten digits. Since quantities of objects were always greater than zero, there was no need to represent numbers less than zero. Leonardo of Pisa introduced Hindu-Arabic notation and algebra to medieval Italy. He also tried to introduce the concept of negative numbers, relating them to losses in business, but without much success. It was not until the seventeenth century that negative numbers were accepted generally.

The numbers which are used in counting (1, 2, 3, ...) are known as the *natural numbers*, or the *counting numbers*. If zero is included with the natural numbers, they are referred to as the *whole numbers* (0, 1, 2, 3, ...). All counting numbers are considered to represent real things and, therefore, are positive in nature. They are called *positive integers*. For each positive integer there is a corresponding number called a *negative integer*. Zero is considered to be neither positive nor negative.

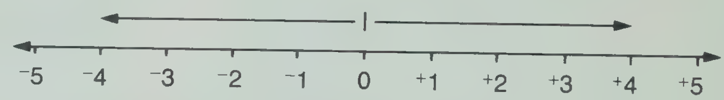
The complete set of integers is thus composed of all the positive numbers and their opposites and zero.

$$(\ldots, {}^-3, {}^-2, {}^-1, 0, {}^+1, {}^+2, {}^+3, \ldots)$$

Positive and negative numbers are named by numerals with the raised symbols $^+$ for positive and $^-$ for negative. These symbols are not symbols for the operations of addition and subtraction. They indicate whether a number is greater than or less than zero. Numbers that are named by numerals with these raised symbols are sometimes called *signed numbers*. If a numeral does not have a sign, it is presumed that it represents a positive number, but the negative symbol is always used to indicate a negative number.

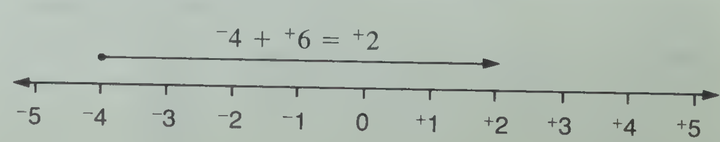
Signed numbers are read using the words "positive" and "negative", such as "positive eight" for $^+8$ and "negative five" for $^-5$. A slight deviation from these forms is found when temperatures below zero are announced on radio and television. For a temperature of $^-6$°C, the wording may be "minus six degrees Celsius" or "six degrees below zero Celsius".

The sequence of counting numbers (positive integers) is an ordered set; that is, the numbers remain in a fixed order, 1, 2, 3, ... Each positive integer is matched with a negative integer, so the negative numbers are also an ordered set. Positive and negative integers having the same digits are equidistant from zero on the number line; on a horizontal number line, $^+4$ is 4 units to the right of zero and $^-4$ is 4 units to the left of zero.

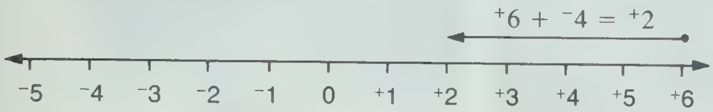


A number line also shows the integers in increasing order from left to right, thus, any number is greater than any other number on its left. For example, $^+2$ is greater than all the integers to the left of it on the number line ($^+1$, 0, $^-1$, $^-2$, ...); $^-3$ is greater than $^-6$; and 0 is greater than $^-5$. Similarly, any number is less than any number on its right. Thus $^-3$ is less than $^-2$, $^-1$, 0, $^+1$, ...; $^-5$ is less than 0; and $^-8$ is less than $^+3$. On a vertical number line, such as a wall thermometer, the greater numbers are toward the top and the lesser numbers toward the bottom.

The operation of addition is associated with the combining of sets to find the total number in the sets. In the addition of two positive integers, such as in $^+3 + {}^+5$, the sum is also a positive integer ($^+3 + {}^+5 = {}^+8$), and in the case of two negative integers, such as in $^-4 + {}^-2$, the sum is a negative integer ($^-4 + {}^-2 = {}^-6$). If one addend is a positive integer and the other is a negative integer, their sum represents the result of combining them, and whether it is a positive integer or a negative integer depends on their relative face values. For example, for $^-4 + {}^+6$, the face value of the positive integer (6) is greater than that of the negative integer (4); the sum will be a positive integer and the face value of the sum will be their difference ($^+2$). For $^-5 + {}^+3$, the face value of the negative integer (5) is greater than that of the positive integer (3); the sum will be a negative integer and the face value of the sum will again be their difference ($^-2$). Each example can be shown on the number line, using an arrow to represent the second addend.



For the first example, the arrow starts at ${}^{-}4$ and points 6 units to the right to represent "positive 6". For the second example, the arrow starts at ${}^{-}5$ and points 3 units to the right to represent "positive 3". It should be noted that the commutative property of addition is also operative in the addition of integers. When the addends in ${}^{-}4 + {}^{+}6 = {}^{+}2$ are interchanged, the second addend is "negative 4", so the arrow starts at ${}^{+}6$ and points 4 units to the left.



Both ${}^{-}4 + {}^{+}6$ and ${}^{+}6 + {}^{-}4$ have ${}^{+}2$ as the sum, so the commutative property is valid in the addition of integers. Although subtraction of integers is not included in this unit, attention is directed here to note how addition and subtraction are inversely related. The addition sentence ${}^{+}6 + {}^{-}4 = {}^{+}2$ calls to mind the subtraction sentence $6 - 4 = 2$. This example indicates that the same result is obtained either by adding a negative number or by subtracting a positive number of the same value.

It is commonly accepted that there are many ways of representing any one number, that is, there are many names or numerals for a number. The value of such an expression as $2 + 4$, 3×2, $8 - 2$, or $24 \div 4$ is quickly identified as the same as that shown by the single numeral 6. A number can sometimes be expressed in terms of a product using the same factor, such as $2 \times 2 \times 2$ for the number 8. The expression $2 \times 2 \times 2$ can be written more concisely as 2^3, in which the factor 2 is shown and the small raised 3 indicates how many times the factor 2 is used to obtain the product 8. The number 81 may be represented by 3^4, since the factor 3 is needed four times ($3 \times 3 \times 3 \times 3 = 81$). The small raised digit to the right of a factor is called an *exponent*. In science very large numbers are needed to represent such things as the age of the earth, the distance to a planet, or the production of energy from an atomic explosion. In such cases, a number is frequently expressed as a product showing an exponent with the factor 10 and another factor. The number 6000, for example, can be represented by 6×10^3 ($6 \times 10 \times 10 \times 10$), and the number 140 000 by 14×10^4, or 1.4×10^5.

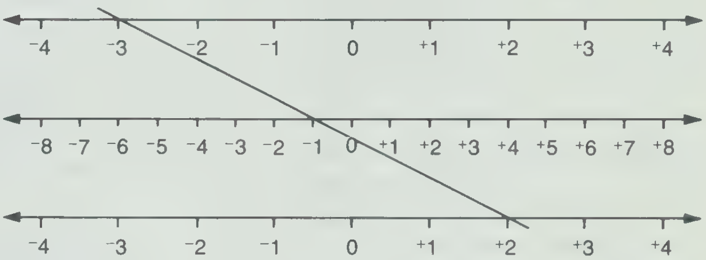
Teaching Strategies

The topic of integers may be beyond the scope of the mathematics program for this level in some schools, but the concepts in this unit are not difficult, nor are the operations. This unit may be used for enrichment or for a change of pace at almost any time in the year's program. Most students are aware of countdowns in space programs and of temperatures below zero and should have little difficulty in completing the exercises.

Probably the most effective teaching and learning aid for use with integers is the number line. This simple device provides a picture of positive and negative numbers and their relative positions to zero. The number line is helpful for showing opposite integers, for ordering integers, and for adding integers. It is suggested that a horizontal number line showing integers from about ${}^{-}30$ to ${}^{+}30$ be displayed prominently in the classroom. A vertical number line may be used along with a demonstration thermometer for the lesson on pages 322 and 323. Students will find it helpful to have individual number lines taped to their desks. Copies of page T 390 should be available for students to use in completing the exercises in the lessons and the *Related Activities*.

On pages 324 and 325 the exercises provide opportunities for students to demonstrate and to check their understanding of integers and their competence in working with them. Exercise 27 is much like a puzzle and can be quite time-consuming. It may be advisable for students to consider it only after completing the other exercises, or even to consider it at another time.

A nomograph is a useful device for showing the sum of two integers. Copies of page T 390 may be used to help prepare three number lines as shown below.



Have the students use a straight edge to align the dot for ${}^{-}3$ on the first number line and the dot for ${}^{+}2$ on the third number line. Point out how the straight edge aligns these dots with the dot for ${}^{-}1$ on the second number line, illustrating the addition sentence ${}^{-}3 + {}^{+}2 = {}^{-}1$.

Using a place-value chart in which the places are shown as powers of 10 may be helpful for the introduction to the use of exponents on page 325. It is relatively easy to write any standard numeral as a product showing a power of 10; a decimal point is placed to the right of the digit which is in the place named in exponential form. For example, in the standard number 390 000, a decimal point may be placed between the 3 and the 9, and the number expressed as 3.9×10^5. Similarly, a large numeral such as 2 496 000 may be expressed in several ways: 2496×10^6, 249.6×10^7, 24.96×10^8, and 2.496×10^9.

billions			millions			thousands					
11^{11}	10^{10}	10^9	10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
						3	9	0	0	0	0
		2	4	9	6	0	0	0	0	0	0

Materials

a number line cut from a copy of page T 390 for each student (optional)
a demonstration thermometer
tracing paper or a copy of page T 396 for each student (optional)
two copies of page T 390 for each student (optional)

Vocabulary

integers
negative integer
positive integer
opposite integers
exponent

Pages 316-317

LESSON OUTCOME

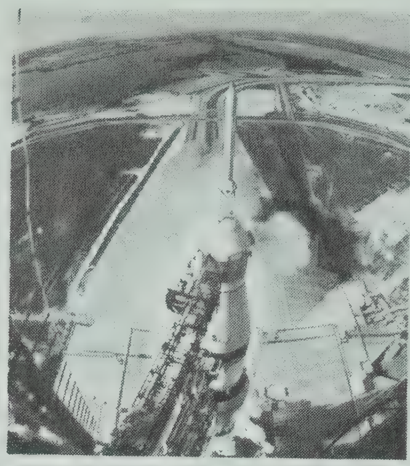
Identify positive integers and negative integers; read and write positive and negative integers; write opposite integers

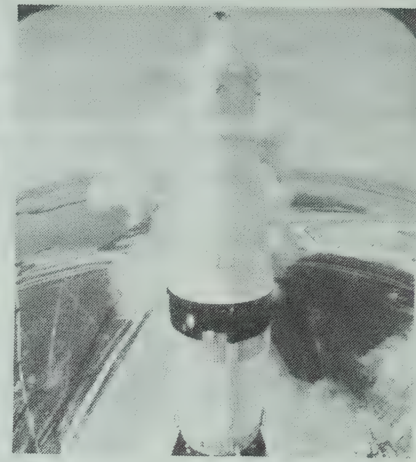
Vocabulary

integers, negative integer, positive integer, opposite integers

16 INTEGERS

Writing Integers





$^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0

The numbers used before "ignition" are **negative integers**.
The numbers used after "ignition" are **positive integers**.

Ignition!

A positive integer is written with the symbol $^+$.

$^+2$ should be read "positive 2".

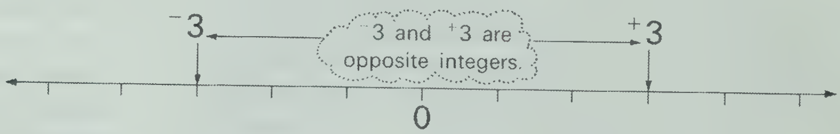
A negative integer is written with the symbol $^-$.

$^-5$ should be read "negative 5".

0 is an integer that is neither positive nor negative.

Each positive integer has an opposite negative integer.
Each negative integer has an opposite positive integer.

On the number line, points for opposite integers are the same distance from 0 but in opposite directions.

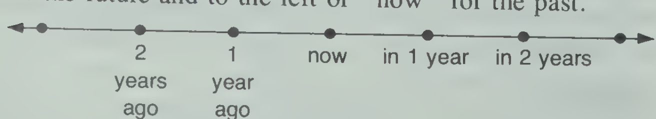


LESSON ACTIVITY

Before Using the Pages

- Ask students to name the opposite of terms such as hot (cold), before (after), under (over), left (right), and future (past).

  Draw a line on the board and mark equally spaced dots along the line. Indicate a point and tell the students that it represents the present year. Print the word "now" below the point. Ask students to indicate points to represent "1 year in the future", "2 years from now", and so on. Ask what the opposite is for "1 year in the future" and "2 years from now", and have students indicate points for these on the line. They will likely show points to the right of "now" for the future and to the left of "now" for the past.

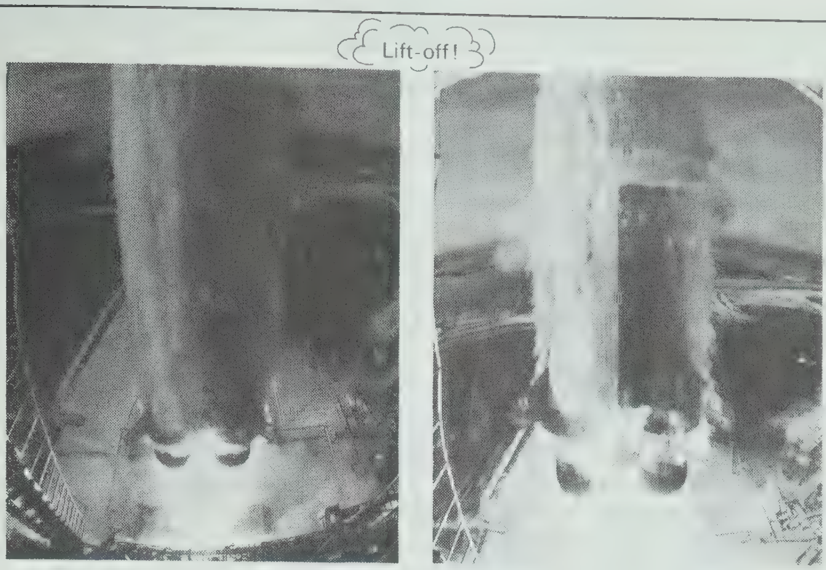


  Ask the students to imagine that they can travel in a time machine to a time 200 years "away". Ask what choice this offers, to elicit that since a direction was not stipulated, they may travel either into the future or into the past.

  Write the numeral 5 on the board and discuss ways of adapting the numeral to show whether "5 years in the future" or "5 years in the past" is indicated, without having to write the phrase "in the future (past)". For example, they may suggest the use of arrows as in $\overrightarrow{5}$ and $\overleftarrow{5}$, the use of letters as in 5^f and 5^p, or the use of the symbols $^+$ and $^-$ as in $^+5$ and $^-5$, to show which direction in time is meant.

Using the Pages

- Begin by asking what is shown in the sequence of four photographs. Contrast the numerals for numbers before "ignition" with those for numbers after "ignition". Introduce the terms *integers*, *negative integers*, and *positive integers*. Emphasize that these numbers are read using the words *positive* and *negative*, and that the symbols $^+$ and $^-$ are raised to distinguish them from the symbols for addition and subtraction. Ask why zero is neither positive



$^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$ $^+8$

Exercises

Use "positive" or "negative" to complete each sentence.

1. $^-8$ is a __negative__ integer.
2. $^+1$ is a __positive__ integer.
3. List the first 10 integers greater than 0. These are the first 10 __positive__ integers. $^+1, ^+2, ^+3, ^+4, ^+5, ^+6, ^+7, ^+8, ^+9, ^+10$
4. List the opposites of the 10 integers listed for Exercise 3. These are __negative__ integers. $^-1, ^-2, ^-3, ^-4, ^-5, ^-6, ^-7, ^-8, ^-9, ^-10$

Jesse and Joanna played a game. Whenever either scored a positive number of points, the other scored the opposite number.

5. When Jesse scored $^+6$ points, how many did Joanna score? $^-6$
6. When Joanna scored $^-2$ points, how many did Jesse score? $^+2$

Solve.

7. On April 30, the lake level was 37 cm above normal. How much would the level need to change to bring it to normal? 37 cm Would it need to rise or fall? fall
8. On September 30, the lake level was 31 cm below normal. How much would the level need to change to bring it to normal? 31 cm Would it need to rise or fall? rise

317

RELATED ACTIVITIES

- Have the students list opposites that suggest negative integers and positive integers. The following are some examples.

Negative	Positive
down	up
left	right
below	above
lose	gain
spend	earn
before	after

- Have each student write one or two statements that suggest integers.
  "Libby walked up four steps."
  "Candice earned $4."
  "Bob lost 2 kg."
  Then have the students exchange papers and write statements that suggest the opposite of the examples.
  "Libby walked down four steps."
  "Candice spent $4."
  "Bob gained 2 kg."
- For the preceding activity, have students write the integers suggested by the statements.
- Prepare a die marked $^-5$, $^-3$, $^-1$, 0, $^+2$, and $^+4$. Have the students work in groups of four. Each student in turn tosses the die. All the students use a tally chart to record the number of times a negative number is shown and the number of times a positive number is shown.
- Students may refer to an atlas to name altitudes above sea level using positive integers and depths below sea level using negative integers.

nor negative. (A negative integer is used before "ignition" and a positive integer is used after "ignition".)

Have students read the statements that introduce the concept of *opposite integers*. Draw attention to the number line at the bottom of page 316 and note that the point for $^-3$ is three units to the left of 0, and the point for $^+3$ is three units to the right of 0. Have students name the opposite of $^+2$ and of $^-5$ and describe the location of each on the number line.

Ask students to read the integers below the four photographs and to identify pairs of opposite integers. Summarize that the integers consist of the positive integers, the negative integers, and zero.

Exercises: Ensure that the students show the $^+$ and $^-$ symbols in raised positions. Before the students begin Ex. 7 and 8, you may wish to discuss why a lake level can be above normal or below normal at different times of the year or in different years. The lake level above normal suggests a positive integer and the lake level below normal suggests a negative integer.

Assessment

Use "positive" or "negative" to complete each sentence.

1. $^+4$ is a __positive__ integer.
2. $^-1$ is a __negative__ integer.
3. List the first 3 integers greater than 0. These are the first 3 __positive__ integers. $^+1, ^+2, ^+3$

Write the opposite integer for each of these.

4. $^-7$ $^+7$ 5. $^+5$ $^-5$

LESSON OUTCOME

Match integers with points on a number line; compare and order integers

Vocabulary

exponent

Comparing and Ordering Integers

The number line can help you compare integers.

$^-9$ $^-8$ $^-7$ $^-6$ $^-5$ $^-4$ $^-3$ $^-2$ $^-1$ 0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$ $^+6$ $^+7$ $^+8$ $^+9$

Numerals for negative integers always show the symbol $^-$.

Numerals for positive integers do not always show the symbol $^+$. Positive integers are the same as whole numbers.

For any two points on a horizontal number line, the integer for the point to the left is less than the integer for the point to the right.

On the number line,

$^+2$ is to the left of $^+4$, so $^+2 < {}^+4$.

$^-2$ is to the left of $^+4$, so $^-2 < {}^+4$.

$^+2$ is to the right of $^-4$, so $^+2 > {}^-4$.

$^-2$ is to the right of $^-4$, so $^-2 > {}^-4$.

Working Together

Complete.

1. $^+1$ $^+2$ ▒ ▒ $^+5$ — +3 +4

2. $^-1$ ▒ ▒ ▒ $^+3$ — 0 +1 +2

3. $^-9$, $^-8$, ▒, ▒, ▒, $^-4$ — −7 −6 −5

4. $^-5$, ▒, ▒, ▒, $^-1$, ▒ — −4 −3 −2 0

Which integer in each pair would be placed farther to the right on the number line?

5. $^+9$, $^+7$
6. $^-1$, $^-3$
7. $^+8$, $^-8$
8. $^-5$, 0

Use > or < to make true statements.

9. $^+9$ ● $^+7$ >
10. $^-1$ ● $^-3$ >
11. $^+8$ ● $^-8$ >
12. $^-5$ ● 0 <

List from least to greatest.

13. $^+3$, $^-3$, $^+7$, $^-7$, $^+1$, $^-1$
−7, −3, −1, +1, +3, +7

List from greatest to least.

14. $^-4$, $^+2$, $^-1$, 0, $^-6$, $^+5$
+5, +2, 0, −1, −4, −6

318

LESSON ACTIVITY

Before Using the Pages

- Draw the following number line on the board, labeling the point for zero, and having students label the points for $^+1$, $^+2$, and $^+3$. Discuss the similarity between this number line and the number line for whole numbers. Note that this number line has arrowheads at both ends. Ask what the arrowheads indicate.

0 $^+1$ $^+2$ $^+3$ $^+4$ $^+5$

Extend the number line to the right and label several more points, noting that the numbers increase in value. Use a pointer to indicate consecutive points from right to left along the line and note that they decrease in value. Ask how the number line can show the opposite of $^+1$, $^+2$, and $^+3$. Have students help to extend the line to the left and label points for $^-1$, $^-2$, $^-3$, and other negative integers.

Using the Pages

- Question the students about the number line at the top of page 318. Ask what number is to the left of $^+5$, to the left of $^-2$, to the right of $^-8$, to the right of $^+3$, between $^+4$ and $^+6$, between $^-9$ and $^-7$, and so on. Review that the numbers on the line increase from left to right and decrease from right to left.

  Ask students to name several numbers that are greater than $^+3$. Note that such numbers are to the right of $^+3$ on the number line. Ask for several numbers to the left of $^+3$ and note that they are less than $^+3$. Ask how the concepts of right and left can help to determine which of two integers is the greater number, for example, $^+5$ and $^+8$, or $^-3$ and 0.

  Have students read the statements and help to explain the examples below the number line. The students can locate the pairs of integers on the number line to verify the examples given. Note that the concepts of right and left apply to a horizontal number line.

Working Together: Ex. 1-4 deal with integers in sequence. The integers for Ex. 5-8 are repeated for Ex. 9-12 to

Exercises

Complete.

1. +2, , , , +6
   +3 +4 +5

2. −2, , , , +2
   −1 0 +1

3. −8, , , , −4
   −7 −6 −5

4. , , −1, 0,
   −3 −2 +1

5. , +5, , +7,
   +4 +6 +8

6. , −6, , −4,
   −7 −5 −3

7. +6, +7, , , , +11
   +8 +9 +10

8. −10, , , −7, ,
   −9 −8 −6 −5

9. , , , +3, , +5
   0 +1 +2 +4

10. , −3, −2, , ,
    −4 −1 0 +1

Use > or < to make true statements.

11. −5 ● +4 <	12. +2 ● 0 >	13. +3 ● +7 <	14. −3 ● −1 <
15. +3 ● −6 >	16. −2 ● +2 <	17. −5 ● 0 <	18. +1 ● −7 >
19. −8 ● −7 <	20. +4 ● +1 >	21. −4 ● −9 >	22. −2 ● −5 >

List from least to greatest.

23. +5, −5, −8, +8, +9, −9
    23 −9, −8, −5, +5, +8, +9

24. +3, +4, −1, −8, −4, +1
    −8, −4, −1, +1, +3, +4

25. 0, −1, +2, −3, +4, −5
    −5, −3, −1, 0, +2, +4

26. +6, −3, +1, +5, −6, −2
    −6, −3, −2, +1, +5, +6

List from greatest to least.

27. −6, +6, −4, +4, −2, +2

28. +2, −7, −5, 0, −2, +3

29. −5, 0, −2, −3, +7, −1

30. 0, +3, −6, +2, +4, −1

27 +6, +4, +2, −2, −4, −6
28 +3, +2, 0, −2, −5, −7
29 +7, 0, −1, −2, −3, −5
30. +4, +3, +2, 0, −1, −6

When all the factors in a product are the same, the product can be shown using **exponents**.

An exponent shows how many times a number is used as a factor.

$5 \times 5 = 5^2$ exponent $3^5 = 3 \times 3 \times 3 \times 3 \times 3$

Show each of these using exponents.

1. $8 \times 8 \times 8$ 8^3 2. $6 \times 6 \times 6 \times 6$ 6^4 3. 4 2×2 2^2 4. 36 6^2 5. 27 3^3

Write the standard form for each. $3^6 = 729$

6. 7^3 $7 \times 7 \times 7$ 343 7. 5^4 625 8. 10^2 100 9. 4^5 1024 10. 2^8 256

try this

319

RELATED ACTIVITIES

• Have students label number lines on copies of page T 390 and mark them to show the integers for one or more of Ex. 11-30 on page 319.

• Prepare a numeral card for each integer from −10 to +10. Have a student draw from six to ten cards, arrange them from least to greatest, and record the integers in this order.

• The cards prepared for the preceding activity may be used for a game by three students. The cards are shuffled and dealt equally. Each student turns up the top card. The student whose card shows the greatest integer claims the three cards. The process continues until one player has no cards left. The player with the most cards is the winner.

• Have students work in pairs using integers from −10 to +10. One student writes an integer. The other student asks questions such as "Is it a negative integer?" "Is it less than −5?" to determine in as few questions as possible what the secret integer is.

• Have each student prepare a number line cut from a copy of page T 390 for integers from −12 to +12. By folding the number line at 0, students can observe that points for opposite integers match. This can also be demonstrated by placing a semitransparent plexiglass mirror at zero on the number line. Students may refer to these number lines for assistance in the lessons of this unit.

emphasize that the positions of two integers on the number line can determine which integer is greater. Discuss ways to complete Ex. 13 and 14 using the number line for assistance and without using the number line. For example, students may choose to order the negative numbers first and then order the positive numbers. Draw attention to the difference in the instructions for Ex. 13 and for Ex. 14.

Exercises: After the students have completed the exercises, have them read some of their answers to review how to read integers. For example, Ex. 11 would be read "negative five is less than positive four". You may wish to summarize that any positive integer is greater than any negative integer (+2 > −7), and that a negative integer closer to zero is greater than a negative integer farther from zero (−2 > −7).

Try This: Read the statement to introduce *exponents* and direct the students' attention to the exponents for 5^2 and 3^5. Have a student read the statement in the "thought cloud". Discuss that the exponent for 5^2 shows that 5 is used two times as a factor; that is, $5^2 = 5 \times 5$, and therefore, 5^2 is another name for 25. Discuss 3^5 in a similar way. The "thought cloud" for Ex. 3 shows how 4 is factored in order to express it using exponents. For Ex. 4 and 5, the students must factor the number and then use exponents to show the numbers. Before the students begin Ex. 6-10, have them multiply with 3 as a factor six times to show that 3^6 represents 729.

Assessment

Complete.

1. −1, , , +2,
   0 +1 +3

2. , , , 0, +1
   −3 −2 −1

Use > or < to make true statements.

3. −6 ● −3 < 4. +2 ● −1 > 5. 0 ● −4 >

List from least to greatest.

6. −1, −5, +3, +1, +2, −4
   −5, −4, −1, +1, +2, +3

List from greatest to least.

7. −3, 0, −7, +6, −2, +4
   +6, +4, 0, −2, −3, −7

Pages 320-321

LESSON OUTCOME

Add integers using a number line; solve related word problems

Materials

a number line cut from a copy of page T 390 for each student (optional)

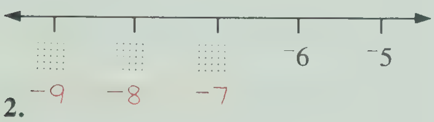
Prerequisite Skills

Match integers with points on a number line

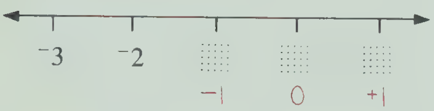
Checking Prerequisite Skills

Complete.

1.



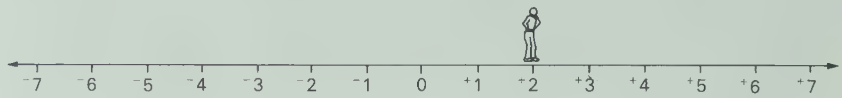
2.



Adding Integers

For the sum of two integers, imagine yourself standing on the number line at the point for the first addend. Face to the right.

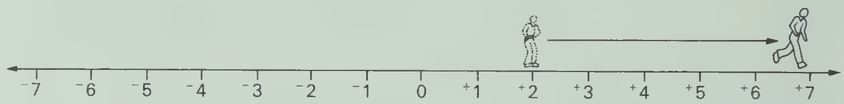
When the first addend is $^+2$, start at $^+2$.



Then walk forward if the second addend is positive.

The second addend is $^+5$. I'll walk forward 5 units.

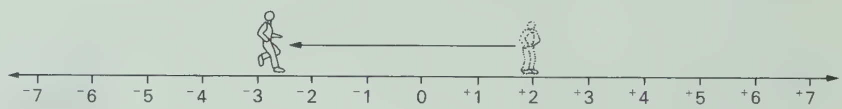
$$^+2 + {}^+5 = {}^+7$$



Walk backward if the second addend is negative.

The second addend is $^-5$. I'll walk backward 5 units.

$$^+2 + {}^-5 = {}^-3$$



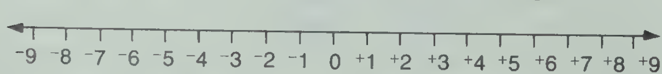
320

LESSON ACTIVITY

Before Using the Pages

- Draw a number line on the board for the whole numbers from 0 to 9, and review how it can be used to find the sum of two whole numbers. For example, write 3 + 5 on the board. Place two chalkboard pointers or two metre sticks at 3 (the first addend) on the number line, and then move one of them 5 units forward (for the second addend) to 8, the sum. Repeat the procedure for 2 + 4. Then have two students show the procedure for 1 + 6 and for 2 + 3. Emphasize that the pointers begin at the number for the first addend.

  Write positive symbols for 1 to 9 on the number line and extend the line to the left to show negative integers to $^-9$.



  Ask students to explain how they think the number line can be used to find the sum of two integers such as $^+4$ and $^+3$, or $^-4$ and $^+3$.

Using the Pages

- The worked example demonstrates how a number line can be used to add two integers. Students can note that addition of two positive integers is the same as for two whole numbers. Read and discuss each step for using the number line to add the integers $^+2$ and $^+5$ as the students refer to the corresponding number lines on the page. Emphasize that the procedure starts at $^+2$ on the number line because $^+2$ is the first addend. The move for $^+5$ is forward because $^+5$ is a positive integer. In contrast, for $^+2 + {}^-5$, the move from $^+2$ is backward 5 units because the second addend is negative.

Working Together: Ex. 1 and 2 emphasize that the first addend names the starting point for addition on a number line. Ex. 3 and 4 emphasize the direction of the move for the second addend. The skills of Ex. 1-4 are applied in Ex. 5 and 6, in which the stopping point identifies the sum. Have each student label a number line from $^-12$ to $^+14$ to help complete Ex. 7-10. Copies of page T 390 or number line

Working Together

Where will you start?

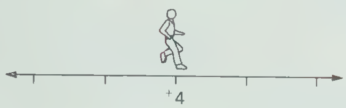
1. $^+7 + ^-3$ 2. $^-3 + ^+7$

Will you walk forward or backward?

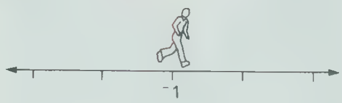
3. $^+4 + ^-5$ backward 4. $^-1 + ^+3$ forward

Where will you stop?

5. $^+4 + ^-5$ $^-1$



6. $^-1 + ^+3$ $^+2$



Add.

7. $^+4 + ^+3$ $^+7$ 8. $^+5 + ^-3$ $^+2$ 9. $^-6 + ^+4$ $^-2$ 10. $^-4 + ^-7$ $^-11$

Exercises

You could use a number line to help you add.

Add.

1. $^+4 + ^-6$ $^-2$	2. $^+3 + ^+2$ $^+5$	3. $^-5 + ^+1$ $^-4$	4. $^-2 + ^-1$ $^-3$
5. $^-3 + ^+7$ $^+4$	6. $^-1 + ^-4$ $^-5$	7. $^+3 + ^-3$ 0	8. $^+5 + ^+5$ $^+10$
9. $^-7 + ^-5$ $^-12$	10. $^+6 + ^-2$ $^+4$	11. $^-8 + ^+8$ 0	12. $^-6 + ^-6$ $^-12$
13. $^+5 + ^+9$ $^+14$	14. $^-1 + ^+6$ $^+5$	15. $^-9 + ^-3$ $^-12$	16. $^+4 + ^-7$ $^-3$

Solve.

17. In a game, Jesse scored $^+7$ points. Then he scored $^-4$ points. How many points did he score in all? $^+3$

18. Joanna scored $^-7$ points. Then she scored $^+4$ points. How many points did Joanna score in all? $^-3$

19. In four rounds, Jesse scored $^+7$, $^-4$, $^-3$, and $^+5$ points. How many points did he score in all? $^+5$

20. In ten rounds, Joanna scored $^-7$, $^+4$, $^+3$, $^-5$, $^-2$, $^+8$, $^-3$, $^-1$, $^-5$, and $^+9$ points. How many points did she score in all? $^+1$

321

strips prepared earlier in *Related Activities* may be used for this purpose and for the exercises that follow.

Exercises: Ex. 19 and 20 involve more than two addends, but the number line can be used in the same way as for two addends. For example, for Ex. 19, the starting point is $^+7$. A move of 4 units backward for $^-4$ is made, from that point ($^+3$) a move of 3 units backward for $^-3$ is made, and from that point (0), a move of 5 units forward is made for a stopping point of $^+5$. When the students have completed the exercises, discuss the results for Ex. 8 and 11. Have students suggest other additions similar to these.

Assessment

Add.

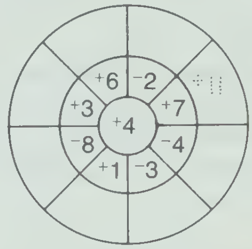
1. $^+1 + ^+5$ $^+6$ **2.** $^-3 + ^+6$ $^+3$ **3.** $^+4 + ^-4$ 0
4. $^-2 + ^-4$ $^-6$ **5.** $^-8 + ^+3$ $^-5$ **6.** $0 + ^-5$ $^-5$

Solve.

7. In two rounds, Jesse scored $^+3$ and $^-4$ points. How many points did he score in all? $^-1$

RELATED ACTIVITIES

- Have students illustrate the sum of 0 and any other integer on a number line.
- For more practice, use copies of page T 391 for number wheels similar to the following.



- Prepare a die marked $^-8$, $^-6$, $^-1$, $^+2$, $^+4$, and $^+9$. Have the students work in groups of four. One student tosses the die and reads the number shown. The next student tosses the die and names the sum of the previous number and the number on the die. Then each student in turn tosses the die and adds the number shown to the previous sum. The procedure continues for several rounds.
- Prepare cards showing integers from $^-9$ to $^+9$ for the game "Greatest Sum" described on page T 379. The required sum for the numbers on a set of cards is zero, for example, $^+4$, $^-3$, and $^-1$, or $^+5$ and $^-5$.
- Ask students to change the order of the addends for several exercises and add. They can discover that the order of adding integers does not affect the sum.

$$^+4 + ^-6 = ^-2$$
$$^-6 + ^+4 = ^-2$$

LESSON OUTCOME

Find the difference between two temperatures and state whether the temperature rose or fell

Materials

a demonstration thermometer

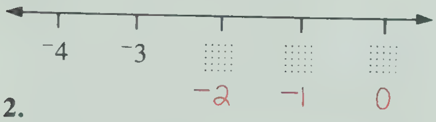
Prerequisite Skills

Match integers with points on a number line

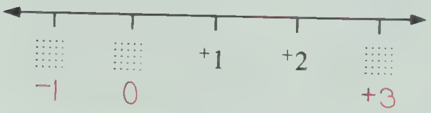
Checking Prerequisite Skills

Complete.

1.



2.



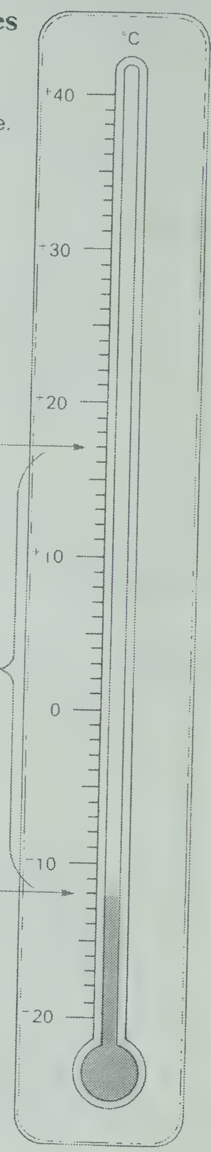
Finding Differences Between Temperatures

Positive and negative integers are used to show temperatures.

The temperature is $^+17$°C.

The temperature is $^-12$°C.

The scale on a thermometer is a vertical number line.



$^+17$°C means seventeen degrees Celsius above zero.

The difference between $^+17$°C and $^-12$°C is twenty-nine degrees Celsius.

$^-12$°C means twelve degrees Celsius below zero.

The temperature changed from $^+17$°C to $^-12$°C.
The temperature fell twenty-nine degrees Celsius.

322

LESSON ACTIVITY

Before Using the Pages

- Display a demonstration thermometer and have students note that the scale shows zero and numbers on either side of zero. This suggests a vertical number line for integers. Develop that temperatures above zero can be represented by positive integers and temperatures below zero by negative integers. Remind the students that positive integers are not always written with the symbol $^+$, but that negative integers are always written with the symbol $^-$. Some demonstration thermometers may show temperatures below zero with the symbol – not raised. This presents an opportunity to explain that the symbol $^+$ and the symbol $^-$ are not always raised. Although they are raised at this time for an introduction to integers, the symbols are lowered in more advanced work.

  Show temperatures on the demonstration thermometer and have students read the temperatures. Then name various temperatures and have students show them on the demonstration thermometer. Some examples are 20°C, $^-15$°C, 6°C, 0°C, and $^-4$°C.

Using the Pages

- Ask a student to read the title at the top of page 322. Direct the students' attention to the photographs and ask them to describe the weather conditions shown for the given temperatures. Ask what is meant by a temperature of $^+17$°C and a temperature of $^-12$°C. Ask what is significant about 0°C and ask why neither a positive symbol nor a negative symbol is shown for 0°C.

  Have the students compare the thermometer illustrated on page 322 with the demonstration thermometer used earlier in the lesson. Have them locate $^+17$°C and $^-12$°C on the thermometer shown on page 322. Ask them to count the number of degrees Celsius between $^+17$°C and $^-12$°C to verify that the difference is twenty-nine degrees Celsius.

  Ask a student to read the statements at the bottom of page 322. Because the temperature went from a higher

Working Together

Is each temperature above or below zero?

1. $^+8$°C above 2. $^-20$°C below

How far is each temperature from zero?

3. $^-14$°C 14°C below zero 4. $^+26$°C 26°C above zero

For each of these, how many degrees did the temperature change?

5. $^+7$°C to $^-3$°C 10 6. $^-14$°C to $^+23$°C 37 7. $^-6$°C to $^-12$°C 6 8. 0°C to $^+15$°C 15

For each of these, did the temperature rise or fall?

9. $^+7$°C to $^-3$°C fall 10. $^-14$°C to $^+23$°C rise 11. $^-6$°C to $^-12$°C fall 12. 0°C to $^+15$°C rise

Exercises

For each of these, tell how many degrees the temperature changed. Then tell whether the temperature rose or fell.

1. $^-7$°C to $^+4$°C rose 11°C 2. $^+10$°C to $^-1$°C fell 11°C 3. $^-2$°C to $^-21$°C fell 19°C
4. $^+6$°C to $^+9$°C rose 3°C 5. $^-8$°C to 0°C rose 8°C 6. $^+25$°C to $^-12$°C fell 37°C
7. $^-10$°C to $^+27$°C rose 37°C 8. $^+32$°C to $^+17$°C fell 15°C 9. $^+3$°C to 0°C fell 3°C
10. $^-20$°C to $^-5$°C rose 15°C 11. 0°C to $^+33$°C rose 33°C 12. $^-4$°C to $^+16$°C rose 20°C
13. $^+19$°C to $^+3$°C fell 16°C 14. $^+23$°C to $^-16$°C fell 39°C 15. 0°C to $^-6$°C fell 6°C

These were the temperatures in Winnipeg for 1978 03 29. What was the change

Time	Temperature (°C)
11:00	$^-6$
12:00	$^-6$
13:00	$^-5$
14:00	$^-4$
15:00	$^-3$
16:00	$^-2$
17:00	$^-1$
18:00	0
19:00	$^-1$
20:00	0

16. from 11:00 to 13:00? rose 1°C
17. from 13:00 to 18:00? rose 5°C
18. from 14:00 to 17:00? rose 3°C
19. from 12:00 to 15:00? rose 3°C
20. from 19:00 to 20:00? rose 1°C
21. from 16:00 to 19:00? rose 1°C
22. from 15:00 to 18:00? rose 3°C
23. from 11:00 to 16:00? rose 4°C
24. from 18:00 to 19:00? fell 1°C
25. from 17:00 to 20:00? rose 1°C
26. from 11:00 to 20:00? rose 6°C

323

RELATED ACTIVITIES

- Students may draw thermometers on copies of page T 390 to illustrate a few exercises from page 323.
- You may wish to have students draw a broken-line graph to show the temperatures in the chart on page 323. Discuss how a graph can be used for finding differences between temperatures. Then have the students use the graph to complete Ex. 16-26 and compare these results with their original answers for the exercises. Graphs prepared for the first of the *Related Activities* on page T 51 can also be used for finding differences in temperatures.
- Students can refer to temperatures shown in newspapers and find the differences between temperatures for two different days or for two different hours.

temperature, $^+17$°C, to a lower temperature, $^-12$°C, emphasize that the temperature fell twenty-nine degrees Celsius.

Working Together: The temperatures for Ex. 5-8 are repeated for Ex. 9-12. Ex. 5-8 involve finding the difference between the temperatures and Ex. 9-12 deal with determining whether the temperature rose or fell. Have students refer to the thermometer shown on page 322, if necessary, to help find the difference between two temperatures. Remind them to pay careful attention to the given temperatures to note whether they are positive, negative, or zero.

Exercises: Have the students record the temperature changes in degrees Celsius because, for any measurement, it is necessary to give the unit. Ex. 16-26 involve reading a chart to find the temperature for each time. You may wish to review the way the date is shown above the chart (year, month, day) and to point out that times in the chart are shown as for a 24-hour clock.

Assessment

For each of these, tell how many degrees the temperature changed. Then tell whether the temperature rose or fell.

1. $^-15$°C to $^+12$°C rose 27°C 2. $^+9$°C to 0°C fell 9°C
3. $^+4$°C to $^+16$°C rose 12°C 4. $^-11$°C to $^-8$°C rose 3°C
5. $^+7$°C to $^-5$°C fell 12°C 6. 0°C to $^-8$°C fell 8°C

Pages 324-325

OBJECTIVE

Demonstrate competence in comparing and ordering integers, in adding integers, and in finding differences between temperatures; solve related word problems

Materials

tracing paper or a copy of page T 396 for each student (optional)

Practice

Complete.

1. $^+3$, ___, ___, ___, $^+7$ — +4 +5 +6
2. $^-5$, ___, ___, $^-2$, ___ — −4 −3 −1
3. ___, ___, 0, $^+1$, ___ — −2 −1 +2
4. ___, ___, $^-6$, ___, $^-4$ — −8 −7 −5

Use > or < to make true statements.

5. 0 ● $^+5$ <	6. $^-8$ ● $^-1$ <	7. $^+3$ ● $^-3$ >	8. $^-6$ ● $^-5$ <
9. $^+7$ ● $^+4$ >	10. $^-7$ ● $^+4$ <	11. 0 ● $^-2$ >	12. $^-4$ ● $^-9$ >

List from least to greatest.

13. 0, $^-3$, $^-6$, $^-9$, $^+3$, $^+6$, $^+9$
−9, −6, −3, 0, +3, +6, +9

List from greatest to least.

14. $^+6$, $^-5$, $^+4$, $^-3$, $^+2$, $^-1$
+6, +4, +2, −1, −3, −5

Add.

15. $^+5 + ^-6$ −1	16. $^+4 + ^+4$ +8	17. $^-1 + ^-8$ −9	18. $^-3 + ^+6$ +3
19. $^-7 + ^-2$ −9	20. $^+2 + ^-2$ 0	21. $^-4 + ^+1$ −3	22. $^-8 + ^-5$ −13
23. $^+5 + ^+2$ +7	24. $^-7 + ^-7$ −14	25. $^+7 + ^-3$ +4	26. $^-6 + ^+6$ 0

Use tracing paper or copy this puzzle. Draw a path from Start to Finish that gives the greatest sum. 71
Do not go through any space more than one time.

27.

Start	$^+3$	4	$^+5$	3	$^+2$	$^-7$	$^+2$	$^-9$
$^+5$	$^-5$	$^+6$	$^-7$	$^+7$	$^-3$	$^+4$	$^-6$	$^+1$
$^-3$	$^+3$	$^-4$	$^+8$	$^-2$	$^+6$	1	$^+3$	$^-8$
$^+6$	$^-8$	$^+5$	$^-2$	$^+4$	$^-5$	$^+5$	$^-3$	$^+1$
$^-2$	$^+2$	$^-1$	$^+3$	$^+4$	$^+1$	$^-2$	$^+7$	$^-1$
$^+9$	$^-4$	$^+7$	$^-5$	$^+7$	$^-7$	$^+9$	$^-8$	$^+8$
$^-9$	$^+1$	$^-6$	$^+3$	$^+3$	$^+6$	1	$^+2$	Finish

324

LESSON ACTIVITY

Using the Pages

- Ex. 15-27 deal with adding integers and Ex. 28-44 involve finding the differences between temperatures. This provides an opportunity to determine whether students have difficulty with either of these or confuse the two processes.

  Give each student a sheet of tracing paper for Ex. 27 or have the students copy the puzzle on a 7-by-9 section of graph paper cut from copies of page T 396. This exercise involves adding integers and considering a strategy. You may prefer to direct the students to draw three paths and write the sum for each path to note which of their three paths gives the greatest sum.

Try This: The concept of exponents was first presented in the *Try This* feature on page 319. Here, the concept is applied in writing numerals for large numbers and this involves 10 as a factor. Discuss the examples in which numerals in standard form are expressed as numerals involving exponents. Pay particular attention to the multiplication symbol in 47×10^6. For each example, have the students note the number of zeros in the standard form, the number of times 10 is used as a factor, and the exponent. The "thought cloud" for Ex. 1 guides the students by showing how many times 10 is used as a factor for 100.

Before the students begin Ex. 13-27, discuss the example in the "thought cloud" for 4.6×10^3. Emphasize that 4.6×10^3 and 4600 name the same number. For examples in which the first factor is a decimal, the exponent does not indicate the number of zeros in the standard numeral.

For each of these, tell how many degrees the temperature changed. Then tell whether the temperature rose or fell.

28. $^{-}4$°C to $^{-}7$°C fell 3°C
29. $^{-}3$°C to $^{+}9$°C rose 12°C
30. $^{+}6$°C to 0°C fell 6°C
31. $^{-}17$°C to $^{+}23$°C rose 40°C
32. 0°C to $^{+}14$°C rose 14°C
33. $^{+}12$°C to $^{+}8$°C fell 4°C
34. $^{+}28$°C to $^{-}14$°C fell 42°C
35. $^{-}16$°C to $^{-}2$°C rose 14°C
36. $^{+}29$°C to 0°C fell 29°C
37. The temperature went from $^{+}8$°C to $^{-}2$°C. fell 10°C
38. The temperature went from $^{-}6$°C to $^{-}8$°C. fell 2°C
39. The temperature went from $^{-}7$°C to $^{+}22$°C. rose 29°C
40. The temperature went from $^{+}3$°C to $^{+}19$°C. rose 16°C

For each of these, give the difference between the two temperatures.

41. One day the low temperature was $^{-}7$°C. The high was $^{+}4$°C. 11°C
42. One day the temperature went from $^{-}5$°C to $^{-}6$°C in 2 h. 1°C
43. A temperature of $^{+}58$°C was once recorded in North Africa. A temperature of $^{-}88$°C was once recorded in Antarctica. 146°C
44. In Canada, a temperature of $^{-}58$°C was once recorded and a temperature of $^{+}41$°C was once recorded. 99°C

Exponents give a short way for writing large numbers.

$1\,000\,000 = 10^6$ $10 \times 10 \times 10 \times 10 \times 10 \times 10$

$47\,000\,000 = 47 \times 1\,000\,000$, or 47×10^6

Show each of these using exponents.

1. 100 10^2 10×10
2. 1000 10^3
3. 10 000 10^4
4. 100 000 000 10^8
5. 2000 2×10^3
6. 60 000 6×10^4
7. 300 3×10^2
8. 90 000 000 9×10^7
9. 7200 72×10^2
10. 94 000 94×10^3
11. 368 000 368×10^3
12. 5 830 000 583×10^4

Write the standard form for each of these.

$4.6 \times 10^3 = 4.6 \times 10 \times 10 \times 10$, or 4.6×1000
$4.6 \times 10^3 = 4600$

13. 10^7 10 000 000
14. 10^9 1 000 000 000
15. 10^5 100 000
16. 8×10^2 800
17. 4×10^3 4000
18. 7×10^6 7 000 000
19. 6×10^8 600 000 000
20. 3×10^4 30 000
21. 2×10^3 2000
22. 6.3×10^3 6300
23. 2.75×10^5 275 000
24. 3.91×10^8 391 000 000
25. 4.2×10^9 4 200 000 000
26. 9.34×10^2 934
27. 5.11×10^6 5 110 000

try this

325

RELATED ACTIVITIES

- Have students make puzzles, similar to but smaller than the puzzle in Ex. 27, for others to find the path that gives the greatest sum. Adapt the puzzle for finding the path that gives the least sum.
- For more practice in using a short way to write larger numbers, have the students use exponents for numbers on pages 5, 7, and 9.
- Name a few integers such as $^{+}4$, $^{-}3$, and 0. For each integer, have the students write three addition exercises that have that integer as the sum. For example, for $^{+}4$, they may write $^{+}2 + {}^{+}2$, $^{+}5 + {}^{-}1$, and $^{-}3 + {}^{+}7$.

  Assign similar exercises for finding differences between temperatures. Provide statements such as "The temperature fell 6°C." Students would write three changes in temperatures, for example, $^{+}9$°C to $^{+}3$°C, $^{+}4$°C to $^{-}2$°C, and 0°C to $^{-}6$°C.
- For enrichment, write addition exercises with several addends, for example, $^{+}6 + {}^{-}9 + {}^{-}3 + {}^{-}8 + {}^{+}4 + {}^{+}5$. Have the students find the sums by
  1. adding from left to right;
  2. adding from right to left;
  3. adding in any order;
  4. adding the positive integers, adding the negative integers, and then adding their sums.

  For some exercises, the third method may involve adding opposite integers first.

OBJECTIVE

Find different solutions for word problems

RELATED ACTIVITIES

- Encourage the students to notice and comment on day-to-day problem situations for which there are several possible solutions. The situations may relate to experiences inside and outside the classroom, or to stories that the students read.
- A discussion of news items may reveal problem situations which might have been solved in other ways if conditions had been different, for example, an increase in the price of an item, a decision by a local committee, or a change in the speed limit.

Thinking About Situations

Thinking about a situation can help you to decide on possible solutions.

Mary wants a bike. She has $65. The bike she wants costs $95.

What could Mary do?

I could earn some money and buy the bike later.

I could buy a secondhand bike.

I could buy a cheaper bike.

I could borrow some money and buy the bike now.

Think of other possible solutions for Mary.

Answers will vary.

Give at least four solutions for each of these.

1. Diana planned to give her friend red beads. When she went shopping, she saw some red beads for $2.25 and some blue beads for $1.98. She has $2.
2. Sylvia hopes to join the track team and also learn to play the flute by joining the school band. They both have practices at the same time. The members of the school band must attend 9 out of every 10 practices.
3. Mabel wants to buy a book that costs $1.75. She also wants to buy a pen that costs $1.29 or a pen that costs $1.69. She has $2.05.
4. It is 17:00. For dinner, Gavin wants to bake a cake at 160°C and cook a casserole at 180°C.

PROBLEM SOLVING

326

LESSON ACTIVITY

Using the Page

- These word problems involve thinking about a situation and considering various possible solutions. Ask students to read the title of the lesson and the word problem at the top of page 326. Note each of Mary's possible solutions and discuss with the students the advantages and disadvantages of each solution. Ask the students to think of other possible solutions and to consider the advantages and disadvantages of these solutions.
- Divide the students into groups of about five and appoint one student in each group as the leader. Direct the groups to discuss the word problems and possible solutions for them. The leader would read a word problem, lead the discussion, and then decide when to continue to the next problem. Dividing the students into small groups provides each student with an opportunity to participate in the discussion for each problem. Move among the groups noting the students' comments and encouraging them to consider several points of view. Later, the leader of each group may share the solutions with the rest of the class.

Checking Up

Which are positive integers?

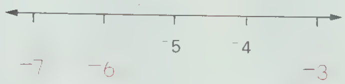
1. $^-6$, $^+3$, 0, $^-3$, $^+6$

Which are negative integers?

2. 0, $^-1$, $^+1$, $^-2$, $^+2$

Complete.

3. 

4. 

Use > or < to make true statements.

5. $^+3$ < $^+6$ **6.** $^+2$ > $^-4$ **7.** $^-1$ > $^-5$ **8.** $^-7$ < $^+3$

List from least to greatest.

9. $^-3$, $^+2$, $^-1$, 0, $^+1$, $^-2$, $^+3$ $-3, -2, -1, 0, +1, +2, +3$

10. $^-7$, $^+6$, $^+3$, $^-5$, $^-2$, $^+1$, $^-4$ $-7, -5, -4, -2, +1, +3, +6$

List from greatest to least.

11. $^-6$, $^-9$, $^+3$, $^-4$, $^+4$, $^+7$, $^-2$ $+7, +4, +3, -2, -4, -6, -9$

12. $^+1$, $^+9$, 0, $^-8$, $^-3$, $^+5$, $^-7$ $+9, +5, +1, 0, -3, -7, -8$

Add. Use a number line if you wish.

13. $^+6 + ^+4$ $+10$ **14.** $^+7 + ^-5$ $+2$ **15.** $^-4 + ^+5$ $+1$ **16.** $^-5 + ^-5$ -10

17. $^-5 + ^+2$ -3 **18.** $^-3 + ^-4$ -7 **19.** $^+1 + ^-6$ -5 **20.** $^-6 + ^-3$ -9

21. $^+4 + ^-4$ 0 **22.** $^-7 + ^+4$ -3 **23.** $^+2 + ^+7$ $+9$ **24.** $^-2 + ^+2$ 0

For each of these, tell how many degrees the temperature changed. Then tell whether the temperature rose or fell.

25. $^+1$°C to $^-15$°C fell 16°C **26.** $^-23$°C to $^+7$°C rose 30°C **27.** $^-4$°C to 0°C rose 4°C

28. $^+9$°C to 0°C fell 9°C **29.** $^+3$°C to $^+8$°C rose 5°C **30.** $^-5$°C to $^-13$°C fell 8°C

31. $^+2$°C to $^-16$°C fell 18°C **32.** 0°C to $^-3$°C fell 3°C **33.** $^+32$°C to $^+24$°C fell 8°C

Solve.

34. In two rounds of a game, Jesse scored $^+3$ points and $^-7$ points. How many points did he score in all? -4

35. In two rounds of a game, Joanna's scores were $^-5$ and $^+8$. What was her score for the two rounds? $+3$

36. The temperature changed from $^-6$°C to $^-3$°C. How many degrees did the temperature change? Did it rise or fall? rose 3°C

37. The coldest temperature was $^-8$°C. The warmest temperature was $^+6$°C. What was the difference between the two temperatures? 14°C

327

Skills	Exercises	Related Pages
Identify positive integers and negative integers	1, 2	T 346-T 347
Match integers with points on a number line	3, 4	T 348-T 349
Compare integers	5-8	T 348-T 349
Order integers	9-12	T 348-T 349
Add two integers	13-24	T 350-T 351
Find the difference between two temperatures and state whether the temperature rose or fell	25-33	T 352-T 353
Solve word problems	34-37	

Page 327

OBJECTIVE

Demonstrate an understanding of the concepts and skills presented in this unit

Materials

two copies of page T 390 for each student (optional)

RELATED ACTIVITIES

- To provide practice in adding integers, prepare number squares similar to the following.

+ →			
↓	$^+3$	$^-8$	$^-5$
	$^-5$	$^-2$	
	$^-2$		

- Have students write and solve word problems suggesting integers, as in the following three examples.

Jake and Bonnie started at the same place. Jake walked 3 blocks to the east and Bonnie walked 2 blocks to the west. How far apart are they?

Wayne got on the elevator on the fourth floor, went down two floors, and then went up six floors. What floor is he on?

Sally earned \$7, spent \$5, and then earned \$2. How much money has she now?

Comments

For assistance in answering Ex. 13-24, and Ex. 34 and 35, students may draw a number line to show integers from $^-10$ to $^+10$. They may draw a thermometer for temperatures from $^-23$°C to $^+32$°C to help in answering Ex. 25-33, and Ex. 36 and 37. You may wish to provide them with copies of page T 390 for showing the number line and the thermometer.

If students have difficulty with the exercises, provide more work with the number line as suggested in the lessons and related activities. Relate positive and negative integers as well as the concept of opposites to students' experiences for east and west, north and south, up and down, and earning and spending.

OBJECTIVE

Demonstrate competence in adding and subtracting whole numbers

Skill Practice – Adding and Subtracting Whole Numbers

Add.

1. 9876 + 5432 = 15 308
2. 9543 + 8523 = 18 066
3. 6865 + 3864 = 10 729
4. 25 699 + 1 569 = 27 268
5. 69 602 + 3 795 = 73 397
6. 19 286 + 92 834 = 112 120
7. 64 551 + 7 387 = 71 938
8. 255 008 + 70 007 = 325 015
9. 660 779 + 808 834 = 1 469 613
10. 987 746 + 712 741 = 1 700 487
11. 377 + 437 + 744 = 1558
12. 488 + 372 + 987 = 1847
13. 599 + 281 + 878 = 1758
14. 98 452 + 7 817 + 43 650 = 149 919
15. 70 342 + 10 818 + 7 763 = 88 923
16. 4476 + 3367 + 1234 + 8327 = 17 404
17. 5051 + 4708 + 6650 + 1231 = 17 640
18. 60 066 + 7 323 + 4 394 + 70 071 = 141 854
19. 3339 + 4448 + 32 + 2567 = 10 386
20. 12 123 + 83 949 + 7 634 + 90 981 = 194 687
21. 48 224 + 782 + 50 863 99 869
22. 78 187 + 9 123 + 10 810 98 120
23. 40 427 + 44 842 + 48 841 134 110
24. 99 112 + 21 087 + 3 343 123 542
25. 32 + 81 143 + 3 434 84 609
26. 477 + 77 477 + 47 78 001
27. 526 + 65 534 + 375 + 56 66 491
28. 832 + 30 232 + 58 + 7 379 38 501

Subtract.

29. 48 − 27 = 21
30. 897 − 670 = 227
31. 6784 − 5434 = 1350
32. 7829 − 7501 = 328
33. 8369 − 4345 = 4024
34. 58 519 − 38 006 = 20 513
35. 76 898 − 26 321 = 50 577
36. 50 978 − 20 852 = 30 126
37. 74 965 − 60 314 = 14 651
38. 99 668 − 94 320 = 5348
39. 4414 − 4325 = 89
40. 6133 − 3616 = 2517
41. 81 189 − 61 389 = 19 800
42. 75 265 − 39 987 = 35 278
43. 69 359 − 40 997 = 28 362
44. 673 432 − 85 675 = 587 757
45. 781 234 − 98 748 = 682 486
46. 325 677 − 268 079 = 57 598
47. 515 256 − 156 167 = 359 089
48. 604 − 559 = 45
49. 8001 − 5632 = 2369
50. 87 010 − 76 408 = 10 602
51. 90 058 − 69 359 = 20 699
52. 50 000 − 47 342 2658
53. 60 050 − 54 664 5386
54. 123 002 − 4 514 118 488
55. 454 003 − 27 028 426 975
56. 800 032 − 249 803 550 229
57. 501 001 − 439 627 61 374

328

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Add two numbers with regrouping, addends with four, five, or six digits	1-10
Add more than two numbers with regrouping, sums with four, five, or six digits	11-20
Write two or three addends in vertical form and add, with regrouping, sums with five or six digits	21-28
Subtract numbers with no regrouping, minuends with up to five digits	29-38
Subtract numbers with regrouping, minuends with up to six digits	39-51
Write two numbers in vertical form and subtract with regrouping for zero in one or more places in the minuend, minuends with five or six digits	52-57

Skill Practice – Multiplying and Dividing Whole Numbers

Multiply.

1. 125 8 1000	2. 375 4 1500	3. 1607 6 9642	4. 4091 7 28 637	5. 9813 9 88 317
6. 4340 6 26 040	7. 9876 3 29 628	8. 54 076 5 270 380	9. 63 047 8 504 376	10. 70 064 9 630 576
11. 8342 40 333 680	12. 7048 50 352 400	13. 3994 60 239 640	14. 4881 70 341 670	15. 8088 80 647 040

16. 100 × 43 4300
17. 400 × 8316 3 326 400
18. 6000 × 947 5 682 000
19. 90 × 1034 93 060
20. 4371 × 800 3 496 800
21. 5088 × 300 1 526 400
22. 7717 × 70 540 190
23. 3919 × 4000 15 676 000

24. 45 28 1260	25. 68 35 2380	26. 73 49 3577	27. 87 78 6786	28. 93 19 1767
29. 412 83 34 196	30. 608 54 32 832	31. 7732 95 734 540	32. 1806 68 122 808	33. 43 933 29 1 274 057
34. 402 383 153 966	35. 648 450 291 600	36. 843 598 504 114	37. 907 306 277 542	38. 494 329 162 526

Divide.

39. 6)54 9	40. 7)56 8	41. 3)29 9 R2	42. 9)81 9	43. 8)72 9
44. 8)68 8 R4	45. 5)34 6 R4	46. 4)27 6 R3	47. 6)57 9 R3	48. 7)50 7 R1

49. 6)2590 431 R4	50. 7)29 561 4223	51. 9)85 644 9516	52. 3)47 477 15 825 R2
53. 6)54 048 9008	54. 8)64 080 8010	55. 5)55 048 11 009 R3	56. 4)64 125 16 031 R1
57. 30)4930 164 R10	58. 90)8190 91	59. 80)47 556 594 R36	60. 60)37 800 630
61. 27)81 3	62. 14)850 60 R10	63. 57)2463 43 R12	64. 43)3040 70 R30
65. 79)41 010 519 R9	66. 83)79 721 960 R41	67. 94)86 260 917 R62	68. 66)51 593 781 R47
69. 137)13 997 102 R23	70. 255)23 985 94 R15	71. 378)68 484 181 R66	72. 717)31 348 43 R517
73. 432)34 432 79 R304	74. 606)23 484 38 R456	75. 598)32 816 54 R524	76. 927)37 594 40 R514

329

OBJECTIVE

Demonstrate competence in multiplying and dividing whole numbers

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Multiply by a one-digit number, multiplicands with up to five digits	1-10
Multiply by a multiple of 10, 100, and 1000, multiplicands with up to four digits	11-23
Multiply by a two-digit number, multiplicands with up to five digits	24-33
Multiply a three-digit number by a three-digit number	34-38
Divide a two-digit number by a one-digit number	39-48
Divide by a one-digit number, dividends with up to five digits, no zeros in the quotient	49-52
Divide by a one-digit number, dividends with up to five digits, zeros in one or more places in the quotient	53-56
Divide by a multiple of ten from 10 to 90, dividends with up to five digits	57-60
Divide by a two-digit number, dividends with up to five digits	61-68
Divide by a three-digit number, dividends with five digits	69-76

OBJECTIVE

Demonstrate competence in adding and subtracting decimals

Skill Practice – Adding and Subtracting Decimals

Add.

1. 43.23 + 72.79 = 116.02
2. 731.7 + 283.9 = 1015.6
3. 9.019 + 3.789 = 12.808
4. 0.6883 + 0.4545 = 1.1428
5. 18.805 + 3.75 = 22.555
6. 0.8 + 0.949 = 1.749
7. 24.4 + 7.588 = 31.988
8. 0.9097 + 1.136 = 2.0457
9. 4.876 + 5.32 + 4.875 = 15.071
10. 0.7 + 5.123 + 6.277 = 12.100
11. 1.832 + 0.941 + 9.041 = 11.814
12. 0.83 + 0.0064 + 0.204 = 1.0404
13. 6.7 + 8.54 + 7.0377 + 0.0187 = 22.2964
14. 0.83 + 21.7 + 18.634 + 0.088 = 41.252
15. 9.74 + 0.9077 + 0.83 + 1.8808 = 13.3585
16. 7.11 + 0.902 + 1.8374 + 2.0126 = 11.8620
17. 1.0084 + 1 + 1.09 + 0.991 4.0894
18. 4 + 0.68 + 1.0432 + 7 12.7232
19. 4.32 + 8 + 0.7 + 0.077 13.097
20. 0.808 + 6 + 7.717 + 2 16.525
21. 7.63 + 14 + 0.118 21.748
22. 23.192 + 0.18 + 9 32.372
23. 45.85 + 23 + 15 + 0.6 84.45
24. 72.112 + 9 + 0.1188 + 6 87.2308

Subtract.

25. 0.4942 − 0.3731 = 0.1211
26. 176.44 − 98.56 = 77.88
27. 43.123 − 42.234 = 0.889
28. 10.004 − 3.095 = 6.909
29. 7.8 − 4.37 = 3.43
30. 19.8604 − 8.477 = 11.3834
31. 0.847 − 0.53 = 0.317
32. 11.1 − 9.247 = 1.853
33. 59.1 − 10.45 = 48.65
34. 46.185 − 3.8 = 42.385
35. 70.112 − 0.9 = 69.212
36. 0.7 − 0.664 = 0.036
37. 1.844 − 1.75 0.094
38. 0.9 − 0.66 0.24
39. 10.4 − 3.58 6.82
40. 10.4 − 1.57 8.83
41. 15.832 − 0.9 14.932
42. 6.8 − 5.95 0.85
43. 4 − 2.85 1.15
44. 23.35 − 4 19.35
45. 10 − 0.8 9.2
46. 19.177 − 12 7.177
47. 17.115 − 8 9.115
48. 9 − 0.99 8.01
49. 3.68 − 2 1.68
50. 4 − 0.01 3.99
51. 33.4 − 4 29.4
52. 17 − 3.876 13.124
53. 19 − 10.1176 8.8824
54. 76.137 − 75 1.137
55. 6.2 − 5 1.2
56. 9 − 4.318 4.682
57. 15 − 0.201 14.799

330

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Add two decimals, from one to four decimal places	1-4
Add two decimals, from one to four decimal places, addends with different numbers of decimal places	5-8
Add three or four decimals, from one to four decimal places, addends with different numbers of decimal places	9-16
Write decimals and whole numbers in vertical form and add, three and four addends with different numbers of decimal places	17-24
Subtract decimals, from one to four decimal places	25-28
Subtract decimals, with different numbers of decimal places from one to four	29-36
Write decimals with one to three decimal places in vertical form and subtract	37-42
Write decimals with one to four decimal places and whole numbers in vertical form and subtract	43-57

Skill Practice – Multiplying Decimals

Multiply.

1. 2.39 4 9.56	**2.** 40.8 5 204.0	**3.** 36.77 6 220.62	**4.** 89.6 7 627.2	**5.** 49.05 8 392.40
6. 89.1 14 12474	**7.** 7.82 25 195.50	**8.** 67.3 36 2422.8	**9.** 2.504 47 117.688	**10.** 4.58 58 265.64
11. 59.8 705 42 159.0	**12.** 9.607 684 6571.188	**13.** 7.76 593 4601.68	**14.** 86.5 484 41 866.0	**15.** 195.4 305 59 597.0
16. 4.6 3.7 17.02	**17.** 9.2 7.4 68.08	**18.** 7.8 9.6 74.88	**19.** 8.7 3.8 33.06	**20.** 5.4 4.5 24.30
21. 12.6 3.7 46.62	**22.** 15.8 5.7 90.06	**23.** 73.7 7.4 545.38	**24.** 98.2 4.7 461.54	**25.** 77.2 5.6 432.32
26. 10.9 7.3 79.57	**27.** 33.7 6.8 229.16	**28.** 56.8 5.7 323.76	**29.** 79.9 4.6 367.54	**30.** 81.8 3.9 319.02
31. 43.55 7.7 335.335	**32.** 26.8 3.78 101.304	**33.** 8.778 8.8 77.2464	**34.** 3.56 4.5 16.020	**35.** 60.07 3.07 184.4149
36. 2.17 7.03 15.2551	**37.** 27.5 1.7 46.75	**38.** 3.084 2.7 8.3268	**39.** 47.31 66.8 3160.308	**40.** 6.01 1.86 11.1786
41. 17.75 8.2 145.550	**42.** 2.808 77.3 217.0584	**43.** 99.1 88.2 8740.62	**44.** 60.87 7.08 430.9596	**45.** 3.08 9.36 28.8288

46. 14.86 × 5.73	**47.** 259.7 × 7.71	**48.** 47.37 × 1.3	**49.** 59.92 × 3.37
50. 33.07 × 13.7	**51.** 33.5 × 685.7	**52.** 5.6 × 56.38	**53.** 3.5 × 80.7
54. 1.08 × 722.3	**55.** 3.7 × 68.57	**56.** 70.17 × 39.9	**57.** 2.7 × 4.586
58. 47.09 × 49.3	**59.** 78.4 × 583.2	**60.** 1.5 × 66.72	**61.** 5.75 × 54.5

62. 0.3 0.2 0.06	**63.** 0.79 0.18 0.1422	**64.** 0.07 0.5 0.035	**65.** 0.76 0.68 0.5168
66. 0.26 0.07 0.0182	**67.** 0.194 0.8 0.1552	**68.** 0.56 0.37 0.2072	**69.** 0.87 0.8 0.696
70. 0.23 0.09 0.0207	**71.** 0.45 0.04 0.0180	**72.** 0.077 0.7 0.0539	**73.** 0.089 0.6 0.0534

331

Page 331

OBJECTIVE

Demonstrate competence in multiplying decimals

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Multiply a one-place or a two-place decimal by a one-digit whole number	1-5
Multiply a decimal with up to three decimal places by a two-digit whole number	6-10
Multiply a decimal with up to three decimal places by a three-digit whole number	11-15
Multiply two one-place decimals	16-30
Multiply a decimal with up to three decimal places by a one-place or a two-place decimal	31-61
Multiply a decimal with up to three decimal places by a one-place or a two-place decimal, products less than one	62-73

46. 85.1478
47. 2002.287
48. 61.581
49. 201.9304
50. 453.059
51. 22 970.95
52. 315.728
53. 282.45
54. 780.084
55. 253.709
56. 2799.783
57. 12.3822
58. 2321.537
59. 45 722.88
60. 100.080
61. 313.375

Page 332

OBJECTIVE

Demonstrate competence in dividing decimals

Skill Practice – Dividing Decimals

Divide.

1. 3)1.374 0.458	**2.** 5)15.9 3.18	**3.** 7)8.75 1.25	**4.** 8)5.4 0.675	**5.** 6)89.4 14.9
6. 4)8.3 2.075	**7.** 9)262.8 29.2	**8.** 2)1.81 0.905	**9.** 6)0.9 0.15	**10.** 8)59.76 7.47
11. 6)0.27 0.045	**12.** 3)739.2 246.4	**13.** 9)88.56 9.84	**14.** 5)6.62 1.324	**15.** 7)5.642 0.806
16. 8)73.4 9.175	**17.** 6)0.477 0.0795	**18.** 7)69.51 9.93	**19.** 4)7.62 1.905	**20.** 4)3.9 0.975

21. 12)14.412 1.201	**22.** 34)816.68 24.02	**23.** 187)3496.9 18.7	**24.** 49)141.12 2.88
25. 88)374 4.25	**26.** 555)188.7 0.34	**27.** 23)14.007 0.609	**28.** 94)56.447 0.6005
29. 678)552.57 0.815	**30.** 56)378 6.75	**31.** 425)343.4 0.808	**32.** 15)10 266 684.4
33. 25)985 39.4	**34.** 934)532.38 0.57	**35.** 264)2138.4 8.1	**36.** 48)4008 83.5
37. 66)168.3 2.55	**38.** 85)59.925 0.705	**39.** 625)4025 6.44	**40.** 72)61.2 0.85
41. 45)3699 82.2	**42.** 118)759.92 6.44	**43.** 37)149.11 4.03	**44.** 452)4135.8 9.15

45. 0.2)3.4 17	**46.** 0.9)3.645 4.05	**47.** 0.6)1.5 2.5	**48.** 0.8)4.4 5.5
49. 0.9)2.88 3.2	**50.** 0.5)73 146	**51.** 0.7)4.417 6.31	**52.** 0.1)7.38 73.8
53. 0.3)6.48 21.6	**54.** 0.8)5.4 6.75	**55.** 0.4)6 15	**56.** 0.6)3 5
57. 4.6)13.11 2.85	**58.** 2.4)148.56 61.9	**59.** 7.5)705 94	**60.** 8.2)844.6 103
61. 1.1)3.597 3.27	**62.** 9.2)6486 705	**63.** 6.7)99.16 14.8	**64.** 5.9)5.074 0.86
65. 3.8)1767 465	**66.** 8.7)37.41 4.3	**67.** 5.4)513 95	**68.** 6.6)201.3 30.5
69. 0.48)31.2 65	**70.** 2.34)71.37 30.5	**71.** 3.95)363.4 92	**72.** 0.24)36 150
73. 0.76)4.94 6.5	**74.** 0.01)2.394 239.4	**75.** 6.85)9179 1340	**76.** 5.03)40.24 8
77. 0.05)302 6040	**78.** 1.28)32 25	**79.** 0.55)41.8 76	**80.** 7.75)7006 904
81. 5.32)321.86 60.5	**82.** 0.064)11.84 185	**83.** 0.001)6.001 6001	**84.** 1.5)99 66
85. 0.008)24.4 3050	**86.** 7.5)228 30.4	**87.** 6.04)25.67 4.25	**88.** 0.234)19.422 83

332

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Divide a decimal with up to three decimal places by a one-digit number, using zeros in the dividend, quotient terminating by the third decimal place	1-20
Divide a whole number or a decimal with up to three decimal places by a two-digit or a three-digit number, using zeros in the dividend	21-44
Divide a whole number or a decimal with up to three decimal places by a one-place decimal less than one	45-56
Divide a whole number or a decimal with up to three decimal places by a one-place decimal	57-68
Divide a whole number or a decimal with up to three decimal places by a two-place decimal	69-80
Divide a whole number or a decimal with up to three decimal places by a decimal with up to three decimal places	81-88

Skill Practice – Multiplying and Dividing by Multiples of 10

Multiply by 10.

1. 4 40	**2.** 0.7 7	**3.** 11 110
4. 87.058 870.58	**5.** 10.1 101	**6.** 3.102 31.02
7. 909 9090	**8.** 1101 11010	**9.** 74.32 743.2
10. 0.9 9	**11.** 0.15 1.5	**12.** 0.001 0.01
13. 1.01 10.1	**14.** 8188 81880	
15. 110.1 1101	**16.** 543.021 5430.21	

Divide by 0.1.

17. 4 40	**18.** 0.7 7	**19.** 11 110
20. 87.058 870.58	**21.** 10.1 101	**22.** 3.12 31.2
23. 3.5 35	**24.** 13.7 137	**25.** 1.05 10.5
26. 0.6 6	**27.** 0.07 0.7	**28.** 0.085 0.85
29. 6.6 66	**30.** 0.007 0.07	
31. 508 5080	**32.** 30.35 303.5	

Multiply by 100.

33. 8 800	**34.** 1.4 140	**35.** 2.01 201
36. 999 99 900	**37.** 1.001 100.1	**38.** 1101 110 100
39. 1011 101 100	**40.** 90.8 9080	**41.** 87 8700
42. 0.9 90	**43.** 0.15 15	**44.** 0.001 0.1
45. 333 33 300	**46.** 70.06 7006	
47. 0.008 0.8	**48.** 20.6 2060	

Divide by 0.01.

49. 8 800	**50.** 1.4 140	**51.** 2.01 201
52. 999 99 900	**53.** 1.001 100.1	**54.** 1101 110 100
55. 3.5 350	**56.** 1.37 137	**57.** 105 10 500
58. 0.6 60	**59.** 0.07 7	**60.** 0.085 8.5
61. 2.091 209.1	**62.** 0.09 9	
63. 747.7 74 770	**64.** 4.043 404.3	

Multiply by 1000.

65. 0.33 330	**66.** 10 10 000	**67.** 1.9 1900
68. 1000 1 000 000	**69.** 101.1 101 100	**70.** 9.99 9990
71. 454 454 000	**72.** 39 39 000	**73.** 3.02 3020
74. 0.9 900	**75.** 15 15 000	**76.** 0.001 1
77. 0.008 8	**78.** 60 60 000	
79. 697.881 697 881	**80.** 7.7 7700	

Divide by 0.001.

81. 0.33 330	**82.** 10 10 000	**83.** 1.9 1900
84. 1000 1 000 000	**85.** 101.1 101 100	**86.** 9.99 9990
87. 3.5 3500	**88.** 13.7 13 700	**89.** 1.05 1050
90. 50 50 000	**91.** 0.07 70	**92.** 0.085 85
93. 10.101 10 101	**94.** 800.8 800 800	
95. 0.43 430	**96.** 88 88 000	

Multiply by 500.

97. 16 8000	**98.** 132 66 000
99. 0.9 450	**100.** 0.15 75
101. 0.001 0.5	**102.** 4.32 2160

Divide by 0.002.

103. 16 8000	**104.** 132 66 000
105. 0.9 450	**106.** 0.15 75
107. 3.5 1750	**108.** 13.724 6862

Multiply by 200.

109. 6312 1 262 400	**110.** 72 14 400
111. 10.1 2020	**112.** 800.818 160 163.6
113. 0.006 1.2	**114.** 4.04 808

Divide by 0.005.

115. 6312 1 262 400	**116.** 72 14 400
117. 10.1 2020	**118.** 800.818 160 163.6
119. 3.5 700	**120.** 13.74 2748

333

Page 333

OBJECTIVE

Demonstrate competence in multiplying by 10, 100, and 1000 and in dividing by 0.1, 0.01, and 0.001

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Multiply by 10, writing only the products	1-16
Divide by 0.1, writing only the quotients	17-32
Multiply by 100, writing only the products	33-48
Divide by 0.01, writing only the quotients	49-64
Multiply by 1000, writing only the products	65-80
Divide by 0.001, writing only the quotients	81-96
Multiply by 500 (extension)	97-102
Divide by 0.002 (extension)	103-108
Multiply by 200 (extension)	109-114
Divide by 0.005 (extension)	115-120

Comments

Use Ex. 1-96 as oral exercises or have the students read the exercises and write only the results. Exercises such as Ex. 1 and 17 show that multiplying by 10 gives the same result as dividing by 0.1. Exercises such as Ex. 33 and 49 show that multiplying by 100 gives the same result as dividing by 0.01. Exercises such as Ex. 65 and 81 show that multiplying by 1000 gives the same result as dividing by 0.001. Ask the students to identify and describe other similar pairs of exercises.

Ex. 97-120 may be assigned after the students have learned to multiply by 100 and to divide by 0.001. For example, to multiply by 500 (Ex. 97-102), they can multiply by 100 and then multiply the result by 5. To divide by 0.002 (Ex. 103-108), they can divide by 0.001 and then divide the result by 2. Note that multiplying by 500 gives the same result as dividing by 0.002, and multiplying by 200 gives the same result as dividing by 0.005. For example, the results for $16 \times 500 = 8000$ (Ex. 97) and $16 \div 0.002 = 8000$ (Ex. 103) are identical. Similarly, the results for $6312 \times 200 = 1\,262\,400$ (Ex. 109) and $6312 \div 0.005 = 1\,262\,400$ (Ex. 115) are identical.

Page 334

OBJECTIVE

Demonstrate competence in dividing by 10, 100, and 1000 and in multiplying by 0.1, 0.01, and 0.001

Skill Practice – Dividing and Multiplying by Multiples of 10

Divide by 10.

1. 40 4 **2.** 162 16.2 **3.** 5.9 0.59
4. 100 10 **5.** 4.07 0.407 **6.** 0.777 0.0777
7. 1 0.1 **8.** 15.3 1.53 **9.** 105 10.5
10. 0.07 0.007 **11.** 0.5 0.05 **12.** 1.834 0.1834
13. 987.65 98.765 **14.** 430.022 43.0022
15. 7000 700 **16.** 40.9 4.09

Multiply by 0.1.

17. 40 4 **18.** 162 16.2 **19.** 5.9 0.59
20. 100 10 **21.** 4.07 0.407 **22.** 0.777 0.0777
23. 10 1 **24.** 1.54 0.154 **25.** 1050 105
26. 3.5 0.35 **27.** 13.723 1.3723 **28.** 1.05 0.105
29. 9.81 0.981 **30.** 3.142 0.3142
31. 28 2.8 **32.** 43.3 4.33

Divide by 100.

33. 4500 45 **34.** 2.8 0.028 **35.** 6700 67
36. 1230 12.3 **37.** 57.75 0.5775 **38.** 105 1.05
39. 6 0.06 **40.** 7.2 0.072 **41.** 1 0.01
42. 0.07 0.0007 **43.** 0.5 0.005 **44.** 1.8 0.018
45. 411.15 4.1115 **46.** 777 7.77
47. 1.6 0.016 **48.** 0.09 0.0009

Multiply by 0.01.

49. 4500 45 **50.** 2.8 0.028 **51.** 6700 67
52. 1230 12.3 **53.** 57.75 0.5775 **54.** 105 1.05
55. 10 0.1 **56.** 1.83 0.0183 **57.** 1050 10.5
58. 3.5 0.035 **59.** 13.7 0.137 **60.** 1.05 0.0105
61. 437 4.37 **62.** 78 0.78
63. 43.6 0.436 **64.** 100.04 1.0004

Divide by 1000.

65. 1000 1 **66.** 5000 5 **67.** 0.3 0.0003
68. 49 0.049 **69.** 6.7 0.0067 **70.** 450 0.45
71. 6 0.006 **72.** 7.2 0.0072 **73.** 1 0.001
74. 3.6 0.0036 **75.** 0.5 0.0005 **76.** 1.8 0.0018
77. 19.8 0.0198 **78.** 199.9 0.1999
79. 24 0.024 **80.** 700 060 700.06

Multiply by 0.001.

81. 1000 1 **82.** 5000 5 **83.** 0.3 0.0003
84. 49 0.049 **85.** 6.7 0.0067 **86.** 450 0.45
87. 0.2 0.0002 **88.** 150 0.15 **89.** 3.8 0.0038
90. 3.5 0.0035 **91.** 13.7 0.0137 **92.** 1.05 0.00105
93. 199.4 0.1994 **94.** 70 707 70.707
95. 1.7 0.0017 **96.** 14.8 0.0148

Divide by 200.

97. 30 000 150 **98.** 400 000 2000
99. 600 3 **100.** 845.48 4.2274
101. 1000.5 5.0025 **102.** 0.06 0.0003

Multiply by 0.005.

103. 30 000 150 **104.** 400 000 2000
105. 6000 30 **106.** 8454.8 42.274
107. 3.5 0.0175 **108.** 13.7 0.0685

Divide by 500.

109. 55 000 110 **110.** 60 0.12
111. 850 000 1700 **112.** 7047 14.094
113. 1798 3.596 **114.** 7485 14.97

Multiply by 0.002.

115. 55 000 110 **116.** 60 0.12
117. 8 500 000 17 000 **118.** 7047 14.094
119. 459 0.918 **120.** 15 0.03

334

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Divide by 10, writing only the quotients	1-16
Multiply by 0.1, writing only the products	17-32
Divide by 100, writing only the quotients	33-48
Multiply by 0.01, writing only the products	49-64
Divide by 1000, writing only the quotients	65-80
Multiply by 0.001, writing only the products	81-96
Divide by 200 (extension)	97-102
Multiply by 0.005 (extension)	103-108
Divide by 500 (extension)	109-114
Multiply by 0.002 (extension)	115-120

Comments

For Ex. 1-96, it is recommended that the students read the exercises and write only the results. These exercises may also be used for oral practice. Exercises such as Ex. 1 and 17 show that dividing by 10 gives the same result as multiplying by 0.1. Exercises such as Ex. 33 and 49 show that dividing by 100 gives the same result as multiplying by 0.01. Exercises such as Ex. 65 and 81 show that dividing by 1000 gives the same result as multiplying by 0.001. Ask the students to identify and describe other similar pairs of exercises.

To complete Ex. 97-120, students can apply the skills of dividing by 100 and of multiplying by 0.001. For example, to divide by 200 (Ex. 97-102), they can divide by 100 and then divide the result by 2. To multiply by 0.005 (Ex. 103-108), they can multiply by 0.001 and then multiply the result by 5. Also, it can be seen that dividing by 200 gives the same result as multiplying by 0.005. For example, the results for 30 000 ÷ 200 = 150 (Ex. 97) and 30 000 × 0.005 = 150 (Ex. 103) are identical.

Skill Practice – Adding and Subtracting Fractions

Add.

1. $\frac{1}{6} + \frac{4}{6}$ = $\frac{5}{6}$
2. $\frac{4}{7} + \frac{2}{7}$ = $\frac{6}{7}$
3. $\frac{2}{10} + \frac{7}{10}$ = $\frac{9}{10}$
4. $1\frac{4}{8} + 2\frac{3}{8}$ = $3\frac{7}{8}$
5. $\frac{6}{12} + \frac{5}{12}$ = $\frac{11}{12}$
6. $3\frac{2}{9} + 4\frac{5}{9}$ = $7\frac{7}{9}$

Subtract.

7. $\frac{5}{6} - \frac{1}{6}$ = $\frac{4}{6}$
8. $\frac{5}{7} - \frac{2}{7}$ = $\frac{3}{7}$
9. $\frac{7}{10} - \frac{2}{10}$ = $\frac{5}{10}$
10. $\frac{6}{8} - \frac{3}{8}$ = $\frac{3}{8}$
11. $3\frac{9}{12} - 1\frac{5}{12}$ = $2\frac{4}{12}$
12. $4\frac{4}{5} - 2\frac{3}{5}$ = $2\frac{1}{5}$

Add. Give the sums in lowest terms.

13. $\frac{3}{5} + \frac{3}{10}$ = $\frac{9}{10}$
14. $\frac{1}{8} + \frac{1}{4}$ = $\frac{3}{8}$
15. $\frac{1}{2} + \frac{1}{6}$ = $\frac{2}{3}$
16. $\frac{1}{12} + \frac{2}{3}$ = $\frac{3}{4}$
17. $\frac{1}{2} + \frac{1}{10}$ = $\frac{3}{5}$
18. $\frac{1}{3} + \frac{4}{9}$ = $\frac{7}{9}$
19. $\frac{2}{3} + \frac{1}{6}$ = $\frac{5}{6}$
20. $\frac{11}{20} + \frac{1}{5}$ = $\frac{3}{4}$
21. $\frac{2}{5} + \frac{3}{20}$ = $\frac{11}{20}$
22. $\frac{2}{3} + \frac{1}{4}$ = $\frac{11}{12}$
23. $\frac{1}{2} + \frac{3}{7}$ = $\frac{13}{14}$
24. $\frac{1}{4} + \frac{5}{8}$ = $\frac{7}{8}$
25. $\frac{3}{20} + \frac{1}{4}$ $\frac{2}{5}$
26. $\frac{1}{3} + \frac{3}{5}$ $\frac{14}{15}$
27. $\frac{3}{10} + \frac{3}{20}$ $\frac{9}{20}$
28. $\frac{3}{4} + \frac{1}{5}$ $\frac{19}{20}$
29. $\frac{1}{2} + \frac{1}{3}$ $\frac{5}{6}$
30. $\frac{2}{9} + \frac{1}{2}$ $\frac{13}{18}$
31. $\frac{1}{8} + \frac{3}{4} + \frac{1}{16}$ $\frac{15}{16}$
32. $\frac{3}{8} + \frac{1}{4} + \frac{3}{16}$ $\frac{13}{16}$
33. $\frac{1}{12} + \frac{1}{4} + \frac{1}{3}$ $\frac{2}{3}$
34. $\frac{1}{5} + \frac{3}{10} + \frac{7}{20}$ $\frac{17}{20}$
35. $\frac{3}{4} + \frac{1}{10} + \frac{1}{20}$ $\frac{9}{10}$
36. $\frac{1}{6} + \frac{1}{8} + \frac{5}{12}$ $\frac{17}{24}$
37. $1\frac{3}{4} + 2\frac{1}{2}$ = $4\frac{1}{4}$
38. $3\frac{3}{8} + 1\frac{15}{16}$ = $5\frac{5}{16}$
39. $4\frac{1}{2} + 2\frac{5}{6}$ = $7\frac{1}{3}$
40. $2\frac{3}{4} + 3\frac{7}{16}$ = $6\frac{3}{16}$
41. $7\frac{5}{9} + 4\frac{2}{3}$ = $12\frac{2}{9}$
42. $5\frac{3}{5} + \frac{7}{10}$ = $6\frac{3}{10}$
43. $1\frac{7}{20} + 2\frac{7}{10}$ = $4\frac{1}{20}$
44. $3\frac{9}{16} + \frac{7}{8}$ = $4\frac{7}{16}$
45. $7\frac{8}{9} + 1\frac{1}{3}$ = $9\frac{2}{9}$
46. $9\frac{1}{2} + 10\frac{7}{10}$ = $20\frac{1}{5}$
47. $6\frac{3}{8} + 1\frac{3}{4}$ $8\frac{1}{8}$
48. $1\frac{9}{10} + \frac{11}{20}$ $2\frac{9}{20}$
49. $2\frac{4}{5} + 3\frac{1}{5}$ 6
50. $6\frac{5}{9} + 1\frac{1}{2}$ $8\frac{1}{18}$
51. $\frac{13}{20} + 1\frac{7}{10}$ $2\frac{7}{20}$
52. $4\frac{8}{9} + 3\frac{1}{3}$ $8\frac{2}{9}$
53. $9\frac{13}{18} + \frac{7}{9}$ $10\frac{1}{2}$
54. $10\frac{4}{7} + 2\frac{1}{2}$ $13\frac{1}{14}$
55. $7\frac{3}{8} + \frac{5}{8}$ 8
56. $5\frac{13}{20} + 3\frac{3}{5}$ $9\frac{1}{4}$
57. $3\frac{3}{4} + 1\frac{11}{20}$ $5\frac{3}{10}$
58. $\frac{2}{3} + 4\frac{4}{5}$ $5\frac{7}{15}$

335

Page 335

OBJECTIVE

Demonstrate competence in adding and subtracting fractions and numbers in mixed form

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Add fractions and numbers in mixed form with like denominators. no regrouping	1-6
Subtract fractions and numbers in mixed form with like denominators, no regrouping	7-12
Add two or three fractions with unlike denominators, no regrouping, sums expressed in lowest terms	13-36
Add fractions and numbers in mixed form, like and unlike denominators, regrouping, sums expressed in lowest terms	37-58

OBJECTIVE

Demonstrate competence in subtracting fractions and numbers in mixed form, in multiplying two fractions, and in multiplying fractions and whole numbers

Skill Practice – Subtracting and Multiplying Fractions

Subtract. Give the differences in lowest terms.

1. $\begin{array}{r} \frac{4}{5} \\ \underline{\frac{7}{10}} \\ \frac{1}{10} \end{array}$
2. $\begin{array}{r} \frac{5}{6} \\ \underline{\frac{2}{3}} \\ \frac{1}{6} \end{array}$
3. $\begin{array}{r} \frac{2}{3} \\ \underline{\frac{7}{12}} \\ \frac{1}{12} \end{array}$
4. $\begin{array}{r} \frac{13}{20} \\ \underline{\frac{2}{5}} \\ \frac{1}{4} \end{array}$
5. $\begin{array}{r} \frac{1}{2} \\ \underline{\frac{3}{16}} \\ \frac{5}{16} \end{array}$
6. $\begin{array}{r} \frac{7}{8} \\ \underline{\frac{7}{16}} \\ \frac{7}{16} \end{array}$
7. $\begin{array}{r} \frac{2}{3} \\ \underline{\frac{1}{9}} \\ \frac{5}{9} \end{array}$
8. $\begin{array}{r} \frac{3}{4} \\ \underline{\frac{3}{8}} \\ \frac{3}{8} \end{array}$
9. $\begin{array}{r} \frac{15}{16} \\ \underline{\frac{7}{8}} \\ \frac{1}{16} \end{array}$
10. $\begin{array}{r} \frac{13}{16} \\ \underline{\frac{3}{4}} \\ \frac{1}{16} \end{array}$
11. $\begin{array}{r} \frac{11}{16} \\ \underline{\frac{1}{2}} \\ \frac{3}{16} \end{array}$
12. $\begin{array}{r} \frac{4}{5} \\ \underline{\frac{11}{20}} \\ \frac{1}{4} \end{array}$
13. $\begin{array}{r} 4\frac{2}{3} \\ \underline{1\frac{11}{12}} \\ 2\frac{3}{4} \end{array}$
14. $\begin{array}{r} 5\frac{3}{5} \\ \underline{4\frac{7}{10}} \\ \frac{9}{10} \end{array}$
15. $\begin{array}{r} 6\frac{1}{3} \\ \underline{5\frac{5}{6}} \\ \frac{1}{2} \end{array}$
16. $\begin{array}{r} 1\frac{1}{16} \\ \underline{\frac{5}{8}} \\ \frac{7}{16} \end{array}$
17. $\begin{array}{r} 7\frac{1}{3} \\ \underline{5\frac{7}{9}} \\ 1\frac{5}{9} \end{array}$
18. $\begin{array}{r} 9\frac{1}{10} \\ \underline{4\frac{1}{2}} \\ 4\frac{3}{5} \end{array}$
19. $\begin{array}{r} 8\frac{3}{10} \\ \underline{3\frac{4}{5}} \\ 4\frac{1}{2} \end{array}$
20. $\begin{array}{r} 7\frac{17}{20} \\ \underline{2\frac{9}{10}} \\ 4\frac{19}{20} \end{array}$
21. $\begin{array}{r} 4\frac{1}{7} \\ \underline{\frac{1}{3}} \\ 3\frac{17}{21} \end{array}$
22. $\begin{array}{r} 3\frac{2}{7} \\ \underline{1\frac{2}{3}} \\ 1\frac{13}{21} \end{array}$
23. $\begin{array}{r} 4\frac{3}{7} \\ \underline{3\frac{1}{2}} \\ \frac{13}{14} \end{array}$
24. $\begin{array}{r} 3\frac{5}{8} \\ \underline{2\frac{2}{3}} \\ \frac{23}{24} \end{array}$

25. $6 - \frac{17}{20}$ $5\frac{3}{20}$
26. $7 - \frac{15}{16}$ $6\frac{1}{16}$
27. $2 - \frac{7}{12}$ $1\frac{5}{12}$
28. $3 - \frac{2}{3}$ $2\frac{1}{3}$
29. $5\frac{1}{12} - 1\frac{1}{2}$ $3\frac{7}{12}$
30. $6\frac{1}{2} - 1\frac{7}{12}$ $4\frac{11}{12}$
31. $3\frac{7}{8} - 1\frac{15}{16}$ $1\frac{15}{16}$
32. $3\frac{1}{4} - 2\frac{13}{16}$ $\frac{7}{16}$

Multiply. Give the products in lowest terms.

33. $\frac{1}{2} \times \frac{3}{4}$ $\frac{3}{8}$
34. $\frac{7}{8} \times \frac{5}{6}$ $\frac{35}{48}$
35. $\frac{3}{5} \times \frac{1}{5}$ $\frac{3}{25}$
36. $\frac{1}{6} \times \frac{5}{6}$ $\frac{5}{36}$
37. $\frac{3}{10} \times \frac{2}{5}$ $\frac{3}{25}$
38. $\frac{5}{12} \times \frac{4}{10}$ $\frac{1}{6}$
39. $\frac{8}{9} \times \frac{5}{12}$ $\frac{10}{27}$
40. $\frac{7}{9} \times \frac{5}{7}$ $\frac{5}{9}$
41. $\frac{1}{4} \times \frac{3}{5}$ $\frac{3}{20}$
42. $\frac{7}{8} \times \frac{2}{3}$ $\frac{7}{12}$
43. $\frac{4}{5} \times \frac{1}{2}$ $\frac{2}{5}$
44. $\frac{3}{7} \times \frac{5}{6}$ $\frac{5}{14}$
45. $\frac{3}{8} \times \frac{2}{9}$ $\frac{1}{12}$
46. $\frac{3}{4} \times \frac{6}{7}$ $\frac{9}{14}$
47. $\frac{1}{9} \times \frac{1}{9}$ $\frac{1}{81}$
48. $\frac{2}{5} \times \frac{4}{7}$ $\frac{8}{35}$
49. $\frac{4}{5} \times \frac{3}{10}$ $\frac{6}{25}$
50. $\frac{3}{7} \times \frac{1}{2}$ $\frac{3}{14}$
51. $\frac{2}{5} \times \frac{1}{3}$ $\frac{2}{15}$
52. $\frac{3}{8} \times \frac{3}{8}$ $\frac{9}{64}$
53. $8 \times \frac{2}{3}$ $\frac{16}{3}$ or $5\frac{1}{3}$
54. $7 \times \frac{5}{7}$ 5
55. $\frac{3}{4} \times 11$ $\frac{33}{4}$ or $8\frac{1}{4}$
56. $15 \times \frac{2}{5}$ 6
57. $\frac{3}{5} \times 3$ $\frac{9}{5}$ or $1\frac{4}{5}$
58. $42 \times \frac{2}{7}$ 12
59. $7 \times \frac{1}{12}$ $\frac{7}{12}$
60. $\frac{5}{9} \times 18$ 10
61. $27 \times \frac{2}{9}$ 6
62. $\frac{7}{12} \times 18$ $\frac{21}{2}$ or $10\frac{1}{2}$
63. $\frac{4}{7} \times 21$ 12
64. $4 \times \frac{3}{8}$ $\frac{3}{2}$ or $1\frac{1}{2}$
65. $\frac{2}{3} \times 4$ $\frac{8}{3}$ or $2\frac{2}{3}$
66. $3 \times \frac{2}{9}$ $\frac{2}{3}$
67. $15 \times \frac{3}{10}$ $\frac{9}{2}$ or $4\frac{1}{2}$
68. $\frac{5}{6} \times 2$ $\frac{5}{3}$ or $1\frac{2}{3}$

336

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Subtract fractions with unlike denominators, no regrouping, differences expressed in lowest terms	1-12
Subtract fractions or numbers in mixed form from numbers in mixed form with unlike denominators, regrouping, differences expressed in lowest terms	13-24 29-32
Subtract fractions or numbers in mixed form from whole numbers	25-28
Multiply two fractions, products expressed in lowest terms	33-52
Multiply fractions and whole numbers, products expressed in lowest terms	53-68

Skill Practice – Dividing Fractions and Percent

Divide. Give the quotients in lowest terms.

1. $\frac{1}{2} \div 8$ $\frac{1}{16}$	**2.** $\frac{3}{4} \div 10$ $\frac{3}{40}$	**3.** $\frac{7}{8} \div 7$ $\frac{1}{8}$	**4.** $\frac{5}{12} \div 15$ $\frac{1}{36}$
5. $6 \div \frac{3}{4}$ 8	**6.** $2 \div \frac{2}{3}$ 3	**7.** $12 \div \frac{5}{6}$ $\frac{72}{5}$ or $14\frac{2}{5}$	**8.** $12 \div \frac{4}{5}$ 15
9. $\frac{1}{2} \div \frac{1}{2}$ 1	**10.** $\frac{1}{4} \div \frac{3}{4}$ $\frac{1}{3}$	**11.** $\frac{9}{10} \div \frac{3}{20}$ 6	**12.** $\frac{4}{7} \div \frac{2}{5}$ $\frac{10}{7}$ or $1\frac{3}{7}$
13. $\frac{7}{12} \div \frac{6}{7}$ $\frac{49}{72}$	**14.** $\frac{9}{10} \div \frac{3}{5}$ $\frac{3}{2}$ or $1\frac{1}{2}$	**15.** $\frac{11}{12} \div \frac{1}{6}$ $\frac{11}{2}$ or $5\frac{1}{2}$	**16.** $\frac{2}{3} \div 4$ $\frac{1}{6}$
17. $\frac{7}{8} \div \frac{7}{8}$ 1	**18.** $8 \div \frac{4}{7}$ 14	**19.** $\frac{5}{7} \div \frac{5}{9}$ $\frac{9}{7}$ or $1\frac{2}{7}$	**20.** $\frac{1}{5} \div \frac{3}{10}$ $\frac{2}{3}$
21. $1 \div \frac{5}{8}$ $\frac{8}{5}$ or $1\frac{3}{5}$	**22.** $\frac{4}{5} \div \frac{2}{3}$ $\frac{6}{5}$ or $1\frac{1}{5}$	**23.** $\frac{3}{8} \div \frac{9}{10}$ $\frac{5}{12}$	**24.** $14 \div \frac{7}{8}$ 16
25. $\frac{9}{10} \div \frac{3}{4}$ $\frac{6}{5}$ or $1\frac{1}{5}$	**26.** $\frac{7}{8} \div 4$ $\frac{7}{32}$	**27.** $10 \div \frac{2}{5}$ 25	**28.** $\frac{4}{7} \div \frac{4}{5}$ $\frac{5}{7}$
29. $\frac{2}{5} \div 4$ $\frac{1}{10}$	**30.** $\frac{5}{6} \div \frac{1}{3}$ $\frac{5}{2}$ or $2\frac{1}{2}$	**31.** $5 \div \frac{6}{7}$ $\frac{35}{6}$ or $5\frac{5}{6}$	**32.** $\frac{2}{5} \div \frac{1}{2}$ $\frac{4}{5}$

Copy and complete.

	Number \ Percent	1%	7%	15%	25%	35%	50%
33.	1000	10	70	150	250	350	500
34.	50	0.5	3.5	7.5	12.5	17.5	25
35.	84	0.84	5.88	12.6	21	29.4	42
36.	18	0.18	1.26	2.7	4.5	6.3	9
37.	17	0.17	1.19	2.55	4.25	5.95	8.5
38.	5	0.05	0.35	0.75	1.25	1.75	2.5
39.	2	0.02	0.14	0.3	0.5	0.7	1
40.	15	0.15	1.05	2.25	3.75	5.25	7.5
41.	48	0.48	3.36	7.2	12	16.8	24
42.	16	0.16	1.12	2.4	4	5.6	8
43.	57	0.57	3.99	8.55	14.25	19.95	28.5
44.	144	1.44	10.08	21.6	36	50.4	72

Find each of these.

45. 3% of 15 0.45 **46.** 18% of 50 9 **47.** 44% of 25 11 **48.** 90% of 32 28.8

337

Page 337

OBJECTIVE

Demonstrate competence in dividing a fraction by a whole number or a whole number by a fraction, and in dividing a fraction by a fraction; demonstrate competence in finding a percent of a number

The skills required for the exercises on this page are indicated in the following chart.

Skills	Exercises
Divide a fraction by a whole number, quotients expressed in lowest terms	1-4, 16, 26, 29
Divide a whole number by a fraction, quotients expressed in lowest terms	5-8, 18, 21, 24, 27, 31
Divide a fraction by a fraction, quotients expressed in lowest terms	9-15, 17, 19, 20, 22, 23, 25, 28, 30, 32
Find a percent of a number	33-48

Page 39 (Working Together)

7.

Name	Number of stamps
Ian	□□□□□
Tim	□□□□□□
Hilda	□□□□▯
David	□□□□□□□□▯
Betty	□□□▯
Each □ stands for 50 stamps.	

Page 39 (Exercises)

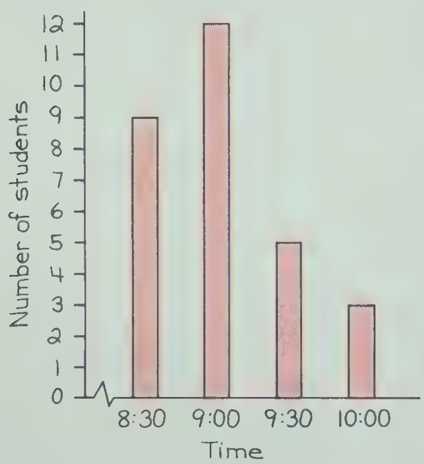
5.

Country	Number of immigrants
Britain	△△△△△△◿
U.S.A.	△△△△△
Portugal	△△△
Hong Kong	△△◿
India	△△
Jamaica	△△
Each △ stands for 10 000 immigrants.	

Page 41 (Exercises)

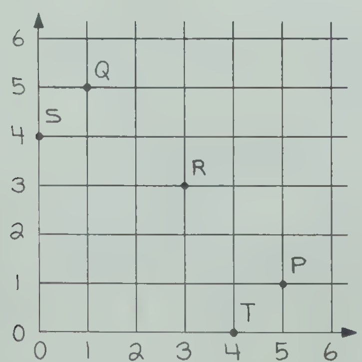
5.

The Time When Students Went to Bed



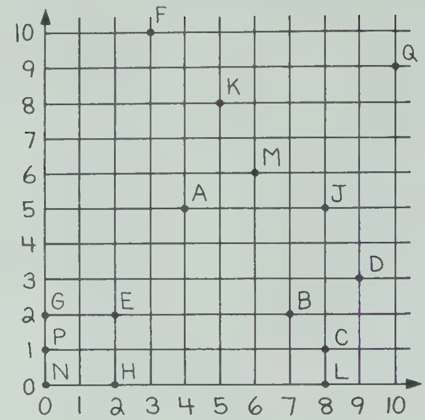
Page 42 (Working Together)

6.-10.



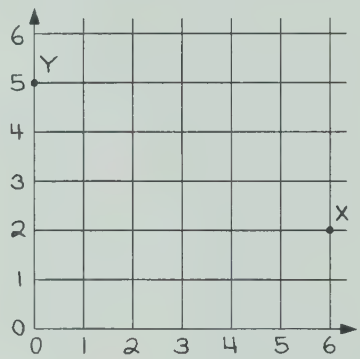
Page 43 (Exercises)

9.-23.



Page T 48 (Checking Prerequisite Skills)

3.-4.

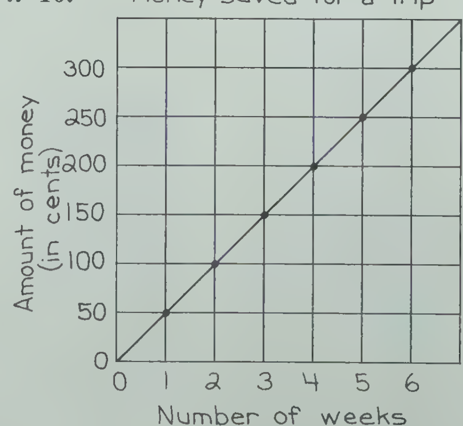


Page 44 (Working Together)

3.

Number of weeks	Amount saved (in cents)	Ordered pair
1	50	(1,50)
2	100	(2,100)
3	150	(3,150)
4	200	(4,200)
5	250	(5,250)
6	300	(6,300)

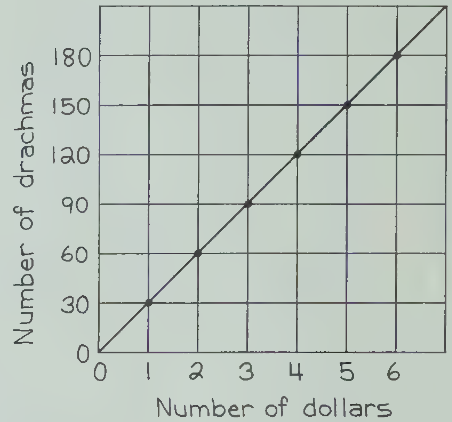
4.-10. Money Saved for a Trip



Page 45 (Exercises)

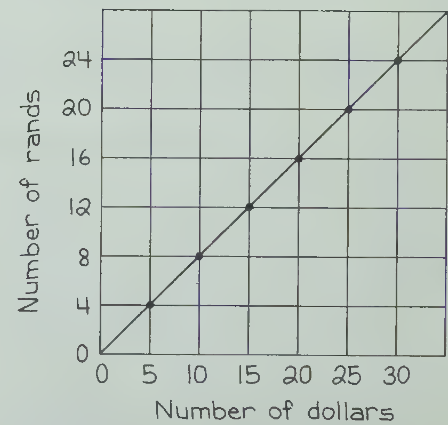
4.

Number of dollars	Number of drachmas	Ordered pair
1	30	(1,30)
2	60	(2,60)
3	90	(3,90)
4	120	(4,120)
5	150	(5,150)
6	180	(6,180)



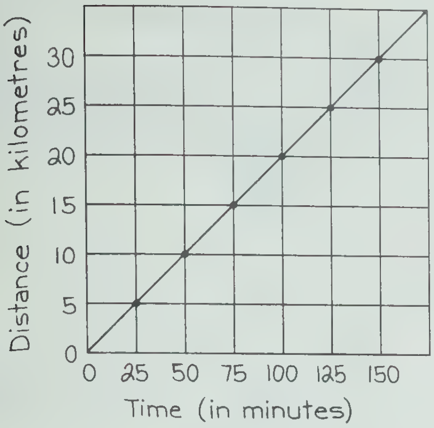
5.

Number of dollars	Number of rands	Ordered pair
5	4	(5,4)
10	8	(10,8)
15	12	(15,12)
20	16	(20,16)
25	20	(25,20)
30	24	(30,24)



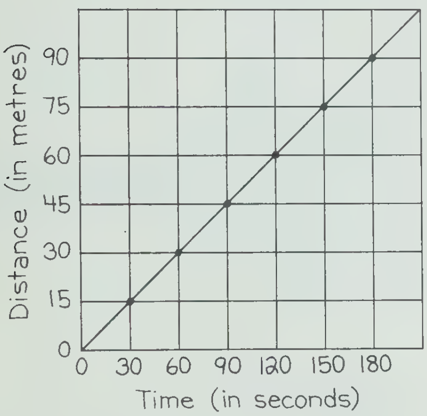
6.

Number of minutes	Number of kilometres	Ordered pair
25	5	(25,5)
50	10	(50,10)
75	15	(75,15)
100	20	(100,20)
125	25	(125,25)
150	30	(150,30)



7.

Number of seconds	Number of metres	Ordered pair
30	15	(30,15)
60	30	(60,30)
90	45	(90,45)
120	60	(120,60)
150	75	(150,75)
180	90	(180,90)

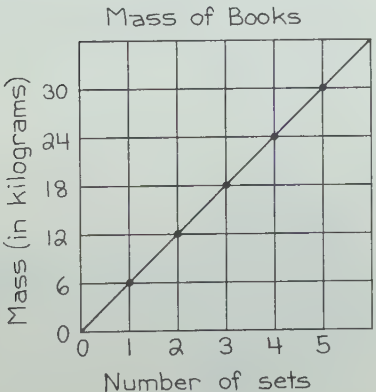


Page T 49 (Assessment)

3.

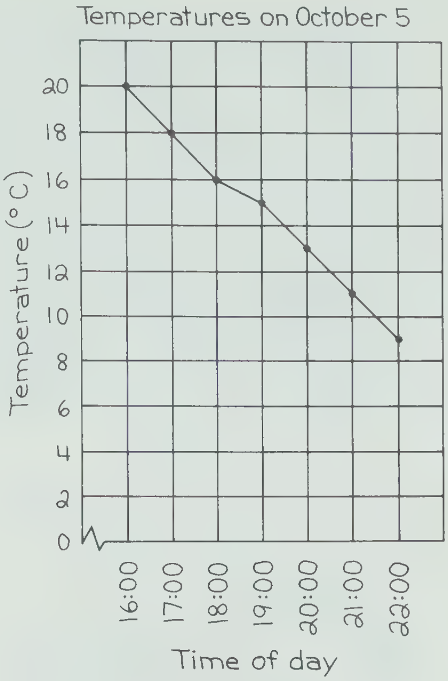
Number of sets	Mass in kilograms	Ordered pair
1	6	(1,6)
2	12	(2,12)
3	18	(3,18)
4	24	(4,24)
5	30	(5,30)

4.



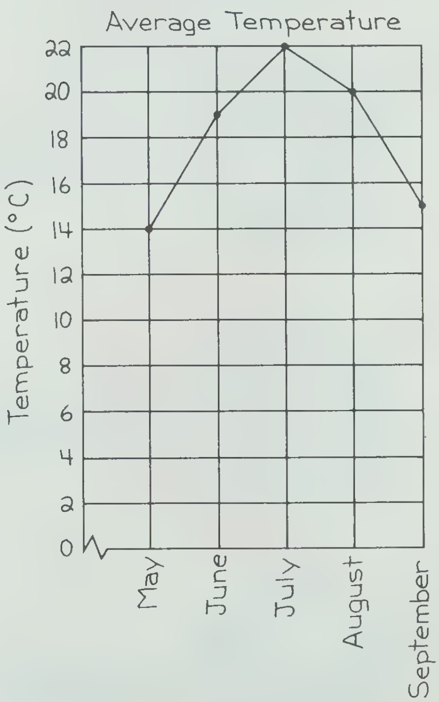
Page 46 (Working Together)

3.-8.



Page T 51 (Assessment)

4.

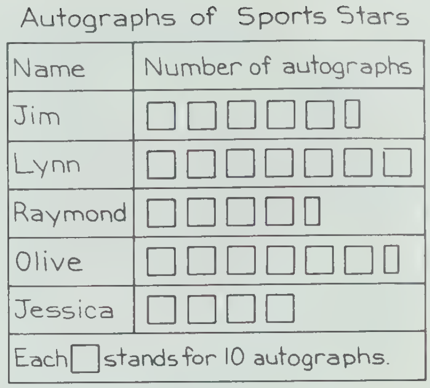


Page 49 (Checking Up)

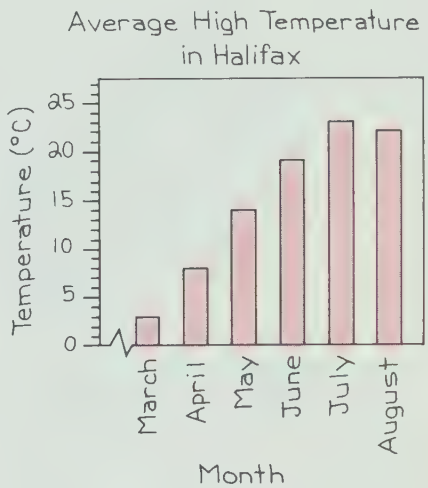
1.

Number of students	Tally	Number of practices
7	||||	4
8	||||	4
9	~~||||~~	5
10	||||	4
11	|||	3
12	||||	4

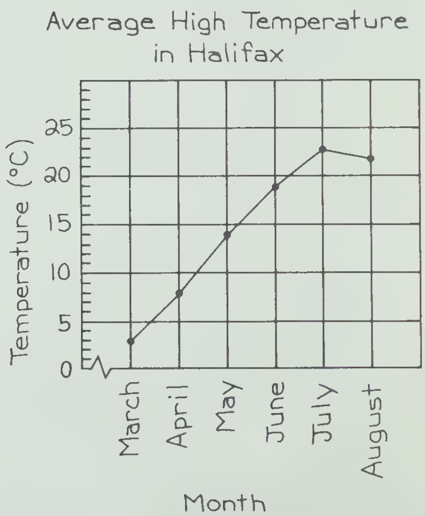
2.



3.



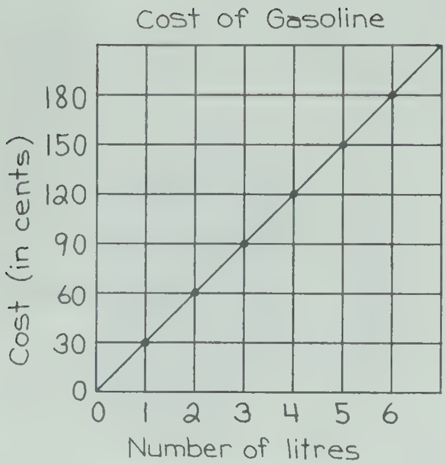
4.



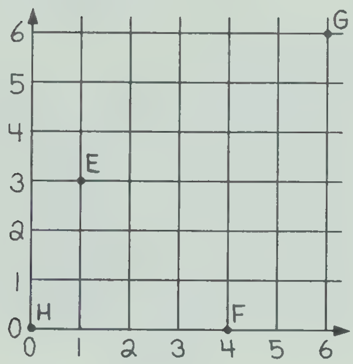
5.

Number of litres	Cost in cents	Ordered pair
1	30	(1,30)
2	60	(2,60)
3	90	(3,90)
4	120	(4,120)
5	150	(5,150)
6	180	(6,180)

6.



11.-14.



Page 53 (Exercises)

25.

625 mL	brown sugar
10 eggs	
375 mL	corn oil
250 mL	orange juice
2500 mL	flour
25 mL	salt
10 mL	baking soda
50 mL	baking powder
1875 mL	mashed ripe bananas
750 mL	chopped walnuts
1000 mL	raisins
25 mL	grated orange rind

Page 117 (Try This)

10.

Base	Height	Perimeter	Area
3 cm	1 cm	8 cm	3 cm²
6 cm	2 cm	16 cm	12 cm²
12 cm	4 cm	32 cm	48 cm²
24 cm	8 cm	64 cm	192 cm²
48 cm	16 cm	128 cm	768 cm²
96 cm	32 cm	256 cm	3 072 cm²
192 cm	64 cm	512 cm	12 288 cm²
384 cm	128 cm	1024 cm	49 152 cm²

Page 169 (Exercises)

15.

Page 171 (Working Together)

1.

2.

3.

4.

Page 171 (Exercises)

6.

7.

8.

9.

10.

11.

Page T 185 (Assessment)

5.

6.

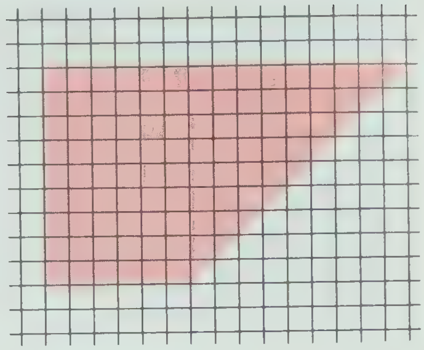
Page 181 (Exercises)

7.

Name of triangle	Kind of triangle	Number of sides of equal length	Number of lines of symmetry	Angle measurements
△ ABC	equilateral	3	3	60°, 60°, 60°
△ DEF	isosceles	2	1	65°, 65°, 50°
△ GHI	scalene	0	0	57°, 97°, 26°
△ JKL	isosceles	2	1	77°, 77°, 26°
△ MNO	scalene	0	0	58°, 35°, 87°
△ PQR	equilateral	3	3	60°, 60°, 60°
△ STU	equilateral	3	3	60°, 60°, 60°
△ VWX	scalene	0	0	35°, 55°, 90°
△ YZA	isosceles	2	1	66°, 48°, 66°

Page T 203 (Assessment)

2.



Page 191 (Exercises)

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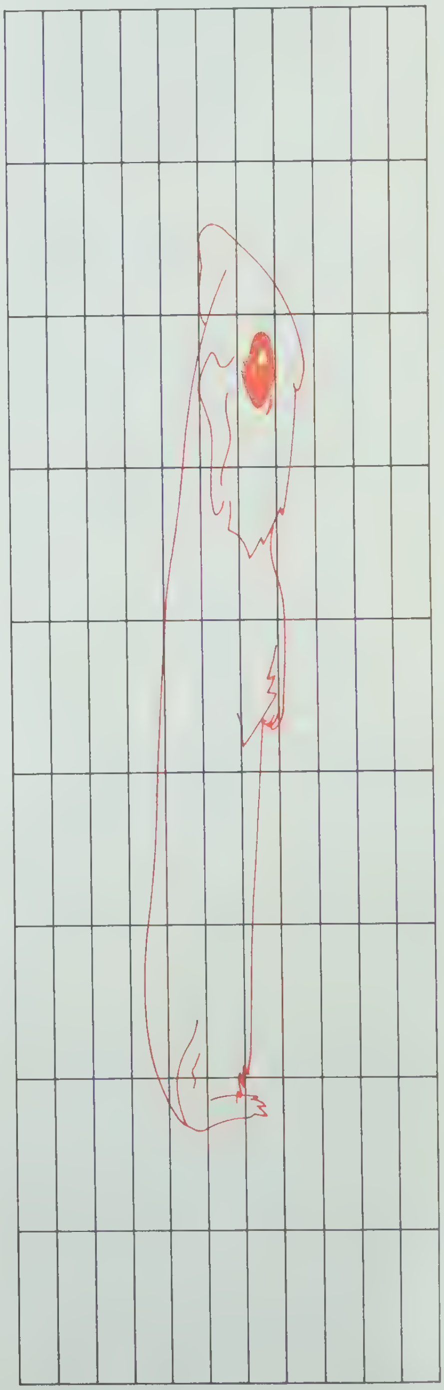

Page 183 (Exercises)

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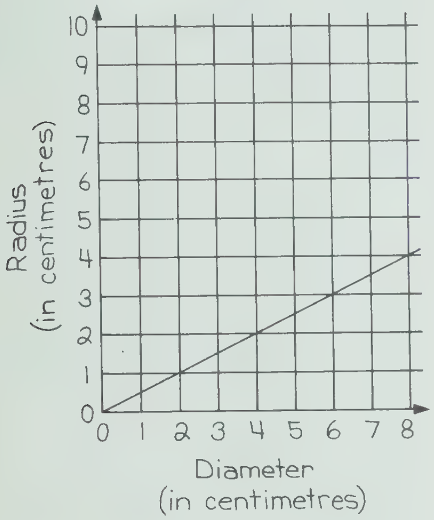

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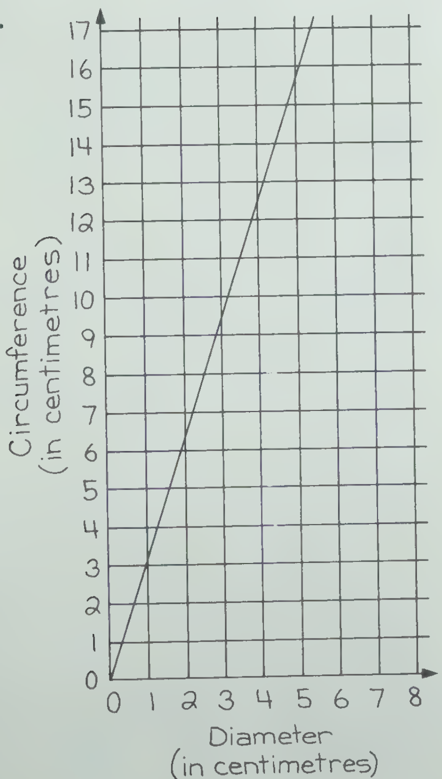

Page 187 (Exercises)

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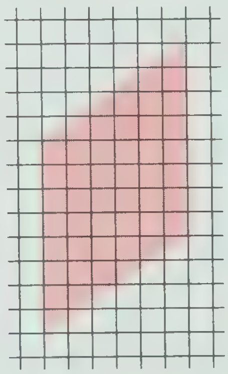

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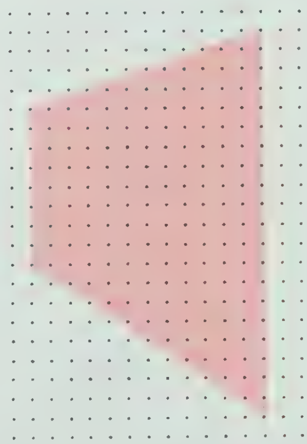

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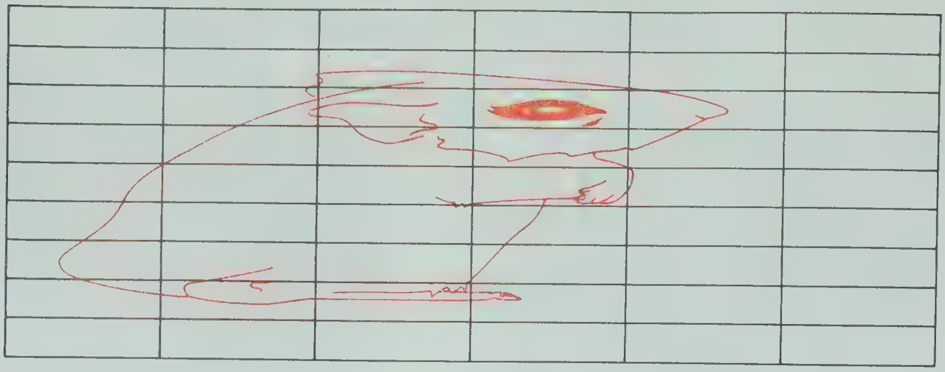

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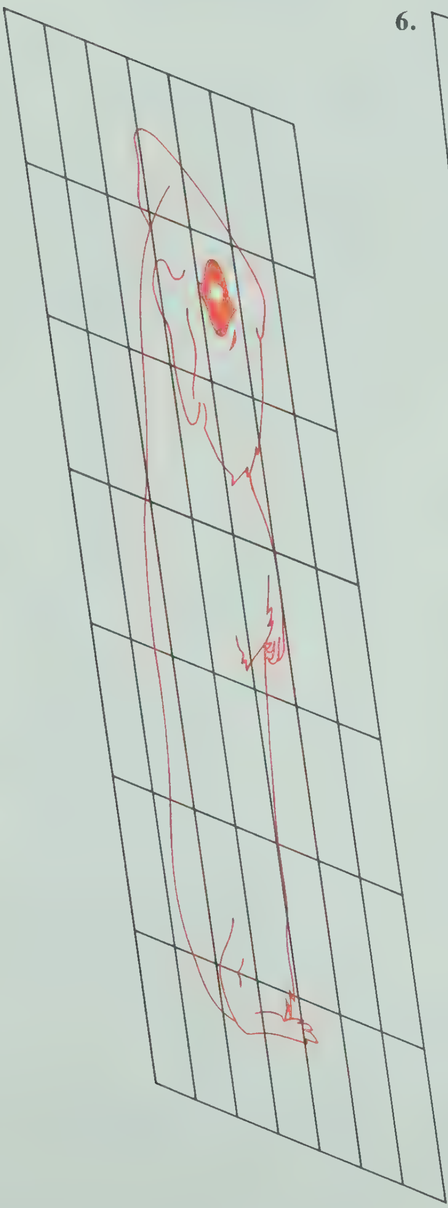

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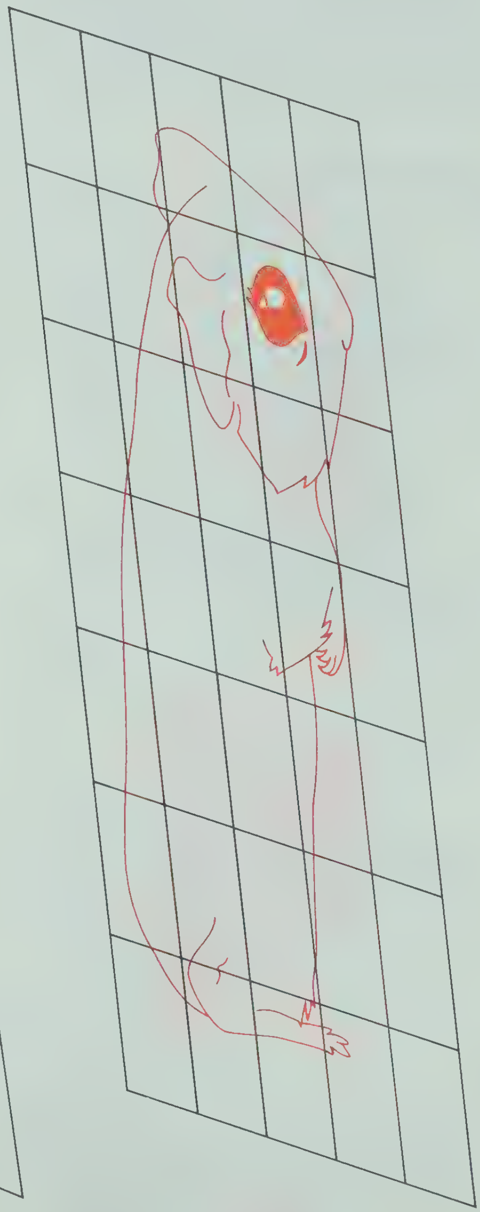

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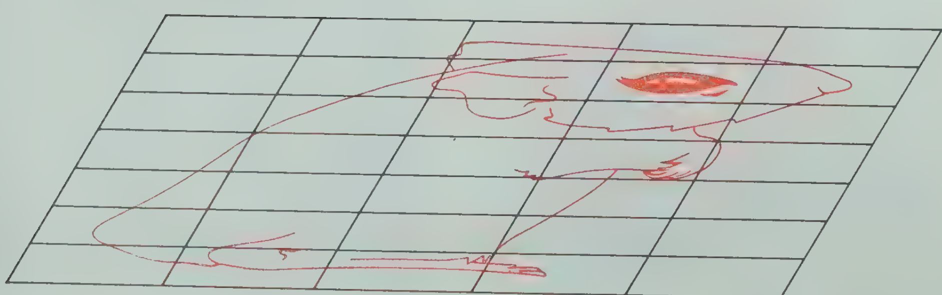

Page 196 (Checking Up)

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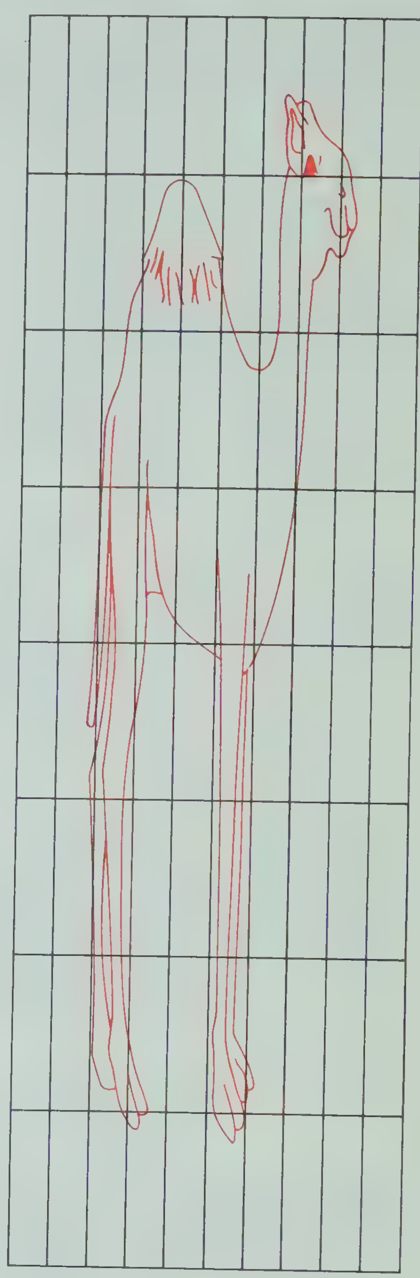


Page 249 (Exercises)

20.

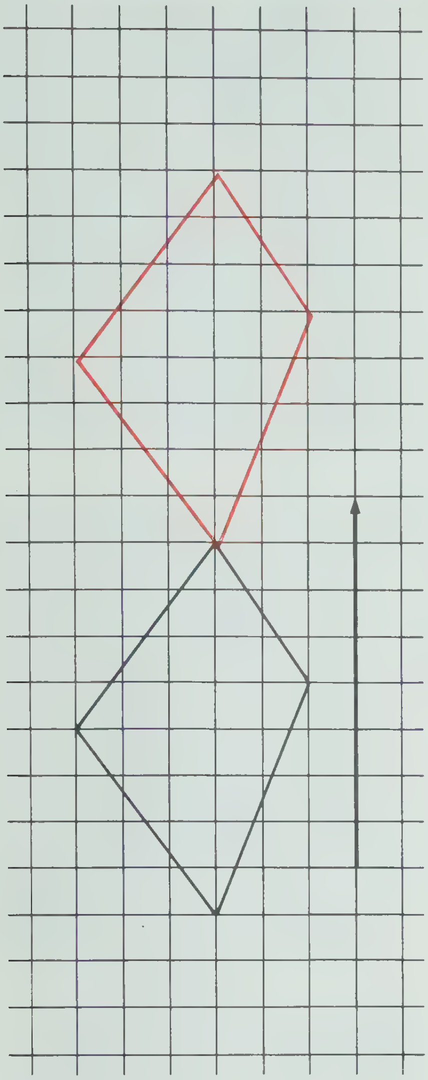
Cartons of white milk	3	6	9	12	15	18
Total number of cartons	6	12	18	24	30	36

Cartons of chocolate milk	2	4	6	8	10	12
Total number of cartons	6	12	18	24	30	36

Cartons of orange drink	1	2	3	4	5	6
Total number of cartons	6	12	18	24	30	36

Page 277 (Exercises)

3.



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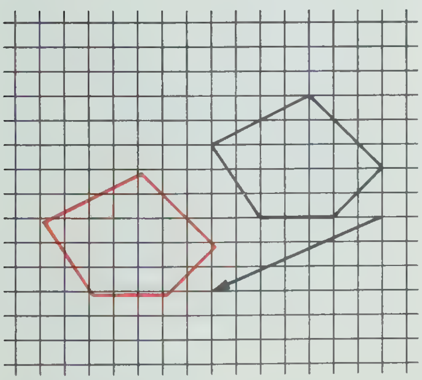

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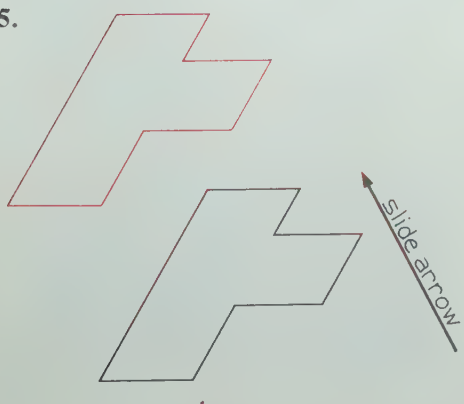

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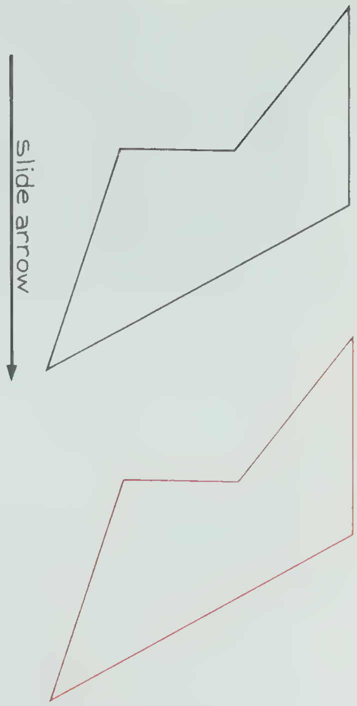

Page T 303 (Assessment)

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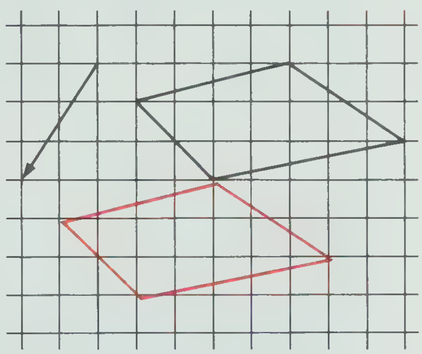

Page 279 (Working Together)

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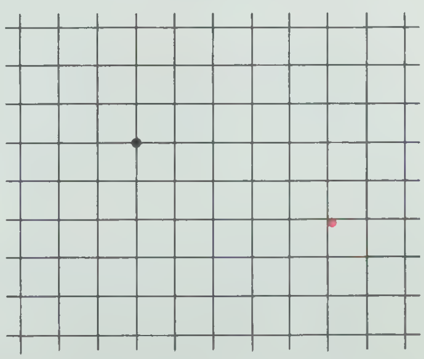

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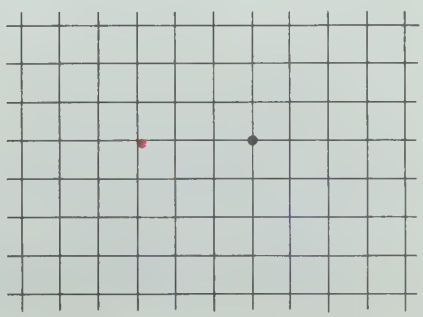

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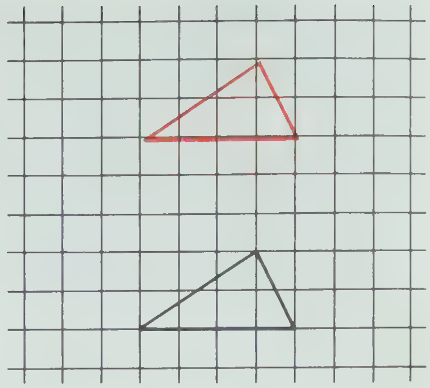

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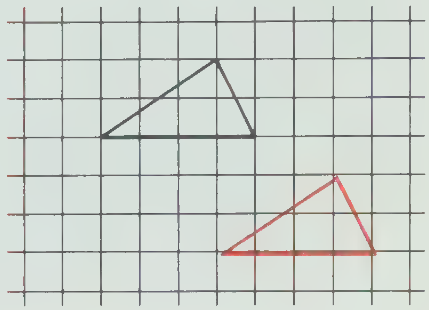

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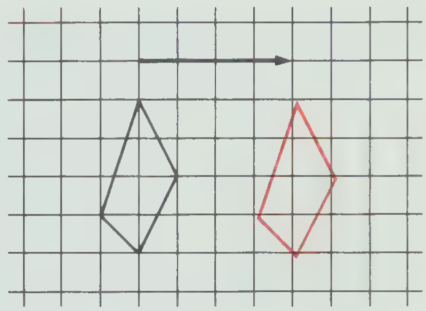

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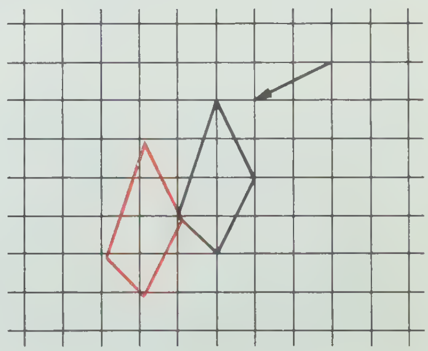

Page 279 (Exercises)

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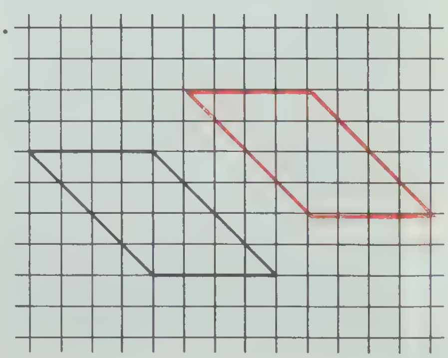

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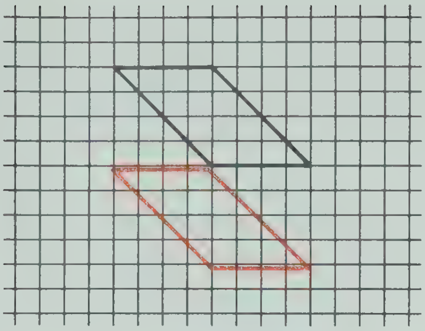

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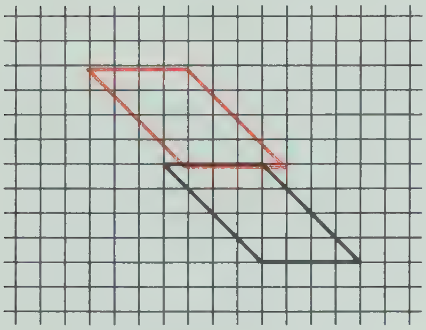

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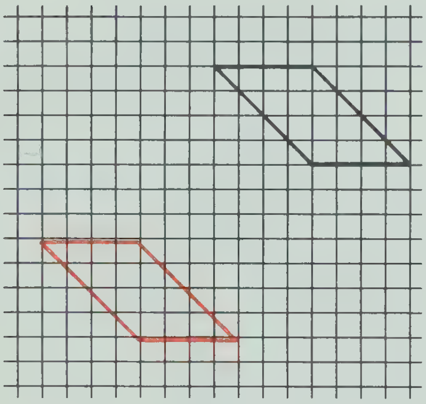

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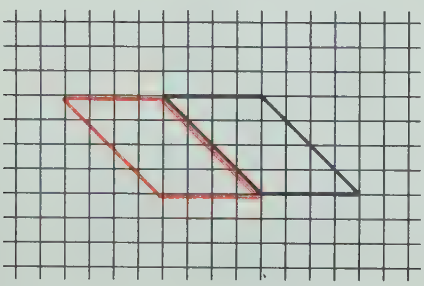

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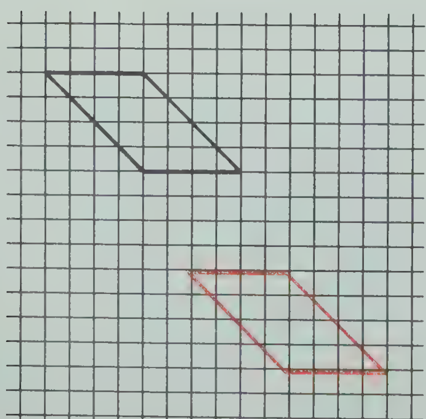

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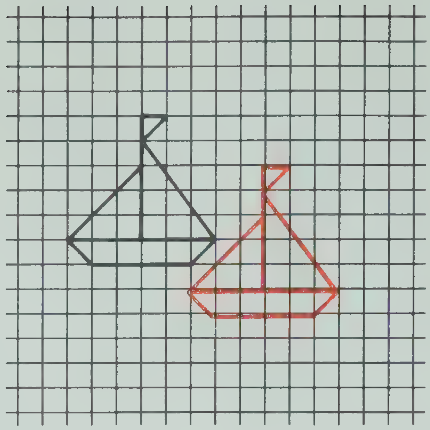

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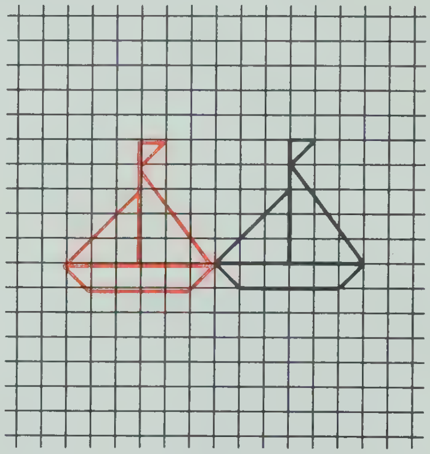

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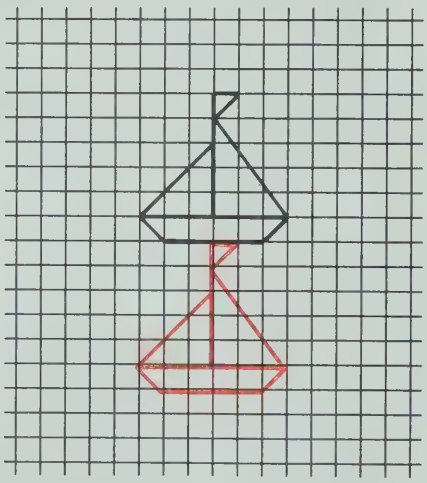

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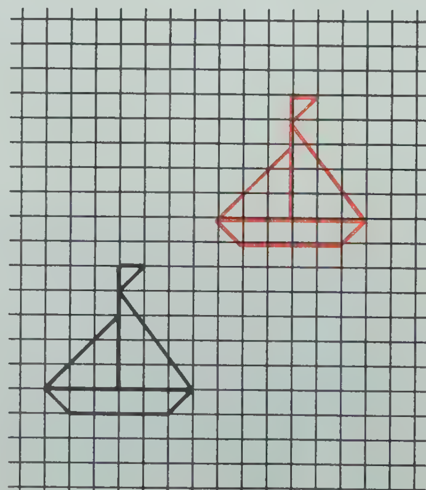

Page T 305 (Assessment)

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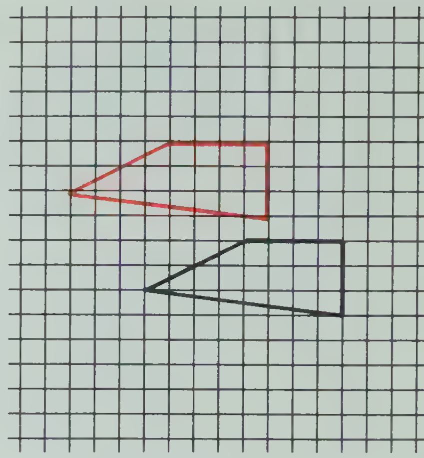

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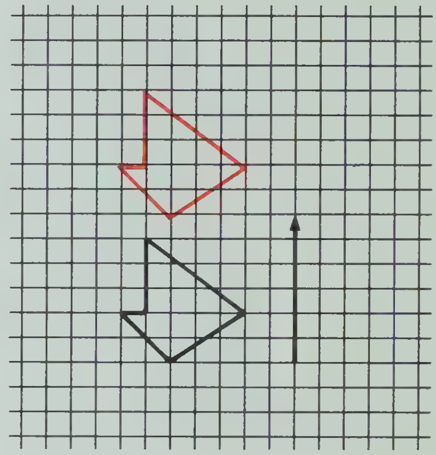

Page 281 (Exercises)

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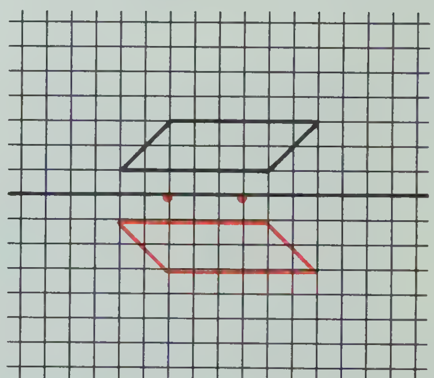

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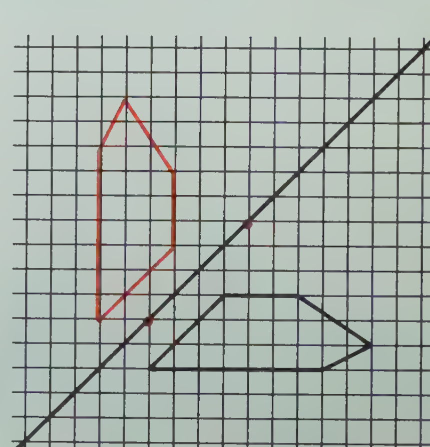

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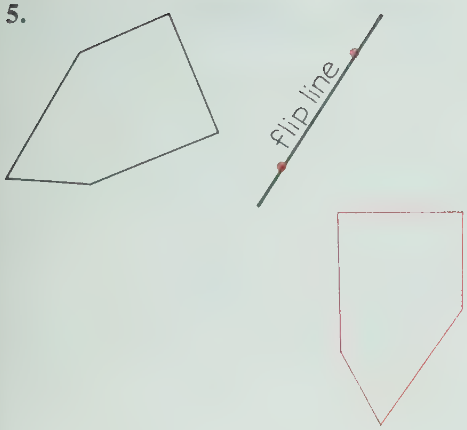

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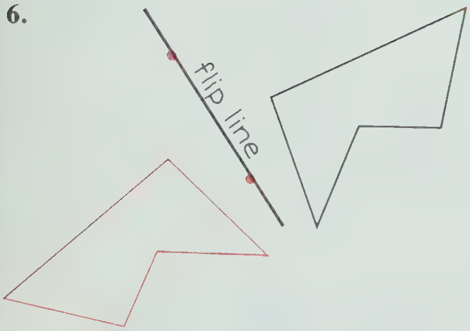

Page T 307 (Assessment)

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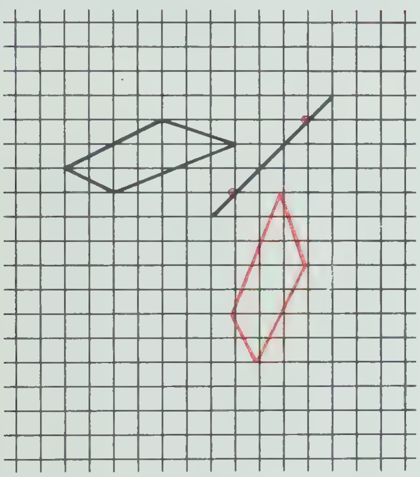

Page 283 (Working Together)

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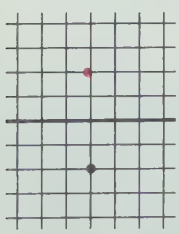

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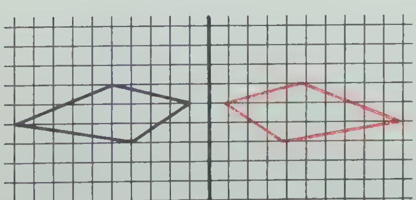

Page 283 (Exercises)

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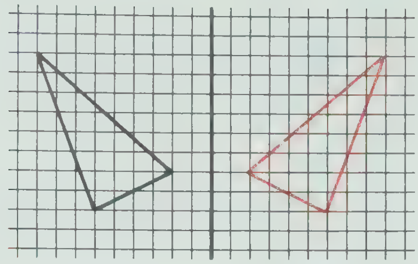

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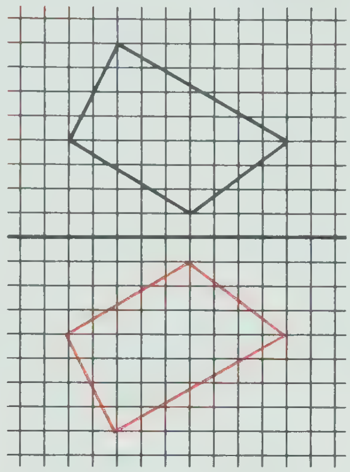

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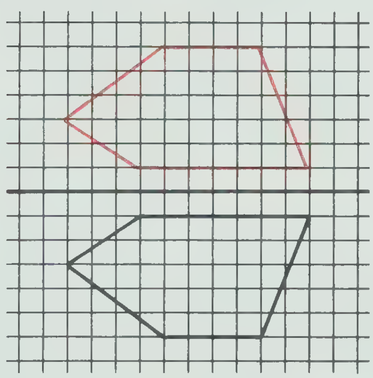

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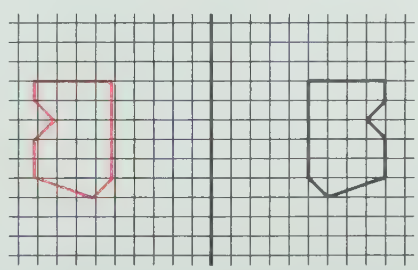

Page 285 (Exercises)

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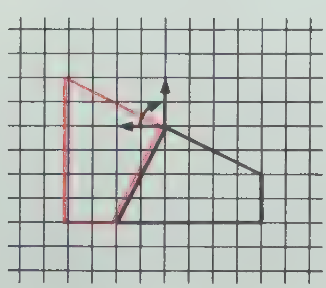

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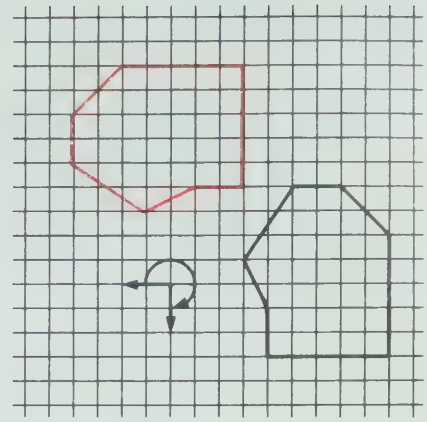

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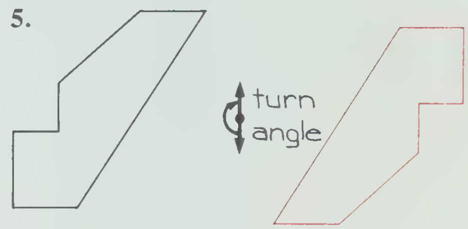

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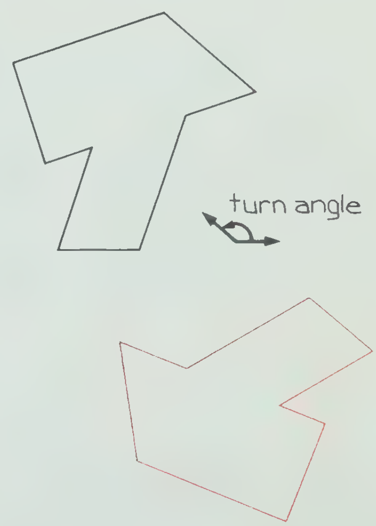

Page T 311 (Assessment)

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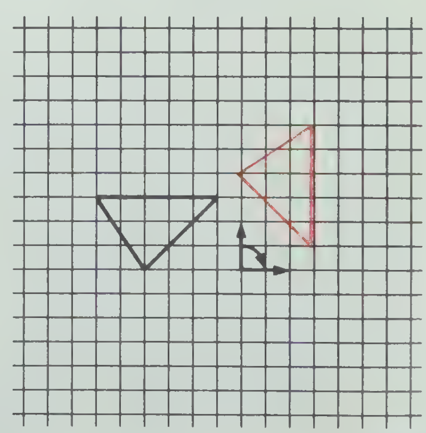

Page 288 (Exercises)

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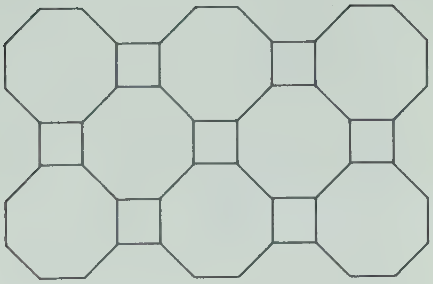

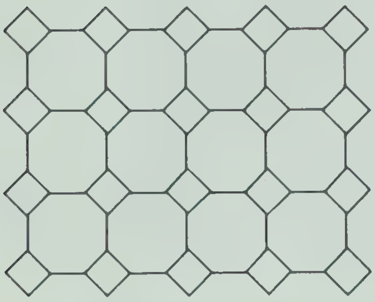

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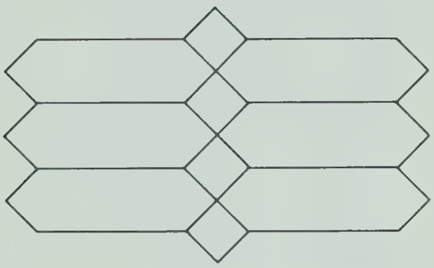

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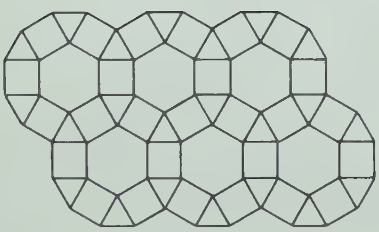

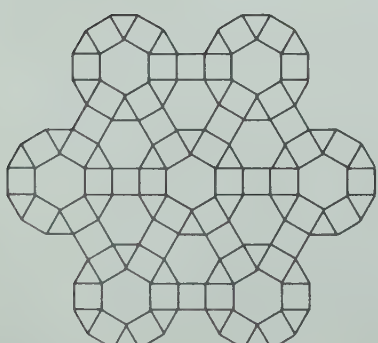

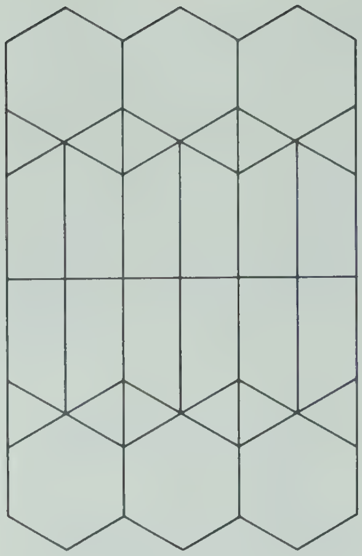

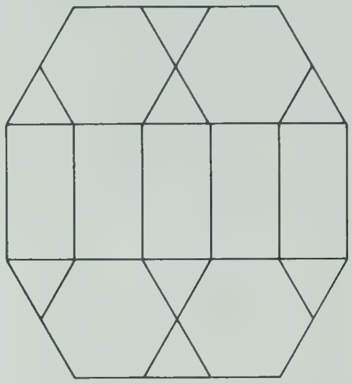

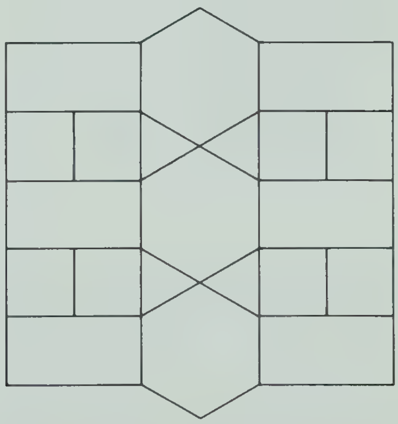

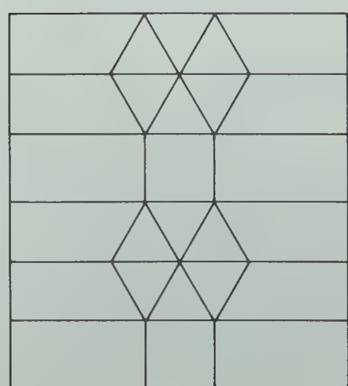

Page T 315 (Assessment)

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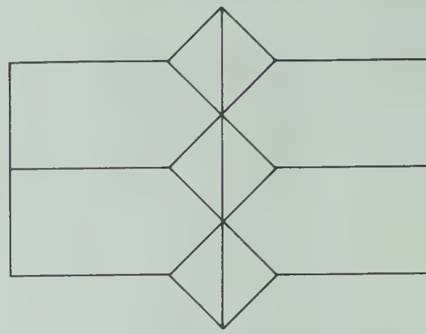

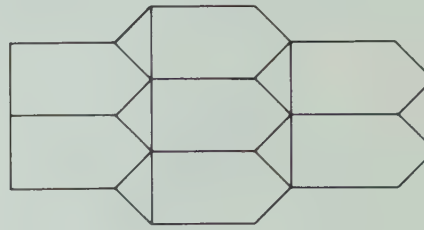

Page 290 (Working Together)

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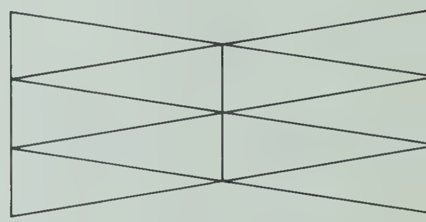

Page 291 (Exercises)

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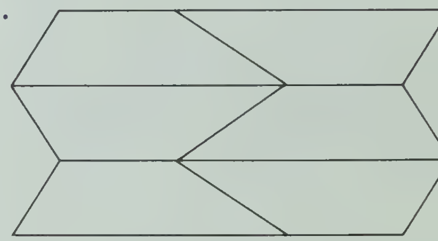

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Page T 317 (Assessment)

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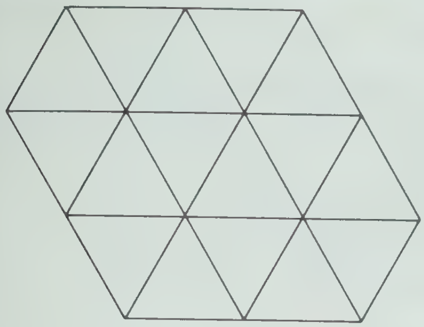

Pages 295-296 (Checking Up)

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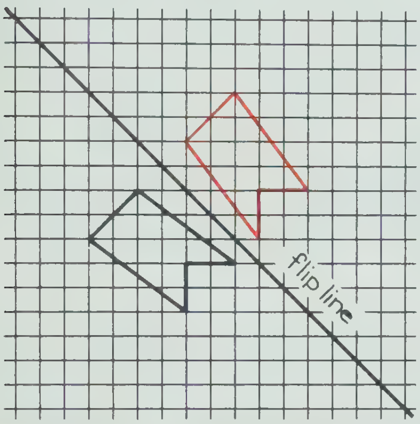

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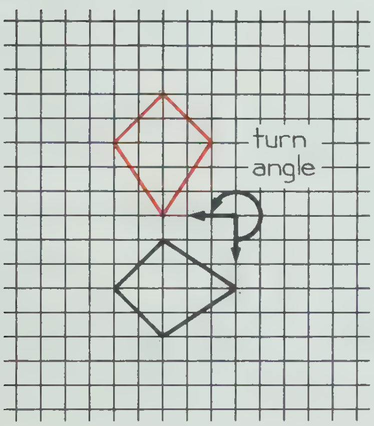

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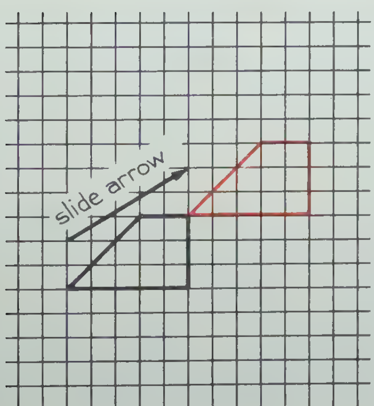

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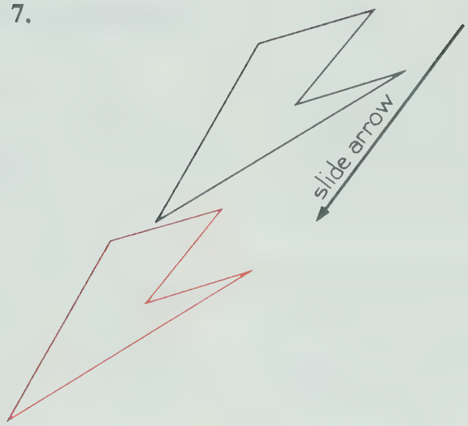

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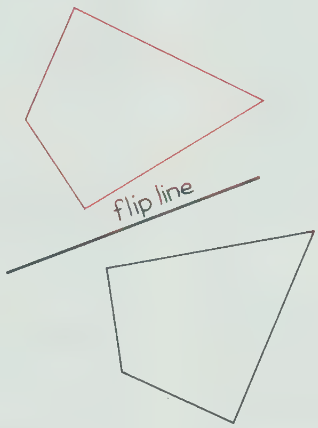

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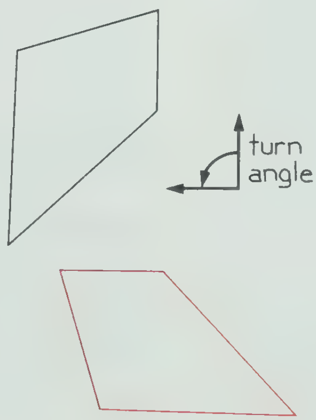

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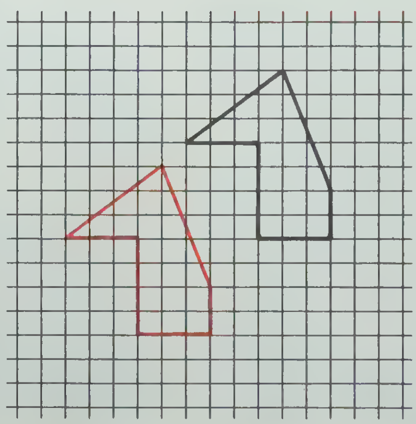

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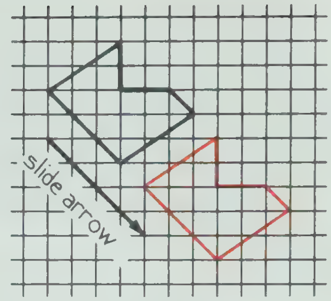

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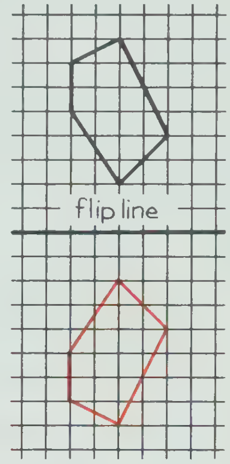
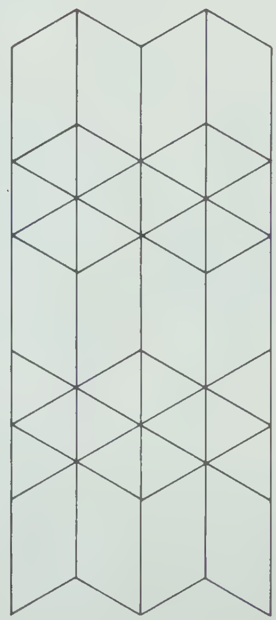


14.



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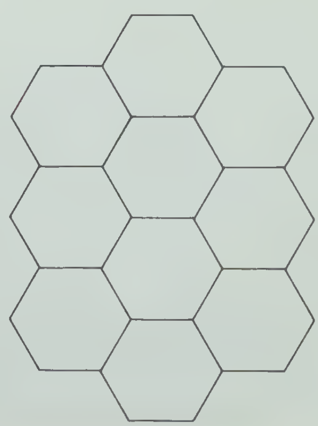


Pages T 379 to T 400 include materials that have been referred to in the teaching suggestions for various lessons. Although suggestions for using many of these materials are given in the related lesson outlines, other suggestions for some of the materials are given below.

Copies of the 10-by-10 grid on page T 382 may be used for preparing individual game boards, for preparing work sheets for activities such as finding the prime numbers less than 100, and for copying pictures from one grid to another. For the work with decimals, copies of this grid may be used for preparing large models for hundredths.

Copies of the 11-by-11 grid on page T 382 may be used for preparing tables of basic addition and multiplication facts. As facts are memorized, students may color inside the appropriate squares on copies of this page. By folding a copy of each table along the appropriate diagonal, students may observe that numbers on one side of the diagonal match numbers on the other side. Students may search for patterns in the tables, for example, even numbers, odd numbers, multiples of five, and square numbers. The tables are useful in demonstrating the commutative properties of addition and multiplication, and in relating inverse operations.

The shapes on pages T 383 to T 385 may be used for making attribute blocks (see page xxiv) since the actual size is shown. The large circle is a suitable size for making individual number spinners. The various polygons lend themselves for work with fractions, perimeter, and area.

The patterns for the three-dimensional shapes on pages T 386 to T 389 are marked with recommended dimensions. You may find it easier to construct some of these shapes if the pattern is outlined first on squared paper (page T 396), using the centimetre grid lines as a guide. Completed shapes may be suspended from the ceiling as mobiles. The names of the shapes may be printed on them to help students in identifying them. Two or more shapes may be pasted together to form another shape. For example, two triangular pyramids may be pasted together to form a shape with six faces, or a square pyramid may be pasted to two opposite faces of a cube to form a shape with twelve faces.

Because the number lines on page T 390 are marked into centimetres and half centimetres, copies of several of these may be pasted together to make "metre tapes" for the students to use in their measuring activities. Copies of the line which is marked in millimetres are useful in measuring small lengths and in relating millimetres and centimetres.

Copies of page T 395 may be used in many different ways. Some suggestions are as follows: for activities similar to those in the *Try This* feature on page 111, Ex. 7 and 8 on page 113, and Ex. 5 and 6 on page 187; for reinforcement for pages 174 to 185 (dots are joined to form line segments, angles, or polygons, and then the students make appropriate measurements); for reinforcement for pages 178 and 179 (dots are joined to form a simple shape having one or more lines of symmetry); for providing arrays for which the students write the multiplication or division sentences; for students to show the size of the arrays for given multiplication and division sentences.

Copies of pages T 396 and T 397 may be used for activities involving slides, flips, and turns; line symmetry; bar graphs, line graphs, and coordinate geometry (naming points on a grid).

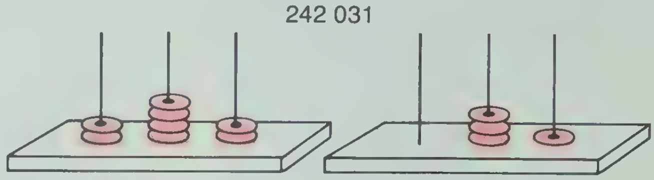
Peg Abacus

A simple peg abacus can be made from a Styrofoam tray, three wires of equal length cut from coat hangers, and plaster of Paris. Fill the tray with a mixture of plaster of Paris. Before it sets, insert the three wires and then allow the plaster to set. Use objects (colored wooden beads, washers, or empty spools) for representing numbers. The wires should be about 5 cm long unless empty spools are to be used, in which case they should be about 30 cm long.

A sturdier peg abacus may be made using wooden dowels on a wooden base. It is important that students view the abacus from the same side and not from opposite sides, which would result in a reversal of the place values.

To show numerals with more than three digits, place one abacus to the left of another.

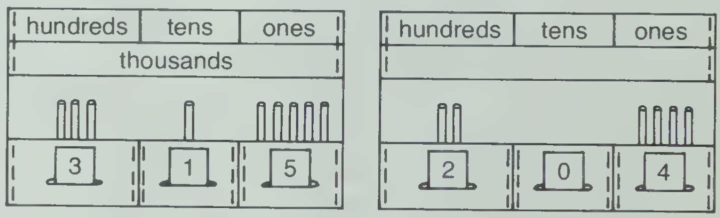
242 031



Place-value Pocket Chart

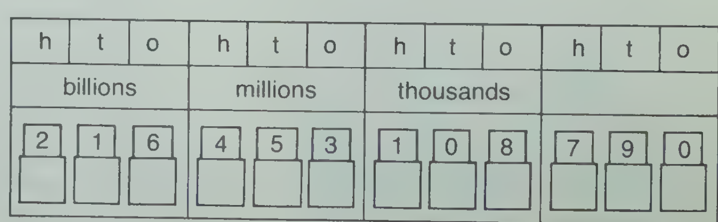
Pocket charts for demonstrating place value may be made in a variety of ways. Two forms are described below. The charts may be fastened to the display board or other convenient location.

Bristol board or thick plastic is cut in one piece, folded, and stitched or stapled where indicated to make three pockets for working with hundreds, tens, and ones. Slits may be made in the pockets to accommodate numeral cards for showing the standard numeral for the number of hundreds, tens, and ones displayed. For showing numerals with more than three digits, one pocket chart may be placed beside another.



315 204

A sheet of Bristol board is cut in half lengthwise. The two pieces are taped together to form a narrow rectangular shape. Library book pockets are attached, lines are drawn, and columns are labeled to make a chart as illustrated. Numeral cards may be placed in the appropriate pockets to represent up to twelve-digit numbers.



Number Sequence (Game for pages 8 and 9)

Materials: a numeral card for each digit from 0 to 9;
a box;
a set of instruction cards to indicate the number of places (from 4 to 12) in a numeral, for example,

| eight-place numeral | ;

a place-value chart ruled on lined paper turned sideways for each player.

Players: three to five

Rules:

1. The instruction cards are placed face down in a pile. One player turns the top card over to indicate the number of places for numerals in the game.
2. The numeral cards are placed in the box. For the first round, each player in turn draws one card, writes the indicated digit in the ones' place of her/his place-value chart, and returns the card to the box. The procedure is repeated for the second round, but the indicated digit is written in the tens' place. The rounds continue until all the digits required for the numeral have been written in sequence from right to left. The number of rounds is determined by the instruction card drawn to start the game.
3. At the end of the required number of rounds, the player with the greatest number is the winner.

Concentration (Game for pages 12 and 13)

Materials: pairs of matching cards

Players: two to four

Rules:

1. The cards are shuffled and placed face down in a rectangular array. The size of the array will depend upon the number of cards used and the ability of the students.
2. The first player selects two cards and shows them to the other players. If the two cards match, the player claims the cards and continues the procedure until the two cards selected do not match. The two cards that do not match are returned face down to their original places as the other players watch. It is then the next player's turn.
3. When all the cards have been claimed, the player with the most cards is the winner.

Total Action (Game for pages 16 and 17)

Materials: numeral cards as described for each version of the game

Players: three to five

Rules:

1. All the cards are shuffled and six cards are dealt to each player. The remaining cards are placed face down as a drawing pile.
2. The players take turns drawing one card from the drawing pile and checking their hands for a set of cards with the required sum. (The required sum depends on the concept that is being reinforced.) A player having a set of cards with the required sum shows them to the other players, places the cards face down in a separate pile, and discards one card from her/his hand. If a player does not have a set of cards with the required sum, it is the next player's turn.
3. When the last card is taken from the drawing pile, the discard pile is shuffled and becomes the drawing pile.
4. The first player without any cards is the winner.

Greatest Sum (Game for pages 22 and 23)

Materials: a numeral card for each digit from 0 to 9;
a box;
a chart similar to the following for each player;
a pencil for each player.
(The spaces in each row on the chart indicate the number of digits possible for the addends. This number may vary to provide different levels of difficulty.)

Players: any number

Rules:

1. The numeral cards are placed in a box. One player draws a card from the box, reads the numeral aloud, and returns the card to the box. Each player writes the numeral in one of the empty spaces as a digit for an addend. Other numeral cards are drawn in turn from the box, and the procedure is repeated until all the spaces for digits of the addends have been filled.
2. The players find the sum of their addends.
3. The player with the greatest sum is the winner.

Dominoes (Game for pages 26 and 27)

Materials: 30 domino cards
(When preparing the domino cards, place the cards in a row and make each card match another card at either end of the row. This ensures that the cards can be matched in the game.)

Players: one to four

Rules:

1. The domino cards are shuffled and five cards are given to each player. One of the remaining domino cards is placed face up and the others are placed face down in a pile.
2. If a player has a domino card that matches either end of the first card, or later in the game matches either end of the row, he/she places the card to continue the row. It is then the next player's turn.
3. If the player does not have a domino card that matches either end of the row, he/she draws a card from the pile. It is then the next player's turn.
4. The first player without any cards is the winner.

Product Search (Game for pages 58 and 59)

Materials: two dice marked 4, 5, 6, 7, 8, 9;
a different color crayon or pencil for each player;
a game board as shown.
(Copies of page T 394 can be used.)

×	4	5	6	7	8	9
4						
5						
6						
7						
8						
9						

Players: two to four

Rules:

1. Each player in turn throws the dice and states the product for the factors shown. If the product is incorrect, the next player has a turn. If the product is correct, the player locates a square for the product on the game board and colors it. When a square has been colored, it cannot be used again. A player may color only one square for each turn. If no square is available, it is the next player's turn.
2. The player who has colored the most squares after an agreed number of rounds is the winner.

To vary the game, show different factors on the dice and adapt the game board accordingly.

Inventory (Game for pages 120 and 121)

Materials: a pencil and a piece of paper for each group (If you wish to display the lists after the game, have each group use a marker and a large piece of paper.)

Players: any number in groups of four or five with a leader for each group

Rules:

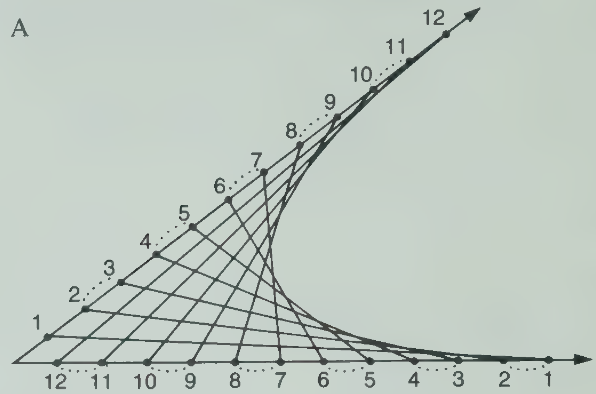
1. Divide the class into groups and give a pencil and a piece of paper to the leader of each group.
2. Allow a specific length of time, for example, five minutes, for the members of each group to think of as many examples as possible for the topic that is being studied and for the leader to list the examples as they are named.
3. At the end of the time limit, each group scores one point for each acceptable example in the list.
4. The winner is the group with the greatest score.

Curve Stitching (Activity for pages 176 and 177)

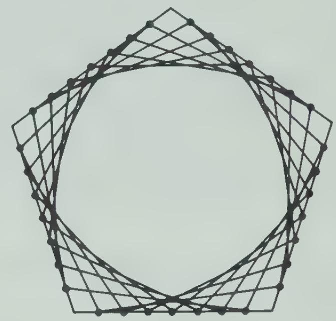
Colorful geometric designs can be made with thread, or yarn, and Bristol board. Each stitch is straight, but the design that is created appears curved.

Mark the same number of equally spaced dots on each ray of an angle. Number the dots as shown in A. Stitch with colored thread, or yarn, to join the dots in the sequence $1 \longrightarrow 1 \longrightarrow 2 \longrightarrow 2 \longrightarrow 3 \longrightarrow 3 \longrightarrow 4 \longrightarrow \ldots \longrightarrow 12 \longrightarrow 12$. If the stitching is done correctly, there will be a curved pattern on one side of the Bristol board. On the opposite side there will be a row of segments on each ray. The procedure may be adapted for polygons as indicated in B.

A



B

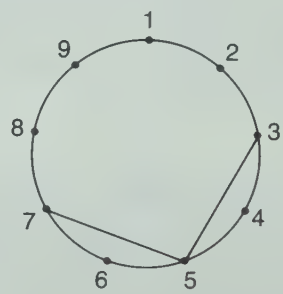


Multipatterns (Activity for pages 195 and 196)

Have students write the products for 7 × 1, 7 × 2, 7 × 3, ..., 7 × 10 in the first row of a table similar to the following. In the second row, have them write the sum of the digits for each product. If a two-digit sum is obtained, the digits of the sum are added to obtain a one-digit number. For example, 7 × 2 = 14, 1 + 4 = 5.

Multiple of 7	7	14	21	70
Sum of digits	7	5	3	7

Prepare large copies of the following diagram and give one to each student.



Have the students use a straight edge to join dots in sequence, following the sequence of numbers 7, 5, 3, ..., 7 for the sums of the digits of the products in the table. Discuss the pattern obtained. Have students explore line symmetry in the pattern using a semitransparent plexiglass mirror. They may also trace the pattern and turn the tracing to explore rotational symmetry in the pattern. Students can color the completed patterns and use a protractor to measure some of the angles. This activity may be repeated for multiples of other numbers.

Name the Fraction (Game for pages 200 and 201)

Materials: 18 cards showing fractions in lowest terms (some cards should show identical fractions);
a list of fractions equivalent to those on the cards.

Players: four to six

Rules:

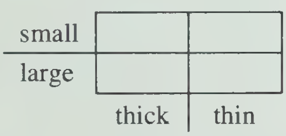
1. The cards are shuffled and three cards are dealt to each player other than the leader.
2. The leader reads a fraction from the list and the player having a card for that fraction displays the card and names a fraction that is equivalent to the one given. The leader uses the list of equivalent fractions to check the player's answer. If the fraction named is correct, the player scores one point. The leader marks the fraction that has been read and then reads another fraction from the list.
3. The first player to score ten points is the winner.

Attribute Challenges (Activity for page 214)

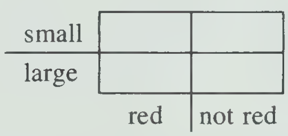
A description of attribute blocks is given on page xxiv. Prepare activity cards similar to the following and have the students work in small groups with the blocks.

Sort the blocks.

1.

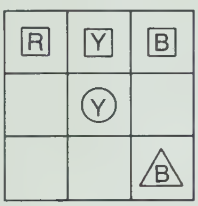


2.



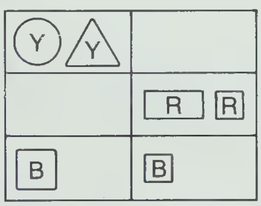
Copy and show blocks to complete the patterns.
Y: yellow B: blue R: red

3.



(thin blocks only)

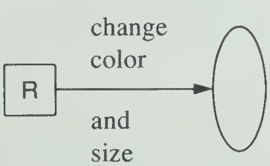
4.



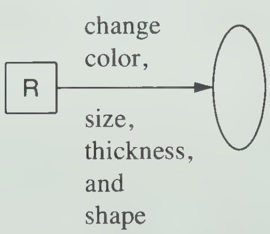
(thick blocks only)

Start with a large, thin, red square each time.
Show which shapes can be placed in the ring.

5.

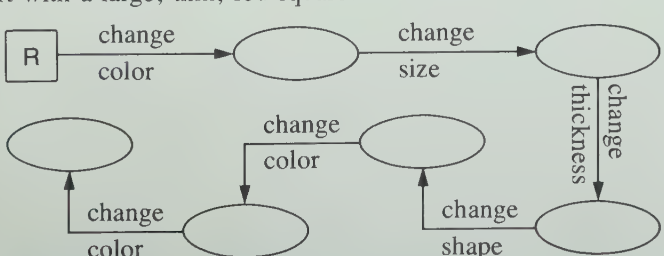


6.



Join "cars" to complete the "train".
Start with a large, thin, red square.

7.



Match Up (Game for page 215)

Materials: 20 sets of cards that match
(A set consists of three cards for the concept being reinforced.)

Players: two to five

Rules:

1. All the cards are shuffled and six cards are dealt to each player. The remaining cards are placed face down as a drawing pile.
2. The players take turns drawing one card from the drawing pile and checking their hands for a set of cards that match. A player having a set of cards that match shows them to the other players, places the cards face down in a separate pile, and discards one card from her/his hand. If a player does not have a set of matching cards, one card is discarded and the next player has a turn.
3. When the last card is taken from the drawing pile, the discard pile is shuffled and becomes the drawing pile.
4. The first player without any cards is the winner.

King of Fractions (Game for pages 220 and 221)

Materials: two cards for each of ten different fractions;
a copy of page T 382 showing the ten fractions as addends (factors) and their sums (products).
(An example is shown below for addition.)

+	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{3}{4}$	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{5}$
$\frac{1}{2}$		$\frac{3}{4}$	$1\frac{1}{4}$	$\frac{5}{6}$	$1\frac{1}{6}$	$\frac{7}{10}$
$\frac{1}{4}$	$\frac{3}{4}$		1	$\frac{7}{12}$	$\frac{11}{12}$	$\frac{9}{20}$

Players: two to four

Rules:

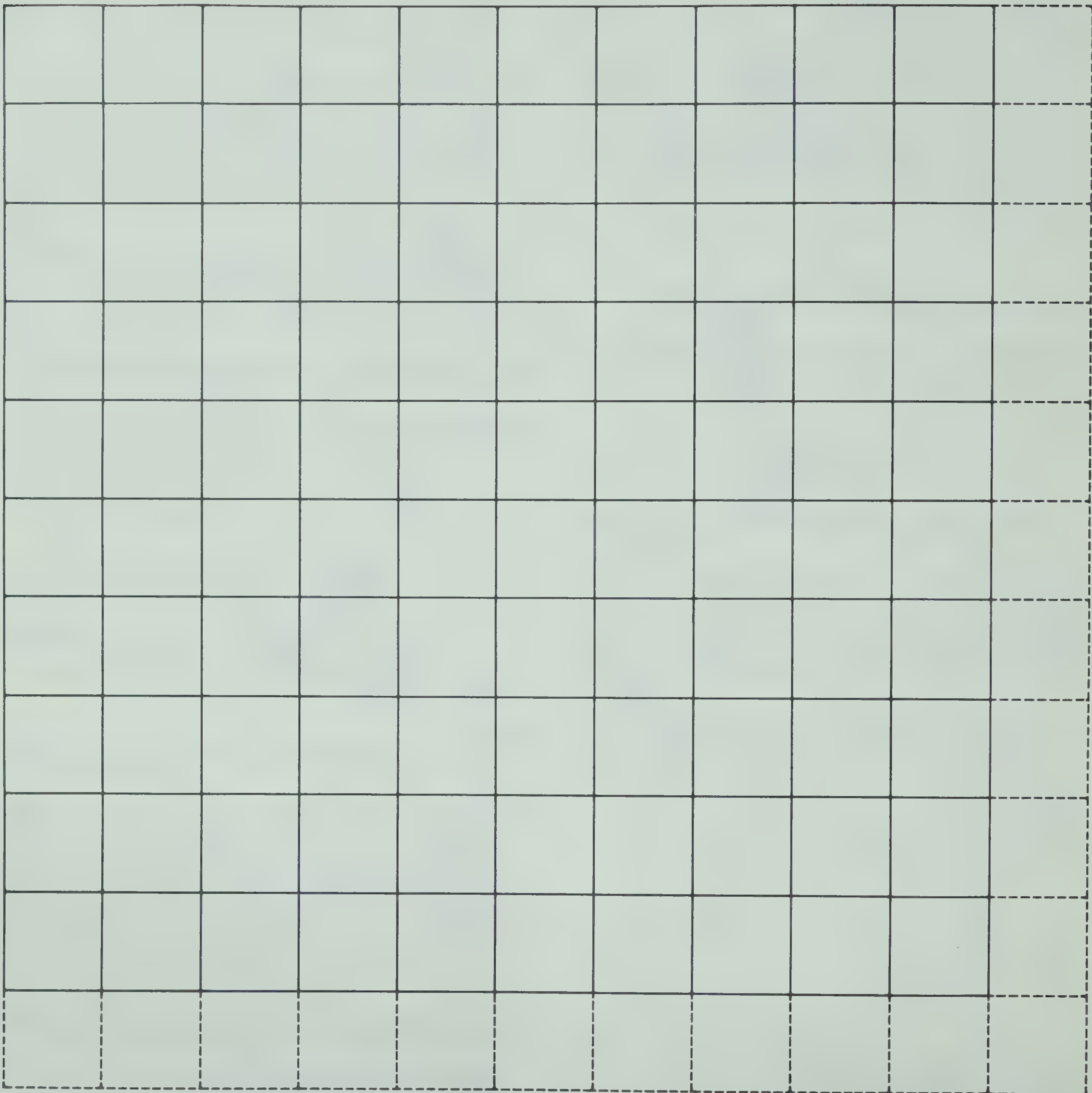
1. The cards are shuffled and placed face down as a drawing pile.
2. Each player in turn draws two cards and finds the sum (product) of the numbers. A pencil and paper may be used. The player on the left of the first player refers to the chart to check the answer. If the answer is correct, the player scores one point.
3. The player with the most points after a specified number of rounds is the winner.

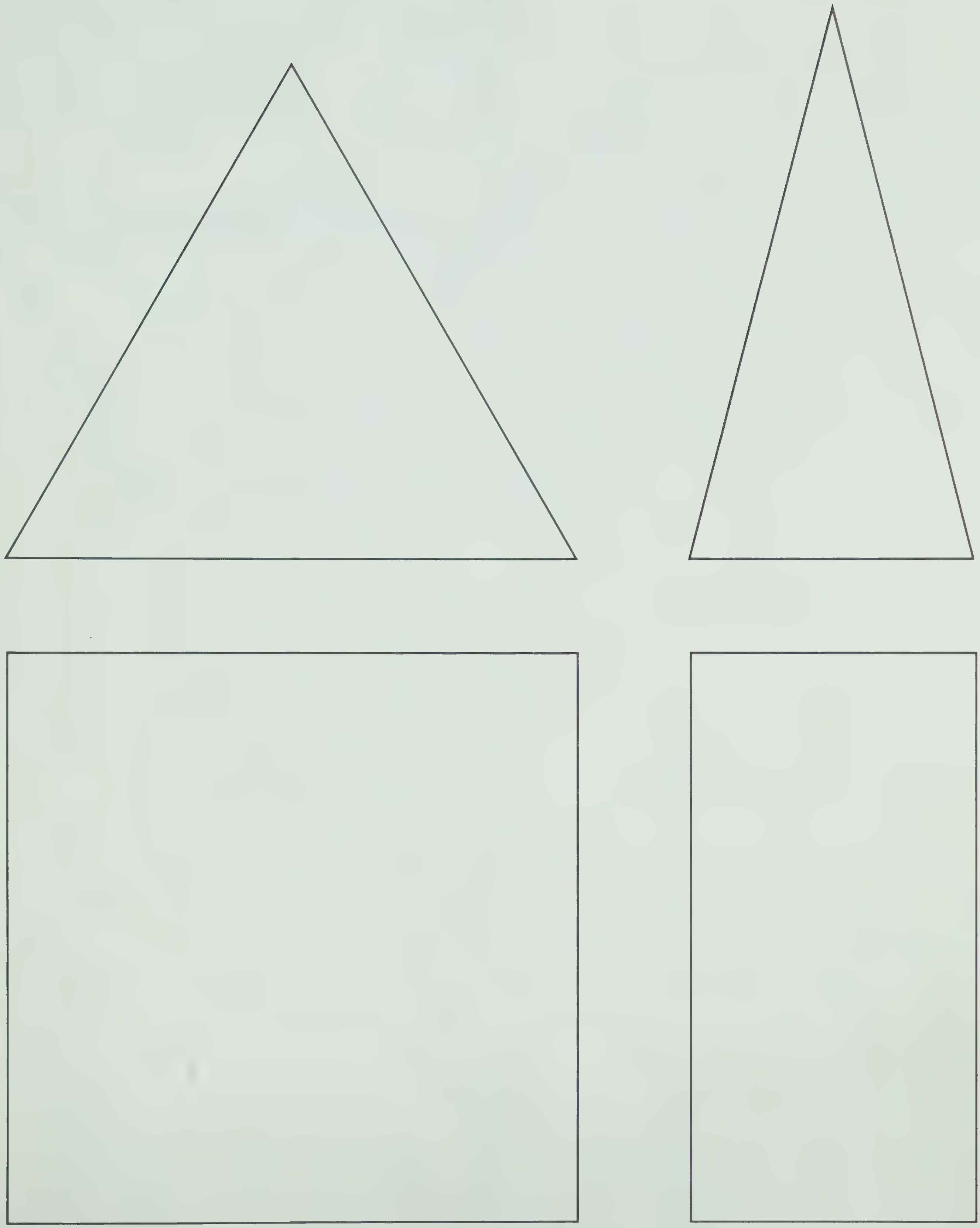
Attribute Challenges (Activity for pages 248 and 249)

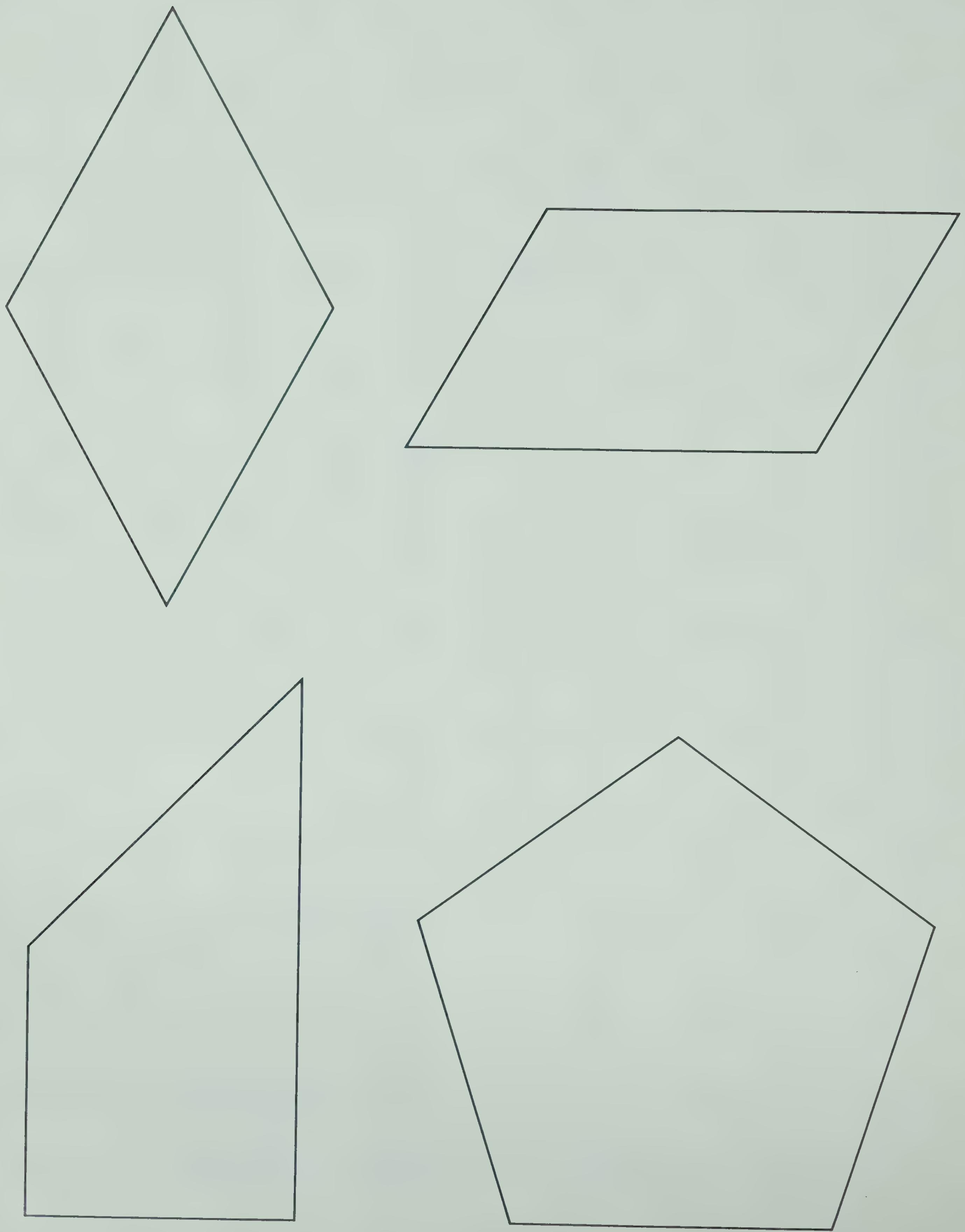
Display some attribute blocks. Have students name ratios that compare the following and other similar examples.

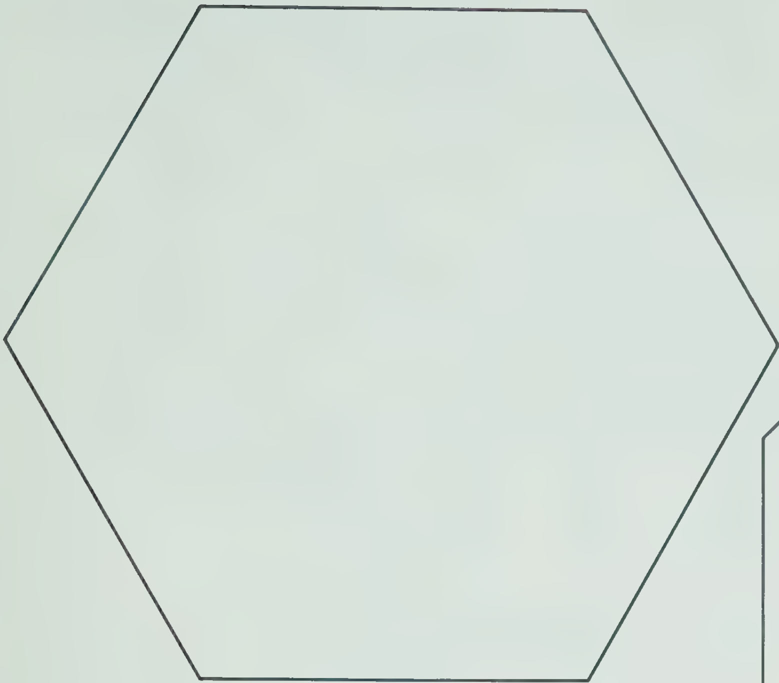
1. the number of yellow blocks to the number of blue blocks
2. the number of thin blocks to the number of thick blocks
3. the number of small, red blocks to the number of large, blue blocks

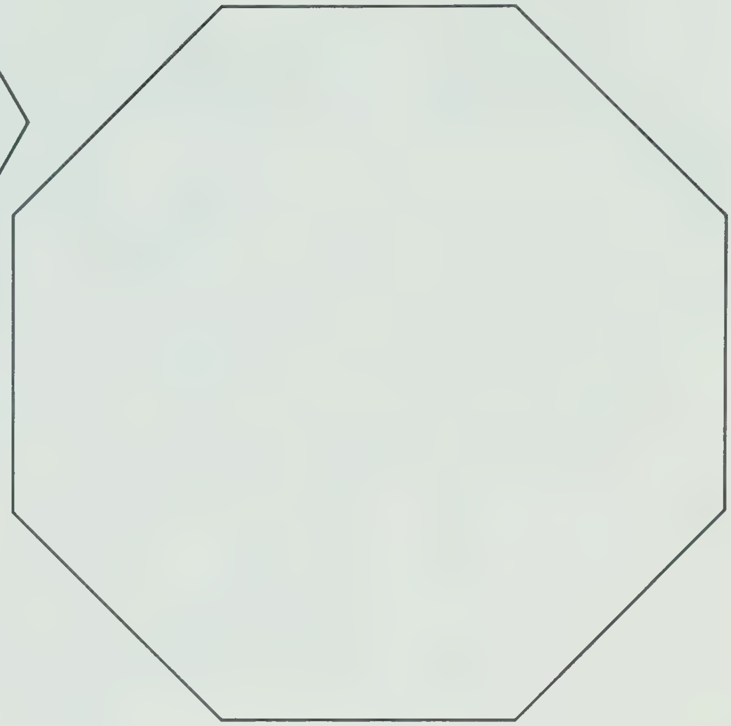
Reverse the procedure by naming a ratio such as 4:7 and having students display the appropriate blocks. Also, display several blocks, name a ratio such as 4:1, and ask students to describe the blocks that are being compared.

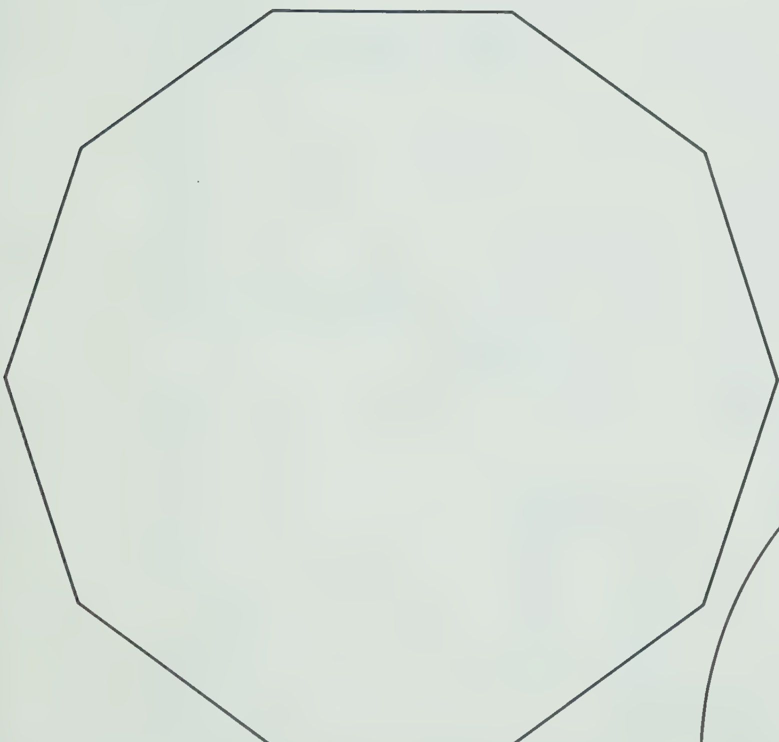


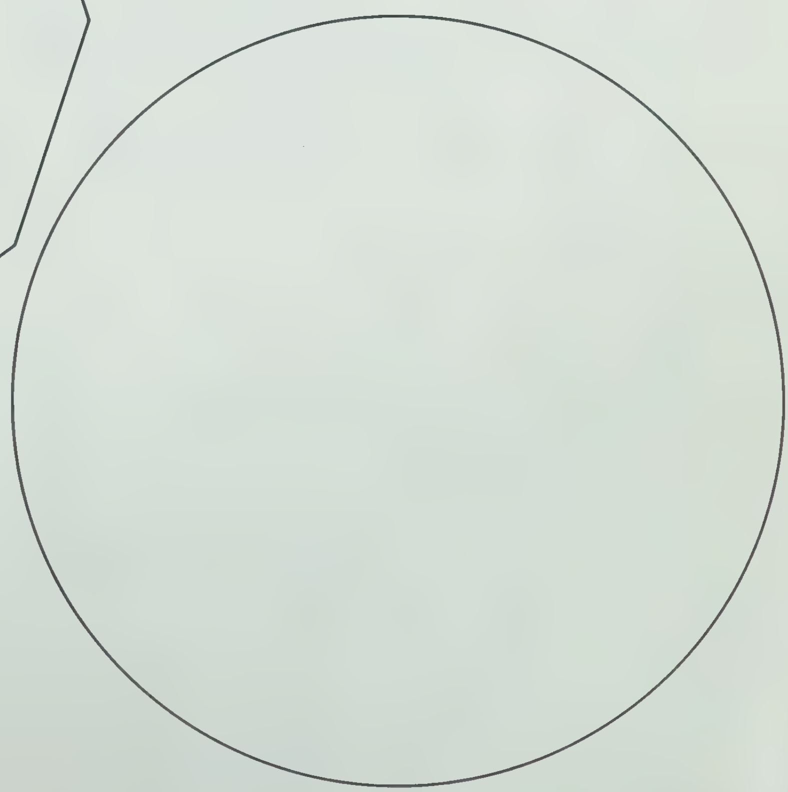




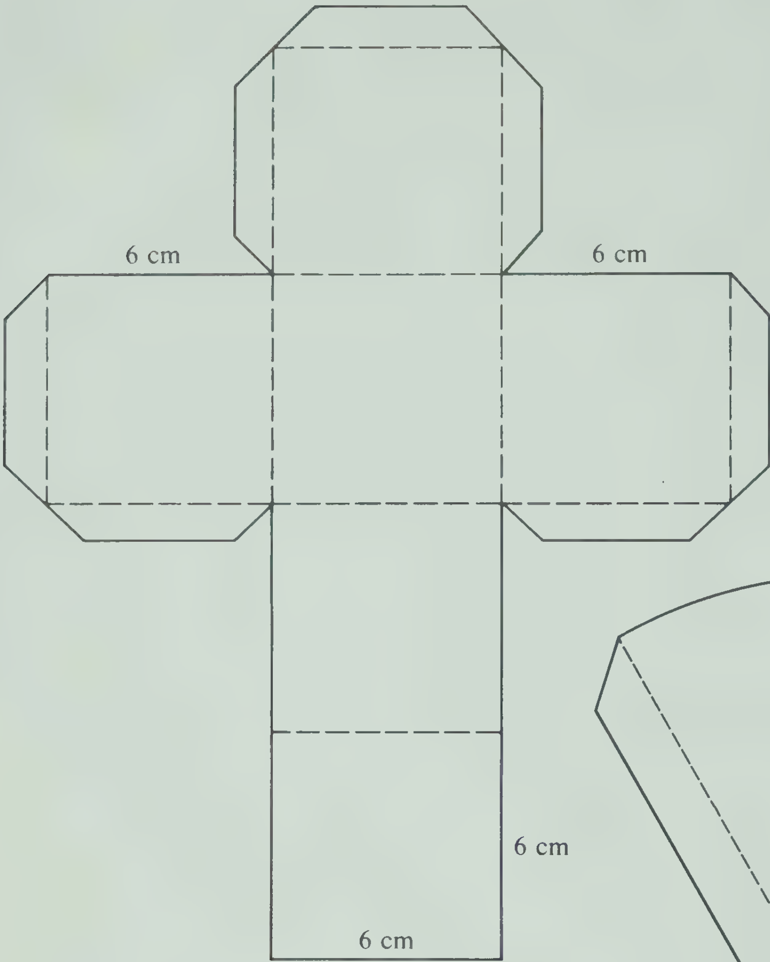




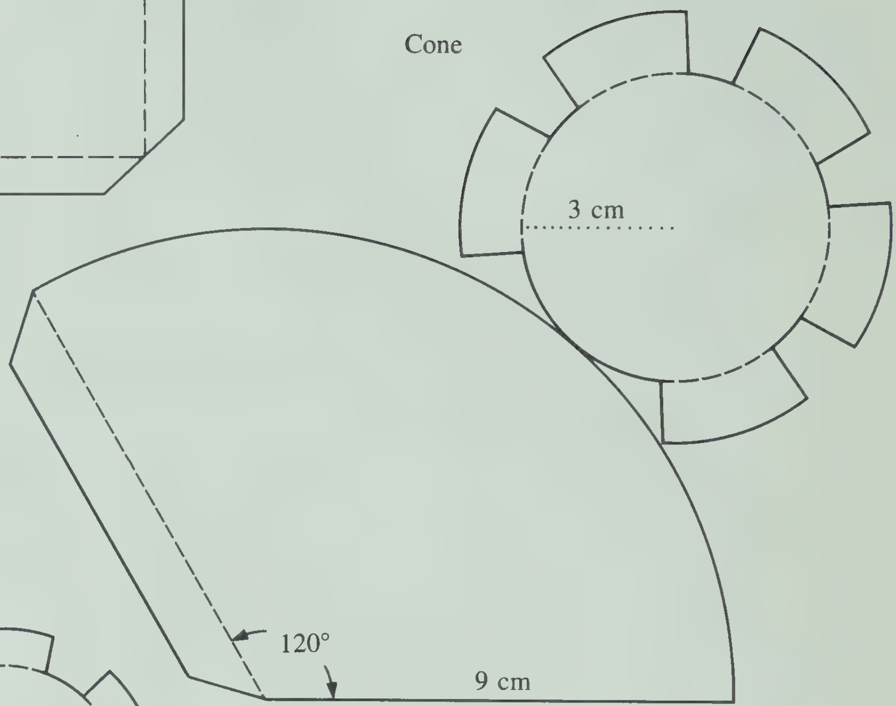




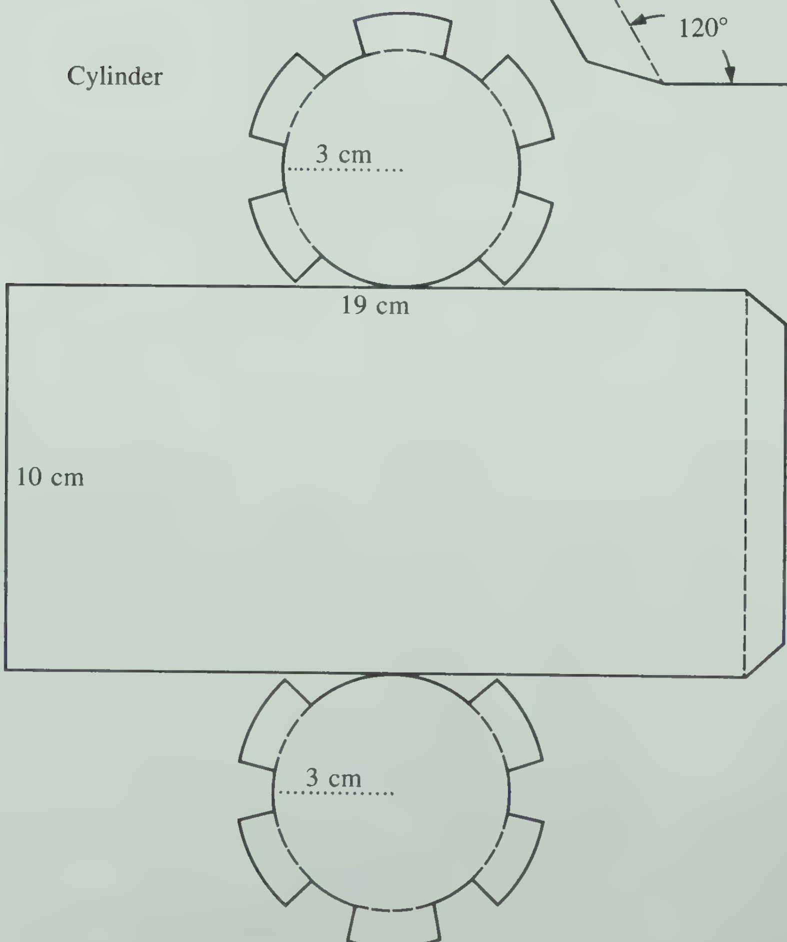
Cube



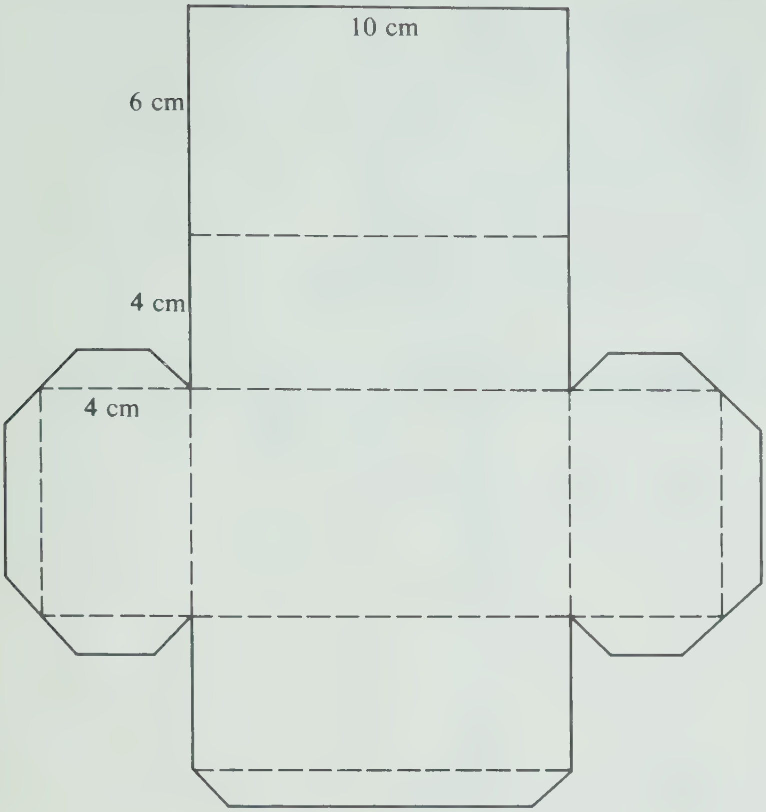
Cone



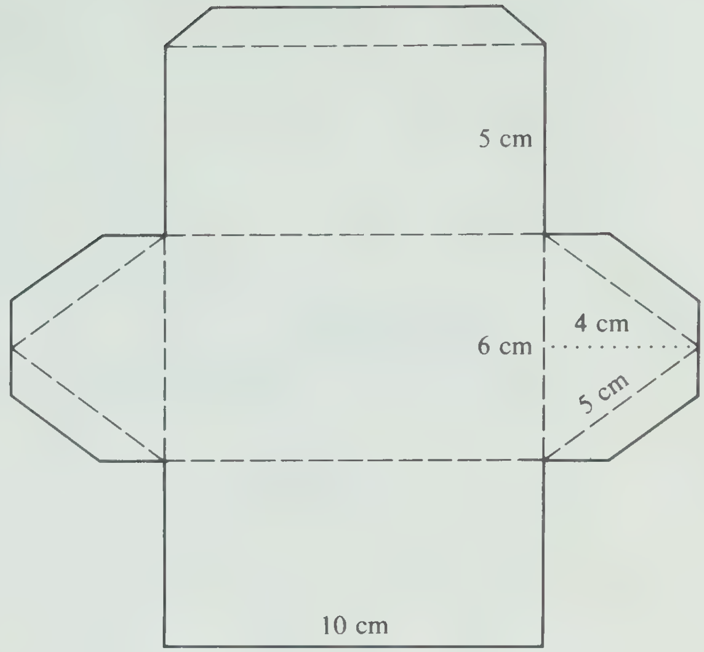
Cylinder



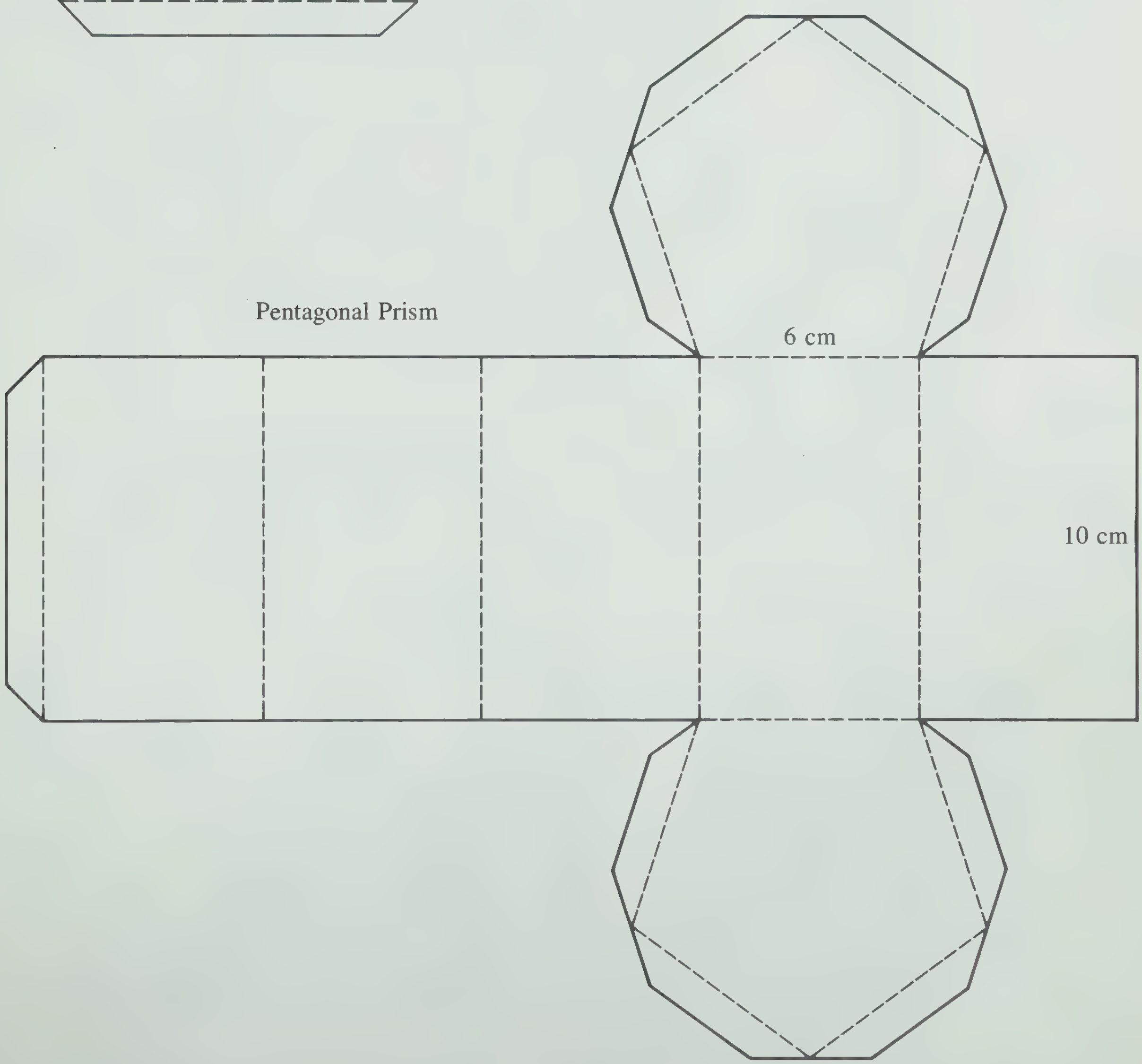
Rectangular Prism



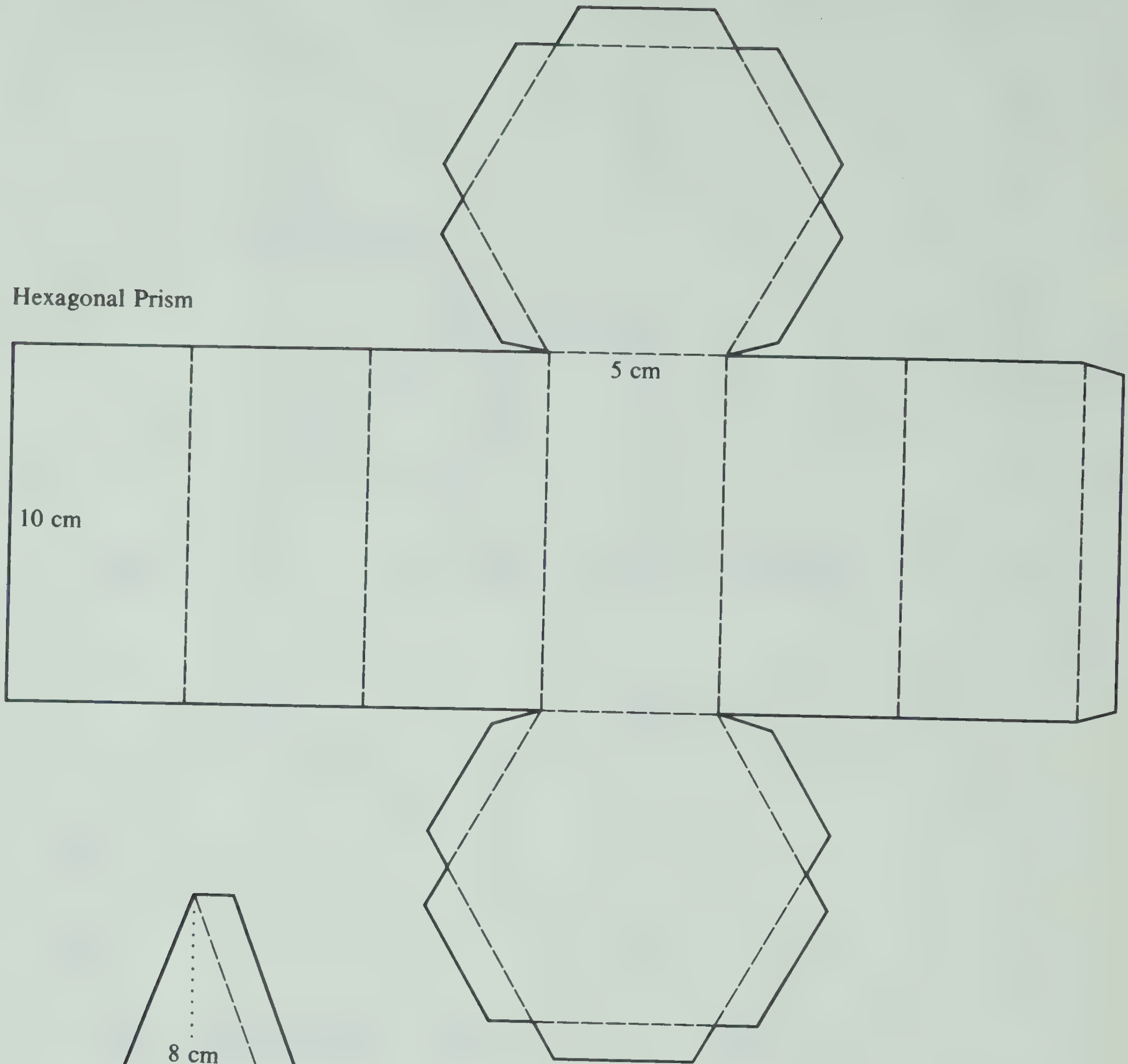
Triangular Prism



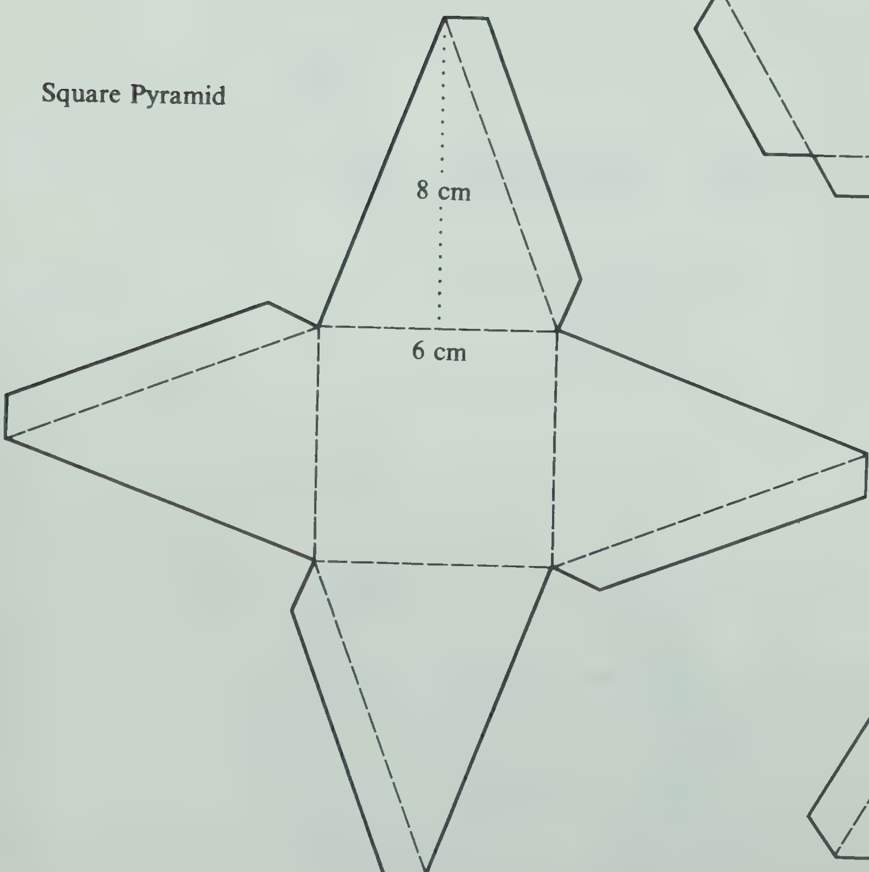
Pentagonal Prism



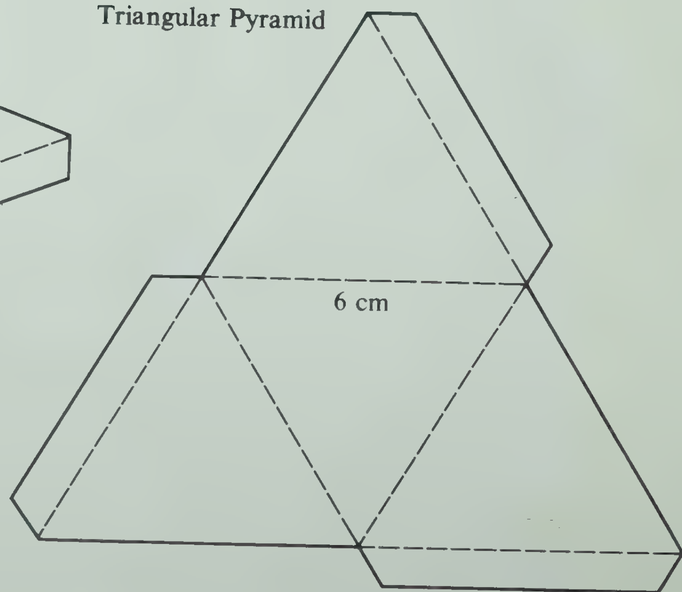
Hexagonal Prism



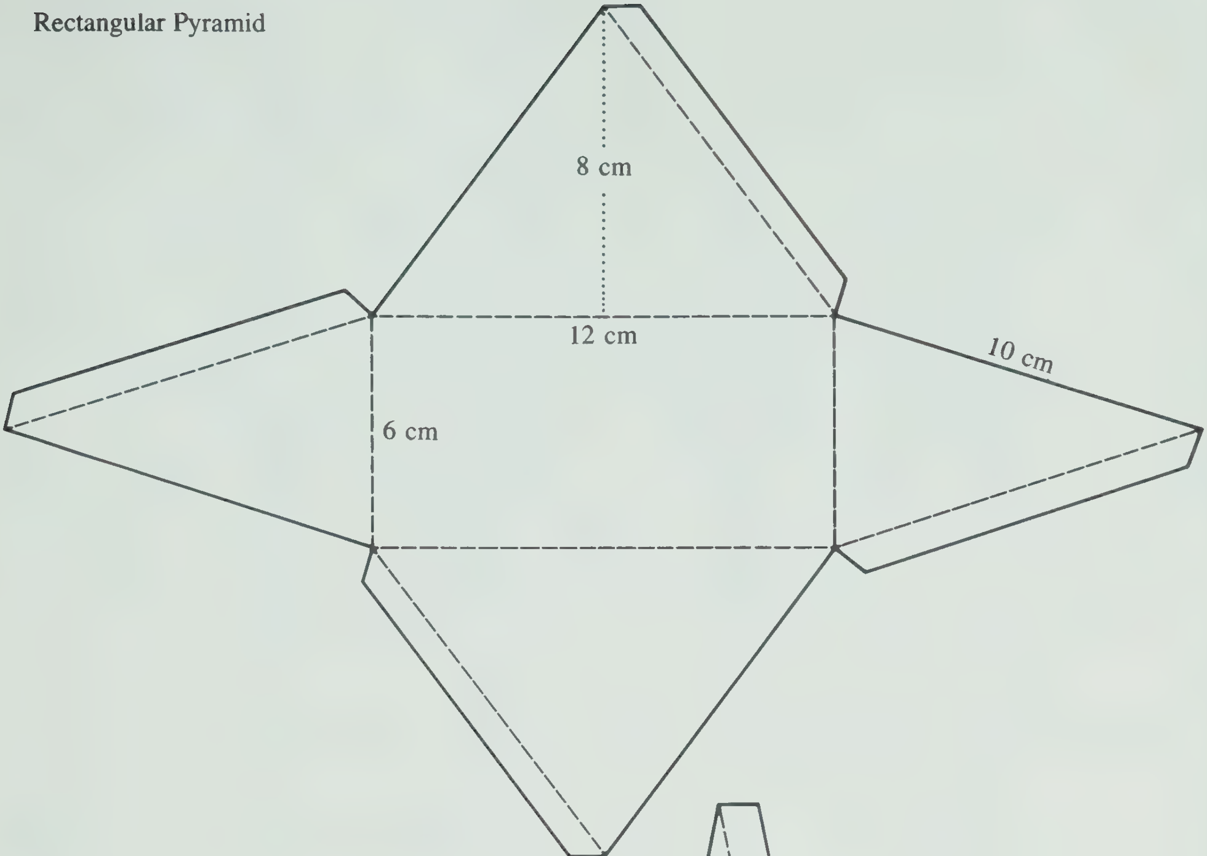
Square Pyramid



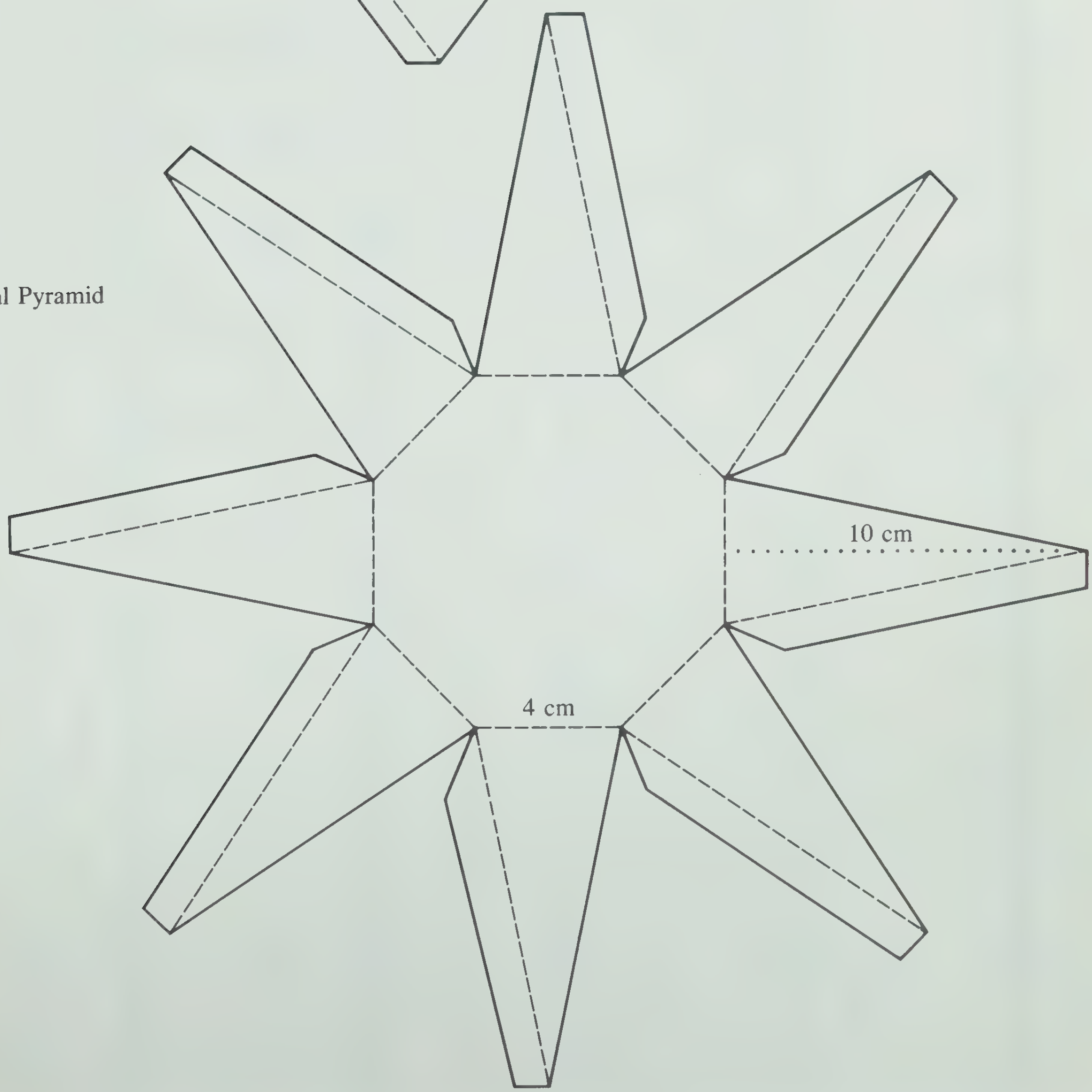
Triangular Pyramid

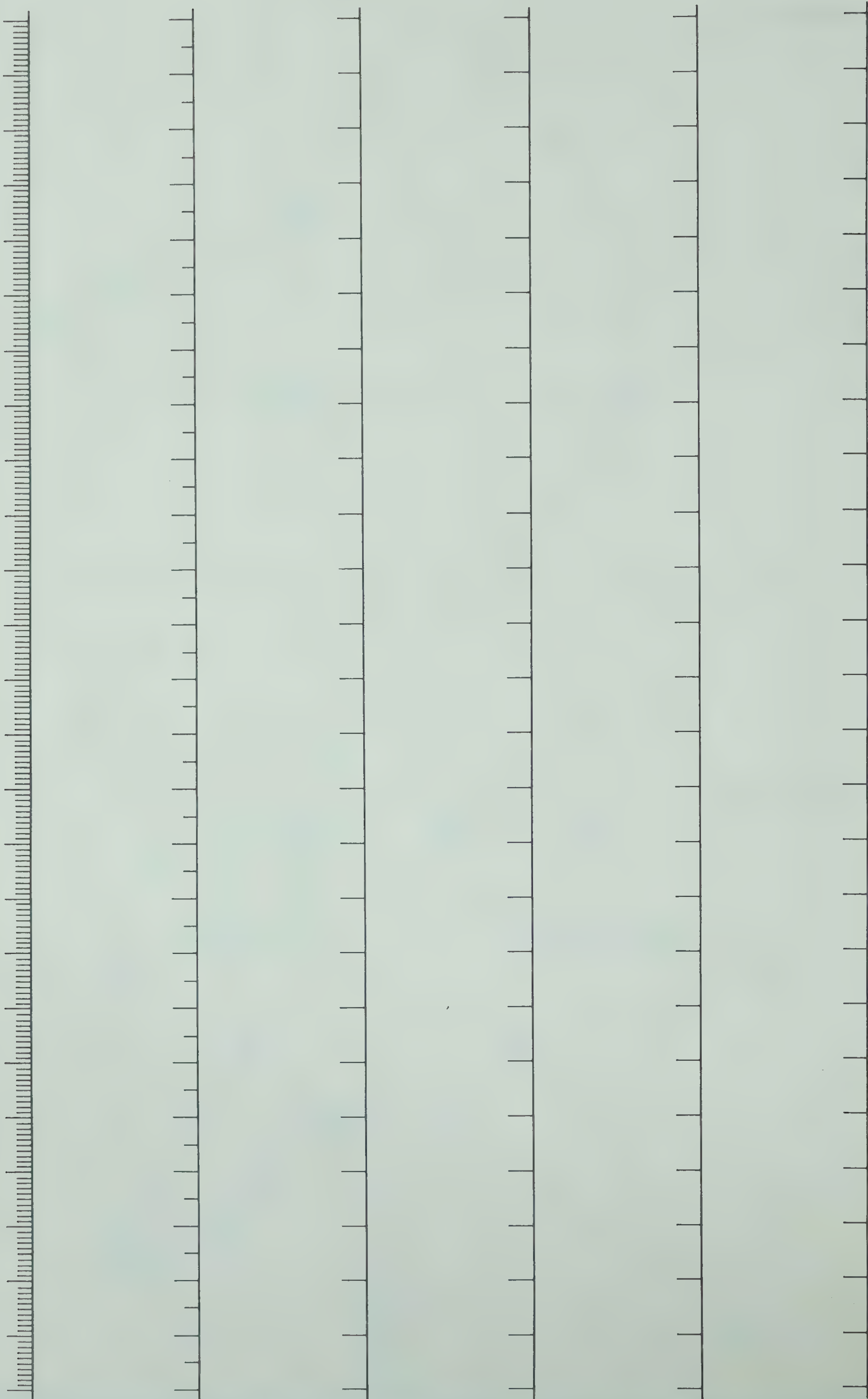


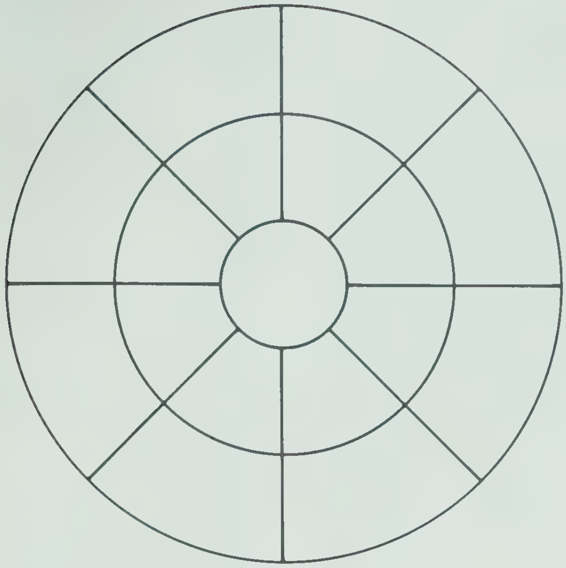
Rectangular Pyramid

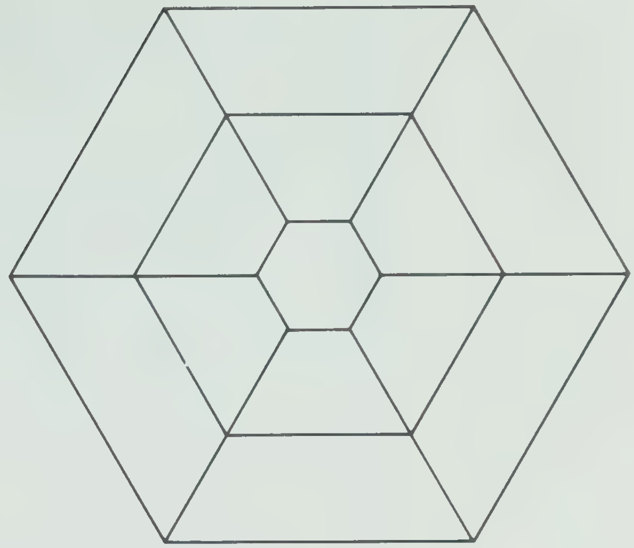


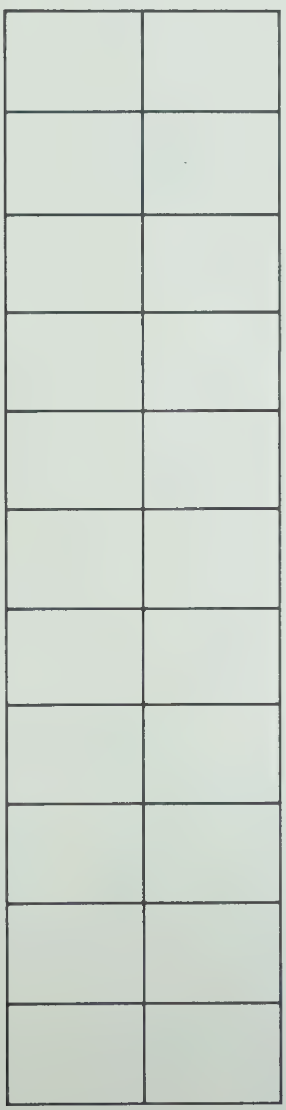
Octagonal Pyramid

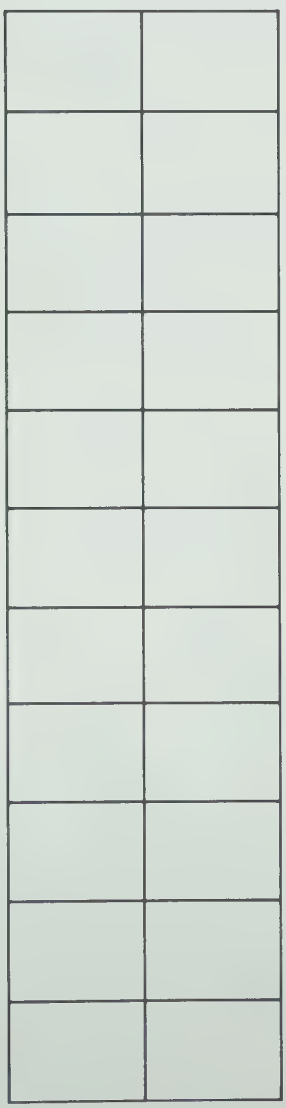


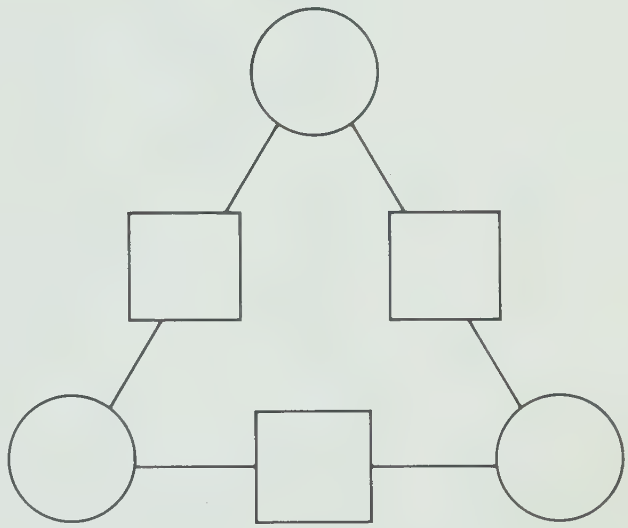


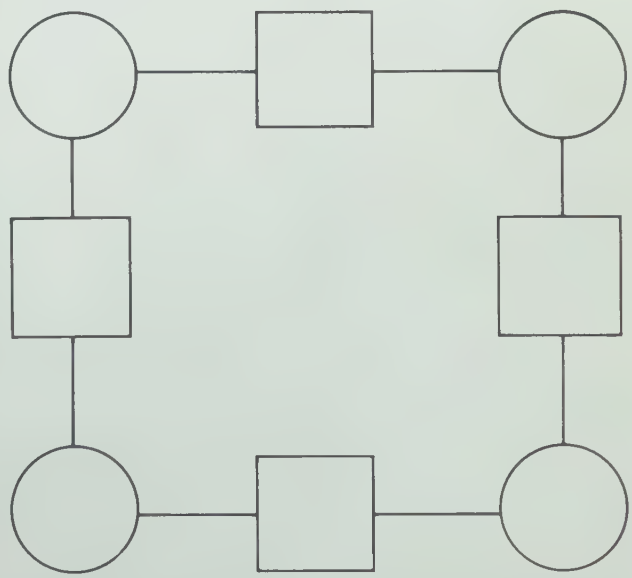


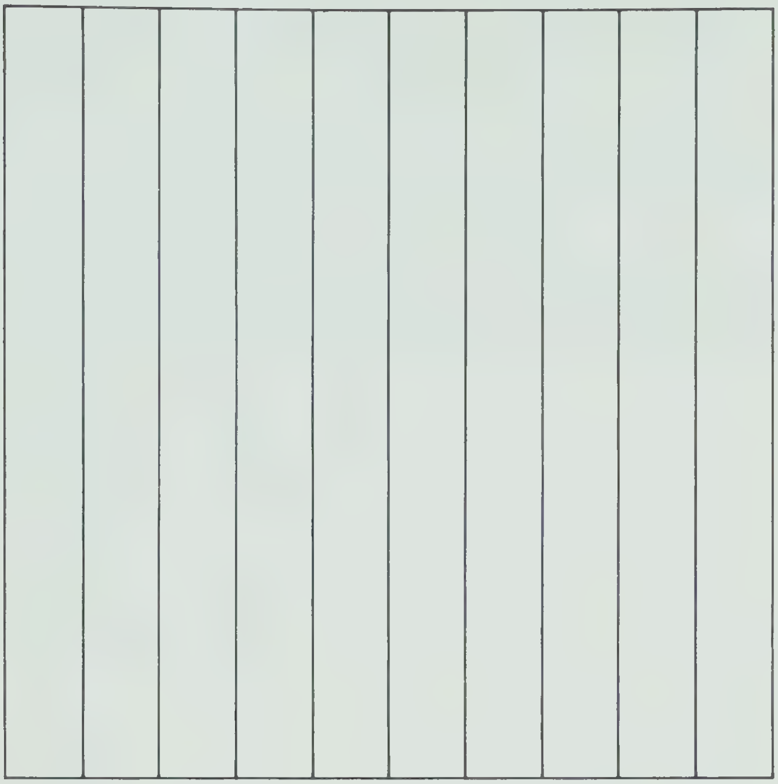


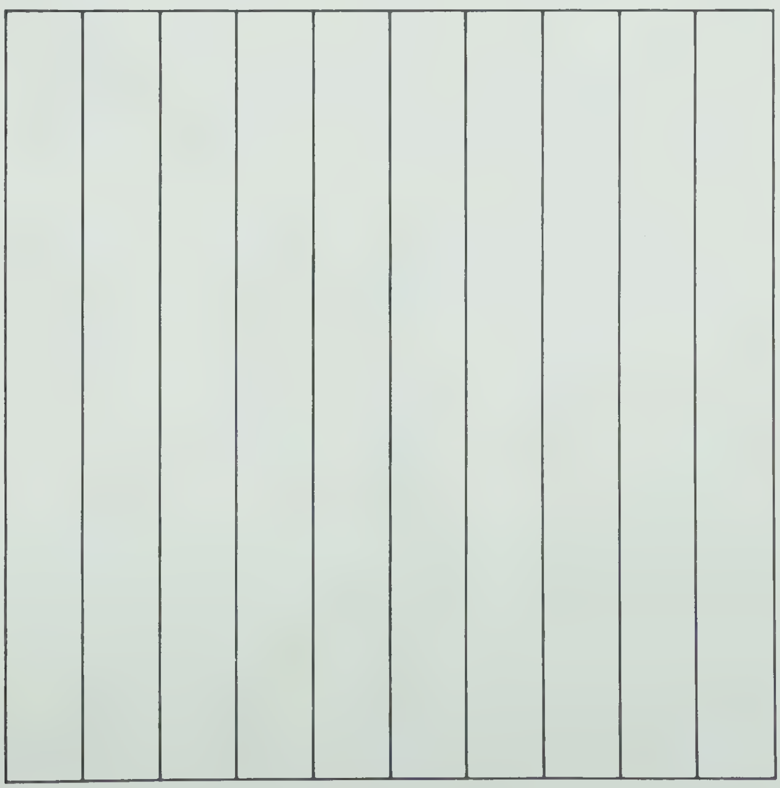


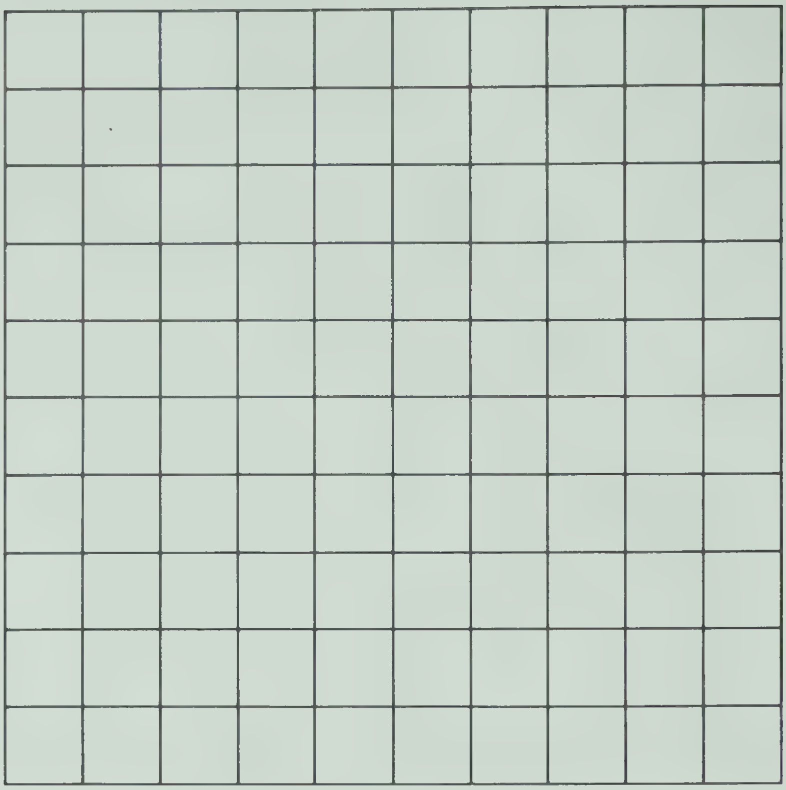


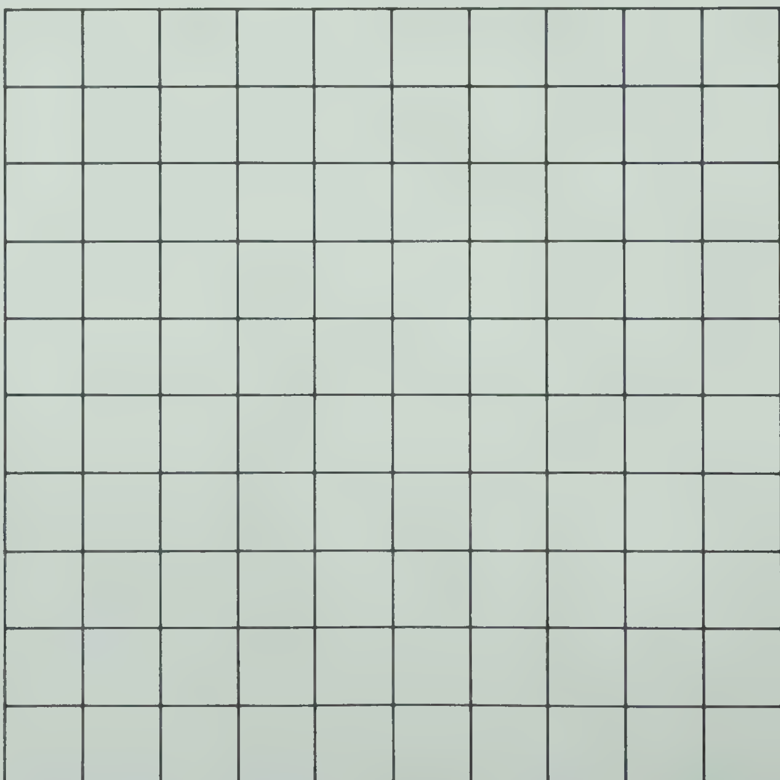


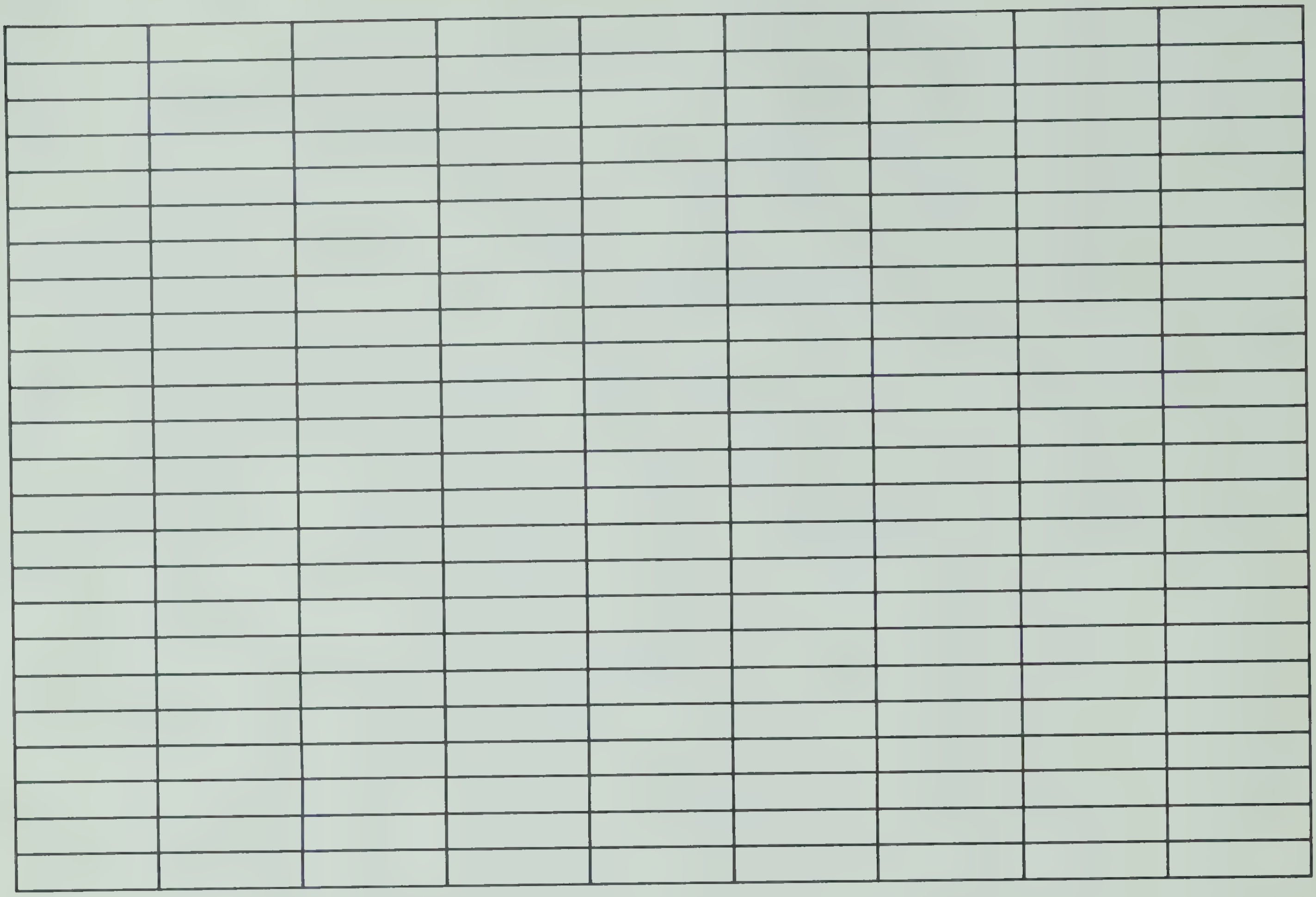


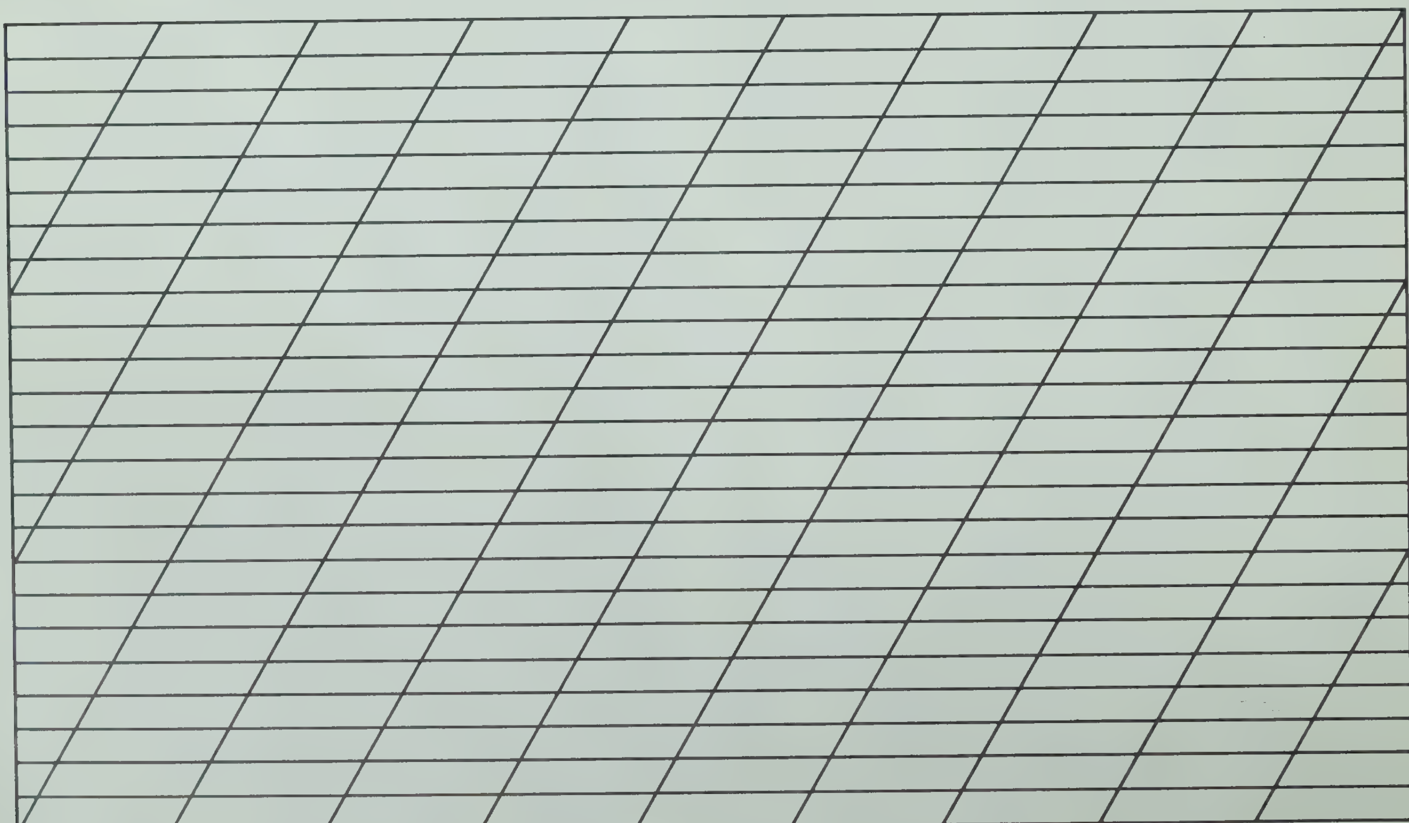


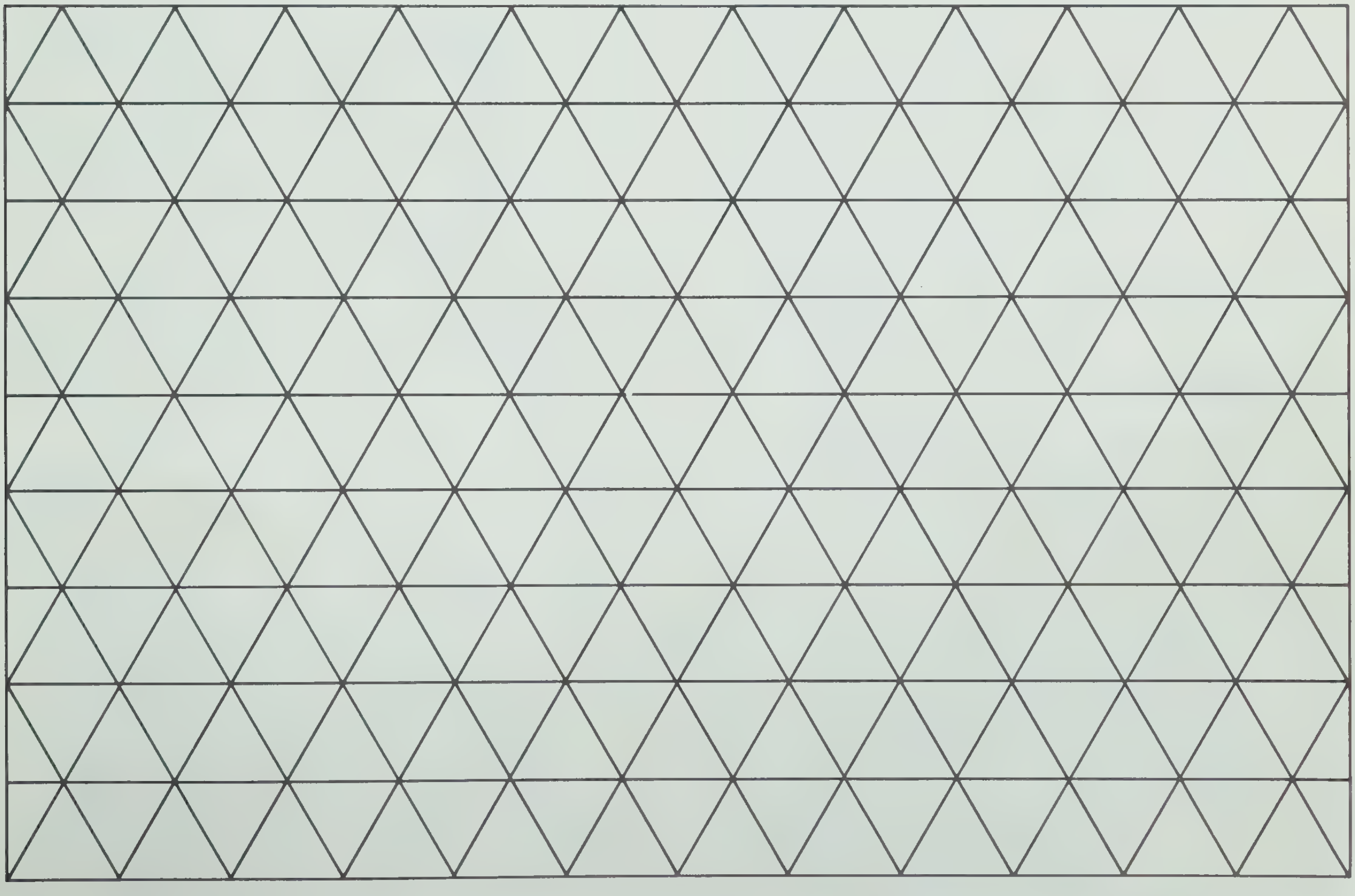




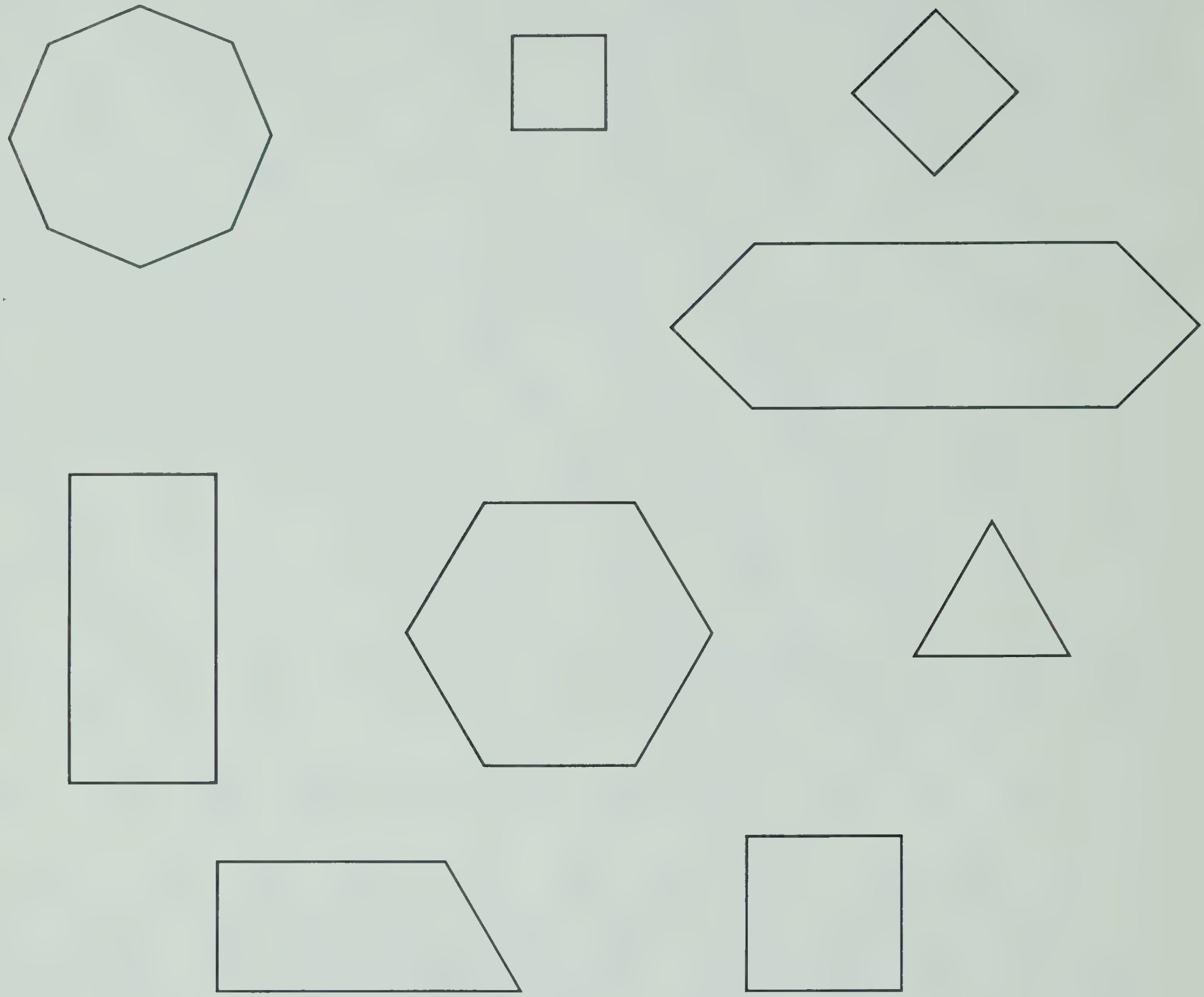








Shapes for pages 288 and 289



Shapes for pages 290 and 291

Year-End Evaluation (See page xv.)

Name ______________________

Numeration

1. Reads/Writes standard numerals to 999 999 999 999 ☐
2. Interprets place value in numerals to 999 999 999 999 ☐
3. Understands/Writes expanded form for numbers to 999 999 999 999 ☐
4. Compares/Orders numbers to 999 999 999 ☐
5. Rounds numbers to the nearest one, ten, or hundred in any period to millions ☐
6. Knows meaning and use of the symbol:
   a. ¢ ☐ b. $ ☐ c. = ☐ d. ≠ ☐
   e. > ☐ f. < ☐ g. + ☐ h. − ☐
   i. × ☐ j. ÷, $\overline{)}$ ☐ k. : ☐ l. % ☐
7. Understands/Uses the term:
   a. greater than ☐ b. less than ☐
   c. equals ☐ d. not equal to ☐
8. Reads/Writes Roman numerals for numbers to 3999 ☐
9. Expresses amounts of money using decimals ☐
10. Reads/Writes decimals to ten-thousandths ☐
11. Interprets place value to four decimal places ☐
12. Compares/Orders decimals to ten-thousandths ☐
13. Rounds decimals ☐
14. Reads/Writes fractions for numbers less than one ☐
15. Reads/Writes numerals for numbers in mixed form ☐
16. Identifies/Finds equivalent fractions ☐
17. Finds the missing term for equivalent fractions ☐
18. Expresses a whole number or a number in mixed form as an improper fraction ☐
19. Expresses an improper fraction as a whole number or as a number in mixed form ☐
20. Identifies/Finds
    a. prime numbers ☐ b. composite numbers ☐
    c. common factors ☐ d. reciprocals ☐
21. Finds like denominators for fractions ☐
22. Compares fractions using common denominators ☐
23. Expresses fractions as decimals ☐
24. Reads/Writes ratios using colon and fraction notation ☐
25. Finds equivalent ratios, rates ☐
26. Finds the missing term in equivalent ratios, rates ☐
27. Finds
    a. unit rates ☐ b. unit prices ☐
28. Reads/Writes percents ☐
29. Converts among decimals, fractions, ratios, percents ☐
30. Reads/Writes integers ☐
31. Compares/Orders integers ☐

Operations

Addition

1. Understands/Uses the term:
   a. add ☐ b. plus ☐ c. addend ☐ d. sum ☐
2. Understands the steps in the algorithm ☐
3. Understands addition properties:
   a. commutative (order) ☐ b. associative (grouping) ☐
4. Adds two or more numbers, up to six digits ☐
5. Adds, sums to ten-thousandths
   a. decimals ☐ b. decimals and whole numbers ☐
6. Estimates sums by rounding when the addends are
   a. whole numbers ☐ b. decimals ☐
7. Adds fractions with
   a. like denominators ☐ b. unlike denominators ☐
8. Adds integers ☐
9. Solves problems using addition ☐

Subtraction

1. Understands/Uses the term:
   a. subtract ☐ b. minus ☐ c. difference ☐
2. Understands the steps in the algorithm ☐
3. Subtracts, up to six-digit numbers, regrouping
   a. without zeros in the minuend ☐
   b. with zeros in the minuend ☐
4. Subtracts, uses addition to check ☐
5. Simplifies number expressions with parentheses ☐
6. Subtracts decimals, minuends to ten-thousandths ☐
7. Subtracts decimals and whole numbers, minuends to ten-thousandths ☐
8. Estimates differences by rounding when
   a. the minuend and the subtrahend are whole numbers ☐
   b. the minuend and the subtrahend are decimals ☐
9. Subtracts fractions with
   a. like denominators ☐ b. unlike denominators ☐
10. Finds differences between temperatures ☐
11. Solves problems using subtraction ☐

Multiplication

1. Understands/Uses the term:
   a. multiply ☐ b. factor ☐ c. product ☐
2. Understands the steps in the algorithm ☐
3. Understands multiplication properties:
   a. commutative (order) ☐ b. associative (grouping) ☐
4. Multiplies by a one-digit number ☐
5. Multiplies by multiples of 10, 100, and 1000 ☐
6. Multiplies by a two-digit number ☐
7. Multiplies by a three-digit number ☐
8. Multiplies a decimal and a whole number, products to thousandths ☐
9. Multiplies a decimal by a decimal, products to ten-thousandths when
   a. the products are greater than one ☐
   b. the products are less than one ☐
10. Multiplies a whole number or a decimal
    a. by 10, 100, or 1000 ☐
    b. by 0.1, 0.01, or 0.001 ☐
11. Estimates products by rounding when
    a. the factors are whole numbers ☐
    b. the factors are decimals ☐
12. Multiplies a fraction
    a. and a fraction ☐ b. and a whole number ☐
13. Identifies/Finds reciprocals ☐
14. Simplifies number expressions with parentheses ☐
15. Finds a percent of a number ☐
16. Calculates interest and discount ☐
17. Solves problems using multiplication ☐

Division

1. Understands/Uses the term:
   a. divide ☐ b. quotient ☐
   c. divisor ☐ d. remainder ☐
2. Understands the steps in the algorithm ☐
3. Uses multiplication to divide ☐
4. Divides by a one-digit number when the quotient
   a. has no zeros ☐ b. has zeros ☐
5. Divides by a multiple of 10 ☐
6. Divides by a two-digit number ☐
7. Divides by a three-digit number ☐
8. Uses multiplication and addition to check results ☐
9. Divides to find an average ☐

10. Divides a decimal
    a. by a one-digit whole number ☐
    b. by a two-digit or a three-digit whole number ☐
    c. by a one-place decimal ☐
    d. by a two-place decimal ☐
    e. by a three-place decimal ☐
11. Divides a whole number or a decimal
    a. by 10, 100, or 1000 ☐
    b. by 0.1, 0.01, or 0.001 ☐
12. Rounds a quotient when dividing a decimal
    a. by a whole number ☐
    b. by a decimal ☐
13. Estimates a quotient by rounding when
    a. the divisor and the dividend are whole numbers ☐
    b. the divisor and/or the dividend are decimals ☐
14. Simplifies number expressions with parentheses ☐
15. Divides
    a. a fraction by a fraction ☐
    b. a fraction by a whole number ☐
    c. a whole number by a fraction ☐
16. Solves problems using division ☐

Measurement

Area

1. Recognizes/Uses the term and its symbol:
    a. square millimetre (mm^2) ☐
    b. square centimetre (cm^2) ☐
    c. square metre (m^2) ☐
    d. square kilometre (km^2) ☐
2. Finds area in square centimetres by counting ☐
3. Uses multiplication to find the area of these polygons:
    a. rectangle ☐ b. square ☐
    c. parallelogram ☐ d. triangle ☐

Capacity

1. Recognizes/Uses the term and its symbol:
    a. millilitre (mL) ☐ b. litre (L) ☐
2. Chooses the best estimate for a capacity ☐
3. Relates millilitres and litres ☐
4. Relates units of volume and capacity ☐

Length

1. Recognizes/Uses the term and its symbol:
    a. millimetre (mm) ☐ b. centimetre (cm) ☐
    c. decimetre (dm) ☐ d. metre (m) ☐
    e. hectometre (hm) ☐ f. kilometre (km) ☐
2. Estimates/Measures length ☐
3. Chooses the preferred unit for measuring a given length ☐
4. Relates units of measurement, using decimals ☐
5. Uses addition to find the perimeter of a shape ☐
6. Uses multiplication and addition to find perimeter ☐

Mass

1. Recognizes/Uses the term and its symbol:
    a. gram (g) ☐ b. kilogram (kg) ☐ c. tonne (t) ☐
2. Chooses the best estimate for a mass ☐
3. Relates grams and kilograms ☐
4. Relates capacity, mass, and volume units for water ☐

Temperature

1. Reads/Records temperatures in degrees Celsius (°C) ☐

Time

1. Reads/Records time to the nearest second ☐
2. Recognizes/Uses numeric dating (year, month, day) ☐

Volume

1. Recognizes/Uses the term and its symbol:
    a. cubic centimetre (cm^3) ☐
    b. cubic decimetre (dm^3) ☐
    c. cubic metre (m^3) ☐
2. Finds volume by counting centimetre cubes ☐
3. Finds volume of a rectangular prism by multiplying
    a. the number of centimetre cubes in one layer by the number of layers ☐
    b. the area of the base and the height ☐

Graphing

1. Collects/Organizes information ☐
2. Draws and interprets
    a. pictographs ☐ b. bar graphs ☐
    c. line graphs ☐ d. broken-line graphs ☐
3. Interprets circle graphs ☐
4. Matches points on a grid with ordered pairs of numbers ☐

Problem Solving

1. Considers ways of estimating ☐
2. Identifies relevant and irrelevant information ☐
3. Finds information needed ☐
4. Recognizes that different situations affect answers ☐
5. Gives reasonable measurements ☐
6. Finds the number of possibilities of an event ☐
7. Solves problems involving two or more steps ☐
8. Writes/Solves an equation for a word problem ☐
9. Thinks logically ☐
10. Restates word problems ☐
11. Considers the chances of an event occurring ☐
12. Uses models to obtain solutions ☐
13. Considers possible solutions ☐

Geometry

1. Identifies/Names/Draws parallel, intersecting, or perpendicular lines, rays, line segments ☐
2. Identifies/Names/Measures/Draws angles ☐
3. Classifies angles as:
    a. acute ☐ b. obtuse ☐
    c. right ☐ d. straight ☐
4. Identifies/Draws lines of symmetry ☐
5. Identifies/Names each of these polygons:
    a. hexagon ☐ b. octagon ☐ c. parallelogram ☐
    d. pentagon ☐ e. quadrilateral ☐ f. rectangle ☐
    g. rhombus ☐ h. square ☐ i. triangle ☐
6. Identifies/Counts vertices and sides of polygons ☐
7. Classifies triangles as:
    a. equilateral ☐ b. isosceles ☐ c. scalene ☐
8. Identifies/Names/Measures parts of a circle:
    a. center ☐ b. radius ☐ c. diameter ☐
    d. chord ☐ e. circumference ☐
9. Identifies congruent shapes ☐
10. Identifies/Draws similar shapes ☐
11. Interprets/Makes scale drawings ☐
12. Copies pictures using grids, including distortions ☐
13. Identifies/Names each of these solids:
    a. cone ☐ b. cube ☐ c. cylinder ☐
    d. prism ☐ e. pyramid ☐ f. sphere ☐
14. Identifies/Counts vertices, edges, faces of solids ☐
15. Uses tracing paper to identify/draw images for
    a. flips ☐ b. slides ☐ c. turns ☐
16. Draws images of shapes for
    a. flips (by counting) ☐ b. slides (by using a rule) ☐
17. Tests for rotational symmetry ☐
18. Makes tiling patterns using one or more shapes ☐
19. Identifies flip, slide, turn images in tiling patterns ☐

starting points in mathematics

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